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DO INCREASED COMMODITY PRICES LEAD TO MORE OR LESS SOIL DEGRADATION?

JEFFREY T. LaFRANCE*
Department of Agricultural and Resource Economics, University of Arizona, Tucson, Arizona 85721, U.S.A.

In this paper, a dynamic economic model is used to analyze the conflicting impacts of crop increasing/land degrading inputs with those of soil conserving/crop reducing inputs in problems of soil degradation in agriculture. Soil is a renewable resource that is generated naturally at a slow, essentially autonomous rate. Cultivation enhances crop production and degrades the soil, while conservation is unproductive for the crop but improves the soil resource. If the effects of cultivation dominate the effects of conservation in the soil dynamics, an increase in the price of the crop accelerates the rate of soil degradation in the short-run and decreases the long-run stock of the soil resource. On the other hand, if the effects of conservation dominate the effects of cultivation, an increase in the price of the crop decelerates the rate of soil degradation in the short run and increases the long-run stock of the soil resource. It is shown that subsidies on conservation activities or taxes on cultivation intensity may well decrease the long-run soil stock, although strong conditions must be satisfied for either of these results to hold. It also is shown that a reduction in the real discount rate or a direct per unit tax on soil losses is certain to increase the long-run soil stock and reduce the short-run rate of soil degradation.

Introduction

Do higher commodity prices in agriculture induce farmers to take better care of the soil? Can the land degradation problems of modern agriculture be solved with subsidies to farm incomes? Or is a major cause of land degradation problems of the developed world the price and income subsidies that are part of current and past agricultural policies? If so, are taxes on agricultural products effective means for reducing land degradation in agriculture? These questions are important and interesting because agricultural incomes are subsidized in

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many developed countries, while environmental concerns are increasing in most of the same countries.

One perspective on this issue can be illustrated with an analogy between farming and the exploitation of natural resources such as mineral deposits, old growth forests, and fish and wildlife populations. Soil is a natural resource that is mined when it is cultivated, like any other natural resource. When a precious metal becomes more valuable due to a once and for all time increase in its market price, mine operators face incentives to increase the current rate of extraction. Since larger quantities of ore are extracted from the mineral deposits in the initial extraction periods, in all future time periods there is less ore remaining to be exploited. Even if some stocks are so low in quality that further extraction is eventually unprofitable, there ultimately will be less ore left *in situ* with a high output price than with a low price. Similarly, if the price of a particular species of fish increases today and is expected to remain at the higher level in all future time periods, fishermen have incentives to increase their level of fishing effort. Since more fish is now being harvested from the current stock, the rate of exploitation of the fishery's parent population is greater. It is well-known (and straightforward to show) that under standard conditions a higher fish price leads to a smaller long-run equilibrium fish stock. Thus, in the case of farming, it might be argued analogously that an increase in agricultural commodity prices results in greater soil degradation as the land is cultivated more intensively and/or extensively. Similar to other cases of natural resource exploitation, the long-run equilibrium stock of the soil resource falls as commodity prices rise. This suggests that one way to get farmers to take better care of their land is by *taxing* agricultural prices.

However, now consider an analogy between soil and produced capital assets. Soil is a capital asset that becomes more valuable when farm commodity prices increase, like any other capital asset. Take the human capital that is created by education, for example. When the return to education capital increases, the rate of investment in education also increases. As a result, the equilibrium level of education in the population increases. Thus, income subsidies to better-educated people lead to greater levels of education in the work force. In the case of farming, if agriculture becomes more profitable, soil is a more valuable asset, and farmers face incentives to make greater investments in soil-improving activities. As a result, the equilibrium level of soil increases. This suggests that one way to get farmers to take better care of their land is by *subsidizing* agricultural prices.

Although they lead us to opposite conclusions, each of these arguments seems intuitively appealing on the surface — each almost certainly contains some element of truth. However, both of these opposing stories cannot reflect the relationship between land degradation and agricultural incomes. Indeed, it is unlikely that either gives a complete picture of the complex issues involved. The first story is

flawed because it ignores the beneficial effects of soil improving activities. But the second story also has flaws.

One weakness with the comparison of soil and education is that there is no degradation effect from the productive use of human capital. Indeed, it is generally agreed that work experience increases the productivity of an educated worker. But this is not the case in farming. In order to obtain a crop from the soil, certain activities are undertaken that degrade the soil resource (Rowell *et al.* 1977). Thus, inputs that increase crop production contribute to land degradation (Burch, Graetz, and Nobel 1987). Examples are numerous and ubiquitous. Ploughing, discing, planting, and cultivating break down the soil structure, which can lead to wind and water erosion. Fertilizer can accelerate the natural process of soil acidification as nitrates and phosphates leach into the soil profile (Adams and Pearson 1969; Bromfield *et al.* 1983; Friesen *et al.* 1982; Cregan *et al.* 1989; Helyar, Hochman, and Brennan 1988; Helyar and Porter 1989; National Research Council 1989). Fertilizer also can contaminate wells and river water as it leaches into the groundwater, flows through the aquifer, and returns to the river in the recharge zone, or is carried directly to the river in surface runoff. The surface runoff of irrigation water often causes the loss of soil material through erosion. Irrigation also can raise the water table and lead to soil salinization as dissolved salts in the groundwater are deposited in the root zone (Burch, Graetz, and Nobel 1987; Musgrave 1990; Quiggen 1988).

A second weakness is that standard treatments of capital investment do not incorporate any direct effect of the current investment level on output, while soil conservation practices reduce current output levels. Examples again are numerous. Planting trees as windbreaks or to draw down the water table results in less surface area available for crops, requires more intensive farming on the remaining land area to obtain the same output level, and limits the ability to manoeuvre farming equipment. Trees also compete with crops for soil nutrients, water, and sunlight. Reduced- and minimum-till cultivation practices can lead to soil compaction, which interferes with sprouting, root development, and the ability to plant seeds at the appropriate depths — all causing current yields to fall — and the additional organic matter on the surface with reduced tillage creates a favourable environment for plant diseases and pests (Gebhardt *et al.* 1985; Hendrix *et al.* 1986; House *et al.* 1984; National Research Council 1989; Phillips *et al.* 1980; Rowell *et al.* 1977). Thus, conservation also reduces the current productivity of cultivation.

