DROUGHTS, FODDER RESERVES AND STOCKING RATES*

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The present article relates to pastoral firms in a drought-prone environment. For such firms, it explores the economic relationship between stocking rates, drought possibilities and fodder reserves.[1] The analysis shows that an optimal stocking rate can be ascertained for any given pattern of drought incidence. Using Queensland data, an empirical application covering a range of fodder, livestock and livestock product prices is presented. Nothing is said of the macro aspects of fodder storage for drought relief; nor of the optimum level of livestock population on an aggregate basis. Likewise, the analytics of supplementary feeding for production—as distinct from drought feeding for survival—are not explored on either a micro or a macro basis. Still, the method of analysis used here is not without relevance to all these topics.

Drought

Agriculturists have tended to approach drought in terms of the "effective rainfall" concept with emphasis on the vegetation rather than on the use of the vegetation.[2] The former approach is quite logical relative to crop production *per se*; but it is unsatisfactory when the land is to be used for pastoral purposes. In other words, distinction must be made between "cropping droughts" and "grazing droughts." The latter interest us here; emphasis is on the use of the vegetation by the grazing animal. Hence we define grazing drought as a period of natural feed shortage such that the supply of vegetation for grazing is inadequate to maintain the *desired* number of livestock without permanent adverse effects on the animals. Under such a definition, drought may be induced by a variety of causes, such as grasshoppers or flooding, as well as by a dearth of rain. Most importantly, the definition recognises that drought incidence and duration are not independent of stocking rate. With a zero stocking rate, grazing drought would never occur! In general, the higher (lower) the stocking rate, the shorter (longer) a given supply of grazing feed will last. Two separate, though related, questions thus arise. First, for a given stocking rate, what quantity of fodder reserves should be stored by the firm at the beginning of any specified production

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[1] Using rather simple models and hypothetical data, the derivation of an optimal fodder reserve for a given stocking rate has been elaborated in a previous article. See Dillon, J. L., and Mauldon, R. G., "Inventory Analysis and the Economics of Fodder Conservation," *Economic Record*, 35 : 209-218, 1959. See also Candler, W. V., "On the Economics of Drought Reserves," *Rev. Mark. Agric. Econ.*, 26 : 215-226, 1958.

[2] For a notable exception, see Everist, S. L., and Moule, G. R., "Studies in the Environment of Queensland. 2. The Climatic Factor in Drought," *Qd. J. Agric. Sci.*, 9 : 185-299, 1952.

period in order to maximise expected profit per animal over the production period? Second, given the land resources available to the firm, what is the stocking rate which maximises expected profit over the specified production period?

Optimal Fodder Storage for a Given Stocking Rate

Fodder is stored to satisfy a future demand for feed induced by the occurrence of drought. For a given future period, however, it is not known with certainty whether or not a drought will occur; nor how long it might last. There are two possible knowledge situations. Either uncertainty prevails or, if a suitable record of prior droughts is available, a probability distribution for drought incidence (and hence for fodder needs) can be calculated. In the present analysis, a probability distribution for drought incidence in the planning period under consideration is assumed to be known. We therefore proceed to formulate, in broad fashion, the expected net revenue function for a firm with a given stocking rate in the face of drought possibilities. The cost and revenue items to be taken into account over a given planning period may be denoted as follows, where y is the quantity of fodder stored:

$c_1(y)$: the acquisition cost of the fodder reserve;

$c_2(y)$: the cost of storing the fodder reserve;

$c_3(y)$: the penalty cost incurred should the fodder reserve be inadequate;

$c_4(y)$: the production cost of the firm exclusive of c_1, c_2 and c_3;

$r_1(y)$: the revenue from the sale of fodder reserves unused at the end of the planned production period;

$r_2(y)$: the revenue from production during the planned production period.

