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A NEW ANALYTICAL FRAMEWORK FOR THE FERTILIZATION PROBLEM

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A NEW ANALYTICAL FRAMEWORK FOR THE FERTILIZATION PROBLEM: PART I, METHODOLOGY

INTRODUCTION

It is well known that, during the last 30 years, soil scientists and agricultural economists analyzed fertilization problems by means of two radically different methodologies. The divergence began during the late 1940's, when the Iowa School of Agricultural Economics proposed a revision of the experimental designs used by agronomists for analyzing yield responses to fertilizers. The new proposal grew out of the empirical applications of the production function concept as it was known at that time. It was justified on the need for numerous combinations of fertilizer treatments in order to estimate with precision the yield response surface postulated, from then on, as a smooth differentiable function, possessing a point maximum, and characterized by substitution among nutrients. In other words, the Iowa school suggested to either reduce or abandon the traditional precaution of soil scientists when conducting yield-fertilizer experiments (they used a few treatments and several replications), in favor of many treatments and few replications. The new methodology was popularized in a series of conferences whose results were published in two volumes prepared by Baum, Heady and Blackmore in 1956, and by Baum, Heady, Pesek and Hildreth in 1957. The main empirical innovation of those works was the adoption of polynomial functions for representing yield response surfaces, thus abandoning almost at once the methodologies of Mitscherlich, Baule, Balmukand and, obviously, of von Liebig. The influence of those two volumes upon the methodological orientation of agricultural economists, in almost any country, was enormous. On the contrary, the influence of the same proposals upon the works of agronomists seems small. After more than 20 years from the initial revelation of the new methodological course proposed by agricultural economists, soil scientists keep conducting their fertilizer experiments according to the scientific and statistical scheme developed at the beginning of the century, that is, analyzing the yield response function with respect to a single nutrient at a time, with few treatments and several replications. At

the same time, they have assumed increasing responsibilities in the preparation of fertilizer recommendation tables. Such tables are used by the agricultural extension agents and recently have been introduced also in the developing countries.

These considerations suggest that the time has come for a critical reevaluation of the methodology proposed by the agricultural economists more than 20 years ago, with the purpose of identifying the reasons for the absence of a common language and of scientific collaboration with soil scientists. It would appear that a major obstacle to such a collaboration is the agricultural economists' lack of understanding of the agronomic principles upon which agronomists base their yield-response analysis. Another handicap seems to have been represented by the agricultural economists' use of relatively complex functional relationships and a statistical methodology rather foreign to soil scientists. It seems convenient, therefore, to attempt the specification of the main agronomic principles in a form and language familiar to agricultural economists and then submit such a formulation to empirical verification. The agronomic principles considered here are the following five: a) the "law of the minimum" of von Liebig, b) the notion of plateau maximum of the yield response function, c) the influence of weather and soil type conditions upon the response function, d) the fertility carry-over effect, and e) the calibration of the soil tests.

The Shape of the Yield Response Surface

Consider first the "law of the minimum" enunciated by von Liebig circa 1840. It states that "the yield of any crop is governed by any change in the quantity of the scarcest factor called the minimum factor, and as the minimum factor is increased the yield will increase in proportion to the supply of that factor until another becomes the minimum. If another factor, not at the minimum, is increased or decreased the yield would not be affected" (Redman and Allen).

It is important to notice that the "law of the minimum" introduced two distinct concepts at once. The first concept was that yields respond in proportion to additions of the limiting nutrient. This linearity was later critized by

Mitscherlich, but it is still defended by some authoritative soil scientists. The second concept introduced by the "law of the minimum" was that of strong complementarity among plant nutrients. The notion that different nutrients play different roles in the plant physiological process and, therefore, cannot substitute for each other, has received a large support from most soil scientists since Liebig, including Mitscherlich. Mitscherlich, in fact, thought that "not only should all factors be present but they should be in balanced proportions so that they may mutually strengthen and support each other, as some of the potency of a factor will remain dormant in the absence of adequate support." (Redman and Allen, p. 458.)

Liebig's theory gave rise to the concept of "essential elements." Nutrient essentiality is rather basic in soil science and can be found in most modern soil science textbooks (Epstein, p. 55-56; Brady, Ch. 2). The concept holds that no element is essential unless its deficiency is <u>specific</u>, that is, the deficiency can not be overcome by the addition of another element to the soil. In the words of other soil scientists: "the impossibility of substitution is a pre-requisite for the essentiality of a given element" (Malavolta <u>et al.</u>, p. 13). Barber (p. 210) is very explicit on this subject: "economists . . . have pointed out that when crops are fertilized to a certain yield level there are a number of different combinations that will give the same yield . . .. But since P cannot substitute for K in the plant, any substitution must be small in amount." Thus, under the postulate of nutrient essentiality, the specification of the multinutrient-yield response function sought by von Liebig, and many other soil scientists after him, may be stated as follows

$$y = \underset{i \in M}{\text{Min}} \; [g_i(b_i + x_i)] \tag{1}$$

where M is the index set of plant nutrients, g_i is the yield response function to the ith nutrient, given that other nutrients are nonlimiting; b_i is the quantity of absorbable nutrient in the soil and x_i is the quantity of nutrient applied during the experiment. In general, the response function g_i is nonlinear and (1) will be called the nonlinear von Liebig response function.

Empirical observation of yields' behavior in response to successive doses of nutrients led soil scientists to postulate that the yield response surface exhibits

a plateau maximum rather than a point maximum. This notion is well established among agronomists. Mitscherlich seems to have been the first to give it a functional representation with the well known specification

$$(2) \qquad y = A(1 - e^{-c(b + x)})$$

where A is the maximum yield attainable. Other examples are the "resistance" function by Balmukand and the linear response and plateau (LRP) function specified more recently by Cate and Nelson. The empirical evidence about the plateau maximum seems to be overwhelmingly in favor of the soil scientists' hypothesis. Even the experimental data analyzed by Heady, Pesek and Brown by means of polynomial functions, exhibit irrefutable plateau maxima. Among agricultural economists, Perrin seems to have been the only author that has performed a test of point maximum versus plateau maximum models. He fitted both a quadratic and a LRP function to a set of experimental data. He then used each estimated relation in the maximization of expected profits subject to the experimental grid associated with an independent set of data. Actual profits of the fertilizer recommendations from each function were then computed directly from the independent set of data. Perrin found that average actual profit associated with fertilizer recommendations from the agronomic LRP function was higher than the average actual profit associated with recommendations from the quadratic function (the difference was not significant). Perrin concluded that it might be "surprising to some that the LRP provides recommendations as valuable as those from the quadratic function" (Perrin, p. 59).

The concept of yield plateau reinforces that of nonsubstitution between nutrients. In view of the wide acceptance of this notion by soil scientists, the specification of the yield response function (1) may be modified as

$$(3) \qquad y = \min_{i \in M, A} [g_i(b_i + x_i), A].$$

<u>The Effect of Weather and Soil Type Variables</u>

Agronomists have recognized long ago that the yield response functions, g_i, are not invariant to location. Weather and soil variables are generally regarded as major determinants of the height and shape of the yield response surface.

Soil scientists, by and large, have followed a proposal originally formulated by Mitscherlich, appropriately modified to reflect the more recent knowledge. Soil variables considered in the ensuing discussion are only soil type variables, that is, the set of physical and chemical properties of the soil that are affected only marginally by fertilizer use. Examples are: percentage of clay, pH, soil depth, moisture holding capacity, redox potential and the like.

The study of the various factors affecting plant growth was highly stimulated by Mitscherlich's theory on the constancy of the proportionality coefficient. Mitscherlich maintained that the coefficient c in (2) was a constant for each nutrient, irrespective of everything else, including crops. Variations in the kind of crop, type of soil and weather conditions would influence only the parameter A, that is, the asymptotic maximum. In the words of Russel (p. 53), "Mitscherlich's work was extraordinarily stimulating and caused a veritable flood of controversy when it was first developed. His equation has been of great practical value though it is certainly not exact. Thus . . . (coefficient c) . . . for a particular nutrient is not a constant, but depends somewhat on the other conditions of growth." Since $y/A = 1 - e^{c(b + x)}$, a natural outgrowth of Mitscherlich's theory was the use of relative (or percentage) yield as a means to standardize response data obtained under different growing conditions. It should also be noticed that, in spite of the shortcomings attributed to Mitscherlich's theory, "most soil laboratories recognize that the fitting of Mitscherlich type relative yield curves to soil analysis is generally the 'best'" (Ryan, p. 25).

Agricultural economists, on the other hand, apparently ignored Mitscherlich's relative yield theory and its implications. They have labored extensively with the introduction of a long series of climatic and environmental variables in the yield response function without being able of reaching a consensus about either the type or the number of variables to use in the estimation. They generally claimed that effects of weather and soil type conditions should be evaluated through the explicit incorporation of weather and soil type variables into "generalized" yield response

functions (Heady, Munson and Doll). The estimation of "generalized" functions was deemed necessary for the design of site specific fertilizer recommendations. In particular, the incorporation of weather variables was required for an assessment of risks associated with fertilizer use (Smith and Parks, de Janvry, Ryan, da Fonseca). It is important to notice, however, that the incorporation of soil type and weather variables into yield response functions is not a search for knowledge on structural relationships. Perrin (footnote, p. 55) is very explicit on this subject: "we do not know and can never know the true specification of the process. If such a thing exists at all, we will misspecify it by omitting variables, choosing the wrong algebraic form and measuring the variables inaccurately. We should, therefore, be more concerned with testing the hypothesis that this theory predicts better than the next best alternative." Yet, as far as the evaluation of weather effects in the yield response is concerned, most of the testing for the "next best theory" seems to have been largely confined to the comparison of weather indexes vs. direct weather measures as alternative regressors in yield response equations (Ryan, pp. 31-36). In particular, the use of weather indexes has been defended on the grounds that they bear some relationship to the process of plant growth.

From an applied research point of view, the difficulties presented by the "generalized yield response function" approach seem to lie mostly on the availability of data for the sites where experimentation took place. For obvious reasons, this kind of limitation is particularly restrictive in less developed countries. In any event, the main objective of economics of yield response research is not to uncover structural yield-soil-weather relationships. In fact, the main objective of microeconomics of yield response research is to provide better fertilizer recommendations for the farmer. In order to easily implement such recommendations, it is very important to rely on a model which includes explicit relationships between yields, fertilizer applications and soil tests. This is so because these variables can be controlled--at least partially--by farmers as decision makers. Weather and soil type, on the other hand, cannot be controlled by the farmer who has already chosen

a given location for his activity. Thus, it is unnecessary to know the levels of weather and soil class variables. Only knowledge of their likely effects on the yield-soil test-fertilizer relationships is required in order to make fertilizer recommendations. Ideally, the required knowledge would be incorporated into a single and readily available index of weather and soil type effects. It is argued that such an index can be obtained under a set of relatively mild assumptions. As it turns out, the result of this analysis is deeply related to Mitscherlich's relative yield theory.