A third weakness is that, while education increases a worker's stock of human capital, most soil conservation activities mitigate but generally do not reverse soil degradation. Windbreaks, contour strips, stubble mulching, and reduced- and minimum-till cultivation practices reduce soil losses, but do not lead to direct increases in the depth of the topsoil, the rate of soil generation, or long term improvements in

the soil resource. For example, applying lime to the surface of the soil does not affect the soil *ph* at deeper levels of the root zone of perennial grasses and legumes and does not reverse soil acidification in these layers (Adams and Pearson 1969; Bromfield *et al.* 1983; Friesen *et al.* 1982; Cregan *et al.* 1989; Helyar, Hochman, and Brennan 1988; Helyar and Porter 1989).

Thus land degradation is a complex problem that involves exploitation of the soil to produce crops as well as efforts to mitigate the effects of cultivation to conserve some of the soil for future periods. Common results of intensive farming practices are changes in soil structure and chemistry, in the same way that gold mining requires the removal of ore from mineral-bearing rocks and fishing from natural fish stocks requires the removal of fish from the parent population. There are possibilities for soil conserving activities, however, which may mitigate the degradation effects of cultivation.

In an effort to increase our understanding of the conflicting incentives to use crop increasing/land degrading inputs as well as soil conserving/crop reducing inputs, in this paper a dynamic economic analysis of soil degradation is presented. A single crop is the output of the production process. There are two variable inputs, cultivation and conservation, and soil is a capital stock. Soil is a renewable resource that is generated naturally at a slow, essentially autonomous rate. Cultivation increases the rate of crop production and degrades the soil, while conservation reduces the rate of crop production and increases the rate of soil growth. These properties of the (obviously stylized) dynamic model capture the essence of the opposing effects of cultivation and conservation on both the current level of crop production (i.e., yields per hectare) and the rate of change in the current stock of soil (i.e., tonnes of soil loss per hectare).

It is first shown that, if the level of conservation activity is treated (artificially) as a fixed input over the entire planning horizon, then the structure of the dynamic economic problem is precisely that of the exploitation of a renewable natural resource. Thus, if conservation activities do not respond to market forces (which is a strong condition), then an increase in the price of the crop, a decrease in the price of cultivation, or an increase in the real discount rate leads to a faster initial rate of soil degradation and a lower soil stock in the long run. On the other hand, if cultivation intensity is held (artificially) fixed throughout the planning horizon, then the problem is precisely that of the optimal accumulation of a produced capital asset. Thus, if cultivation intensity does not respond to market forces (also a strong condition), then an increase in the price of the crop, a decrease in the price of conservation, or a reduction in the real discount rate leads to a slower initial rate of soil degradation and a higher soil stock in the long run.

These special cases clearly illustrate the opposing effects of cultivation and conservation in the production function and in the state

equation for the soil stock. As a result, in the general soil degradation problem where both cultivation intensity and the level of conservation activity respond to market forces, it is not possible to reach general, unequivocal conclusions regarding the short- or long-run effects of changes in input or output prices on the soil resource. However, all of the possible outcomes are not equally likely in the most general case analyzed. Under plausible conditions, for example, an increase in the crop price, a decrease in the price of cultivation, or an increase in the price of conservation accelerates the rate of soil degradation in the short run and decreases the long-run stock of the soil resource. Consequently, under conditions where cultivation is the dominant influence in the soil degradation problem (in a sense that is defined precisely below), policies that subsidize crop prices or the prices of soil degrading inputs (e.g., irrigation water) may contribute to, rather than mitigate, land degradation in agriculture.

In general, it is shown that either subsidies or taxes on product prices are less likely to be effective than subsidies on conservation or taxes on cultivation as incentives for soil conservation. It is also shown that there are conditions (that is, values of the model parameters) where a tax or subsidy on output or either type of input has the opposite effect to the one which might have been expected. However, if direct measurements of changes in the soil stock are feasible, a per unit tax on soil losses, or equivalently, a per unit subsidy on soil growth, is shown to decrease the short-run rate of soil degradation and increase the long-run stock of the soil resource.

In the next section the dynamic economic model is developed and the role of the assumptions of the model are discussed. In the third section the issue of land degradation and prices is analyzed. In the fourth section is a summary of the conclusions that can be reached from this analysis. Almost all of the qualitative results are the result of straightforward, but extensive and tedious, mathematical manipulations. These derivations are contained in an appendix that is available from the author upon request.

The Dynamic Economic Model

Let $q(t)$ be the rate of crop production in period t, let $x(t)$ be the rate of cultivation, which increases output and degrades the soil, let $y(t)$ be the rate of conservation, which decreases output and saves the soil, and let $s(t)$ be the stock of the soil resource. The crop production function is

$$q(t) = f(x(t), y(t), s(t)), \tag{1}$$

and the equation governing the rate of change in the stock of soil over time is

$$\dot{s}(t) = g(x(t), y(t), s(t)), \tag{2}$$

where $\dot{s}(t) \equiv ds(t)/dt$ is the time rate of change of the soil stock, and $s(0) = s_o$ is given as a fixed quantity at the initial date in the planning horizon. The objective of the rational farmer is assumed to be to maximize the discounted net present value of the commodity prices from crop production,

$$J = \int_0^\infty e^{-rt}[pq(t) - wx(t) - vy(t)]dt, \tag{3}$$

where p, w, and v are the market prices of q, x, and y, respectively, and r is the real rate of time discount. The constraints on the decision problem are equations (1) and (2) and the non-negativity conditions $(x(t),y(t),s(t)) \geq (0,0,0)$ for all $0 \leq t < \infty$.