With the exception of the fodder acquisition cost, all these costs and revenues are influenced by the occurrence of drought. Hence they must be evaluated on an expectational basis relative to the probability of drought incidence. Thus the discounted expected net revenue, $R_i(y)$, is given by (1) where the subscript i specifies the given stocking rate; d is the relevant discount factor; and E denotes expectation.

$$(1)\ R_i(y) = (d)[-c_1(y) + E[r_1(y) + r_2(y) - c_2(y) - c_3(y) - c_4(y)]]$$

Since the stocking rate is specified in (1) and the land base of the firm is assumed to be fixed, $R_i(y)$ may be assessed on a livestock unit basis without loss of generality. The value of y, say y*, which maximises (1) for a given value of i, is then the optimal quantity of fodder to store per livestock unit on an *ex ante* basis.

Derivation of the Optimal Stocking Rate

The usual cause of drought is a dearth of rain. Accordingly, we conceptualise the stocking rate problem in terms of rainfall. Denote by:

i: the length of the continuous period without effective rain that elapses before drought conditions commence;

$D(i)$: the stocking rate which determines i;

$D(\bar{i})$: the predetermined stocking rate in use at the start of the planning period;

h: the price of a livestock unit at the start of the planning period;

$\overline{R}_i(y)$: the discounted expected net revenue per acre over the planned production period with a reserve of y fodder units per animal.

The stocking rate $D(i)$ bears an inverse relationship to i. Assuming this function to take the simplest possible form, we may postulate that

$$D(i) = m/i \tag{2}$$

where m is an empirically determined constant such that $D(i)$ is in livestock units per acre.[3] The discounted expected net revenue per acre over the planned production period when the firm has a drought reserve of y fodder units per animal and shifts from a stocking rate of $D(\bar{i})$ to $D(i)$, is then given by

$$\overline{R}_i(y) = [R_i(y)\ m/i] + h\ [D(\bar{i}) - m/i] \tag{3}$$

With only the land resource limitational, the optimal stocking rate and fodder reserve is given by that value of i, say i^*, which, in association with its y^* value, maximises (3).[4] Moreover, from the size of i^* relative to $\bar{i}$, the entrepreneur can tell whether he should dispose of or acquire livestock.

If $i^* > \bar{i}$, he should sell $[D(\bar{i}) - D(i^*)]$ livestock units per acre.

If $i^* < \bar{i}$, he should buy $[D(i^*) - D(\bar{i})]$ livestock units per acre.

Some Limitations of the Model

Even though the model has been presented in extremely broad form, with full scope left for the artistic specification of the costs and revenues of (1), it is not without flaws. Most seriously, no allowance is made for the fact that an entrepreneur may quite rationally consider higher moments of the probability distribution of net revenue. For instance, the variance and skewness of this distribution are pertinent to any *ex ante* consideration of the possible divergence between *ex post* and expected net revenue. However, to take account of these higher moments is operationally unfeasible without knowledge of the decision-maker's preferences.[5] Also in conflict with reality is the assumption that land is the only limiting resource. Agistment may be an available alternative. As well, non-land resources may be limitational; in particular, working capital and livestock watering facilities. The appropriate method to take account of such opportunities and restrictions is programming. A feasible procedure would be to represent a range of

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[3] Equation (2) is not based on any scientific evidence. Still, it is thought to be reasonable within the relevant range of stocking rates.

[4] While the maximisation of (1) relative to y is a problem of differential calculus, the derivation of the optimal plan from (3) in terms of i and y is a problem in the calculus of variations. See Allen, R. G. D., *Mathematical Analysis for Economists.* Macmillan and Co., London, 1938. Ch. 20.

[5] See Tintner, G., "A Contribution to the Non-static Theory of Choice," *Quart. J. Econ.*, 56 : 274-306, 1942; Allais, M., "Le comportement de l'homme rationnel devant le risque," *Econometrica,* 21 : 503-546, 1953; Dorfman, R., *et al., Linear Programming and Economic Analysis.* McGraw-Hill, New York, 1958, p. 431; also Lloyd, A. G., "Fodder Conservation in the Southern Tablelands Wool Industry," *Rev. Mark. Agric. Econ.*, 27 : 5-50, 1959.

stocking rates as alternative activities, the optimal fodder reserve associated with each of these activities being ascertained via inventory analysis. The extent to which these various activities were selected, given the prevailing resource opportunities, would then define some overall stocking rate for the firm.[6]