Assumption 1: For predictive purposes, the generalized yield nutrient relationships can be adequately represented by a weakly separable function with respect to the partition (set of nutrients; set of other factors of growth) such as:

$$y = f(w, s)\, g(b + x) \tag{4}$$

where y is yield, w is a random vector of weather variables, s is a vector of soil type variables, b is the vector of the nutrient quantities available in the soil prior to fertilization and x is a vector of nutrient quantities applied as fertilizers.

Assumption 2: There exists a set of weather, soil type and nutrient conditions, w^*, s^*, $(b + x)^*$, such that:

$$f(w^*, s^*)\, g(b + x)^* = A^* \geq f(w, s)\, g(b + x) \tag{5}$$

where A^* is the maximum attainable yield.

The implications of assumptions 1 and 2 are as follows: dividing and multiplying the right-hand side of (4) by A^* one gets:

$$y = A^*\, h(w, s)\, g(b + x) \tag{6}$$

where $h(w, s) = f(w, s)/A^*$. Since, by assumption 2, $y \leq A^*$, equation (6) implies that $0 \leq h(w, s)\, g(b + x) \leq 1$.

It can be assumed, without any loss of generality, that:

$$0 \leq h(w, s) \leq 1 \tag{7}$$

$$0 \leq g(b + x) \leq 1. \tag{8}$$

From (6), (7), (8) and the definition of A*, it is clear the $h \to 1$ as $(w,s) \to (w^*, s^*)$. Also, $g \to 1$ as $(b + x) \to (b + x)^*$. In particular, by letting $A_{ws} = A^* h(w,s)$ one can rewrite (6) as:

$$y = A_{ws}\, g(b + x). \tag{9}$$

Equation (9) is an obvious generalization of Mitscherlich's theory. It also identifies the index to be used for pooling experimental data under different soil class and weather conditions. This location index is A_{ws}, that is, the yield plateau of each experiment. An intuitive defense for such an index could be as follows: "after all, the plant experiences and integrates the same weather recorded only in part by our instruments as well as the complex plant-soil-weather interactions and the side effects of insects and diseases" (Dale, p. 179). However, the justification for the separability assumption which, incidentally, uncovers the nature of the relative yield concept, is not restricted to intuition only. As stated before, Mitscherlich's assertion that the coefficient c was a constant for each nutrient has been subject to criticism. Yet, in the words of an authoritative modern soil scientists, "the development of the significance of (Mitscherlich) equation in its relation to soil fertility problems represents an important contribution to soil-plant relationships. The subsequent controversy over its application has unfortunately dealt more with its shortcomings than with the reasons for its shortcomings" (Bray, 1958, p. 314). Bray conducted a large number of empirical researches in order to establish the conditions under which the relative yield principle seems to hold. The observations of Bray led him to formulate the "nutrient mobility concept" (see Bray, 1954, 1958, 1963). Although a technical description of the "nutrient mobility concept" is beyond the scope of this paper, it is important to notice that Bray's findings strongly support Mitscherlich's relative yield theory for nutrients such as phosphorus and potassium. The conditions under which the relative yield concept seems to hold are as follows: (a) the form of the nutrient must be kept constant; (b) the distribution pattern of the nutrient in the soil relative to plant distribution must be kept constant; (c) the kind of plant must be

kept constant and (d) the planting pattern and rate of planting must be kept constant. Condition (a) refers to the kind of fertilizer employed, such as rock phosphate or superphosphate. It also refers to the form of the nutrient available in the soil. Conditions (b) and (d) refer to agricultural techniques, such as band and broadcast fertilizer application and number of plants per acre. Condition (c) says that the relative yield response function for a given crop should not be expected to be the same as for another crop. In this context, it is important to notice that although--in principle--nitrogen is not considered a relatively immobile nutrient, some authors have had considerable success in pooling large sets of nitrogen experiments under the assumption that coefficient c (in Mitscherlich's equation) is a constant (see, for example, Hanway and Dumenil). Moreover, Hildreth found no significant interaction between soil type, weather and nitrogen levels when corn yields were expressed in logs on a discrete response model fitted to a large set of experimental data. This "confirmed the a priori belief that equal percentage effects were a more plausible assumption than equal absolute effects" (Hildreth, p. 68). Therefore, the conditions under which the relative yield concept is currently believed to hold are not very restrictive. Yet, this concept has been largely neglected by most agricultural economists. Furthermore, one cannot even argue that the relative yield concept has been restricted to the theoretical body of soil science. In fact, this concept is frequently employed by soil scientists for the design of fertilizer recommendation tables (Rouse, p. 6).

Fertility Carry-Over and Control; The Soil Test Calibration

"As fertilizer is applied in increasing quantities, it becomes apparent that increased attention must be given to the value of carry-over. In many cases the cost of fertilization is charged to the crop treated. However, carry-over fertilizer is like money in the bank and is part of fertilizer economics. Hence, it is apparent that if we are to make a critical evaluation of fertilizer use, the carry-over value must be considered" (Tisdale and Nelson, p. 538).

Although agricultural economists have written less than a handful of papers on this subject (Kennedy, Whan, Jackson and Dillon; Stauber, Burt and Linse; Fuller), the

above quotation clearly indicates the importance agronomists attribute to fertilizer carry-over effects. Thus, fertilizer recommendations designed by soil scientists are not the result of a static yield maximization process. In most soil laboratories they are designed to build and to maintain the level of soil fertility at the minimum level required to achieve the yield plateau (Rouse, p. 16-17). The agronomists' recommendations for driving the system to this steady state and keeping it there must be based on observations of soil fertility as measured by soil tests. Therefore, it is surprising to realize that, while agronomists in general do not have a clear notion and grasp of dynamic functional models, nevertheless they organize their fertilizer experiments and analyses as if they would intend to solve an optimal control problem.

These considerations provide a key for interpreting the fundamental differences between the research framework and objectives of agronomists and agricultural economists. For the latter group, the main goal of fertilizer analyses is the proper estimation of the response function to use later in the profit maximizing scheme. The nature and formulation of this problem is clearly static. For agronomists, on the contrary, the principal objective of fertilizer experiments is to identify the optimal level of soil fertility and the best strategy to reach it.

The determination of the optimal fertility level is made relative to specified crops or rotations. But how is soil fertility to be measured? By means of a series of soil tests. Soil scientists consider that a correct interpretation of soil test measurements is of paramount importance for the design of fertilizer recommendations (Walsh and Beaton, partic. Chs. 1, 2, 4, and 14) because, in many instances, most of the information concerning a particular farming site comes embodied into a single soil sample. It is from the chemical analysis of such a small piece of "information" that fertilizer recommendation are usually derived. In the terminology of optimal control theory, soil test measurements are "sensor measurements signals" of a dynamic-stochastic physical process (Athans). Soil testing chemical methods, on the other hand, constitute the "sensors" themselves. Ideally, the soil sensors should produce measurements expressed in the same units employed to measure fertilizer quantities,

irrespective of everything else. However, such an idealized set of soil sensors has not as yet been produced by soil scientists.

Therefore, the quantity, b, of a given nutrient available in the soil is to be measured by soil tests, b*, which soil scientists assume to be proportional to the true value of b. In other words, $b = \lambda b^*$, where b* is the result of the soil sample's analysis and λ is the proportionality factor. The conditions required for the validity of the proportionality assumption may be considered stringent but under the present state of the arts the procedure used by agronomists seems widely accepted with the appropriate adjustments. For example, one of the conditions for the validity of the assumption is that the chemical form of the nutrient in the soil be constant. In practice this is not so and the proportionality coefficient λ varies somewhat according to soil type. The pragmatic approach followed by soil scientists, then, is to classify soils into "homogeneous" soil types in terms of their λ values. This procedure ensures that the chemical form of the nutrient in the soil is approximately the same for those soils included in a given "homogeneous" group. Hence, estimates of λ can be obtained from the following simple formula:

$$\lambda = x/(b_o^* - b_t^*) \tag{10}$$

where b_o^* is a soil test measurement taken prior to fertilizer application and b_t^* is a soil-test measurement taken some time after a given quantity of fertilizer (x) has been incorporated to the soil. The process of typifying the soils according to the magnitude of their λ coefficients is called "calibration" in soil terminology. Indeed, the λ coefficient "calibrates," or filters the sensor signals (b*) with respect to the actual signals (b). In practice, however, the "calibration" process is undertaken via the comparison of the response functions of a given crop or rotation cultivated in different soil types (Rouse, Cope and Rouse).

For these reasons, the λ coefficients depend on at least some of the soil type variables, so that the single nutrient yield response function (9) must be adjusted to read:

$$y = A_{ws}\, g(\lambda_s b^* + x) \tag{11}$$

where λ_s means that the proportionality coefficient λ is a function of some soil type variables. The other symbols retain their previous meaning. In view of (11), a truly "generalized response function" ought to incorporate the λ_s functions. This procedure, however, requires that soil type measurements be made in addition to conventional soil test measurements. The alternative, of course, is the "calibration" approach currently used, which aggregates soil types into groups for which the range of the λ_s function's value is relatively narrow. The choice of approach depends obviously, on economic considerations. On one hand, there are the costs of additional soil analyses and research in order to develop the λ_s functions. On the other hand, there are potential gains to be realized from a "finer tuning" than that allowed by the calibration procedure. In any event, it is clear that the λ coefficients (or λ_s functions) play a crucial role in the formulation of fertilizer recommendations. To some agricultural economists, however, "there does not seem to be any particular advantage in attempting to measure a λ factor when the aim is to develop a generalized response function which will accurately estimate the importance of currently available soil nutrient measurements in yield response" (Ryan, p. 20; emphasis added).

The discussion of residual fertility undertaken in this section illustrates the need for casting the analysis of fertilizer recommendations into a dynamic framework. First of all, one must estimate the optimal stock of soil fertility which maximizes the stream of discounted expected net returns. Secondly, there is the control problem of achieving and maintaining the fertility stock at its optimal level by the exogenous supply of fertilizer inputs. Control rules can be developed from the knowledge of past fertilizer applications and carry-over rates or functions. Thus, the design of fertilizer recommendations requires updated estimates of the level of extractable nutrients in the soil. From the point of view of the soil laboratory that makes fertilizer recommendations such updated estimates do not, necessarily, require the knowledge of past fertilizer applications and weather records. Instead, updated estimates of the level of extractable nutrients in the soil are to be obtained from the chemical analysis of soil samples. Thus, the dynamic extension of the nonsubstitution relative-yield model is as follows:

$$(12) \qquad y_t = A_{w_t s} \underset{j\ M}{\mathrm{Min}}\{g_j(\lambda_{js} b^*_{jt} + x_{jt})\}$$

$$(13) \qquad b^*_{jt} = h_{js}\ (y_{t-1};\ b^*_{jt-1} + \lambda^{-1}_{js} x_{jt-1})$$

where y_t is crop yield in period t; $A_{w_t s}$ is the yield plateau given w_t (weather conditions in period t) and s (soil type); g_j is the relative yield response to nutrient j ($0 \leq g_j(\cdot) \leq 1$; j M); λ_{js} is the proportionality factor which allows for the addition of soil test and applied fertilizer for nutrient j given soil type s; b^*_{jt} is the soil test level for nutrient j in period t; x_{jt} is the quantity of the jth nutrient incorporated to the soil in period t; h_{js} is the carry-over function for the jth nutrient given soil type s; and M is the set of macro nutrients.