The following assumptions are made concerning the crop production technology:

P1 $f_x > 0, f_y < 0, f_s > 0;$

$f_{xs} > 0, f_{xy} < 0,$ and $f_{sy} < 0;$

$f \in C^2$ is strictly concave in (x,y,s); and

$f(x,y,0) \equiv 0 \supset (x,y) \in \mathrm{R}^2_+$,

where subscripts denote partial derivatives throughout the paper, e.g., $f_x \equiv \partial f(x,y,s,)/\partial x$. In words, hypothesis P1 means that cultivation and soil are productive inputs to crop production; conservation is unproductive for crop production; more soil increases the marginal productivity of cultivation; more conservation reduces the marginal productivity of cultivation and soil; there are diminishing marginal returns to all of the inputs and in total in the crop production technology on a given unit of land; and soil is essential for crop production. These properties are intuitively appealing and weak conditions.

The following assumptions are made concerning the equation of motion for the soil resource:

P2 $g_x < 0, g_y > 0, g_{xy} > 0;$

$g_s \approx 0;$

$g \in C^2$ is concave in (x,y,s) and strictly concave in (x,y); and

$g(0,y,s) > 0 \supset (y,s) \in R^2_+$.

That is, cultivation degrades the soil, conservation saves the soil, and conservation mitigates the soil degrading effects of cultivation; soil growth is essentially independent of the soil stock, so that $\dot{s} \approx g(x,y)$ provides a reasonable approximation over the observable range of values for (x,y,s); there are diminishing marginal returns to the inputs and in total in the equation of motion for the soil resource; and soil growth is positive with a zero cultivation level. As in the case of the crop production function, these properties are intuitively appealing

and weak, except perhaps for the condition that soil growth is independent of the soil stock. The latter hypothesis simplifies the analysis and discussion greatly, but it is much stronger than necessary. It can be shown that all of the results of the paper can be generalized to situations where soil growth depends explicitly on the soil stock so long as the terms of g_s, g_{xs}, and g_{ys} (whether positive or negative) do not dominate all of the other terms in the algebraic expressions where they appear.

The aim is to determine the impact of changes in the input and output prices and the real discount rate on (i) current production, input use, and soil losses, and (ii) the long-run equilibrium levels of production, input use, and the stock of soil. Therefore, all prices are assumed constant throughout the planning period. This greatly simplifies the analysis, but the results can be generalized readily to time varying prices following the arguments in LaFrance and Barney (1991). In essence, perfect foresight is assumed with respect to all prices and the real discount rate.

The focus of the model is entirely on the intensive margin — there is a fixed area of land that is continuously cultivated for crop production. This permits the effects of price changes on land that is currently under production to be isolated, thereby avoiding any complicating effect of land degradation that results from the development of marginal ground for crop production when farming becomes more profitable.

Another aspect of the model structure is that technology is held constant throughout the planning horizon. This is necessary to avoid confusion between changes in the technology of farming and changes in the physical or chemical structure of the soil resource. Many technological advances in agriculture, such as improved plant varieties and superphosphate fertilizers, have increased the productivity of soil with a given set of characteristics. At the same time, many of the soil's productive characteristics have deteriorated, e.g., loss of physical soil matter, increased soil salinity or acidity, and so forth. As a result, the effects of improvements in technology are often confused with the effects of changes in the soil resource. It is sometimes argued that the soil has not been degraded but rather improved since it is now more productive than in the past. But this is an obvious violation of the *ceteris paribus* property necessary for the effective evaluation of the net effects of market forces on natural resources. Holding technology constant is essentially equivalent to the assumption that, in the long run at least, we will not be able to count on a perpetual technological fix for problems of environmental degradation. While this may or may not be the case, this assumption allows economic effects to be isolated from technological effects on the soil resource.

Finally, although there are off-farm effects of soil erosion and other types of land degradation (Bunce 1942; CiriacyWantrup 1952; Clark, Haverkamp, and Chapman 1985; Crosson 1982; Crosson and Brubaker

1982; Moore and McCarl 1987; National Resource Council 1989; USDA 1980; Wade and Heady 1978), there is no private economic incentive for farmers to pay attention to these off-farm impacts. Therefore, in the analysis the off-farm externalities due to land degradation are ignored. This does not imply that the analysis of this model is not useful for considering the design of policies to internalize or control the off-site effects of soil degradation. Indeed, the model can be used to indicate what types of policies — input, output, or emissions taxes or subsidies, or standards on the rate of soil degradation — are likely to work effectively to modify the incentives and actions of individual farmers.

The nature of the optimal solution to the dynamic economic problem that is faced by the wealth-maximizing farmer is now considered. Substituting $f(x,y,s)$ for q in the profit function, the current value Hamiltonian for this optimal control problem is

$$H = pf(x(t),y(t),s(t)) - wx(t) - vy(t) + \lambda(t)g(x(t),y(t)), \tag{4}$$

where $\lambda(t)$ is the current value shadow price for the soil state equation. The time index is dropped for notational parsimony whenever this creates no confusion. The first-order necessary conditions for an interior optimal solution (that is, an optimal solution that satisfies $(x,y,s,\lambda) \gg (0,0,0,0)$ over the entire planning horizon) are (Pontryagin, *et al.* 1962)

$$H_x = pf_x - w + \lambda\, g_x = 0, \tag{5}$$

$$H_y = pf_y - v + \lambda g_y = 0, \tag{6}$$

$$H_s = pf_s = r\lambda - \dot{\lambda}, \lim_{t\to\infty} e^{-rt}\lambda(t) = 0, \tag{7}$$

$$H_\lambda = g = \dot{s}, s(0) = s_0. \tag{8}$$

For optimal control problems of this type, it is well-known that the shadow price, $\lambda(t)$, is positive-valued and satisfies $\lambda(t) \equiv \partial J^*/\partial s_0$, where J^* is the maximum discounted present value of profits (see, for example, Benveniste and Scheinkman (1979)). That is to say, the shadow price for the soil resource reflects the marginal increase in the present value of profits due to an increase in the initial stock of the soil resource. As such, the shadow price is like other prices and must be positive. It follows from this and hypotheses P1 and P2 that H is strictly concave in (x,y,s). It is also well-known that if the Hamiltonian is strictly concave, then the first-order conditions (5) through (8) are necessary and sufficient for an optimal solution and the optimal path is unique (Arrow and Kurz 1970; Hadley and Kemp 1971; Kamien and Schwartz 1971; Seierstad and Sydsaeter 1987). To further simplify the analysis and the discussion, it also is assumed that the optimal path is an interior solution along the entire optimal path. This means, in particular, that the farmland is not abandoned as part of the optimal solution. However, it can be shown that hypotheses P1 and P2, which

are reasonable and intuitively appealing conditions, are sufficient conditions to preclude long-run abandonment of farmland as part of the optimal path.