Another disadvantage of the pre-drought model is that it only gives answers that are optimal *ex ante*. That the answers derived may be wrong *ex post,* however, simply reflects the fact that the entrepreneur operating in a real-world environment can never remove all his risks and still behave as *homo œconomicus*. Moreover, when faced with an actual within-drought situation of exhausted fodder stocks, the firm is free to buy additional fodder and to buy or sell livestock (or to just let them die). The inventory model is still the appropriate decision mechanism, provided that account be taken of the additional information that is available about prices and drought length.[7]

Empirical Application of the Model

A few applications of the model will now be presented. They relate to the Hughenden area of north central Queensland. This area was chosen because it is a pastoral one in which droughts are important. It has an average annual rainfall of 20.5 inches.

Drought Probabilities

A monthly record of effective rainfall at Hughenden has been tabled by Everist and Moule for the years from 1893 to 1950.[8] From this record, the probability was calculated that the planning period following a non-drought month would include a completed drought of specified length as the first drought occurring within the planning period; the planning period being considered as a constant period into the future from the current time point. Only planning periods following a non-drought month were considered because the model is completely *ex ante*; planning periods following a drought month relate to the within-drought decision problem.

The length of the planning period was taken as the length of the longest drought that might be expected with a given stocking rate. By definition, therefore, the length of the planning period varies with the stocking rate that is being used. Only completed droughts falling within possible planning periods were considered because an assessment of the revenue and penalty cost implications of partially completed droughts would make the model too complicated for expository purposes; bearing in mind also that the influence of uncompleted droughts is a minor one.

Drought probabilities were calculated for two stocking rates, D(4) and D(5). These stocking rates imply, respectively, that drought conditions commence after 4 and after 5 consecutive months not

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[6] See Heady, E. O., "Economic Concepts in Directing and Designing Research for Programming of Range Resources," *J. Farm Econ.*, 38 : 1604-1616, 1956.

[7] Within-drought decision rules are discussed in Dillon and Mauldon, *op. cit.*, pp. 217-218.

[8] Everist and Moule, *op cit.*, p. 281. Given the expository aim of the present analysis, the cost of obtaining data for the years since 1950 was not deemed worthwhile.

receiving effective rain.[9] Under these stocking rates, the longest recorded drought at Hughenden is 13 months for D(4) and 12 months for D(5). The actual probabilities, as listed in Table 1, were derived from the frequencies of drought occurrence in sequences of 13 and 12 months, respectively, that were preceded by a non-drought month.[10] The subsequent use of these probabilities assumes that droughts of various lengths occur in random sequence and not in cycles. This assumption is supported by the conclusions of Everist and Moule, at least for cycles of less than 57 years.[11] A notable feature of the probabilities is their bimodality. To fit a continuous probability function to such distributions would be difficult, necessitating the use of compound functions. In consequence, we use the probabilities in their discrete form.

Type of Firm

The firm considered is a typical pastoral property producing wool.[12] The grazier is assumed to have no control over the cost of his inputs at any particular time. We also assume that there is no capital restriction in determining optimal fodder reserves for a given stocking rate. Initial fodder stocks are taken to be zero and no allowance is made for a time lag between ordering and delivery of fodder.

Table 1

PROBABILITY DISTRIBUTION OF DROUGHT LENGTH AT HUGHENDEN, QUEENSLAND, UNDER ALTERNATIVE STOCKING RATES

Drought length (t) in months	*Probability*	
	D(4) stocking rate	*D(5) stocking rate*
0	.321	.581
1	.260	.082
2	.082	.022
3	.022	.082
4	.082	.128
5	.128	.069
6	.069	.034
7	.034	
.		
.		
12		.002
13	.002	

[9] How many sheep equivalents per acre these stocking rates imply we do not know. Nor do we need to know for present purposes. Certainly they are within the relevant range.

[10] The ideal procedure would be to derive a separate probability distribution relative to each calendar month as the start of the planned production period. Specific decision rules could then be drawn up for each month. However, such a detailed approach seems unwarranted. Still, it might be legitimately argued that our probabilities are too gross. Perhaps a set of probabilities for each season of the year would be an operational compromise. Certainly it would provide a fair measure of protection against dependence in the data used to derive the probabilities; while still providing a reasonable sample of observations for the calculation of probabilities.

