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MEASURING STOCHASTIC TECHNOLOGY: THE CASE OF TULARE MILK PRODUCTION

by

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Research in production economics has increasingly focused on the role of production uncertainty in farm management (see, e.g., Hazell, Pope, Robison). It has been shown in theory that if farm managers are risk averse their production decisions should depend on the relationship between input use and the probability distribution of output. Thus, knowledge of the relationship between inputs and the stochastic structure of production is important to farm managers as well as production economists, extension specialists, and policy makers.

Despite the potential importance of the relationship between production decisions and the stochastic structure of production processes, it seems reasonable to say that production economists, farm management specialists, and farmers all have difficulty assessing these relationships, and in fact very little research has been aimed at their measurement. For example, consider the effect on milk production of an increase in a dairy's milking capacity. Most production economists and dairymen would agree that the effect on average output would be nonnegative. But how many economists or farmers would be willing to wager what the effect of an increase in milking capacity would be on the "riskiness" of output due, for example, to the effects of new milking machines on the herd's health? It is safe to say that few would be confident of their answer, especially if we were to ask more specifically for the effects on the variance and skewness of the probability distribution of output. Yet, many of these individuals would probably agree that production risk or uncertainty is important to farm decision making.

Perhaps our state of ignorance concerning the stochastic structure of production processes stems from the lack of reliable statistical methods for measuring and testing the relevant relationships with actual production data. The methods that have been used to study stochastic production range from the elicitation of subjective output distributions (e.g., Bessler, Herath et al.), to the "method of moments" applied to experimental data (Day, Anderson, Roumassett) and econometric production function models (de Janvry, Moscardi and de Janvry, and Just and Pope). Each of these approaches has severe limitations for estimating and testing the stochastic structure of production with actual production data.

In this paper an alternative moment-based approach to specifying, estimating, and testing stochastic production models is used to measure the stochastic structure of large-scale dairy production. This approach, recently developed by Antle (1983b), provides a statistical framework for estimating the functional relationship between inputs and the moments of the probability distribution of output. It is hypothesized that factors such as capital equipment, herd size and quality, management ability, veterinary services, and herd health affect not only the "mean" or "average" rate of milk production, as would be done in usual production function studies, but also the dispersion and skewness of the probability distribution of milk production are hypothesized to be functions of inputs and production practices.

Our empirical analysis is based on a data sample from nine Tulare County, California, dairies. We find that the mean output as well as second and third moments about the mean of milk production are statistically significant functions of inputs. We use the empirical results to answer a series of questions concerning the stochastic structure of the production process.

First, we test for the validity of the multiplicative error production function and the model proposed by Just and Pope (1978, 1979). The milk data reject both specifications because they impose untenable restrictions across the moments of output.

Second, we consider the implications for dairy management. Using an approximate negative exponential utility function, we find that risk averse decision makers who are aware of the stochastic structure of the production process would feed their cows more and operate at a lower capital intensity than would a risk-neutral decision maker.

Third, we compare the implications of different utility functions for optimal input decisions. We find that there may be substantial differences between a mean-variance criterion and a more general criterion which also accounts for the effects of inputs on skewness of the output distribution.

The first section of the paper discusses the theoretical foundations and empirical methods for the moment-based approach to the study of stochastic technologies. In this section we survey previous studies based on the "method of moments" and production function models. The statistical model used in this paper is outlined and we show that it overcomes some other models' limitations. The second section discusses the empirical specification of the model, the milk production data, and the empirical results.

The Moment-Based Approach to Production Economics

Theoretical Foundations

A number of studies have used the moments of the probability distribution of output to represent the stochastic structure of an agricultural production process, but little attention has been paid to the theoretical foundations of this approach. In this section we discuss why stochastic production

technologies can be meaningfully represented in terms of the moments of the probability distribution of output, and how these moments relate to the production decisions of farm managers.

The theoretical foundations for the moment-based approach to production economics have been provided by Antle (1983b). It can be shown that output distributions are unique functions of their moments, and it follows that the behavior of rational farm managers can always be defined in terms of the moments of the output distribution. Thus, the moments of the output distribution can be used both to uniquely identify, and to approximate to the desired degree, the stochastic structure of technology.