Since both p and λ are positive, hypotheses P1 and P2 are sufficient to determine the signs of all second-order partial derivatives of H with respect to (x,y,s). The only term that is not immediately obvious from P1 and P2 is the sign of $H_{xy} \equiv pf_{xy} + \lambda g_{xy}$ since the terms f_{xy} and g_{xy} oppose each other in determining its sign. However, from the first-order condition (6) we have $\lambda = (pf_y - v)/g_y$, which implies

$$H_{xy} = pf_y(f_{xy}/f_y + g_{xy}/g_y) - vg_{xy}/g_y < pf_y(f_{xy}/f_y + g_{xy}/g_y) < 0$$

by P1 and P2. Thus, the following list summarizes the signs of the second-order derivatives of the Hamiltonian for this problem:

$$H_{xx} < 0;\ H_{xy} < 0;\ H_{xs} > 0;\ H_{yy} < 0;\ H_{ys} < 0;\ H_{ss} < 0.$$

Let $\alpha \equiv [p,w,v,r,s_0]'$ be the vector of parameters in the problem; let $x^*(\alpha,t)$ and $y^*(\alpha,t)$ be the optimal controls for cultivation and conservation, respectively; let $s^*(\alpha,t)$ and $\lambda^*(\alpha,t)$ be the optimal solutions for the soil resource and the dynamic shadow price for the soil stock; let $q^*(\alpha,t) \equiv f(x^*(\alpha,t),y^*(\alpha,t),s^*(\alpha,t))$ be the optimal level of crop production; let $\dot{s}^*(\alpha,t) \equiv g(x^*(\alpha,t),y^*(\alpha,t))$ be the optimal time rate of change in the soil resource; and let $J^*(\alpha)$ be the maximum present value of profits from crop production,

$$J^*(\alpha) \equiv \int_0^\infty e^{-rt}[pq*(\alpha,t) - wx^*(\alpha,t) - vy^*(\alpha,t)]dt\,. \tag{9}$$

The objective of the analysis is to utilize the first-order conditions and the curvature properties of f, g, and H to study the influences of changes in the input and output prices on the optimal choice functions $\{q^*(\alpha,t),x^*(\alpha,t),y^*(\alpha,t),s^*(\alpha,t),\lambda^*(\alpha,t)\}$ in the short and long run and intertemporally over the planning horizon. This requires an analysis of the relationships among four groups of properties of the solution to the optimal control problem: (a) the short-run responses of the choice variables to changes in the problem parameters; (b) the intertemporal relationships among the choice variables; (c) the structure of the long-run equilibrium; and (d) the monotonicity, homogeneity, and curvature properties of the present value of profits function. Although many of these individual properties have been analyzed elsewhere (Araujo and Scheinkman 1979; Caputo 1989, 1990a, 1990b; Epstein 1978, 1981, 1982; Hadley and Kemp 1971; LaFrance and Barney 1991; McLaren and Cooper 1980; Oniki 1972), a comprehensive analysis of them all along with the relationships among them has not been presented. (However, the mathematical appendix to this paper, referred to in the introduction and available from the author on request, contains a detailed analysis of the short-run, intertemporal, and long-run structure of the current problem when the equation of motion depends upon the soil stock.) In the next section the effects of changes in market prices on soil degradation are analyzed.

The Effects of Commodity Price Changes

In this section, the impacts of changes in prices and the real discount rate on the output supply, input use, and soil loss are analyzed within the model framework of the previous section. First, the artificially constrained optimization problem where conservation is held fixed at some arbitrary level throughout the planning horizon is considered to get a firm understanding of the properties of the resource exploitation or 'mining' problem. Then the opposite situation where cultivation is held fixed at some artificial level throughout the planning horizon is analyzed to better understand the nature of the 'produced capital' problem. Lastly, the general problem, where both cultivation and conservation are assumed to respond to market forces, is analyzed.

The simple dynamics of soil and cultivation

In this subsection, we suppose that conservation activities are held fixed at some level, $y_0 \geq 0$, throughout the planning horizon. The only control variable in this constrained optimization problem, then, is the level of cultivation intensity. Let the optimal solutions for the constrained problem be

$$\{x^y(\alpha,t), q^y(\alpha,t), s^y(\alpha,t), \lambda^y(\alpha,t)\} \text{ for all } 0 \leq t < \infty.$$

Also let the long-run steady state solutions be

$$\{\tilde{x}^y(\alpha), \tilde{q}^y(\alpha), \tilde{s}^y(\alpha), \tilde{\lambda}^y(\alpha)\},$$

where the long-run equilibrium satisfies equations (5), (7), and (8) and $\dot{s} = \dot{\lambda} = 0$.

Note that properties P1 and P2 together with the necessary conditions for an optimal long-run steady state solution preclude the exhaustion of the soil stock in the long-run equilibrium. This is because P1 implies $\lim_{s \to 0} f_x(x,y_0,s) \equiv 0$ for all $x \geq 0$. Since the marginal cost of crop production is $w/f_x(x,y_0,s)$, this implies that the marginal cost increases without bound for any positive level of cultivation intensity as the soil stock approaches exhaustion. The first-order condition for the optimal choice of x is $p = (w - \lambda g_x(x,y_0))/f_x(x,y_0,s)$. Thus, since $f_x > 0$, $g_x < 0$, and $\lambda^y > 0$ it must be the case that $p \geq w/f_x(s,y_0,s)$ along the entire optimal path, including the long-run equilibrium. But this excludes $\tilde{s}^y(\alpha) = 0$. As a result, although land degradation is similar to mining in the sense that if $s_0 > \tilde{s}^y(\alpha)$ then $\dot{s}^y(\alpha,0) < 0$, soil differs from a purely non-renewable resource because there is a long-run economic equilibrium with a positive soil stock and a positive rate of crop production. It also follows that in the more general situation where both cultivation intensity and conservation activity respond to market forces, the long-run equilibrium will not be characterized by an exhausted soil resource and abandonment of the farm.