The quantity $(\lambda_{js} b^*_{jt} + x_{jt})$ in equation (12) represents the total supply of nutrient j in period t as measured in fertilizer units (e.g., lb of P_2O_5 per Acre). On the other hand, the term $(b^*_{jt-1} + \lambda^{-1}_{js}\ x_{jt-1})$ in equation (13) represents the total supply of nutrient j in period t-1 as measured in terms of soil test units (e.g., ppm of P).

The Economics and the Design of Fertilizer Recommendation Tables

It is assumed that farmers would like to follow a fertilization strategy that maximizes the expected stream of discounted profits from fertilizer use. Hence, the problem of optimal fertilizer use can be written as:

$$(14) \qquad \mathrm{Max}\ \pi = \sum_{t \in T} (1 + i)^{-t} (P_{yt} y_t - \sum_{j \in M} P_{jt} x_{jt})$$

subject to $y_t - A_{w_t s} g_j(\lambda_{js} b^*_{jt} + x_{jt}) \leq 0; \quad j \in M,\ t \in T$

$$b^*_{jt} - h_{js}(y_{t-1},\ b^*_{jt-1} + \lambda^{-1}_{js} x_{jt-1}) = 0; \qquad j \in M,\ t \in T$$

$$b^*_{j1} = \text{given}; \quad j \in M; \quad x_{jt} \geq 0; \quad j \in M,\ t \in T$$

where i is the interest rate; T is the set of periods for planning purposes; P_{yt} is the output price on period t; P_{jt} is the jth fertilizer price in period t, and other symbols are as before.

This multistage programming formulation assumes that both the prices (P_{it}; $t \geq 1$) and yield plateaus ($A_{w_t s}$; $t \geq 1$) are known in advance. Since this is not a

reasonable assumption in general, these parameters are better understood as expected values. Capital constraints are easily introduced in this programming formulation. For example, they can be useful for examining institutional credit conditions. The b^*_{j1} represent known soil test levels at the beginning of the first planning period. The programming formulation generates a fertilization strategy for the planning horizon T. Such a strategy is obviously dependent upon the initial conditions b^*_{j1}. In practice, however, farmers periodically acquire new information on fertility levels, by means of soil tests. They can and probably use this information for updating their fertilization strategy. A conceptual modification of the problem is then required to accommodate this situation. Nevertheless, the programming formulation can be maintained as the basic tool for decisions. The incorporation of updating information into the programming formulation can be achieved by the "moving horizon" concept. Under this concept, "every decision made is a first-period decision corresponding with a (finite) horizon" (Theil, 1968, p. 155).

The control rules obtained from model (14) can be summarized as follows: for a given set of expected prices there will be an optimum stock of soil fertility to be maintained by means of periodic supply of fertilizer inputs. The stock of soil fertility present at any given point in time is measured via soil tests. The optimum quantity of fertilizer (control) to apply in any given period will be the difference between the current soil fertility level and its desired stock level (target).

Lastly, it should be noticed that the mathematical programming problem specified in (14) is likely to have important nonlinearities. In this case, the computation of exact solutions may be very difficult and approximations are in general required in empirical applications.

The Estimation of the Response Functions

The formulation of the yield response function (12) is novel and its estimation requires a suitable procedure. The specification is related to recent models of market disequilibrium (Goldfeld and Quandt, Maddala and Nelson) and is discussed in detail by Lanzer, Paris and Green. The sample information utilized in this study,

however, is not suitable to estimate relation (12) in its generality. The scientific tradition of soil scientists prescribes only single nutrient experiments, with all other nutrients kept at nonlimiting levels. An experienced agronomist has little difficulty in keeping the supply of all nutrients, but one, at nonlimiting levels. Thus, for example, the data from an experiment where the supply of phosphorus and potassium is known to be nonlimiting can be used to estimate the yield response to nitrogen. It is important to notice that, when this condition prevails, the parameters of the individual nutrient response function can be estimated by conventional regression techniques. The individual yield response functions so estimated, are combined in the form prescribed by the nonsubstitution model. In this way, an estimate for the multiple nutrient response surface is obtained. The single nutrient yield response function can be indicated as:

$$(15) \quad y_{pe} = A_{w_e s_e} \, g_n(X^T_{npe}) + v_{npe}, \qquad p = 1,\ldots I, \quad n = P,K,N \quad e = 1,\ldots,E$$

where y_{pe} is the yield observed on the p-th plot of the e-th experiment concerning the n-th nutrient; $A_{w_e s_e}$ is the yield plateau of the e-th experiment; g_n is the relative yield response to the n-th nutrient; $X^T_{npe} = \lambda_{ns_e} b^*_{npe} + x^a_{npe}$, is the total amount of nutrient available in the soil, and it is composed of the quantity applied during the experiment, x^a_{npe}, and of the quantity available before, $\lambda_{ns_e} b^*_{npe}$; v_{npe} is white noise. The quantity X^T_{npe} can be measured as soon as estimates of λ_{ns_e} are available.

The functional form of the yield response function (15) has been the target of considerable efforts among soil scientists as well as agricultural economists, without the comfort of any robust consensus. Thus, rather than commiting oneself to any given functional specification it seems most appropriate to employ the notion and the technique based on spline functions. This technique is well establised in the engineering literature, but was only recently brought to the attention of economists. It is based upon the idea of approximating any nonlinear function by polynomial segments of any desired degree, taking care of joining the extreme points

of the various segments. The points where two segments join are called knots. Poirier, Erterl and Fowlkes, Suits, Mason and Chan give a detailed and very intelligible exposition of the spline procedures.

The yield response function may be taken to be a concave function without loss of generality. Hence, it can conveniently be represented by linear splines to any desired degree of approximation. The procedure can be easily illustrated by reference to Figure 1. Given the knots 0, X_1, X_2, ..., X_K, which are simply known levels of nutrients, the spline function corresponding to Figure 1 is:

$$(16) \qquad y_{ij} = A_j\left(\sum_{m=1}^{K} \beta_m Z_{mij}\right) + e_{ij}$$

where $Z_{1i_j} = X^T_{ij}$; $Z_{2ij} = \max[X^T_{ij} - X_1; 0]$, ...,; $Z_{Kij} = \max[X^T_{ij} - X_{K-1}; 0]$; and $X^T_{ij} = \lambda_x b^*_{ij} + x^a_{ij}$ is the total availability of nutrient X for the i-th treatment and the jth location. Model (16) assumes that A_j, the expected yield plateau of the jth experiment, is observable. The maximum observed yield of the jth experiment (M_j), on the other hand, is an order statistic that is likely to overestimate A_j, that is, the <u>expected</u> maximum yield of the jth experiment. Thus, (16) has been modified to:

$$(17) \qquad y_{ij} = \alpha M_j\left(\sum_{m=1}^{K} \beta_m Z_{mij}\right) + e_{ij}$$

where parameter α can be considered as the expectation of a random coefficient a_j such that $M_j = a_j^{-1} A_j$. In words: the highest order statistic for the yields of a given experiment is assumed to be (stochastically) proportional to the expected maximum of that experiment. Under this random coefficient assumption, e_{ij} must now be viewed as a heteroskedastic error term.

Furthermore, notice that $\max[\Sigma\beta_m Z_m] = 1$, that is, the maximum for relative yields equals 1 (or 100 percent). Consider, then, Figure 1. It depicts the relative yield response (yr) to the total supply of nutrient X (X^T). Notice that

$$(18) \quad yr_m = \beta_1 X_1 + (\beta_1 + \beta_2)(X_2 - X_1) + \ldots + (\beta_1 + \ldots + \beta_j)\ (X_m - X_{m-1})$$

where the X_m are known fixed knots and $\beta_1 = tg\gamma_1$, $(\beta_1+\beta_2) = tg\gamma_2$ etc. Concavity

of the yield response requires that $\beta_m \leq 0$ for $m \geq 2$. Suppose that it were known that yr equals to its maximum for some range of X^T starting at X_m ($m \geq 4$) and extending to X_4 at least. In this case, the spline function must satisfy:

$$(19) \quad \beta_1 X_1 + (\beta_1 + \beta_2)(X_2 - X_1) + \ldots + (\beta_1 + \ldots + \beta_4)(X_4 - X_3) = 1.$$

After some simple algebraic manipulations, (19) can also be written as:

$$(20) \quad \beta_1 = \frac{1}{X_4} - \frac{(X_4 - X_1)}{X_4}\beta_2 - \frac{(X_4 - X_2)}{X_4}\beta_3 - \frac{(X_4 - X_3)}{X_4}\beta_4.$$

Equation (20) is a linear constraint on the parameters of (17). This constraint ensures that relative yields attain a maximum which equals 1 (or 100 percent) at X_m (for $m \leq 4$) and extends to X_4 at least (provided that $\beta_m \leq 0$ for $m \geq 2$). By substituting (20) into (17)--assuming $k = 4$--and rearranging terms one arrives at:

$$(21) \quad Y_{ij} = \alpha M_j \{X_4^{-1} + \sum_{m=2}^{m=4} \beta_m (Z_{mij} - \frac{(X_4 - X_{m-1})}{X_4} Z_{1ij})\} + e_{ij} \qquad \text{or,}$$

$$(22) \quad Y_{ij} = \alpha Z^*_{1ij} + \alpha\beta_2 Z^*_{2ij} + \alpha\beta_3 Z^*_{3ij} + \alpha\beta_4 Z^*_{4ij} + e_{ij}$$

where $Z^*_{1ij} = M_j X_4^{-1}$; $Z^*_{2ij} = M_j (Z_{2ij} - \frac{(X_4 - X_1)}{X_4} Z_{1ij})$, etc.

The parameters of (22) can be estimated by conventional linear regression methods (parameter β_1 is recovered with the help of (20)). In the present research, however, it was decided to estimate (21) directly with a nonlinear least-squares procedure. Such a procedure allowed for further constraining the β_k estimates ($k > 2$) to be nonpositive, a requirement for ensuring the resulting spline function to be a quasi-concave function (with the possibility of an initial range of increasing returns if both β_1 and β_2 are positive). The convenience and the importance of using a spline approach for the estimation of response functions is further enhanced by the adoption of a separable programming framework (as explained in Part II) for the economic evaluation of fertilizer recommendations. Thus, the knots of the spline approach naturally generate the grid required by separable programming.