The moments of the probability distribution of output are related to the farm manager's decisions. To see this, define the probability density of output as $f(Q|x)$, where Q is output and x is a vector of decision variables. The density function is defined for a given set of decision variables, and therefore the moments of the distribution can be written in general form as functions:

$$\mu_1(x) = \int Qf(Q|x)dQ$$
$$\mu_i(x) = \int (Q-\mu_1)^i f(Q|x)dQ, \; i \geq 2. \tag{1}$$

Thus, the moments are functions of x, and express the functional relationship between the stochastic structure of the production process and input decisions.

Consider a model of farm decision making in which inputs are chosen to maximize the mathematical expectation of utility, and utility is a function of profit. Assume that the distribution of profit can be expressed as a function of the mean and the first m-1 moments about the mean. Following Anderson, Dillon, and Hardaker, we can write expected utility as a function of the

moments of profit. To illustrate, consider a third-order approximation to the negative exponential utility function. Assuming utility is a function of profit π the utility function is

$$U(\pi) = a - be^{-c\pi}, \quad a,b,c > 0.$$

We assume output price is nonrandom, as is the case with milk price supports. Letting normalized input prices be r_i (input price divided by output price) we can write normalized profit as $\pi = Q - \sum_{i=1}^{n} r_i x_i$. Letting $E(\cdot)$ denote the mathematical expectation operator,

$$\bar{\pi} \equiv E(\pi) = \mu_1 - \sum_{i=1}^{n} r_i x_i.$$

Expanding $U(\pi)$ in an mth order Taylor series about $\bar{\pi}$ and taking expectations we obtain:

$$EU(\pi) = a - be^{-c\bar{\pi}} - be^{-c\bar{\pi}} \sum_{i=2}^{m} \frac{(-c)}{i!} \mu i,$$

where the μ_i are defined in equation (1). Letting m = 3 we obtain the following first-order condition for expected utility maximization:

$$\frac{\partial\mu_1}{\partial x_k} + \delta^{-1} \frac{(-c)}{2} \frac{\partial\mu_2}{\partial x_k} + \delta^{-1} \frac{(-c)^2}{6} \frac{\partial\mu_3}{\partial x_k} = r_k, \quad k=1,\ldots,n,$$

$$\delta = 1 + \frac{(-c)^2}{2} \mu_2 + \frac{(-c)^3}{6} \mu_3.$$

The above equation can be written in elasticity form as:

(2) $$\eta_{1k} + \delta-1 \frac{(-c)}{2} \frac{\mu_2}{\mu_1} \eta_{2k} + \delta-1 \frac{(-c)^2}{6} \frac{\mu_3}{\mu_1} \eta_{3k} = \frac{r_k x_k}{\mu_1}.$$

where

(3) $$\eta_{ik} = \frac{\partial\mu_i}{\partial x_k} \cdot \frac{x_k}{\mu_i}.$$

We use equation (2) to analyze decision-making under uncertainty as follows. The first term on the left-hand side represents the expected production elasticity which, under risk-neutrality (or under a first-order appoximation to the utility function), equal the input cost share in equilibrium. If the decision maker uses a mean-variance criterion (second-order approximation to the utility function) the first two terms on the left-hand side measure the expected marginal benefit of the input in percentage terms and equal the factor cost share. However, if the decision maker also takes the skewness of the distribution into account, i.e., if he is "downside risk averse" (see Menezes et al. 1980), then all three terms on the left-hand side represent the marginal benefit of the input. Thus equation (2) can be used to compare the marginal value of an input under risk neutral (RN) preferences, and under mean-variance (MV) and mean-variance-skewness (MVS) decision criteria. By specifying the single parameter c, which is the Arrow-Pratt risk aversion coefficient, the moment function estimates can be used to compute these marginal values.

We conclude that moments of the probability distribution of output can be used to provide a meaningful representation of stochastic technologies. These moments can be interpreted in terms of statistical decision theory.

Measuring Moment Functions: Literature Review

The studies involving the estimation of output distribution moments as functions of inputs are of two types, those based on the statistical procedure known as the "method of moments" (Kendall and Stuart, Ch. 18) and those based on econometric production function models. We briefly consider each of these and discuss their methodological limitations.

The first attempt to explicitly characterize the agricultural production process in terms of a probability distribution appears to be Day's study of yield distributions of field crops. Day used experimental data on yields and fertilizer applications, and the method of moments, to estimate the moments of yield distributions for different levels of nitrogen application. While Day did find that the yield distributions changed systematically with the level of fertilizer applications, and that these distributions were skewed, he did not develop a statistical model to explain such functional relationships or subject them to statistical test. The method of moments procedure requires many observations per individual production unit and thus requires a long time series of cross-section data. Experimental data is sometimes available with enough observations of each production unit over time to make the method of moments reliable, but such survey data are nonexistent.