It can be shown that the following short- and long-run qualitative responses characterize this constrained optimization problem:

Short-run:	$x_p^y(\alpha,0) > 0$	$q_p^y(\alpha,0) > 0$	$\dot{s}_p^y(\alpha,0) < 0$	$\lambda_p^y(\alpha,0) > 0$
	$x_w^y(\alpha,0) < 0$	$q_w^y(\alpha,0) < 0$	$\dot{s}_w^y(\alpha,0) > 0$	$\lambda_w^y(\alpha,0) < 0$
	$x_r^y(\alpha,0) > 0$	$q_r^y(\alpha,0) > 0$	$\dot{s}_r^y(\alpha,0) < 0$	$\lambda_r^y(\alpha,0) > 0$
Long-run:	$\tilde{x}_p^y(\alpha) = 0$	$\tilde{q}_p^y(\alpha) < 0$	$\tilde{s}_p^y(\alpha) < 0$	$\tilde{\lambda}_p^y(\alpha) < 0$
	$\tilde{x}_w^y(\alpha) = 0$	$\tilde{q}_w^y(\alpha) > 0$	$\tilde{s}_w^y(\alpha) > 0$	$\tilde{\lambda}_w^y(\alpha) < 0$
	$\tilde{x}_r^y(\alpha) = 0$	$\tilde{q}_r^y(\alpha) < 0$	$\tilde{s}_r^y(\alpha) < 0$	$\tilde{\lambda}_r^y(\alpha) < 0$

With a fixed level of conservation activity, the level of cultivation intensity increases with the crop price and the real discount rate, which results in a greater level of output and a more rapid rate of soil degradation in the short run. Cultivation intensity decreases with its own price, which results in a lower level of output, and a slower rate of soil degradation in the short run. In the long run, the level of cultivation intensity is independent of all prices. Output and the soil stock decrease with the crop price and the real discount rate and increase with the price of cultivation. Thus, when the level of conservation activity is a fixed input, taxes on cultivation intensity or the rate of crop production or subsidies on the real discount rate reduce the initial rate of soil degradation and increase the long-run stock of the soil resource.

These conclusions are illustrated in figure 1, where

$\alpha_0 = [p_0, w_0, v, r_0, s_0]'$ and $\alpha_1 = [p_1, w_1, v, r_1, s_0]'$, , with $p_0 < p_1, w_0 > w_1, r_0 < r_1$

or any combination of these inequalities, and $s_0 > \tilde{s}^y(\alpha_0)$.

The simple dynamics of soil and conservation

Suppose that the level of cultivation intensity is held fixed at some level, $x_0 \geq 0$, over the entire planning horizon and the only control variable is the level of conservation activity. Let the optimal solutions for the constrained problem be

$$\{y^x(\alpha,t), q^x(\alpha,t), s^x(\alpha,t), \lambda^x(\alpha,t)\} \text{ for all } 0 \leq t < \infty$$

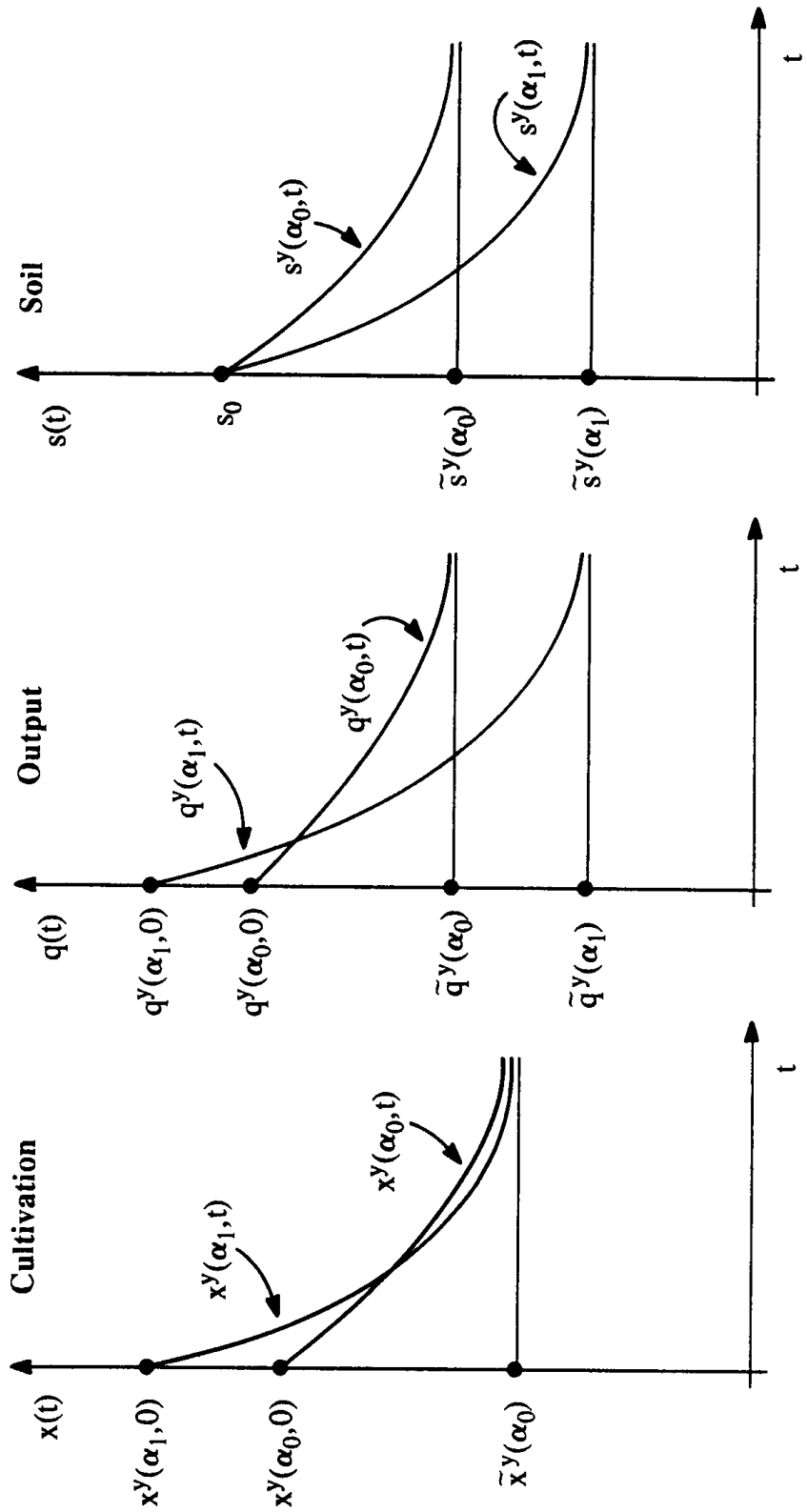
and let the long-run steady state solutions be