The Estimation of the Carry-over Functions

The econometric specification of the carry-over function adopted in this study postulates a condition of additive separability between yields and fertilizer levels,

that is:

$$(23)\quad b^*_{jt} = \Theta_j(b^*_{jt-1} + \lambda^{-1}_{js}\, x^a_{jt-1}) + \beta_j y_{t-1} + u_{jt}.$$

Notice that (23) is the "reduced form" of the following distributed lags model:

$$(24)\quad b^*_{jt} = \sum_{i=1}^{n} \Theta_j^{i-1}\, (\Theta_j \lambda^{-1}_{js}\, x^a_{jt-i-1} + \beta_j y_{t-i-1}) + e_{jt}$$

so that the parameter Θ_j is a rate of geometric decline of the availability of the jth nutrient from one period to another. It is expected that $0 < \Theta_j < 1$. Notice also that $u_{jt} = e_{jt} - \Theta_j e_{jt-1}$. Thus, consistent estimation of (23) by Ordinary Least Squares (OLS) would require the assumption that $e_{jt} = \Theta_j e_{jt-1} + v_{jt}$ and where v_{jt} is assumed to be a white noise.

The assumption that the autoregression coefficient is equal to the geometric decline coefficient seems rather implausible. A less restrictive assumption on the error term of (23) is: $u_{jt} = r_j u_{jt-1} + v_{jt}$, $|r_j| < 1$, where v_{jt} is assumed to be a white noise. In this case, consistent estimates of the parameters of (23) can be obtained from conditional OLS regressions on:

$$(25)\quad b^*_{jt} - r_j b^*_{jt-1} = \Theta_j(b^*_{jt-1} - r_j b^*_{jt-2}) + \Theta_j\, \lambda^{-1}_{js}(x^a_{jt-1} - r_j x^a_{jt-2})$$

$$+ \beta_j(y_{t-1} - r_j y_{t-2}) + v_{jt}.$$

The coefficient r_j is made to vary until the sum of squared residuals is minimized. Under the normality assumption, this procedure produces maximum likelihood estimates of Θ_j, λ_{js} and β_j (Theil, pp. 414-424).

The estimates of the proportionality factors obtained from the estimation of the carry-over functions can then be used in the estimation of the yield response equations.

A NEW ANALYTICAL FRAMEWORK FOR THE FERTILIZATION PROBLEM: PART II, EMPIRICAL RESULTS

The model presented in Part I was applied to the economic analysis of fertilizer recommendations for the wheat-soybeans cropping system in southern Brazil. Most of the data used in this study were provided by Dr. João Mielniczuck of the Department of Soil Science of the Federal University of Rio Grande do Sul (DS/UFRGS) and by Engeniero Agronomo Otávio Siqueira of the National Wheat Research Center (CNPT). A smaller part of the information was collected from reports published by the DS/UFRGS, by the CNPT, and by the Institute of Agricultural Research (IPAGRO) of the Agricultural Department of the State of Rio Grande do Sul.

A total of 37 independent experiments carried out in the wheat-soybean producing area of Rio Grande do Sul provided the observations used in this research. The duration of each experiment varied from one single cropping period--either wheat or soybeans--to eight consecutive cropping periods--double cropping system. The period of experimentation ranged from 1968 to 1976. In most cases, only the means of three to five replications were available. Soil acidity had been corrected through the use of lime in all observations selected for this research. The source for P was superphosphate (triple), whereas the source for K was potassium chloride and the source for N was urea. The fertilizers were broadcast, and high yielding varieties were used in all experiments. The North Carolina soil test extractant (H_2SO_4.0025 N + HCl .005 N) was used to evaluate the levels of P and K in the soil. The soil test method for N, on the other hand, was an indirect measure through the organic matter percentage content of the soil (H_2SO_4 + $Na_2Cr_2O_7$/oxidation). Some of the experiments had a factorial design with three to five levels of N, P and K. However, since the model allowed for the independent estimation of yield responses to N, P and K, a number of experiments where only one of those elements had been tested was also included. For these experiments, the supply of all nutrients, except one, had been set at nonlimiting levels.

Fertility Carry-over Functions

The data used to estimate the carry-over functions came from a set of seven experiments conducted by the National Wheat Research Center. All seven experiments were initiated in 1973 and terminated in 1976. Five of those experiments were located on soils having a 20 to 40 percent clay. Such soils are classified as "type 2." The other two experiments were located on soils having more than 40 percent clay. Agronomists classified such soils as "type 1." This soil classification is considered of paramount importance as far as soil tests for phosphorous are concerned. More than 10 years of research led the local soil scientists to conclude that a soil test level of x ppm of P for a soil type 1 reflects the same availability of P as a soil test level of 2x ppm of P for a soil type 2 (UFRGS, p. 3). This information was entered as a constraint on the estimation of the λ parameters of the phosphorous carry-over functions. The data available for the estimation of carry-over functions had two important limitations. First, the straw of every crop was removed rather than incorporated into the soil as is done in practice. This, of course, may lead to an underestimation of carry-over fertilizer. Second, fertilizers were applied only for wheat; soybeans were carried as a "residual" crop. Thus, it became impossible to estimate separate carry-over functions for wheat and for soybeans. Under such circumstances, it did not seem interesting to also include lagged yields in the carry-over functions, but rather to estimate "average" carry-over functions for the two crops.

Phosphorous

The carry-over model adopted for phosphorous was as follows:

$$P_t^s = \Theta_P(P_{t-1}^s + \lambda_p^{-1} P_{t-1}^a) + u_t \tag{26}$$

where P_t^s is the soil test level for P at the beginning of cropping period t (in ppm of P); P_t^a is the quantity of phosphate fertilizer applied in period t (in kg of P_2O_5 per ha); Θ_p is the geometric decline parameter; λ_p is the proportionality factor for phosphorous and u_t is a random error ($u_t = \rho_p u_{t-1} + v_t$; $v_t \{v \sim N(0, \sigma_p^2 I)$).

As the value of λ_p was known to vary with soil type, agronomists in southern Brazil considered that $\lambda_{p1} = 2\ \lambda_{p2}$ (or $\lambda_{p1}^{-1} = \frac{1}{2}\ \lambda_{p2}^{-1}$). Since this information is held with a high degree of confidence by local soil scientists, it was incorporated into the carry-over function.

Thus, a combined carry-over function for the two soil types can be written as:

$$(27) \quad P_t^s = \Theta_p \ P_{t-1}^s + \lambda_{p1}^{-1} (P_{t-1}^a + P_{t-1}^a D) \quad + u_t$$

where D is a dummy variable which equals zero for soil type 1 and equals 1 for soil type 2.

A Gauss-Newton algorithm was used to estimate the parameters of model (27) in the autoregressive version of equation (25). The statistical results are summarized in Table 1.

The results shown in Table 1 indicate that a satisfactory fit for the phosphorous carry-over function can be obtained from the available data. Also, from the results shown in Table 1, one concludes that $\bar{\lambda}_{p1} = 48.26$ and that $\bar{\lambda}_{p2} = 24.13$. In other words: one ppm of P, for a soil type 1, is estimated to be equivalent to a fertilizer application of 48.26kg of P_2O_5 per ha. The same soil test unit is equivalent to a fertilizer application of 24.13kg of P_2O_5 per ha for type 2 soils. These estimates will be required later for the estimation of yield response functions. An unconstrained model was also fitted to the available data. The estimates obtained in this case were $\bar{\lambda}_{p1} = 51.69$ and $\bar{\lambda}_{p2} = 22.80$. The MSE for the unconstrained model was 85.96. It is also interesting to notice that the estimated geometric decline coefficient is relatively high, i.e., close to unity. In this case, the level soil phosphorous is seen to decrease at a relatively low rate across time. This means that phosphate fertilizers possess highly significant carry-over effects in southern Brazilian soils.

Potassium

For potassium no soil classification is currently made by southern Brazilian agronomists. The results from the nonlinear least-squares fit of model (25) are

summarized in Table 2. The R^2 statistic in Table 2 does not indicate a fit as satisfactory as the one obtained for the phosphorous carry-over (Table 1). Yet, the asymptotic standard deviations are small for all three parameter estimates of the potassium carry-over function. Thus, it appears that such point estimates can be used with a high degree of confidence. From Table 2 one concludes that $\bar{\lambda}_k = 3.73$, i.e., each ppm of K is estimated to be equivalent to an application of 3.73 kg of K_2O per ha. This estimate will be used later for the estimation of yield response functions to potassium.

Yield Response Functions

Soybeans

Two yield response functions--one for phosphorous and one for potassium--were estimated for soybeans using the spline procedure described in Part I. The knots for phosphorous were chosen as follows:

X_1 = 75kg of P_2O_5/ha (approx. 1.5 (3.0) ppm of P for soil type 1(2))

X_2 = 225kg of P_2O_5/ha (approx. 4.5 (9.0) ppm of P for soil type 1(2))

X_3 = 375kg of P_2O_5/ha (approx. 7.5 (15.0) ppm of P for soil type 1(2))

X_4 = 525kg of P_2O_5/ha (approx. 10.5 (21.0) ppm of P for soil type 1(2))

X_5 = 675kg of P_2O_5/ha (approx. 13.5 (27.0) ppm of P for soil type 1(2)).

Since southern Brazilian agronomists consider that a soil test level of 9 (18) ppm of P for soil type 1(2) is "high," the spline function was constrained to attain a maximum of 1 (or 100 percent) at--or before--the level of 675kg of P_2O_5/ha.

Model (17) was fitted to the empirical data of soybeans and phosphorous with the help of the nonlinear least-squares procedure already referred to. The parameter α was constrained to the interval (0.5, 1.5) and parameters β_m (m = 3,4,5,6) were constrained to be nonpositive (in order to ensure concavity of the response function for levels of P^T equal to or above 225kg of P_2O_5/ha). The statistical results of the regression are summarized in Table 3.

The results of Table 3 indicate that a satisfactory fit was obtained for the soybean response to total phosphorous. The estimated grid of points for the relative

yield (yr) spline function is thus:

at P^T = 0kg of P_2O_5/ha → yr = 0.000 (00.0 percent of the max. expected yield)

at P^T = 75kg of P_2O_5/ha → yr = 0.549 (54.9 percent of the max. expected yield)

at P^T = 225kg of P_2O_5/ha → yr = 0.868 (86.8 percent of the max. expected yield)

at P^T = 525kg of P_2O_5/ha → yr = 0.997 (99.7 percent of the max. expected yield)

at P^T = 675kg of P_2O_5/ha → yr = 1.000 (100 percent of the max. expected yield).

Notice that no change of inclination occurred at the level of 375kg of P_2O_5/ha (the nonpositive constraint of β_4 was binding). Also, the estimate obtained for β_6 indicates that yields tend to decrease at a very small rate after the level of 675 of P_2O_5/ha. This result, together with the relative yield estimate of 97.7 percent for P^T = 525kg of P_2O_5/ha, strongly suggests the presence of a yield plateau.

The knots chosen to estimate the soybeans response to potassium were as follows: X_1 = 40, X_2 = 110, X_3 = 185, X_4 = 260, X_5 = 410kg of k_2O/ha. The spline function was constrained to attain a maximum of 1 (or 100 percent) at--or before--the level of 410kg of K_2O. This was because agronomists consider a soil test level of 60 ppm of K as "high," or nonlimiting.