Anderson (1973) proposed a technique which would allow the method of moments to be utilized with "sparse data," that is, with only a few observations per production unit. If successful, this technique would reduce the demanding data requirements of the method of moments. However, the reliability of this "sparse data" technique is questionable, because it involves an ad hoc, subjective method of approximating moments. However, Anderson made an important contribution by hypothesizing that the output distribution's moments are explicit functions of inputs. He postulated linear relationships between moments and inputs, and regressed mean, variance, and standardized measures of skewness and kurtosis of experimental crop yields on nitrogen and phosphorous inputs. While this approach has appeal as a descriptive tool, Anderson did not provide justification for attributing desirable statistical properties to the parameter estimates. In fact, the

discussion below shows that such moment regressions are heteroscedastic, so that standard errors of parameter estimates computed with least squares formulae are biased.

Roumasset (1976) employed a method similar to Anderson's to study the role of risk in rice production. Using survey data from the Philippines, Roumasset stratified the data by input use levels and found no apparent relationship between the moments and nitrogen fertilizer input. This finding led Roumasset to conclude that survey data with few time-series observations are not adequate for studying output distributions. He also used experimental data to compute moments and regressed them on inputs, and found that the first four moments of the output distribution vary systematically with nitrogen fertilizer input.

The method of moments has been successfully used to study the relationship between output distribution moments and production inputs only with experimental data on yield as a function of fertilizer input. This approach has serious limitations: it requires many observations per individual production unit to estimate a probability distribution for each unit; the regression of sample moments on inputs does not produce parameter estimates with desirable statistical properties, so that valid statistical tests of hypotheses are not possible; and results from experimental trials must be used to draw inferences about actual production conditions.

The second class of models which can be used to estimate output distributions is econometric production function models. The models in the literature impose _ad hoc_ restrictions on the relationship between inputs and moments of output. Just and Pope showed that the conventional multiplicative error model of the form:

(4) $$Q = f(x,\beta)u,$$

where $f(x,\beta)$ is a deterministic production function and u is a random error, forces the variance of output to be increasing in x if marginal products are positive. Just and Pope propose a more general model with an additive heteroscedastic error of the form

(5) $$Q = f(x,\beta) + h(x,\gamma)\varepsilon$$

where ε is independently and identically distributed with zero mean. Just and Pope show that this model allows inputs to have distinct effects on the mean $f(x,\beta)$ and the variance $h^2(x,\gamma)E(\varepsilon^2)$ of output and is thus more general than (4).

Antle (1983b) shows that when higher moments of output are considered, both (4) and (5) impose arbitrary restrictions on the relationship between inputs and moments. Defining the elasticities of moments with respect to inputs as in equation (3), Antle shows that model (4) implies, for $\mu_i \neq 0$,

(6) $$\eta_{ik} = i\eta_{1k},\ i \geq 2,$$

and that model (5) implies

(7) $$\eta_{ik} = \frac{i}{2}\eta_{2k},\ i > 2.$$

There is no theoretical justification for the existence of such restrictions. For example, (6) implies that the input elasticities of μ_2 and μ_3 are 2 and 3 times the mean production elasticity; (7) implies that the percentage effect of an input on μ_3 is 1.5 times its effect on μ_2. Referring to equation (2), these arbitrary restrictions on moments clearly impose arbitrary restrictions on the firm's behavior under uncertainty.

Rather than employing models which embody restrictions on higher moments, it would be preferrable, on both theoretical and methodological grounds, to utilize a model which imposes fewer restrictions and which allows the

researcher to test for restrictions on the stochastic structure of the production process.

The Flexible Moment-Based Approach

In this section we outline the flexible moment-based approach, introduced by Antle (1983b), for estimating and testing the moment functions defined in equation (1). The model is based on the hypothesis that a linear-in-parameters relationship exists between moments of the output distribution and the farm manager's decision variables. This approach has the advantage of being flexible in the sense that distinct parameters can be estimated for each moment-input relationship, thus avoiding the restrictions imposed by the econometric production function models discussed above. In addition, this approach can be used with a single cross section (or time series) of data, and the parameter estimates have known asymptotic distributions which can be used to construct tests of hypotheses about the structure of the technology. Thus, the flexible moment-based approach also overcomes the limitations of the method of moments identified above.