[11] Everist and Moule, *op. cit.*, p. 185.

[12] See *Australian Sheep Industry Survey, 1954. Pastoral Zone.* B.A.E., Canberra. 1956. pp. 24-38.

Specification of the Model

The following preliminary comments are pertinent:

(a) To give generality, the analysis is made on a sheep equivalent basis with fodder specified only in starch equivalent terms. It is taken for granted that the fodder contains a satisfactory level of protein.

(b) Pre-drought planning is on the basis that fodder will not be bought once a drought commences. Still, the grazier is free to buy additional fodder whenever faced with a within-drought situation of exhausted fodder stocks.

(c) It is assumed in planning that should fodder stocks tend to prove inadequate, only some and not all sheep will die or be sold. In other words, the grazier may plan to discontinue feeding some sheep after a certain stage so that he might have sufficient feed for the remainder of his flock. We therefore consider the expected fraction of sheep deaths ensuing from drought.

(d) The revenue obtained from selling a drought-stricken sheep is taken as equal to the revenue obtainable from the sheep's dead wool if it were allowed to die.

(e) The presence of one or more shearings during the planning period complicates the formulation of the expected net revenue function. When fodder supplies are adequate to maintain sheep through drought, production revenue comes only from live wool. Such wool will have been produced from both natural grazing during non-drought and from fodder fed during drought. In the case where fodder supplies are inadequate and shearing does not occur before the sheep die, revenue comes from dead wool produced from both natural grazing during non-drought and stored fodder fed until supplies ran out, plus live wool grown on replacement sheep bought after the drought terminates and produced from natural grazing. If shearing occurs before fodder reserves are exhausted, account must also be taken of live wool produced up until the time of shearing. The probability distribution of drought length gives no indication of the probable time of occurrence of drought. In determining whether non-drought wool production should be assessed at the live or dead wool prices in the event of fodder reserves being inadequate, we should know the quantity of live wool already removed through shearing. This can only be known in a probability sense. Let the expected time after shearing of the commencement of a drought of any length be j months. Then, in the event of fodder reserves being inadequate, there is a probability of $j/12$ that wool growth during the production period should be valued at the dead wool price and a probability of $(12-j)/12$ that it should be valued at the live wool price.

(f) Due to a lack of suitable information, wool production per sheep equivalent during non-drought is evaluated at a constant figure irrespective of stocking rate. For the same reason, a constant figure for production costs per sheep equivalent (exclusive of fodder acquisition and holding costs, and penalty costs) is used regardless of whether drought or non-drought conditions prevail.

(g) For simplicity, the discount factor associated with expected net revenue is taken as unity.

Bearing the above preliminaries in mind, the model might be specified as follows:[13]

T_i : length of the planning period in months with a stocking rate of $D(i)$;
$p_i(t)$: probability of a drought of length t months occurring in the planning period with stocking rate $D(i)$, $o \leqslant t \leqslant T_i$;
j : expected time in months after shearing of the commencement of a drought;
y : quantity of fodder stored in starch equivalents per sheep equivalent;
λ : fraction of a sheep equivalent that is expected to die, $0 \leqslant \lambda \leqslant 1$;
k_1 : non-drought price of fodder per starch equivalent;
k_2 : price of shorn wool per lb.;
k_3 : pre-drought price of a sheep equivalent;
$g(t)$: difference between the pre-drought and end-of-drought replacement price of a sheep equivalent;
p_r : end-of-drought replacement price of a sheep equivalent, $p_r = k_3 + g(t)$;
k_4 : physical deterioration coefficient of fodder over the planning period;
k_5 : dead wool price per lb.;
k_6 : minimum fodder requirement per sheep equivalent in starch equivalents per month;
k_7 : wool production during drought in lb. per starch equivalent;
k_8 : non-drought wool production in lb. per month per sheep equivalent;
k_9 : production cost per sheep equivalent per month.