The statistical results of the regression are summarized in Table 4. Again, they seem satisfactory. The R^2 statistic and the relatively small standard deviation of the estimated parameters indicate that the fitted equation has a high predictive power. The estimated grid of points for the soybean response to potassium, computed from the information presented in Table 4 is:

at K^T = 0kg of K_2O/ha → yr = 0.000 (or 00.0 percent of the max. expected yield)

at K^T = 40kg of K_2O/ha → yr = 0.534 (or 53.4 percent of the max. expected yield)

at K^T = 110kg of K_2O/ha → yr = 0.721 (or 72.1 percent of the max. expected yield)

at K^T = 185kg of K_2O/ha → yr = 0.915 (or 91.5 percent of the max. expected yield)

at K^T = 260kg of K_2O/ha → yr = 0.970 (or 97.0 percent of the max. expected yield)

at K^T = 410kg of K_2O/ha → yr = 1.000 (or 100 percent of the max. expected yield).

The small numerical estimate for β_6 indicates that relative yields tend to decrease very slowly after K^T = 410kg of K_2O/ha. Again, the existence of a yield plateau is strongly suggested.

Wheat

Three yields response functions were estimated for wheat: one for phosphorous, one for potassium and one for nitrogen. The methods employed for the first two functions are identical to those already discussed for soybeans (including the choice of knots). Thus, only the results are presented for phosphorous and potassium. The case of nitrogen will then be presented in detail.

Table 5 presents the statistical results obtained from the spline of the wheat response to phosphorous. The statistical results shown in Table 5 indicate that a satisfactory fit was obtained. It is interesting to notice that $\hat{\beta}_2 > 0$ implies increasing returns for relatively low levels of phosphorous. This has not been observed for the case of soybeans. The estimated grid of points for the wheat response to phosphorous is:

at P^T = 0kg of P_2O_5/ha → yr = 0.000 (00.0 percent of the max. expected yield)

at P^T = 75kg of P_2O_5/ha → yr = 0.197 (19.7 percent of the max. expected yield)

at P^T = 225kg of P_2O_5/ha → yr = 0.799 (79.9 percent of the max. expected yield)

at P^T = 375kg of P_2O_5/ha → yr = 0.905 (90.5 percent of the max. expected yield)

at P^T = 675kg of P_2O_5/ha → yr = 1.000 (100 percent of the max. expected yield).

Table 6 summarizes the statistical results obtained for the spline fit of the wheat response to potassium. The results of Table 6 again indicate that a satisfactory fit was obtained. The estimated grid of points for the wheat response to potassium computed from the information of Table 6 is as follows:

at K^T = 0kg of K_2O/ha → yr = 0.000 (00.0 percent of the max. expected yield)

at K^T = 40kg of K_2O/ha → yr = -0.83

at K^T = 110kg of K_2O/ha → yr = 0.553 (55.3 percent of the max. expected yield)

at K^T = 185kg of K_2O/ha → yr = 0.885 (88.5 percent of the max. expected yield)

at K^T = 260kg of K_2O/ha → yr = 0.970 (97.0 percent of the max. expected yield)

at K^T = 410kg of K_2O/ha → yr = 1.000 (100 percent of the max. expected yield).

The estimate of -0.83 at K^T = 40kg of K_2O/ha is meaningless, of course. It is due to the fact that no observation was available in the range of 0 to 40kg of K_2O/ha.

In this case, the first spline segment "floated" free in order to link the lower point estimate for the second spline segment. For all practical purposes, the knot [40; -0.83] can be excluded from the analysis.

The estimation of wheat response to nitrogen did not follow the same methods employed for phosphorous and potassium for two main reasons. First, the soil test method used by the Brazilian agronomists to evaluate the soil supply of N is not a direct measurement as in the case of P and K. Nitrogen is indirectly evaluated through the organic matter percentage content of the soil. Secondly, nitrogen carry-over is not important in the wheat-soybeans double cropping system of southern Brazil. This is because soybeans can produce their own nitrogen via the Rhizobium bacteria and rain leaches abundantly the soil. Therefore, a carry-over function was not estimated for N. Instead, it was decided to estimate the relation between soil nitrogen, applied nitrogen and yields directly via the yield response function. The model adopted in this case was based on the work of Bray:

$$y_{ij} = A_j(1 - e^{-c^*N^s_{ij} - cN^a_{ij}}) + e_{ij} \tag{28}$$

where y_{ij} is the yield obtained on the ith treatment of the jth experiment (in kg of wheat per ha); A_j is the expected asymptotic yield plateau of the jth experiment (as before, it is assumed that $M_j = a_j^{-1}A_j$ and $E(a_j)=\alpha$; M_j is the maximum yield observed on the jth experiment); N^s_{ij} is the soil test level for nitrogen (in percent content of organic matter); N^a_{ij} is the quantity of applied nitrogen fertilizer (in kg of N/ha); e_{ij} is assumed to be a white noise term; and c* is a parameter of the model such that $c^* = c\lambda_n$, where λ_n is the proportionality factor between percentage organic matter and kg of N/ha. Notice that, in (28), the term in parentheses is the relative yield response of wheat (yr) to total nitrogen (N^T). Total nitrogen, in turn, is defined as $N^T = \lambda_n N^s + N^a$ measured in kg of N/ha. Model (28) was estimated by the nonlinear least squares procedure already referred (αM_j being substituted for A_j). The statistical results are summarized in Table 7.

The statistical results of Table 7 indicate that a satisfactory fit was obtained for the wheat response to nitrogen. The estimated proportionality factor for

nitrogen is $\lambda_n = 13.13$ (=c*/c), that is, each percentage unit of organic matter is estimated to be equivalent to an application of 13.13kg of N/ha.

Separable Programming and Economic Analysis

Model (14) constitutes the basis for the economic analysis of the fertilization problem. A separable programming approach was chosen to implement it. It was also decided to apply the moving horizon concept for a planning period of four years (or eight consecutive cropping periods). Therefore, only the optimum levels of nutrients computed for the first two cropping periods are of interest for the analysis. These levels constitute "soil fertility targets" or, alternatively, optimum nutrient stocks, to be maintained for wheat and soybeans.

According to the National Wheat Research Center, the expected yield plateaus for wheat and soybeans in southern Brazil are 1800kg/ha and 2800kg/ha, respectively. The prices used in the programming model were (Cr$/kg) 5.61 for N, 7.06 for P_2O_5, 2.49 for K_2O, 2.03 for wheat and 1.84 for soybeans. They are 1976 average prices for southern Brazil. They do not include a 40 percent subsidy for fertilizers which was in effect from 1974 to early 1977. The prices used will be referred to as "current prices."

The programming formulation used to establish the optimum levels of soil fertility for the wheat-soybeans double-cropping system was as follows:

Objective Function

$$\text{(29)} \qquad \text{Max PV} = \text{PVR} - \text{PVVC}$$

where PV is present value of (expected) net revenues, PVR is present value of (expected) total revenues, and PVVC is present value of (expected) fertilizer costs.

Constraints

Present value of expected total revenues:

$$\text{(30)} \quad \text{PVR} = \frac{3654}{1.03} yr_1 + \frac{5152}{1.03^2} yr_2 + \frac{3654}{1.03^3} yr_3 + \frac{5152}{1.03^4} yr_4 + \ldots + \frac{5152}{1.03^8} yr_8$$

where yr_j is the relative yield of the crop cultivated during the jth period, j = 1, ..., 8. The coefficient 3654 (=1800kg of wheat/ha times Cr$ 2.03 per kg of wheat)

represents the expected revenue plateau per ha of wheat (in Cr$/ha). The coefficent 5152 (=2800kg of soybeans/ha times Cr$ 1.84 per kg of soybeans) represents the expected revenue plateau per ha of soybeans (in Cr$/ha). An interest rate of three percent per semester has been used to discount future revenues. This rate is officially adopted for savings accounts ("cadernetas de poupança") in Brazil.

Present values of expected fertilizer cost:

$$(31)\quad PVVC = 5.61\ N_1^a + 7.06\ P_1^a + 2.49\ K_1^a + \frac{7.06}{1.03} P_2^a + \frac{2.49}{1.03} K_2^a + \frac{5.61}{1.03^2} N_3^a + \frac{7.06}{1.03^2} P_3^a + \frac{2.49}{1.03^2} K_3^a + \ldots + \frac{2.49}{1.03^7} K_8^a$$

where x = N, P, K and x_j^a is the quantity of nutrient X added to the soil at the beginning of the jth cropping period. Notice that no nitrogen is applied for soybeans.

Total nitrogen:

$$(32)\quad N_1^s = \text{given},\ N_j^T = 13.13\ N_1^s + N_j^a,\ j = 1, 3, 5, 7$$

where N_j^T represents the total quantity of nitrogen available for the plants (in kg of N/ha) in the jth cropping period, N_1^s is the percentage content of organic matter content of the soil (assumed to be constant because of lack of information) and N_j^a is nitrogen applied to the soil as fertilizer in the jth cropping period (in kg of N/ha).

Applied and total phosphorous:

$$(33)\quad P_1^s = \text{given};\ P_{j+1}^s = 0.8895\ (P_j^s + 0.02072\ P_j^a),\ P_j^T = 48.26\ P_j^s + P_j^a,\ j = 1, \ldots, 8.$$

The above set of equations describes the phosphorous supply over time. P_j^T is the quantity of phosphorous available for the plants in the beginning of the jth cropping period (in kg of P_2O_5/ha). P_j^s is the soil test level for phosphorous in the beginning of the jth cropping period (in ppm of P). P_j^a is the quantity of phosphorous applied in the beginning of the jth cropping period (in kg of P_2O_5 per ha). Notice that the coefficients for soil type 1 were used in the analysis; since the only

difference between soils types 1 and 2 is the "exchange rate" factor λ_p, the adjustment of results is straightforward.

Applied and total potassium:

(34) K_1^s = given, $K_{j+1}^s = 0.8139\ (K_j^s + 0.2682\ K_j^a)$, $K_j^T = 3.73\ K_j^s + K_j^a$, $j = 1, \ldots, 8$.

The above set of equations describes the dynamics of potassium supply. K_j^T is the quantity of potassium available for the plants in the beginning of the jth cropping period (in kg of K_2O/ha). K_j^s is the soil test level of potassium in the beginning of the jth cropping period (in ppm of K). K_j^a is potassium fertilizer applied in the beginning of the jth cropping period (in kg of K_2O/ha).