For the jth observation define Q_j as output and $x_j = (x_{1j}, \ldots, x_{nj})$ as the input vector. The moment functions given in equation (1) are written in linear form as

$$\mu_{1j}(x_j) = x_j\gamma_1$$
$$\mu_{ij}(x_j) = x_j\gamma_i, \; i \geq 2.$$

Below we apply a quadratic parameterization of the moment functions. In general, the moment functions can be specified as any linear-in-parameters functional form (see Fuss, McFadden, and Mundlak). Output is random and $E(Q_j) = \mu_{1j}$, so we can write the mean production function or first moment function as the regression equation

(8) $$Q_j = x_j\gamma_1 + u_j,\ E(u_j) = 0,$$

where u_j is assumed to be independently distributed. Similarly, noting that

$$E(Q_j - \mu_{1j})^i = E(u_j{}^i) \equiv \mu_{ij},\ i\geq 2,$$

we write the ith <u>moment function</u> as the regression equation

(9) $$u_j{}^i = x_j\gamma_i + v_{ij},\ E(v_{ij}) = 0,\ i\geq 2.$$

The goal is to estimate the γ_i parameters which relate inputs to moments. It is well known that the least squares estimate $\hat{\gamma}_1$ of γ_1 is consistent. In addition, it can be shown that with the residual $\hat{u}_j = Q_j - x_j\hat{\gamma}_1$, the least squares regression of $\hat{u}_j{}^i$ on x_j produces a consistent estimate $\hat{\gamma}_i$ of γ_i, $i\geq 2$. However, the least squares formulae for the standard errors of the $\hat{\gamma}_i$ are not valid because (8) and (9) are heteroscedastic. To see this, observe that the variance of Q_j is

$$E(u_j^2) = \mu_{2j} = x_j\gamma_2$$

and the variance of $u_j{}^i$ is

$$E(v_{ij}{}^2) = \mu_{2i,j} - \mu_{ij}{}^2,\ i\geq 2.$$

Since $\hat{\gamma}_i$ is a consistent estimator of γ_i it follows that $w_{1j}{}^2 = x_j\hat{\gamma}_2$ is a consistent estimator of μ_{2j} and in general

$$w_{ij}{}^2 = x_j\hat{\gamma}_{2i} - (x_j\hat{\gamma}_i)^2,\ i\geq 2,$$

is a consistent estimator of $E(v_{ij}{}^2)$. Therefore, a feasible GLS estimator for γ_1 can be obtained by the weighted regression

$$Q_j/w_{1j} = x_j\gamma_1/w_{1j} + u_j/w_{1j}$$

and a feasible GLS estimator for any γ_i, $i\geq 2$, can be obtained by the weighted regression

$$\hat{u}_j^i/w_{ij} = x_j\gamma_i/w_{ij} + v_{ij}/w_{ij}.$$

It can be shown that the estimators thus obtained are asymptotically equivalent to the true GLS estimators. In large samples, under the statistical assumptions required for the central limit theorem, the parameter estimates obtained from this procedure are asymptotically normally distributed. Therefore, standard large-sample test statistics can be used.

In applications of the above estimation procedure, another problem must be taken into account. When estimates of the regression variances w_{ij}^2 are computed it is possible, if not likely, that some will be negative. Variances are positive, by definition, but either sampling error or small sample bias in the parameter estimates may cause negative variance estimates. Fortunately, this problem can be solved using standard nonlinear programming techniques and existing computer software. To consistently estimate γ_2 subject to the constraint that the estimated variance $w_{1j}^2 = x_j\hat{\gamma}_2 \geq 0$, solve the problem

$$\min_{\gamma_2} \sum_{j=1}^{N} [\hat{u}_j^2 - x_j\gamma_2]^2 \text{ subject to } x_j\gamma_2 \geq 0.$$

Similarly, to estimate any γ_{2i}, $i \geq 2$, choose γ_{2i} to solve

$$\min_{\gamma_{2i}} \sum_{j=1}^{N} [\hat{u}_j^{2i} - x_j\gamma_{2i}]^2 \text{ subject to } [x_j\gamma_{2i} - (x_j\hat{\gamma}_i)^2] \geq 0,$$

where $\hat{\gamma}_i$ has been obtained from a previous regression. The latter inequality constraint simultaneously forces $\hat{\gamma}_{2i}$ to satisfy the requirement that $\mu_{2i,j} \geq 0$ as well as the requirement that the variance w_{ij}^2 is nonnegative. In the following section we use this procedure, with the MINOS nonlinear optimization program developed by Murtaugh and Saunders, to estimate the linear moment model.