The fodder required per sheep equivalent for maintenance through a drought of length t is k_6t. For a given t and $y \leqslant k_6t$, the fraction of a sheep equivalent which can be carried through a drought is therefore y/k_6t. The fraction which would be disposed of is thus $(1-y/k_6t)$. Hence, in rather naive fashion, the possibility of saving some sheep at the expense of others might be handled by way of (4).

$$(4)\quad \lambda = \sum_{t=o}^{T_i} (1 - yu/k_6t)p_i(t), \text{ where } \begin{cases} u = o \text{ if } y > k_6t \\ u = 1 \text{ if } y \leqslant k_6t \end{cases}$$

The cost and revenue items that were left unspecified in (1) may now be formulated. Thus we have:

$$\begin{array}{lll}
\text{fodder acquisition cost} & : k_1y, & \\
\text{fodder holding cost} & : k_1k_4(y - k_6t), & y > k_6t \\
\text{drought penalty cost} & : (1 - y/k_6t)\,[k_3 + g(t)], & y < k_6t \\
\text{fodder salvage revenue} & : k_1(1 - k_4)(y - k_6t), & y > k_6t \\
\text{production revenue} & : \begin{cases} k_8(T_i - t)[jk_5/12 + (12 - j)k_2/12] \\ \quad + k_7y\,[k_5(1 - y/k_6t) + k_2y/k_6t], \\ k_2k_6k_7t + (T_i - t)k_2k_8, \end{cases} & \begin{array}{l} y < k_6t \\ \\ y \geqslant k_6t \end{array} \\
\text{production cost} & : k_9T_i &
\end{array}$$

[13] The specification of the model that follows must be recognised as only covering the broader aspects of the drought inventory problem. Still, it suffices for expository purposes.

Making the appropriate substitutions into (1) the expected net revenue function per sheep equivalent over the period T_i becomes:

$$
\begin{aligned}
(5) \qquad R_i(y) = & -k_1 y - k_9 T_i + \sum_{t=o}^{y/k_6} [k_2 k_6 k_7 t + (T_i - t) k_2 k_8 \\
& - k_1 k_4 (y - k_6 t) + k_1 (1 - k_4)(y - k_6 t)] \, p_i(t) \\
& + \sum_{t = 1 + y/k_6 t}^{T_i} [k_7 y [k_5 (1 - y/k_6 t) + k_2 y/k_6 t] \\
& + k_8 (T_i - t) [j k_5/12 + (12 - j) k_2/12] \\
& - (1 - y/k_6 t) [k_3 + g(t)]] p_i(t)
\end{aligned}
$$

This function may now be used to illustrate optimal stocking rate and fodder reserve considerations. The values attached to the coefficients are outlined below. They are in units such that $R_i(y)$ is in £ per sheep equivalent.

Based on the work of Briggs and co-workers, the minimum food requirement per sheep equivalent, k_6, is taken to be three starch equivalents per week or 13 per month.[14] Wool production in lb. per starch equivalent, k_7, is taken to be 0.05. This figure is also in line with the findings of Briggs.[15] On the basis of B.A.E. figures, the monthly non-drought production of wool per sheep equivalent, k_8, is taken to be one lb.[16] Production costs per sheep equivalent per month are evaluated at £0.19.[17] This figure includes fodder storage charges other than deterioration costs on the assumption that fodder is stored in a permanent structure whose depreciation is independent of fodder quantity. The physical deterioration of fodder held from one production period to the next, k_4, is taken to be five per cent.[18] The price of dead wool per pound, k_5, is assumed to be £0.1. When i is four months, j is 4.0 months and T_4 is 13 months. When i is five months, j is 4.2 months and T_5 is 12 months.

Substituting these coefficients in (5), expected net revenue per sheep equivalent can be written as a function of fodder, sheep and shorn wool prices. Thus, in general form:

$$(6) \qquad R_i(y) = F_i[k_1, k_2, k_3, g(t)]$$

Optimal Drought Fodder Reserves

Equation (6) was evaluated, with k_1, k_2, k_3 and $g(t)$ left unspecified, for levels of fodder reserve ranging from zero to the amount necessary for the longest expected drought. The incremental change considered was that amount of fodder necessary to maintain a sheep through one month, 13 starch equivalents. The prices of fodder (k_1) per starch

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[14] Briggs, P. K., *et al.*, "Maintenance Rations for Merino Sheep," *Aust. J. Agric. Res.*, 8 : 75-82, 1957.