Relative yield response function for wheat:

$$yr_j \leq 0w1_{nj} + 0.724w2_{nj} + 0.924w3_{nj} + 0.978w4_{nj} + 1.000w5_{nj}$$

$$N_j^T = 0w1_{nj} + 30w2_{nj} + 60w3_{nj} + 90w4_{nj} + 120w5_{nj}$$

$$1 = w1_{nj} + w2_{nj} + w3_{nj} + w4_{nj} + w5_{nj}$$

$$yr_j \leq 0w1_{pj} + 0.197w2_{pj} + 0.799w3_{pj} + 0.905w4_{pj} + 1.000w5_{pj}$$

$$P_j^T = 0w1_{pj} + 75w2_{pj} + 225w3_{pj} + 375w4_{pj} + 675w5_{pj}$$

$$1 = w1_{pj} + w2_{pj} + w3_{pj} + w4_{pj} + w5_{pj}$$

$$yr_j \leq 0w1_{kj} + 0.553w2_{kj} + 0.885w3_{kj} + 0.970w4_{kj} + 1.000w5_{kj}$$

$$K_j^T = 0w1_{kj} + 110w2_{kj} + 185w3_{kj} + 260w4_{kj} + 410w5_{kj}$$

$$1 = w1_{kj} + w2_{kj} + w3_{kj} + w4_{kj} + w5_{kj}, \qquad j = 1, 3, 5, 7.$$

The set of restrictions above describes the relative yield of wheat in the first, third, fifth and seventh cropping periods, as a function of the available supplies of N, P and K.

Relative yield response function for soybeans:

$$yr_j \leq 0w1_{pj} + 0.549w2_{pj} + 0.868w3_{pj} + 0.997w4_{pj} + 1.000w5_{pj}$$

$$P_j^T = 0w1_{pj} + 75w2_{pj} + 225w3_{pj} + 525w4_{pj} + 675w5_{pj}$$

$$1 = w1_{pj} + w2_{pj} + w3_{pj} + w4_{pj} + w5_{pj}$$

$$yr_j \leq 0w1_{kj} + 0.534w2_{kj} + 0.721w3_{kj} + 0.915w4_{kj} + 0.970w5_{kj} + 1.00w6_{kj}$$

$$K_j^T = 0w1_{kj} + 40w2_{kj} + 110w3_{kj} + 185w4_{kj} + 260w5_{kj} + 410w6_{kj}$$

$$1 = w1_{kj} + w2_{kj} + w3_{kj} + w4_{kj} + w5_{kj} + w6_{kj},\ j = 2, 4, 6, 8.$$

The set of restrictions above describes the relative yield of soybeans in the second, fourth, sixth, and eighth cropping periods, as a function of the available supplies of P and K.

The programming model was completed with nonnegativity constraints on all variables. In total there were 196 variables (excluding slacks) and 127 constraints. The computations were carried out by conventional linear programming procedures. No special separable programming algorithm was required because the model exhibited the appropriate convexities.

Of special interest in the analysis was the evaluation of the stability of the optimum soil fertility targets (N_1^T, P_1^T, K_1^T, P_2^T, and K_2^T) with respect to changes in input prices. It was expected that the results of this analysis would provide useful information for improving the fertilizer recommendation tables currently in use in southern Brazil. The results indicate that parametrization of the initial soil fertility conditions (N_1^s, P_1^s and K_1^s) did not affect the computed optimum soil fertility targets of the first two periods (N_1^T, P_1^T, K_1^T, P_2^T and K_2^T). These, in turn, were as follows:

a) optimum soil fertility targets for wheat (current prices)

N_1^T = 57.15kg of N/ha or 4.35 percent organic matter content

P_1^T = 375.0kg of P_2O_5/ha or 7.8 (15.6) ppm of P for soils type 1(2)

K_1^T = 202.6kg of K_2O/ha or 54.3 ppm of K

b) optimum soil fertility targets for soybeans (current prices)

P_2^T = 456.0kg of P_2O_5/ha or 9.4 (18.8) ppm of P for soils type 1(2)

K_2^T = 260.0kg of K_2O/ha or 69.7 ppm of K

Next, the input prices were parametrized within the interval of .6 to 1.4 times the current prices. The results of the analysis are reported in Table 8. As Table 8 indicates, the optimum fetility targets (stocks) for both wheat and soybeans are relatively stable with respect to changes in fertilizer prices (particularly with respect to increases in fertilizer prices). For example: a decrease of 30 percent in fertilizer prices, ceteris paribus, increases the nitrogen target for wheat by only 4.9 percent, whereas the phosphorous and potassium targets for the same crop are increased by 16.0 and 8.3 percent, respectively. The relative changes in output levels would be even smaller. Such stability is possibly due to the high carry-over effect of both phosphorous and potassium fertilizers. In any event, it seems that the optimum fertility targets computed at current prices can be viewed as solid lower bounds for the purpose of making fertilizer recommendations. Therefore, it appears that such recommendations are relatively well protected against the possibility of small errors in the estimates of the coefficients of the programming model (particularly against a possible overestimation of the expected yield plateaus for wheat and soybeans).

In view of the above results, the analysis turns to a critical evaluation of the fertilizer recommendation tables currently used for the southern Brazilian wheat-soybeans double cropping system. Such tables do not make a distinction between soil fertility targets for wheat and for soybeans. In either case, soil scientists recommend that a level of 9 (18) ppm of P for soils type 1(2) and of 60 ppm of K be maintained in the soil (UFRGS). Thus, it appears that only minor modifications are required on the tables as far as target levels for phosphorous and potassium are concerned (the relative differences from the table targets to the computed optimum are in the range of 5 to 15 percent). In the analysis to follow, it will be assumed

that the current targets of the tables are satisfactory approximation to the actual optima. It is worthwhile to mention, at this point, that a static (one period) optimization for wheat and for soybeans (independently) leads to soil fertility targets that are 20 to 50 percent below the targets computed under the moving horizon concept. The static optimization targets for wheat, under current prices, are as follows: N^T = 41.3kg of N/ha; P^T = 225.0kg of P_2O_5/ha and K^T = 165.6kg of K_2O/ha. The static optimization targets for soybeans, under current prices, are as follows: P^T = 225.0kg of P_2O_5/ha and K^T = 167.2kg of K_2O/ha. This illustrates further the need for considering fertilizer carry-over in the economic analysis of fertilizer data.

In order <u>to maintain</u> the levels of soil fertility for P and K at their desired levels, the soil scientists in southern Brazil recommend that an application of 75kg of P_2O_5/ha and 40kg of K_2O/ha be used in each cropping period. Such "maintenance recommendation" can be evaluated by the carry-over equations (33) and (34). By treating (33) deterministically, letting $P^s_{t-1} = P^s_t = P^s_*$ (where P^s_* is thus defined as the desired target level for P, in ppm units) and solving for $P^a_{t-1} = P^a_*$ (where P^a_* is thus defined as the maintenance application of P in kg of P_2O_5/ha/cropping period), one finds that the estimated fertilizer application to maintain the level of soil phosphorous at any level P^s_* is given by $P^a_* = 5.996\ P^s_*$ for soil type 1, and $P^a_* = 2.998\ P^s_*$ for soil type 2.

Similarly, from equation 34 by letting $K^s_{t-1} = K^s_t = K^s_*$ (where K^s_* is thus defined as the desired target for K in ppm units) and solving for $K^a_{t-1} = K^a_*$ (where K^a_* is thus defined as the maintenance application of K in kg of K_2O/ha/cropping period), one finds that the fertilizer application required to maintain the level of soil K at its desired level is given by $K^a_* = 0.874\ K^s$ for soil types.

The soil fertility targets associated with current recommendations are $P^s_* = 9$ (18) ppm of P for soils type 1(2) and $K^s_* = 60$ ppm of K for all soil types. By substituting these values into the equations for P^a_* and K^a_*, just derived, one finds that the maintenance application levels for P and K should be, approximately, $P^a_* =$

54kg of P_2O_5/ha/cropping period and K_*^a = 52.4kg of K_2O/ha/cropping period. These levels contrast with the maintenance levels currently recommended in southern Brazil, which are 75kg of P_2O_5/ha/cropping period and 40kg of K_2O/ha/cropping period. Therefore, the current recommendations for "maintenance P" are overestimated by 48 percent (approx.) whereas the recommendations for "maintenance K" are underestimated by 31 percent (approx.). Thus, the maintenance fertilizer recommendations of southern Brazilian tables do not seem to be consistent with their own soil fertility targets. If those recommendations were followed for a long period of time, it seems likely that soil phosphorous would be built up to a level which is above the 9 (18) ppm target for soils type 1(2). For potassium, on the other hand, the maintenance recommendation of the tables seem to be insufficient for keeping the level of soil K at its desired target (60 ppm of K for all soil types). In short, even though the target levels for P and K are close to the computed optima (the relative differences were in the range of 5 to 15 percent), it seems that major modifications of the tables are required as far as "maintenance P" and "maintenance K" are concerned (the relative differences found here are in the range of 30 to 50 percent). Table 9 summarizes these results.

The carry-over equations can also provide the information on how much fertilizer is required to change the soil test level found at the beginning of cropping period t to the desired target at the beginning of cropping period t + 1. The fertilizer used for the purpose of this change is generally called "corrective fertilizer" (in contrast to the "maintenance fertilizer," or the amount required to keep the soil fertility at the desired target across the time). To attain soil fertility targets of 9 ppm of P (for soil type 1) and 60 ppm of K in the beginning of cropping period t, given that the soil test levels of period t-1 were P^s_{t-1} and K^s_{t-1}, the "corrective fertilizer" requirements are given by:

$$P^a_{corr} = 488.3 - 48.26\ P^s_{t-1} \tag{35}$$

and,

(36) $$K^a_{corr} = 274.9 - 3.73\ K^s_{t-1}$$

(equation (35) and (36) were computed from equations (33) and (34) by setting P^s_t and K^s_t at their target levels and solving for the applied fertilizer quantity). However, since a maintenance recommendation is also made for each period (54.0kg of P_2O_5/ha/cropping period and 52.4kg of K_2O/ha/cropping period), the corrective recommendations should be diminished by that amount. Thus, the corrective recommendation for soil targets of $P^s_* = 9.0$ ppm of P (soil type 1) and $K^s_* = 60$ ppm of K should be modified to:

(37) $$P^a_{corr} = 434.3 - 48.26\ P^s_{t-1}$$

and

(38) $$K^a_{corr} = 222.5 - 3.73\ K^s_{t-1}.$$

The discussion so far has centered around fertilizer recommendations for phosphorous and potassium only. The optimum target level computed for nitrogen was 57.2kg of N/ha/cropping period of wheat. The same level of N is equivalent to a soil test level of 4.3 percent of organic matter content (the estimated proportionality factor for N was 13.13). Current recommendations of nitrogen for wheat are: 15kg of N/ha for a percentage of organic matter above 5.0 percent, 30kg of N/ha for 2.5-5.0 percent of organic matter; 45kg of N/ha for 0-2.5 percent of organic matter. The optimum nitrogen recommendations for the same crop are declining at a faster rate than the current recommendations. They indicate 47kg of N/ha for 0.5 percent of organic matter; 30kg of N/ha for 2.0 percent of organic matter; and 15kg of N/ha for 3.2 percent of organic matter.