$$\{\tilde{y}^x(\alpha), \tilde{q}^x(\alpha), \tilde{s}^x(\alpha), \tilde{\lambda}^x(\alpha)\},$$

where in this case the long-run equilibrium satisfies (6)–(8) and $\dot{s} = \dot{\lambda} = 0$. The following short- and long-run qualitative responses characterize the properties of this constrained optimization problem:

Short-run:	$y_p^x(\alpha,0) > 0$	$q_p^x(\alpha,0) < 0$	$\dot{s}_p^x(\alpha,0) > 0$	$\lambda_p^x(\alpha,0) > 0$
	$y_v^x(\alpha,0) < 0$	$q_v^x(\alpha,0) > 0$	$\dot{s}_v^x(\alpha,0) < 0$	$\lambda_v^x(\alpha,0) > 0$
	$y_r^x(\alpha,0) < 0$	$q_r^x(\alpha,0) > 0$	$\dot{s}_r^x(\alpha,0) < 0$	$\lambda_r^x(\alpha,0) < 0$

FIGURE 1
Constrained Optimal Paths for Cultivation, Output, and Soil



Long-run:	$\tilde{y}_p^x(\alpha) = 0$	$\tilde{q}_p^x(\alpha) > 0$	$\tilde{s}_p^x(\alpha) > 0$	$\tilde{\lambda}_p^x(\alpha) > 0$
	$\tilde{y}_v^x(\alpha) = 0$	$\tilde{q}_v^x(\alpha) < 0$	$\tilde{s}_v^x(\alpha) < 0$	$\tilde{\lambda}_v^x(\alpha) > 0$
	$\tilde{y}_r^x(\alpha) = 0$	$\tilde{q}_r^x(\alpha) < 0$	$\tilde{s}_r^x(\alpha) < 0$	$\tilde{\lambda}_r^x(\alpha) < 0$

With a fixed level of cultivation intensity, the initial level of conservation increases, the initial level of crop production decreases, and the initial rate of change in the soil resource increases with an increase in the price of the crop, a decrease in the price of conservation, or a decrease in the real discount rate in the short run. In the long run, the level of conservation activity is independent of all prices. Output and the equilibrium level of the soil stock increase with the price of the crop, and decrease with the price of conservation and the real discount rate. These conclusions are illustrated in figure 2, where

$\alpha_0 = [p_0, w, v_0, r_0, s_0]'$ and $\alpha_1 = [p_1, w, v_1, r_1, s_0]'$, with $p_0 < p_1, v_0 v_1, > r_0 > r_1$,

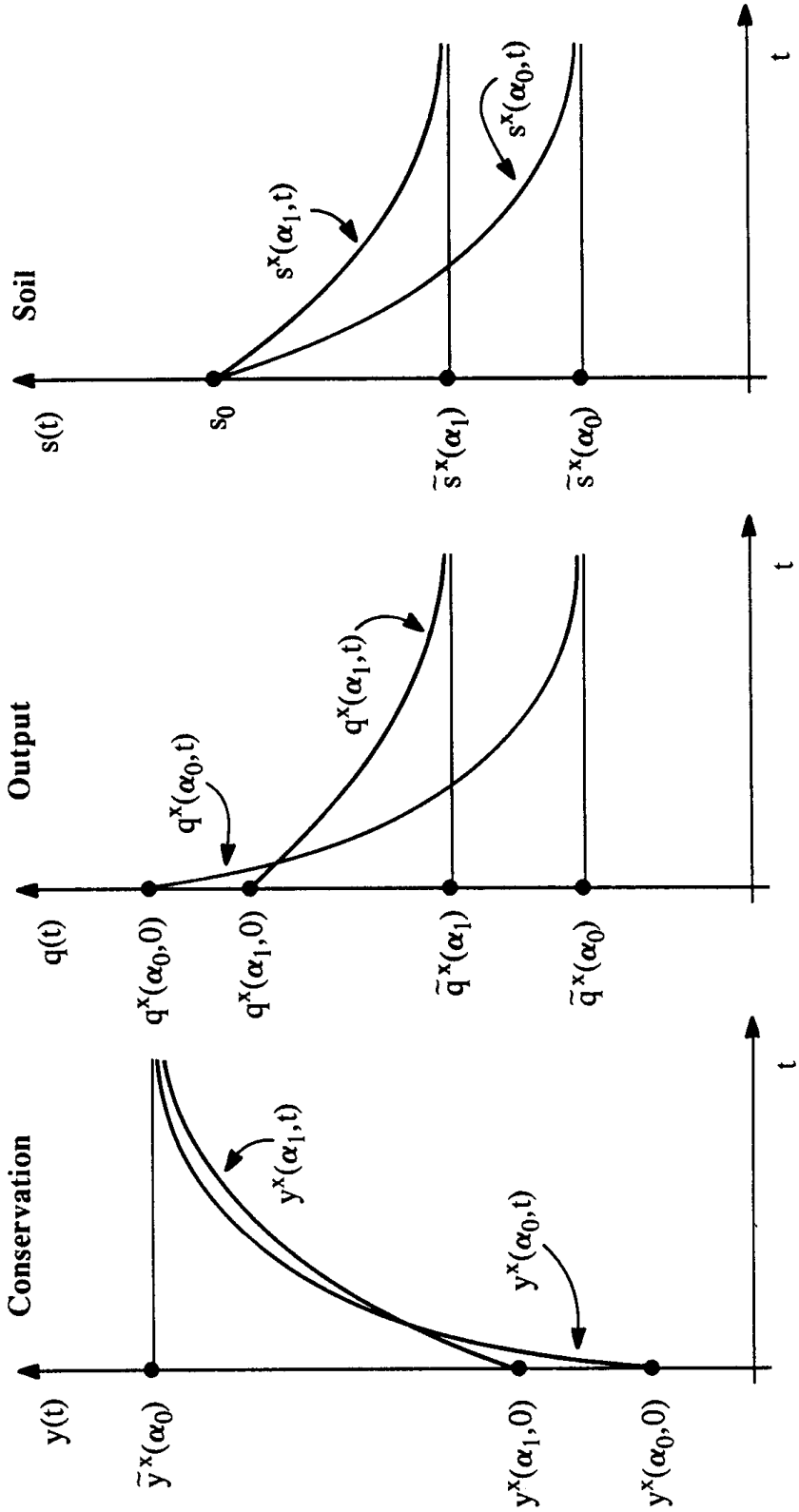
or any combination of these possibilities, and $s_o > \tilde{s}(\alpha_1)$. Thus, with a fixed level of cultivation, a subsidy on the price of either conservation or output reduces the initial rate of soil degradation and increases the long-run equilibrium of the soil resource.