[15] *Loc cit.*

[16] *Australian Sheep Industry Survey. 1954: Queensland.* B.A.E., Canberra. 1956. p. 11.

[17] A guess guided by B.A.E. figures for 1954 adjusted to 1959 price levels. *Ibid.*, pp. 19-21.

[18] See Franklin, M. C., "Losses in Haymaking." Canterbury Chamber of Commerce, *N.Z. Agric. Bul.* 78. 1935; also Lloyd, *op. cit.*, pp. 49-50.

equivalent, shorn wool (k_2) per lb., and sheep (k_3) per head were allowed to assume the following values:[19]

k_1 : 2d., 8d.
k_2 : 50d., 100d.
k_3 : £2, £4.

The replacement price of a sheep at drought's end was allowed to increase at rates of £0.1 and £0.4 per month. Thus g(t) assumed values of 0.1t and 0.4t.

Schedules of expected net revenue per sheep equivalent at each level of fodder storage were calculated for each of the 32 possible combinations of the k_1, k_2, k_3, g(t) and D(i) levels. Four of these 32 expected net revenue schedules are plotted in Figure 1. They relate to the (k_1, k_2, k_3) combinations of (2d., 100d., £4) and (8d. 50d., £4) at a g(t) of 0.1t and at stocking rates of D(4) and D(5). As Figure 1 shows, expected net revenue increases rapidly over the range from zero to four months' supply of fodder. This is also true for the other

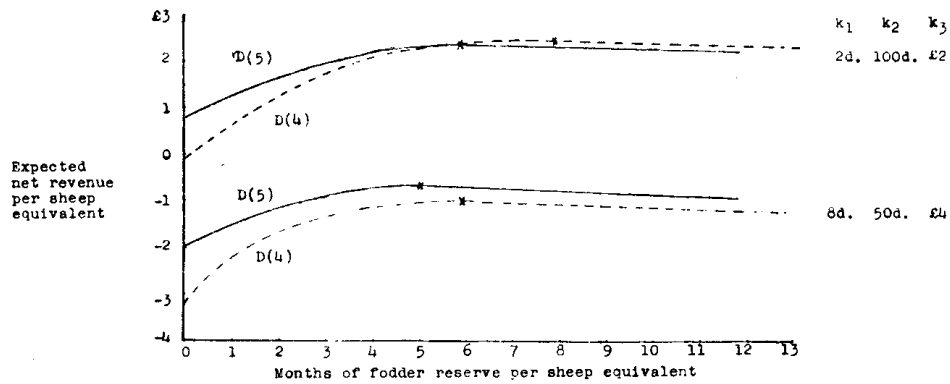

Figure 1. Expected net revenue per sheep equivalent for alternative levels of fodder reserve and stocking rates at Hughenden

28 revenue schedules not shown in the figure. Such an effect occurs because the probability of $o < t \leqslant 4$ is relatively high. For fodder reserves progressively beyond the optimal level (marked by a cross on the graph), expected net revenue declines at a very much slower rate than the net revenue decline from having insufficient fodder. It is safer, therefore, to have too large a fodder reserve than to have too little. This is further illustrated by the data of Table 2 showing expected net revenues under zero, optimal and maximal fodder reserves for D(4) and g(t) of 0.4t.

"Droughts Can Be Beaten," *J. Aust. Inst. Agric. Sci.*, 24 : 93-96. 1958; and Skerman, P. J., "Cropping for Fodder Conservation and Pasture Production in the Wool-growing Areas of Western Queensland," *Qd. Univ. Dept. Agric. Papers* 1 : 135-136. 1958. It should be noted that while fodder and sheep prices are known at the start of the planning period, the entrepreneur has to decide in advance what wool price he believes to be relevant to his planning.