Hence, current nitrogen recommendations for wheat overestimate the computed optimum. The differences accentuate as the level of organic matter increases from zero to five percent. It also seems advisable to make a finer division of classes of soil organic matter percentage content in the tables: only three classes are currently adopted. The closeness of recommendation for very low levels of soil

organic matter imply that the level of total nitrogen supply adopted as a target is similar for both current and computed optimum recommendations. The difference noticed for higher levels of soil organic matter is caused by a difference of the estimates of nitrogen supply capacity from soil organic matter: in this study it was estimated that each percentage point of soil organic matter can supply 13.13kg of N/ha/cropping period of wheat. The estimate adopted in the tables, however, is more conservative: approximately 8kg of N/ha/cropping period of wheat for each percentage point of soil organic matter. The difference among such estimate is probably due to the fact that soil pH correction (through limestone applications) has been made for all observations included in this study: when soil pH is corrected, the supply of nitrogen produced by any given amount of organic matter is significantly increased. The recommendations made by the agronomists, on the other hand, do not assume that soil pH has been necessarily corrected through the use of lime. Under such circumstances, a more conservative estimate for the equivalence factor between soil organic matter and applied nitrogen appears to have been made by the agronomists. In any event, since pH correction itself is also recommended by the tables, consistency requires that this point be taken into account for the purpose of making fertilizer recommendations.

In conclusion, it seems that some adjustments on fertilizer recommendation tables for the wheat-soybean system would be highly worthwhile. The maintenance levels currently recommended for P were found to be overestimated: a reduction from 75 to 55kg of P_2O_5/ha/cropping period is strongly suggested. For potassium, on the contrary, it appears that the maintenance recommendation should be increased from 40 to 50kg of K_2O/ha/cropping period (and perhaps even a little more for soybeans). For nitrogen it was found that the current recommendations could be somewhat reduced, particularly for soils where lime has been applied. It is also suggested that the number of classes of soil organic matter be increased from the current number of three. The above suggestions are based on the assumption that soil fertility targets are kept at the levels currently adopted on the tables, as

these levels were found to be relatively close to the computed optima (see Table 9). Nevertheless, it is also suggested that future improvements of the tables make a distinction between fertility targets for wheat and for soybeans. Moreover, as the optimum target levels of P and K for soybeans were found to be higher than those computed for wheat, it seems reasonable to recommend higher maintenance levels for soybeans and no P and K at all for the sequential wheat crop.

Finally, it is important to evaluate the probable gains from the changes recommended above. First, since the suggestions assume that current fertility targets will be maintained, no change in output levels is bound to occur. Thus, the gains from the changes would come from a reduction in fertilizer costs. Table 10 summarizes the likely results of the change.

Under current recommendations, the yearly fertilizer costs of one ha of wheat-soybeans are approximately Cr$ 1,426. If the suggested changes are implemented, this cost can be reduced to Cr$ 1,138. The reduction in costs of fertilization is Cr$ 288/ha/year or US$ 27.30/ha/year (at July 1976 exchange rate). The relative decrease in yearly fertilization costs would be around 20 percent, a very significant amount. The costs of changing the tables, on the other hand, are small and, in practice, can be assumed to be insignificant.

The soil scientists of Rio Grande do Sul, who are responsible for the formulation of the fertilizer recommendation tables have declared their intent to adjust the tables according to the indications of this study.

jma 3/30/79

REFERENCES

Athans, M., "The Discrete Time Linear-Quadratic-Gaussian Stochastic Control Problem," Annals Econ. Soc. Measur., 1(1972): 449-491.

Balmukand, B. H., "Studies in Crop Variation - V. The Relation Between Yield and Soil Nutrients," J. Agric. Sci., 18(1978): 602-627.

Barber, S. A., "The Changing Philosophy of Soil Test Interpretations," in Soil Testing and Plant Analysis, Walsh, L. M. and J. D. Beaton (eds.), Soil Sci. Soc. Amer., Inc., 1973.

Baule, B., "Zu Mitscherlich's Gesetz der Physiologishen Beziehungen," Landwirtschaftliche Jahrbücher, 51(1918): 363-385.

Baum, E. L., E. O. Heady, and J. Blackmore, eds., Methodological Procedures in the Economic Analysis of Fertilizer Use Data, Ames: Iowa State College Press, 1956.

Baum, E. L., E. O. Heady, J. T. Pesek, and C. G. Hildreth, eds., Economic and Technical Analysis of Fertilizer Innovations and Resource Use, Ames: Iowa State College Press, 1957.

Brady, N. C., The Nature and Properties of Soils, 8th ed., London, McMillan, 1974.

Bray, R. H., "A Nutrient Mobility Concept of Soil-Plant Relationships," Soil Sci., 78(1954): 9-23.

_____, "The Correlation of a Phosphorous Soil Test with the Response of Wheat Through a Modified Mitscherlich Equation," Soil Sci. Soc. Amer. Proc., 22(1958): 314-317.

_____, "Confirmation of the Nutrient Mobility Concept of Soil-Plant Relationships," Soil Sci., 95(1963): 124-131.

Cate, R. B., Jr., and L. A. Nelson., "A Simple Statistical Procedure for Partitioning Soil Test Correlation Data into Two Classes," Soil Sci. Soc. Amer. Proc., 35 (1971): 858-860.

Cope, J. T., and R. D. Rouse, "Interpretation of Soil Test Results" in Soil Testing and Plant Analysis, Walsh and Beaton (eds.), Rev. Ed., Soil Sci. Soc. of America, Inc., 1973.

da Fonseca, V., "Análise Econômica de Doses e Fontes de Nitrogênio na Cultura do Trigo, sob Condições de Risco, RS," IEPE/UFRGS, Tese n° 47, Porto Alegre, Brasil (Mimeo), 1976.

Dale, R. F., "Discussion: Use of Weather Factors in Short-Run Forecasts of Crop Yields," J. Farm Econ., 43(1961): 1179-1182.

de Janvry, A., "Optimal Levels of Fertilization Under Risk: the Potential for Corn and Wheat Fertilization Under Alternative Price Policies in Argentina," Amer. J. Agric. Econ., 54(1972): 1-10.

Epstein, E., Mineral Nutrition of Plants: Principles and Perspectives, New York: John Wiley and Sons, 1972.

Erterl, J. E., and E. B. Fowlkes, "Some Algorithms for Linear Spline and Piecewise Multiple Linear Regression," J. Am. Stat. Ass., 71(1976): 640-648.

Fuller, W. A., "Stochastic Fertilizer Production Functions for Continuous Corn," J. Farm Econ., 47(1965): 105-119.

Goldfeld, S. M., and R. E. Quandt, "Estimation in a Disequilibrium Model and the Value of Information," J. Econometrics, 3(1975): 325-348.

Hanway, J. and L. Dumenil, "Predicting Nitrogen Fertilizer Needs of Iowa Soils. III: Use of Nitrate Production Together with Other Information as a Basis for Making Nitrogen Fertilizer Recommendation for Corn in Iowa," Soil Sci. Soc. Amer. Proceedings, 19(1955): 77-80.

Heady, E. O., J. Pesek, and W. Brown, Crop Response Surfaces and Economic Optima in Fertilizer Use, Iowa State College Agric. Exp. Sta. Res. Bull. 424, 1955.

Heady, E. O., "Methodological Problems in Fertilizer Use," in Methodological Procedures in the Economic Analysis of Fertilizer Use Data, Baum, Heady, and Blackmore (eds.), Ames: Iowa State College Press, 1956.

Hildreth, C. G., "Discrete Models with Qualitative Restrictions," in Methodological Procedures in the Economic Analysis of Fertilizer Use Data, Baum, Heady and Blackmore (eds.), Ames, Iowa State College Press, 1956.

Kennedy, J. O. S., I. F. Whan, R. Jackson, and J. L. Dillon, "Optimal Fertilizer Carry-over and Crop Recycling Policies for a Tropical Grain Crop," Aust. J. Agric. Econ., 17(1973): 104-113.

Lanzer, E. A., Q. Paris, and R. Green, "A Maximum Likelihood Estimator for Switching Regressions with Common Gaussian Noise," working paper no. 78-8, Department of Agricultural Economics, UC Davis, 1978.

Liebig, J. von, Organic Chemistry in its Application to Agriculture and Physiology, London, Playfair, 1840.

Maddala, G. S., and F. D. Nelson, "Maximum Likelihood Methods for Models of Markets in Disequilibrium," Econometrica, 42(1974): 1013-1030.

Malavolta, E., H. P. Haag, F. A. F. Mello, and M. O. C. Brasil Sobrº, La Nutricion Mineral de Algunas Cosechas Tropicales, Berna, Inst. Internac. de la Potasa, 1964.

Mitscherlich, E. A., "Das Gesetz des Minimums und das Gesetz des abnehmenden Bodenertrages," Landwirtschaftliche Jahrbücher, 38(1909): 537-552.

Munson, L. D. and J. P. Doll, "The Economics of Fertilizer Use in Crop Production," in Advances in Agronomy, Vol. XI, New York: Academic Press, 1959.

Perrin, R. K., "The Value of Information and the Value of Theoretical Models in Crop Response Research," Am. J. Agric. Econ., 58(1976): 54-61.

Poirier, D. J., "Piecewise Regression Using Cubic Splines," J. Am. Stat. Ass., 68 (1973): 515-524.

Redman, J. C., and S. Q. Allen, "Interrelationships of Economic and Agronomic Concepts," J. Farm Econ., 36(1954): 453-465.

Rouse, R. D., "Soil Test Theory and Calibration for Cotton, Corn, Soybeans, and Coastal Bermudagrass," Bull. No. 375, Agric. Exp. Station, Auburn University, Alabama, 1968.

Russel, J., Soil Conditions and Plant Growth, 10th Ed., London: Longmans, 1973.

Ryan, J., "A Generalized Crop Fertilizer Production Function," (unpublished Ph.D. dissertation), North Carolina State University at Raleigh, 1972.

Smith, W. G., and W. L. Parks, "A Method for Incorporating Probability into Fertilizer Recommendations," J. Farm Econ., 49(1967): 1511-1515.

Stauber, M. S., O. R. Burt, and F. Linse, "Economic Evaluation of Nitrogen Fertilization of Grasses when Carry-over is Significant," Amer. J. Agric. Econ., 57 (1975): 463-471.

Suits, D. B., A. Mason, and L. Chan, "Spline Functions Fitted by Standard Regression Methods," Rev. Econ. Stat., (1978): 132-139.

Tisdale, S. L., and W. L. Nelson, Soil Fertility and Fertilizers, ~~7th~~ 3rd Ed., ~~London~~ New York: MacMillan Co., 197~~1~~5.

Theil, H., Optimal Decision Rules for Government and Industry, Amsterdam: North Holland, 1968.

_____, Principles of Econometrics, New York: John Wiley and Sons, 1971.

UFRGS, "Tabelas de Adubação Corretiva e Adubação de Manutenção para os Solos e Culturas dos Estados do Rio Grande do Sul e Santa Catarina," Departamento de Solos, Universidade Federal do Rio Grande do Sul, Porto Alegre, 1973.