The Stochastic Structure of Milk Production

In this section we use the flexible moment-based approach to study the stochastic structure of milk production in Tulare County, California. Besides its importance as an agricultural commodity, the milk production process is especially suited for our analysis of the stochastic structure of production. One useful attribute is that milk production is a true single-product process.[1] In addition, all of the inputs are chosen before the production process begins (except perhaps veterinary services), can therefore be treated as exogenous variables, and are not subject to simultaneous equation bias as would be the case with sequential decision making (see Antle 1983a).

Data Sample

The Tulare milkshed is located in Tulare County of the central valley of California. Dairy operations are characterized by large scale, the predominance of Holstein stock, and a relatively high enrollment in a Dairy Herd Improvement (DHI) program. In 1978 the average herd size was 440 cows. In Tulare County 57 percent of the dairies are enrolled in the electronic data processing system of Agri-Tech Analytics for DHI production records. The computerized records formed the basis for the data used. The DHI program dairies have more investment, larger labor forces, higher gross receipts and expenditures, larger farm incomes, more milk production, and are otherwise larger than most dairies in Tulare County.

In Goodger's study of the determinants of herd health, for which these data were originally collected, the emphasis was given to precise, valid data on a few dairies in order to isolate the relatively small and certainly complex influences of veterinary services. By concentrating on a small sample of high-quality data, it was believed that more credible, although less

generalizable results could be obtained. This "clean" data would then provide a testing ground for methodological advances and a starting place for further data collection.

A four-year cost study conducted under the auspices of Agri-Tech Analytics provided detailed, precise data for the preliminary sampling frame. Dairymen on 20 dairies were trained to collect and record feed, labor and capital costs accurately. Of the 20 dairies, nine had data that came closest to satisfying the objectives of the overall research program of which this paper reports a part. The nine dairies had 5,052 Holstein cows, representing 5.6 percent of the milking cows in Tulare County. All nine dairies received veterinary services. The data on mastitis and days open were collected for the nine dairies monthly for the period of July 1976 through December 1978 along with data on production inputs.

The Role of Veterinary Services and Herd Health

In the past, the effects of veterinary services could be seen in reduced mortality and morbidity associated with infectious diseases. A number of such diseases are caused by single agents, allowing them to be controlled efficiently and in a clearly economical way. These diseases are now largely under control, and standard well-known treatments exist to deal with outbreaks. However, as management increased production efficiency of each cow, a new group of diseases, called "production diseases," began to emerge. These diseases affect the profits of the dairy enterprise by reducing production efficiency rather than causing mortality.

Production diseases typically are not well understood and have multiple interacting determinants related to management, genetic, technological, and environmental factors. In today's high producing dairy cows, production

disease may be aggravated. These animals are subjected to a considerable amount of stress because management systems are designed to increase output per animal resulting in higher milk yields. Stress can result in breeding problems such as lowered fertility at first heat or service, a longer interval to first heat and postpartum ovulations without observed heat. The most important of the production diseases in dairying are mastitis and infertility.

Description of the Variables

A detailed discussion of the data is in Goodger. Here we briefly describe the variables used to estimate the moments of milk production.

The output variable is average monthly pounds of milk per dairy, based on daily milk shipments. Feed (F) is pounds of roughages and concentrates fed to the herd. Allowance was made for evaporation.

The herd (H) variable measures the quality-adjusted stock of animal capital. H is the product of the herd size and an animal capital index. This index was obtained by weighting the cows in each lactation by factors derived from the relative milk production that would be expected in each lactation and a maturity factor that corrects for the cow's age and month of the year at calving. The index adjusts the herd size to represent its potential for milk production relative to a "standard" Holstein herd of the same size in Tulare County in which all cows calve at a mature age, in an average month, with all cows in the third lactation.

Equipment capital (C) is represented by the type of milking parlor and its degree of mechanization. Throughput in cows per hour was derived from performance standards of manufacturers and the expert opinion of suppliers.

Performance standards depended on parlor design (e.g., herring bone, polygon) and extent of mechanization.

An index of management performance (M) was developed based on the expert opinion. Twelve management practice categories and a total of 48 indicators were defined in an iterative process with experts, and the experts then rated each of the nine dairies. Examination of summary statistics on the dairy scores and management practice categories indicate that the resulting management score is a valid measure of overall management performance.