[19] In selecting these values we were guided by the following: "Drought Feeding of Sheep." C.S.I.R.O., Melbourne. *Leaflet Ser.* No. 23. 1958. p. 30; Franklin, M. C.,

Table 3 presents the optimal level of fodder storage in terms of months of fodder per sheep equivalent for the 32 price and stocking rate combinations studied. It must be emphasised that these quantities are only optimal *ex ante*. Perusal of Table 3 leads to the following comments about normative pre-drought fodder reserves for the Hughenden district:

(a) For a given stocking rate, the normative demand for fodder is highly inelastic with respect to the prices of wool and sheep and only slightly responsive to fodder price changes.

(b) At D(5) stocking rate, a grazier would never be far wrong in his pre-drought planning if he kept a six-months supply of fodder under any normal range of fodder, wool and sheep prices. At D(4) the corresponding figure is eight months' supply with fodder from 2d. to 4d. per starch equivalent; and six months' supply with fodder at 4d. to 8d.

(c) Using Table 1 in conjunction with Table 3, there is only about one chance in ten that the optimal *ex ante* reserve will prove inadequate. However, the optimal level of reserves is only about half the level that would be required should a drought of maximum length occur. Moreover, as the data of Table 2 illustrates, expected net revenue per sheep equivalent would not be appreciably effected if "no-risk" reserves, i.e. 12 or 13 months' fodder supply, were kept. In other words, for the Hughenden area, conservationists have tended to assess the situation fairly correctly in recommending maximum fodder reserves. At the same time, it might be said that farmers have also tended to act rationally in keeping somewhat less than maximum reserves.

Stocking Rate Comparisons

The comparative efficiency of the two stocking rates studied, D(4) and D(5), may be gauged from Table 4. This table shows expected net revenue per acre per month under D(5) with optimal fodder reserves, $\bar{R}_5(y^*)$, as a fraction of the corresponding revenue with D(4), $\bar{R}_4(y^*)$. Since variations in sheep equivalent prices appear to have no significant influence on these ratios, the table only covers changes in fodder and wool prices. The $\bar{R}_i(y^*)$ values were derived by way of (7) which allows for the fact that the planning period, by definition, varies between stocking rates.

$$(7) \qquad \bar{R}_5(y^*)/\bar{R}_4(y^*) = 13\ R_5(y^*)/15\ R_4(y^*)$$

When price conditions are such that both stocking rates lead to a negative net profit, a ratio of less than unity indicates that the (D5) stocking rate is to be preferred.[20] When both stocking rates lead to positive profits, a ratio of less than unity indicates that the D(4) stocking rate is to be preferred. Thus Table 4 shows D(5) (the lighter stocking rate) to be much better than D(4) when wool is at 50d. per lb. With

[20] Whether or not the firm should continue to operate when alternative net revenues are negative is a longer run problem that is not considered here.

Table 2

EXPECTED NET REVENUE PER SHEEP EQUIVALENT OVER THE PLANNING PERIOD AT HUGHENDEN UNDER ALTERNATIVE PRICE REGIMES WITH ZERO, OPTIMAL AND 13 MONTHS OF FODDER RESERVES PER SHEEP EQUIVALENT

Fodder per Starch Equivalent		2d.		8d.		*Months of Fodder per Sheep Equivalent*
Wool per lb.		50d.	100d.	50d.	100d.	
		£	£	£	£	
Sheep per Head	£2	–2.67	–0.86	–2.67	–0.86	0
		–0.21	2.35	–1.03	1.50	optimal
		–0.26	2.30	–1.30	1.26	13
	£4	–4.02	–2.22	–4.02	–2.22	0
		–0.20	2.35	–1.04	1.50	optimal
		–0.26	2.30	–1.30	1.26	13

Table 3

OPTIMAL FODDER RESERVES AT HUGHENDEN IN MONTHS OF FODDER PER SHEEP EQUIVALENT UNDER ALTERNATIVE PRICE REGIMES

Fodder per Starch Equivalent		2d.		8d.		*Stocking Rate**	g(t)**
Wool per lb.		50d.	100d.	50d.	100d.		
		Mths.	Mths.	Mths.	Mths.		
Sheep per Head	£2	6	6	5	6	D(5)	0.1t
		7	8	5	6	D(4)	
		6	6	5	6	D(5)	0.4t
		8	8	6	6	D(4)	
	£4	6	6	5	6	D(5)	0.1t
		8	8	6	7	D(4)	
		6	6	6	6	D(5)	0.4t
		8	8	6	7	D(4)	

* Denoted by D(i) where i is the number of months without effective rain that elapse before drought prevails.