Note that when the level of cultivation intensity is fixed, the short-run supply curve is backward-bending. This is consistent with casual observation — conservation requires substitution away from current production and towards future production in order to save the soil resource for future periods. However, a backward-bending short-run supply curve is counter to empirical evidence. (See, e.g., Askari and Cummings (1977) for a survey of short- and long-run agricultural product supply elasticities.) In the United States of America during the mid 1970s, for example, agricultural commodity prices were high and farmland was cultivated more intensively during this period than any other period in recent history. The 'swampbuster', 'sodbuster', and 'conservation reserve' provisions of recent farm legislation in the United States of America are reactions by the Federal government to this aspect of the soil degradation problem.

The dynamics of soil, cultivation, and conservation

Now, the more general problem is explored where both the level of cultivation intensity and conservation activity are choice variables. The main issue is whether cultivation or conservation is the dominant factor in soil degradation. This is clear from the above discussion — the qualitative effects of each individual input, holding the other input fixed, are in direct opposition. This is the fundamental nature of soil degradation that differs from the traditional models of both natural resource exploitation and capital growth and investment. In soil degradation, cultivation plays essentially the same role that the harvest rate plays in natural resource exploitation, while conservation plays essentially the same role that the investment rate plays in capital

FIGURE 2
Constrained Optimal Paths for Conservation, Output, and Soil



growth. When both types of input are present, without prior knowledge regarding which input exerts the dominating influence on soil losses and crop production rates, it is unclear whether an increase in the price of the output, say, will lead to improvements in or deterioration of the soil stock in either the short or the long run. The objective in this subsection is to characterize the qualitative nature of these issues as completely as possible.

Let $\{\tilde{x}(\alpha),\tilde{y}(\alpha),\tilde{q}(\alpha),\tilde{s}(\alpha),\tilde{\lambda}(\alpha)\}$ denote the steady state values for the choice variables, where in this case, the steady state satisfies (5)–(8) and $\dot{s} = \dot{\lambda} = 0$. The present aim is to identify conditions that will allow the signs of the long-run responses of the soil stock to changes in the input and output prices, $\tilde{s}_p(\alpha)$, $\tilde{s}_w(\alpha)$, and $\tilde{s}_v(\alpha)$, to be determined. These responses can be shown to be

$$\tilde{s}_p(\alpha) = [r(wH_{yy} - vH_{xy})g_x + r(vH_{xx} - wH_{xy})g_y - vH_{ys}g_x^2 + (wH_{ys} + vH_{xs})g_xg_y - wH_{xs}g_y^2]/\tilde{\Delta}\,, \tag{10}$$

$$\tilde{s}_w(\alpha) = [-g_x(rH_{yy} + g_yH_{ys}) + g_y(rH_{xy} + g_yH_{xs})]/\tilde{\Delta}, \tag{11}$$

$$\tilde{s}_v(\alpha) = [g_x(rH_{xy} + g_xH_{ys}) - g_y(rH_{xx} + g_xH_{xs})]/\tilde{\Delta}, \tag{12}$$

where $\tilde{\Delta}$ is the Jacobian for the implicit functions (5)–(8) with respect to (x,y,s,λ) given the equilibrium conditions $\dot{s} = \dot{\lambda} = 0$. This determinant is given by

$$\tilde{\Delta} = g_x[r(H_{xy}H_{ys} - H_{xs}H_{yy}) - g_x(H_{yy}H_{ss} - H_{ys}^2) + g_y(H_{xy}H_{ss} - H_{xs}H_{ys})] + g_y[r(H_{xy}H_{xs} - H_{xx}H_{ys}) + g_x(H_{xy}H_{xs} - H_{xs}H_{ys}) - g_y(H_{xx}H_{ss} - H_{xs}^2)] < 0. \tag{13}$$

Consider the equilibrium response of the soil stock to changes in the price of the crop. The main issue is whether cultivation or conservation dominates the soil dynamics. This is clear from the fact that the numerator in the expression for $\tilde{s}_p(\alpha)$, denoted by $Q(g_x,g_y)$, is a quadratic function of the equilibrium levels of the marginal effects of cultivation and conservation on the soil dynamics,

$$Q(g_x,g_y) = r(wH_{yy} - vH_{xy})g_x + r(vH_{xx} - wH_{xy})g_y - vH_{ys}g_x^2 + (wH_{ys} + vH_{xs})g_xg_y - wH_{xs}g_y^2. \tag{14}$$

Clearly, $\tilde{s}_p(\alpha) \gtreqless 0$ if and only if $Q(g_x,g_y) \lesseqgtr 0$.