The veterinary service variable (V) represents an attempt to account for two types of services, emergency and scheduled. For the period over which a dairy had each type of services the monthly expenditures on the service were averaged. Whereas monthly values of V are likely to be endogenous in the production system due to sequential decision making this averaged variable can be interpreted as a measure of each dairy's long-term policy towards veterinary services. Thus, V is assumed exogenous to monthly milk production.

Days open (DO) is computed as the number of days between calving and a breeding that is followed by a declaration of pregnancy. DO is averaged over all pregnant cows in the current and preceding lactation and is usually between 90 and 120 days. Longer DO is generally presumed to be an indication of infertility. Although this variable has certain limitations (Goodger, pp. 148-150), according to a number of sources days open is the best available summary measure of reproductive efficiency.

The California Mastitis Test was developed to measure the leukocyte content of milk as an indication of subclinical mastitis. Using the data, the mastitis score for each cow tested was weighted to create "trace-equivalent

cows." The CMTA variable is the trace-equivalent cows as a proportion of the cows tested. As the value of CMTA decreases, udder health improves.

Temperature (T) is a measure of the mean monthly temperature at the weather station nearest to each dairy.

Model Specification

To test the general hypothesis that the mean, variance, and third moment are functions of inputs, we employed the following quadratic representation of the moment functions:

$$\mu_i = \alpha_{i0} + \sum_{k=1}^{n} \alpha_{ik} x_k + \frac{1}{2} \sum_{k=1}^{n} \sum_{\ell=1}^{n} \alpha_{ik\ell} x_k x_\ell, \quad i=1,2,3. \tag{10}$$

This function has the feature that the derivatives are linear functions:

$$\frac{\partial \mu_i}{\partial x_k} = \alpha_{ik} + \sum_{\ell=1}^{n} \alpha_{ik\ell} x_\ell. \tag{11}$$

The standard errors of these derivatives can be calculated at any data point. We calculated the derivatives at the sample means of the data. The elasticities of moments are also linear functions of the parameters when calculated at the sample means of μ_i and x_k.

One problem encountered with the quadratic function is that a large number of parameters must be estimated if there are very many inputs. To help overcome this problem we specified the moment functions as quadratic in F, C, H, and M, and linear in the herd health variables V, DO, and CMTA. In principle, there may be interactions between production inputs and herd health variables, as well as interactions among the herd health variables themselves. We found this specification explains milk production as well as a full

quadratic expansion while limiting the parameters to a manageable number.

Empirical Results

The data are monthly time series for nine dairies so the problem of autocorrelation was suspected. Thirty or fewer complete observations were available for each dairy, and it was not judged that a sufficient number of observations per dairy were available to consistently estimate the autocorrelation coefficient. Because the dairies all have similar weather and climate, it is reasonable to assume the first order autocorrelation coefficient is constant across dairies. After estimating the autocorrelation coefficient and transforming the data for the mean regression given in general form by equation (8), the estimation procedure proceeded as follows: residuals from the autocorrelation-adjusted mean equation were used to obtain consistent estimates of the heteroscedastic error variances for the mean, variance, and third moment functions, subject to the required inequality constraints on variances. These variance estimates were then used to compute feasible GLS estimates of the parameters for the mean function using the weighted regression method described above. To increase estimation efficiency, and to facilitate testing cross-equation restrictions, the second and third moments were estimated jointly, as described in Antle (1983b).

The parameter estimates for the quadratic moment functions, based on the GLS estimation scheme, are in Table 1. To test for the significance of the regressions we computed χ^2 statistics (Theil, Ch. 8) for the null hypothesis that all slope coefficients equal zero. The χ^2 statistics for these regressions indicate rejection of the hypothesis of zero slope coefficients for all three moments at all conventional significance levels. A majority of

the parameters of the model are statistically significant. These results provide strong empirical support for the hypothesis that higher moments of the output distribution are functions of inputs, and strong rejection of the hypothesis that output follows a normal or symmetric distribution.