** g(t) is the rate at which the end-of-drought sheep replacement price increases with length of drought, t.

Table 4

$\bar{R}_5(y^*)$ *AS A FRACTION OF* $\bar{R}_4(y^*)$ *UNDER ALTERNATIVE PRICE REGIMES AT HUGHENDEN.* [a]

Price of Wool per lb.	*Price of Fodder per Starch Equivalent*	
	2d.	8d
50d.	.39	.59
100d.	.85	.97

(a) $\bar{R}(y^*)$ is expected net revenue per acre per month. As used here it is unadjusted for sheep acquisition or disposal activities.

wool at 100d., D(4) (the heavier stocking rate) is to be preferred, although only slightly so when fodder is at 8d. per starch equivalent. Of course, in order to ascertain the optimal stocking rate for a given price regime it would be necessary to compare an array of stocking rates. We have only considered two stocking rates because of computational restrictions.

Concluding Comments

The empirical analysis relates only to the Hughenden area. For other areas, distinctive drought probabilities will generally be the rule. Moreover, the model as specified has been elaborated mainly as an expository device. No attention has been paid to a number of real-world considerations that may be important. Still, the analysis indicates that the problem of drought in relation to stocking rates and fodder reserves can be approached empirically in an economic framework.

The more important limitations of the empirical analysis might be noted. It is felt that all of them could be handled satisfactorily if the additional complexity were deemed worthwhile and, most importantly, if the required data were available. Perhaps the most serious limitation is the implicit assumption that droughts, while differing in length, do not differ in severity per unit of time. This simplification shows up most markedly in the assumption that there is an instantaneous change from non-drought to drought conditions. In reality, the change from non-drought to full drought conditions is gradual. To take account of this gradation, the fodder input coefficient necessary for livestock maintenance would have to be allowed to enter the model at levels varying from zero (at the start of drought) to a maximum after a specified period (when full drought conditions emerged). Likewise, no allowance has been made for the effect of prior droughts and stocking rates on current drought possibilities. Provided data were available, this could be done by introducing lag effects into the model.

A further limitation is the lack of consideration of seasonal fluctuations in the grazing supply. To some extent, the seasonal pattern might be accounted for by the use of production coefficients based on average performance over the year. Likewise, seasonal effects might be

accounted for in part by the effective rainfall criteria on which the drought probabilities are based. Such an adjustment may be all that is warranted for situations in which the normal year-to-year variations due to drought exceed the within-year seasonal effects. However, for environments in which the within-year variations predominate, special account would have to be taken of the seasonal cycling of the grazing feed supply. For such situations, the gains to be derived from ascertaining normative intra-year patterns of pasture utilisation are far greater than the gains that might be obtained from normative studies of the inter-year utilisation problem.[21] Still, both sources of variation should be taken into account for an ideal normative analysis.

Another major simplification of the model, as presented, is the assumption of but a single product. Were the firm capable of following a number of enterprises, account would have to be taken of the full range of possible enterprises. A special case of the multi-enterprise situation is that in which one of the enterprises is fodder production. Possibilities then exist for both storing and selling home produced fodder. The model would also have to recognise that the placing of an order for home produced fodder entails a time lag in delivery; and that the yield of home produced fodder is a stochastic variable.

Mention should perhaps be made of the implicit assumption that constant returns prevail with respect to sheep numbers and acres under a given stocking rate. Should empirical evidence indicate the assumption to be wrong, scale factors would need to be inserted in the model.

Finally, a rather arbitrary feature of the model is the selection of the longest drought that might be expected as the length of the planning period. This was done because it presents the simplest method of assessing the expected costs and revenues. However, any alternative planning period of practical significance could be used. Recognition might also be given to the problem of profit maximisation and resource conservation over the long run.

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[21] See Lloyd, *op. cit.*