Walsh, L. M., and J. D. Beaton, (eds.), Soil Testing and Plant Analysis, Rev. Ed., Soil Sci. Soc. of America, Inc., 1973.

FIGURE 1

Relative Yield (yr) Spline Response to Total Supply of Nutrient X (X^T)

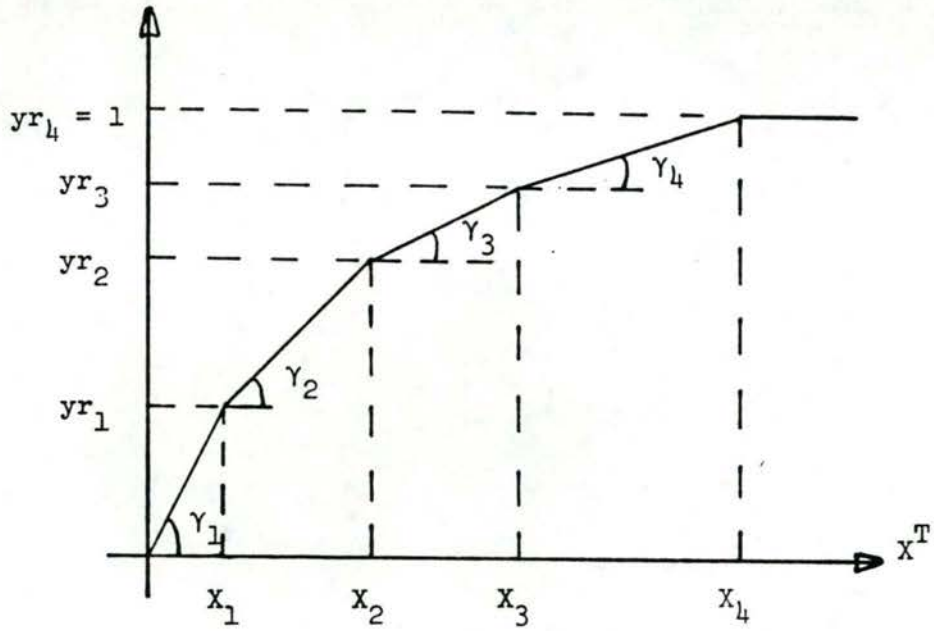

TABLE 1. Statistical Results for the Phosphorous Carry-over Function

Parameter Name	Point Estimate	Asymptotic Std. Dev.
Autocorrelation Coefficient (ρ_p)	-0.67470	0.041630
Geometric Decline Coeff. (Θ_p)	0.88950	0.020900
Reciprocal of Proportionality Constant (λ_{p1}^{-1})	0.02072	0.002304

R^2 = 0.7881; MSE = 86.45; N = 345 observations

TABLE 2. Statistical Results for the Potassium Carry-over Function

Parameter Name	Point Estimate	Asymptotic Std. Dev.
Autocorrelation Coefficient (ρ_k)	-0.4154	0.04876
Geometric Decline Coeff. (Θ_k)	0.8139	0.01210
Reciprocal of Proportionality Constant (λ_k^{-1})	0.2682	0.03653

R^2 = 0.3542; MSE = 797.50; N = 420 observations

TABLE 3. Spline Regression Results for the Soybean Response to Phosphorous

Parameter Name	Symbol	Point Estimate	Asymptotic Std. Dev.
scaling factor	α	0.906100	0.013332
rel.yield incl.in the range P^T= [0;75]kg of P_2O_5/ha	β_1	0.007324£	-
change in rel.yield inclin. at P^T= 75 kg of P_2O_5/ha	β_2	-0.005194	0.000604
change in rel.yield inclin. at P^T= 225 kg of P_2O_5/ha	β_3	-0.001706	0.000299
change in rel.yield inclin. at P^T= 375 kg of P_2O_5/ha	β_4	0.000000	-
change in rel.yield inclin. at P^T= 525 kg of P_2O_5/ha	β_5	-0.000397	0.000248
change in rel.yield inclin. at P^T = 675 kg of P_2O_5/ha	β_6	-0.000060	0.000180

R^2 = 0.7885 ; MSE = 125.300 ; N = 340 observations; £ - computed according to equation (20).

TABLE 4. Spline Regression Results for the Soybean Response to Potassium

Parameter Name	Symbol	Point Estimate	Asymptotic Std. Dev.
scaling factor	α	0.91240	0.01001
rel.yield incl. in the range K^T = [0;40]kg of K_2O/ha	β_1	0.01335£	-
change in rel.yield incl. at K^T= 40 kg of K_2O/ha	β_2	-0.01068	0.003481
change in rel.yield incl. at K^T = 110 kg of K_2O/ha	β_3	-0.00008	0.000003
change in rel.yield incl. at K^T= 185 kg of K_2O/ha	β_4	-0.00186	0.000975
change in rel.yield incl. at K^T = 260 kg of K_2O/ha	β_5	-0.00053	0.000394
change in rel.yield incl. at K^T= 410 kg of K_2O/ha	β_6	-0.00028	0.000198

R^2 = 0.8819 ; MSE = 50.960 ; N = 273 observations; £ computed according to equation (20).

TABLE 5. Spline Regression Results for the Wheat Response to Phosphorous

Parameter Name	Symbol	Point Estimate	Asymptotic Std. Dev.
scaling factor	α	0.883700	0.017720
rel.yield response incl. in the range P^T = [0;75] kg P_2O_5/ha	β_1	0.002630 £	-
change in rel.yield incl. at P^T = 75 kg P_2O_5/ha	β_2	0.001386	0.001091
change in rel.yield incl. at P^T = 225 kg P_2O_5/ha	β_3	-0.003316	0.000653
change in rel.yield incl. at P^T = 375 kg P_2O_5/ha	β_4	-0.000380	0.000413
change in rel.yield incl. at P^T = 525 kg P_2O_5/ha	β_5	0.000000	-
change in rel.yield incl. at P^T = 675 kg P_2O_5/ha	β_6	-0.000313	0.000146

R^2 = 0.9009 ; MSE = 59.070 ; N = 179 observations ; £ computed according to equation (20).

TABLE 6. Spline Regression Results for the Wheat Response to Potassium

Parameter Name	Symbol	Point Estimate	Asymptotic Std. Dev.
scaling factor	α	0.88230	0.01916
rel.yield response incl.in the range K^T = [0;40] kg of K_2O/ha	β_1	-0.02074£	-
change in rel.yield incl. at K^T = 40 kg of K_2O/ha	β_2	0.04048	0.03755
change in rel.yield incl. at K^T = 110 kg of K_2O/ha	β_3	-0.01531	0.01525
change in rel.yield incl. at K^T = 185 kg of K_2O/ha	β_4	-0.00331	0.00308
change in rel.yield incl. at K^T = 260 kg of K_2O/ha	β_5	-0.00092	0.00148
change in rel.yield incl. at K^T = 410 kg of K_2O/ha	β_6	-0.00032	0.00037

R^2 = 0.9485 ; MSE = 34,440 ; N = 125 observations; £ computed according to equation (20).

TABLE 7. Regression Results for the Wheat Response to Nitrogen

Parameter Name	Symbol	Point Estimate	Asymptotic Std. Dev.
Scaling factor	α	0.8507	0.01328
Response coeff. to N^s	c*	-0.5634	0.05571
Response coeff. to N^a	c	-0.0429	0.02990

R^2 = 0.8823; MSE = 60.040; N = 158 observations

TABLE 8. Stability of Optimum Soil Fertility Stocks (Targets) for Wheat and Soybeans with Respect to Fertilizer Price Changes

Percentage Fertilizer Change in Prices	Targets for Wheat				Targets for Soybeans		
	N[1]	P[2]	K[3]	yr[4]	P[2]	K[3]	yr[4]
- 40%	60.0 (4.6)	435.0 (9.0)	219.4 (58.8)	92.4	525.0 (10.9)	410.0 (109.0)	100.0
- 30%	60.0 (4.6)	435.0 (9.0)	219.4 (58.8)	92.4	525.0 (10.9)	410.0 (109.0)	100.0
- 20%	60.0 (4.6)	435.0 (9.0)	219.4 (58.8)	92.4	489.0 (10.1)	331.0 (88.7)	98.4
- 10%	57.2 (4.3)	375.0 (7.8)	202.6 (54.3)	90.5	489.0 (10.1)	331.0 (68.7)	98.4
00%	57.2 (4.3)	375.0 (7.8)	202.6 (54.3)	90.5	456.0 (9.4)	260.0 (69.7)	97.0
+ 10%	57.2 (4.3)	375.0 (7.8)	202.6 (54.3)	90.5	454.0 (9.4)	258.8 (69.4)	96.9
+ 20%	57.2 (4.3)	375.0 (7.8)	202.6 (54.3)	90.5	454.0 (9.4)	258.8 (69.4)	96.9
+ 30%	57.2 (4.3)	375.0 (7.8)	202.6 (54.3)	90.5	421.6 (8.7)	239.3 (64.2)	96.9
+ 40%	54.2 (4.1)	346.7 (7.8)	185.0 (54.3)	88.5	421.6 (8.7)	239.3 (64.2)	95.5

1. the first entry is in kg of N/ha and the entry in parenthesis is in percentage content of organic matter
2. the first entry is in kg of P_2O_5/ha and the entry in parenthesis is in ppm of P for soil type 1(for soil type 2: multiply the entry by 2)
3. the first entry is in kg of K_2O/ha and the entry in parenthesis is in ppm of K
4. predicted relative yields, in terms of percentage of the expected yield plateau.

TABLE 9. Target Levels and Maintenance Applications Recommended for Phosphorous and Potassium

Nutrient	Current Recommend.		Adj.Curr.Recommed.		Optima for Wheat		Optima for Soybeans	
	Target	Mainten.	Target	Mainten.*	Target	Mainten.*	Target	Mainten.*
Phosphorus	9.0	75.0	9.0	54.0	7.8	46.8	9.4	56.4
Potassium	60.0	40.0	60.0	52.4	54.3	47.4	69.7	60.9

- target levels in ppm units (soil type 1 units for P)
- maintenance levels in kg of P_2O_5/ha/cropping period for P and kg of K_2O/ha/cropping period for K

* computed from equations 33 and 34.

TABLE 10. Yearly Fertilization Costs Required to Maintain the Soil Fertility at Desired Targets for the Wheat-Soybean Double Cropping System (1976 prices)

Item	Current Recommendation		Suggested Recommendation	
	Quantity[1]	Cost[2]	Quantity[1]	Cost[2]
Maintenance N for wheat[3]	30.0	168.30	20.0	112.20
Maintenance P for wheat	75.0	529.50	55.0	388.30
Maintenance K for wheat	40.0	99.60	50.0	124.50
Maintenance P for soybean	75.0	529.50	55.0	388.30
Maintenance K for soybean	40.0	99.60	50.0	124.50
Total Cost	-	1,426.50	-	1,137.80

1 - in kg of N/ha, kg of P_2O_5/ha and kg of K_2O/ha for N, P and K respectively.

2 - in Cr$/ha

3 - assuming a 3% organic matter content.