Table 2 presents the elasticities η_{ik} defined above equation (6), calculated at the sample means of the data. The first column contains the elasticities of mean output with respect to inputs; these can be interpreted as conventional production elasticities. The other two columns contain the elasticities of the second and third moments (around the mean) with respect to the inputs.[2] Note that these elasticities do not satisfy restrictions (6) and (7) implied by the multiplicative error model and the Just-Pope model, since several elasticities of the second and third moments are negative and do not satisfy the necessary proportionality. To test these restrictions, we used the fact that (6) and (7) both imply $\eta_{3k} = 3\eta_{2k}/2$. This in turn implies, using (11), that

$$\alpha_{3k} = 3\mu_3\alpha_{2k}/2\mu_2$$

$$\alpha_{3k\ell} = 3\mu_3\alpha_{2k\ell}/2\mu_2, \; k,\ell = 1,\ldots, n.$$

To test these cross-equation restrictions, the second and third moment functions were jointly estimated. The resulting test statistic $\chi^2(18) = 76.84$ clearly indicates rejection of the restrictions (5 percent critical value is 28.67). Therefore, both the multiplicative error model and the Just-Pope model were rejected by the data. Note that rejection of (6) means that the conventional lognormal error model is inconsistent with the data.

Several features of the individual parameter estimates are worth noting. One is the role capital and management play in the dairy production process. Table 1 shows C and M interact strongly and significantly in each moment.

This result supports the hypothesis that high-quality management is especially important to capital-intensive dairies, as suggested by Matulich. Moreover, the elasticities in table 2 show that both C and M have positive effects on the mean, and negative effects on the variance and third moment (see footnote 1). We discuss below the implications of these effects for dairy management.

Also notable are the effects of the health variables. If we were to consider only the mean production function, we would find only V has a marginally significant effect on milk production. However, considering the higher moments dramatically alters the picture. While V has a positive mean effect, it also increases the variance of milk production. Days open reduces not only mean production but also variance and skewness. The mastitis index CMTA has an anomalous positive (but insignificant) effect on mean production, and also a highly significant positive effect on the variance of production.

However, these results for the health variables should be interpreted with caution. There is evidence from other studies that different types of veterinary services have different productivity effects. There is also the possibility that health variables are endogenous to output because the decision to use veterinary services is often made sequentially in response to randomly occurring health problems. Some questions also have been raised about the use of days open to represent udder health (Goodger and Kushman).

To evaluate the implications for dairy management we used equation (2) which was derived from the approximate negative exponential utility function. Table 3 presents the marginal values of the inputs and health variables in percentage terms using first, second, and third-order approximations to the utility function (i.e., the first, first plus second, and all three terms on

the left-hand side of equation (2). The risk aversion coefficient is set at 0.01, a reasonable value according to utility function estimates (French and Buccola, Moscardi and de Janvry, Nikiphoroff). Two remarkable results emerge from table 3.

First, the marginal values based on risk-neutral (RN) and mean-variance (MV) criteria are almost identical. Even though the variance is a significant function of inputs, the magnitude of the marginal effects of inputs on variance is small. However, this is not true for the third moment; indeed, the RN and mean-variance-skewness (MVS) criteria produce very different marginal values. The marginal effects of inputs on the third moment are relatively large. An important implication, for the general analysis of decision making under uncertainty, is that use of the MV criterion indicates that the effects of uncertainty are not important, whereas incorporating skewness and downside risk aversion with the MVS criterion shows uncertainty has a large effect on optimal decisions.

The second implication of table 3 concerns dairy management. It is clear that the risk-averse dairyman would feed much more intensively and utilize less of all other inputs, especially capital and management, than would the RN dairyman. Thus, it appears that intensive feeding is risk-reducing when its effect on skewness is accounted for. We can rationalize this result in terms of the health problems that capital-intensive dairies may have. The physical capital variable measures milking capacity in terms of cows per hour. High speed, mechanized milking operations are known to aggravate udder health problems. In addition, if "good" managers who scored high on the management scale tend to increase milk production by using practices which stress the animals, the result may well be an output distribution which has a higher mean

but also is more risky. Thus, these results suggest that production diseases may be manifested in increased production risk.

Conclusion

We have discussed limitations of the method of moments and econometric production functions for measuement of the relationship between inputs and the moments of the output distribution. After discussing the more flexible linear moment model, which overcomes these limitations, we applied it to data from the Tulare milk shed. The parameter estimates support the general hypothesis that higher moments are functions of inputs, and they reject the restrictions implied by the multiplicative error model and the Just-Pope model. Using a Taylor series approximation to the negative exponential utility function we found that the implied behavior of a risk averse dairy manager changes dramatically when effects of inputs on the skewness of the distribution are included.

Our findings suggest that more detailed research is needed to identify the effects of individual production and health practices. We feel it would be unwise to generalize about the effects of such broad categories as "management," "capital," and "veterinary services." What these results do show, however, is that there may be important tradeoffs between higher mean productivity and the riskiness of the production process. To confidently answer the kind of question we raised at the outset of this paper, more studies are needed to establish empirical regularities about the stochastic structure of agricultural technologies.

Footnotes

[1]The literature on estimation of milk response functions, based on experimental data, shows there are important differences in the components of raw milk (Paris *et al.*). In this analysis we do not consider these differences.

[2]Because the mean value of the third moment is negative, the reader should note that the signs of the derivatives given in (9) are opposite of the signs of the elasticities in Table 2.

Table 1

GLS Estimates of Quadratic Moment Functions

	Moment		
	First	Second	Third
Intercept	3.523	-4.787	47.565
	(12.145)	(22.846)	(89.100)
F	19.282***	26.459***	10.221
	(7.308)	(7.966)	(50.793)
C	-128.360***	29.686*	381.570***
	(43.878)	(17.563)	(104.320)
H	107.420***	-76.526***	-233.150
	(40.881)	(24.425)	(143.560)
M	81.647	-19.058	-512.770*
	(188.800)	(76.243)	(294.180)
F^2	-3.633**	-8.219***	-7.559
	(1.508)	(1.939)	(15.793)
C^2	-8.821	.342	29.775
	(10.197)	(5.279)	(18.469)
H^2	-31.789*	-76.572***	29.981
	(18.536)	(19.148)	(122.560)
M^2	-215.070	47.371	815.790***
	(141.880)	(61.007)	(268.430)
F•C	-3.257	-21.366***	29.970
	(3.709)	(5.246)	(28.604)
F•H	21.559**	51.736***	46.178
	(8.740)	(11.967)	(81.268)
F•M	-22.233**	-7.505	-59.455
	(10.703)	(9.005)	(54.355)
C•H	.594	52.005***	-123.770
	(25.151)	(12.977)	(79.275)
C•M	234.660***	-49.959*	-664.190***
	(71.223)	(27.133)	(177.170)
H•M	-71.529	40.490	410.000***
	(57.206)	(32.200)	(171.780)
V	.015*	.034**	-.041
	(.008)	(.015)	(.047)
DO	-2.607	-2.109*	-14.483**
	(1.681)	(1.242)	(6.576)
CMTA	.141	.482***	2.522
	(.176)	(.172)	(1.914)
T	4.308***	1.871**	-9.107**
	(.878)	(.835)	(4.132)
$\chi^2(18)$	7130***	855.5***	67.7***

Note: Standard errors in parentheses.
*Significant at 10 percent level.
**Significant at 5 percent level.
***Significant at 1 percent level.

Table 2

Mean Elasticities of Moments with Respect to Inputs

Elasticity With Respect to:	Moment		
	First	Second	Third
Feed (F)	.059**	.448	-33.276***
	(.027)	(.779)	(12.238)
Physical Capital (C)	.678***	-4.051***	153.102***
	(.311)	(1.580)	(44.584)
Animal Capital (H)	.877***	1.057	8.414
	(.065)	(.992)	(16.727)
Management (M)	.230	-1.557	75.269***
	(.291)	(2.241)	(32.726)
Veterinary Services (V)	.010**	.471**	1.729
	(.005)	(.208)	(1.982)
Days Open (DO)	-.096	-1.682*	35.170**
	(.062)	(.991)	(15.968)
Mastitis (CMTA)	.008	.567***	-9.031
	(.010)	(.202)	(6.854)
Temperature (T)	.091***	.863**	12.585**
	(.018)	(.379)	(5.710)

Note: Standard errors in parentheses.
*Significant at 10 percent level.
**Significant at 5 percent level.
***Significant at 1 percent level.

Table 3

Marginal Value of Inputs for Risk Neutral and Risk Averse Decision Criteria
(in percent)

Decision Criterion	Input						
	Feed	Physical Capital	Animal Capital	Management	Veterinary Services	Days Open	CMTA
Risk Neutral	5.9	67.8	87.7	23.0	1.0	- 9.6	.8
Mean-Variance	5.8	68.6	87.5	23.3	0.9	- 9.3	.7
Mean-Variance-Skewness	13.1	34.9	85.6	6.7	0.5	-17.0	2.7

Note: Based on equation (11) with c = .01

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