Note that strict concavity of the Hamiltonian requires H_{xy} to be greater than either vH_{xx}/w or wH_{yy}/v (recall that $H_{xy} < 0$). Otherwise $H_{xy}^2 \geq H_{xx}H_{yy}$ and the Hamiltonian cannot be strictly concave. Therefore, for symmetry and to simplify the discussion, assume hereafter that vH_{xx}–wH_{xy} and wH_{yy}–vH_{xy} are both non-positive.

Suppose that $g_y \approx 0$, so that the marginal effect of conservation activity on soil growth is small. Then we have

$$(15) \qquad Q(g_x,g_y) \approx r(wH_{yy} - vH_{xy})g_x - vH_{ys}g_x^2 > 0$$

for all $g_x < 0$. On the other hand, if $g_x \approx 0$, so that the marginal effect of cultivation on soil growth is small, then

$$(16) \qquad Q(g_x,g_y) \approx r(vH_{xx} - wH_{xy})g_y - wH_{xs}g_y^2 < 0$$

for all $g_y > 0$. Thus, if conservation has a small effect on the soil stock relative to cultivation, then the steady state stock of soil is a decreasing function of the price of the crop. If cultivation has only a small effect on the soil stock relative to conservation, then the steady state stock of soil is an increasing function of the price of the crop. These are important results because they illustrate that the boundary curve, $Q(g_x,g_y) = 0$, does not cross either axis in the third quadrant of the equilibrium (g_x,g_y) plane (that is, the set of equilibrium values for (g_x,g_y) such that $g_x \leq 0$ and $g_y \geq 0$) except at the origin.

A precise statement is possible for the conditions under which $\tilde{s}_p(\alpha)$ can be signed. It can be shown that $Q(g_x,g_y)$ is a rectangular hyperbola in the equilibrium (g_x,g_y) plane that passes through the origin and has negative slope in the third quadrant. As a result, the relevant boundary for determining the sign of $\tilde{s}_p(\alpha)$ is obtained by solving $Q(g_x,g_y) = 0$ for the larger root for g_y as a function of g_x. This gives

$$(17) \qquad \hat{g}_y(g_x) = \Big[r(vH_{xx} - wH_{xy}) - (wH_{ys} + vH_{xs})g_x + \sqrt{[r(vH_{xx}-wH_{xy})-(wH_{ys}+vH_{xs})g_x]^2 - 4wH_{xs}[r(wH_{yy}-vH_{xy})g_x - vH_{ys}g_x^2]}\Big] \Big/ 2wH_{xs}.$$

It follows that $\tilde{s}_p(\alpha) \gtreqless 0$ if and only if $g_y \lesseqgtr \hat{g}_y(g_x)$ at their steady state values.

Next consider the equilibrium response of the soil stock to changes in the price of cultivation. From (11) we have $\tilde{s}_w(\alpha) \gtreqless 0$ if and only if

$$(18) \qquad g_x \lesseqgtr (rg_yH_{xy} + g_y^2H_{xs})/(rH_{yy} + g_yH_{ys}).$$

Finally, consider the equilibrium response of the soil stock to changes in the price of conservation. From (12) we have $\tilde{s}_v(\alpha) \gtreqless 0$ if and only if

$$(19) \qquad g_y \lesseqgtr (rg_xH_{xy} + g_x^2H_{ys})/(rH_{xx} + g_xH_{xs}).$$

The conditions under which the long-run stock of the soil resource is an increasing or decreasing function of each of the relevant prices can now be stated. This statement is facilitated by a definition of

conditions where one input 'dominates' the other in the long-run equilibrium. Denote *weak domination* and *strong domination* by the binary operators *wd* and *sd*, respectively. In words, x *wd* y means 'cultivation weakly dominates conservation', while x *sd* y means 'cultivation strongly dominates conservation'. Formally, the definition that is required is as follows.

Definition 1:

(a) x *wd* y if $(rg_xH_{xy} + g_x^2H_{ys})/(rH_{xx} + g_xH_{xs}) \leq g_y \leq \hat{g}_y(g_x)$;

(b) x *sd* y if $g_y < (rg_{xy} + g_x^2H_{ys})/(rH_{xx} + g_xH_{xs})$;

(c) y *wd* x if $\hat{g}_y(g_x) \leq g_y$ and $g_x \leq (rg_yH_{xy} + g_y^2H_{xs})/(rH_{yy} + g_yH_{ys})$;

(d) y *sd* x if $(rg_yH_{xy} + g_y^2H_{xs})/(rH_{yy} + g_yH_{ys}) < g_x$.

It can be shown that these cases are mutually exclusive and exhaustive throughout the interior of the third quadrant of the equilibrium (g_x,g_y) plane. That is, the boundaries $\tilde{s}_p(\alpha) = 0$, $\tilde{s}_w(\alpha) = 0$, and $\tilde{s}_v(\alpha) = 0$ do not intersect one another in the third quadrant of the (g_x,g_y) plane except at the origin. This gives the following fundamental result.

Proposition 1:

(a) If x *sd* y, then $\tilde{s}_p(\alpha) < 0$, $\tilde{s}_w(\alpha) > 0$, and $\tilde{s}_v(\alpha) > 0$;

(b) if x *wd* y, then $\tilde{s}_p(\alpha) \leq 0$, $\tilde{s}_w(\alpha) > 0$, and $\tilde{s}_v(\alpha) \leq 0$;

(c) if y *wd* x, then $\tilde{s}_p(\alpha) \geq 0$, $\tilde{s}_w(\alpha) \geq 0$, and $\tilde{s}_v(\alpha) < 0$;

(d) if y *sd* x, then $\tilde{s}_p(\alpha) > 0$, $\tilde{s}_w(\alpha) \leq 0$, and $\tilde{s}_v(\alpha) < 0$;

A complete list of the qualitative short- and long-run responses under the conditions established by Proposition are presented in table 1.

Proposition 1 is illustrated graphically in figure 3, which is constructed as follows. The units of measure for the choice variables are scaled so that, at the steady state, the marginal value product of soil is one dollar, $pf_s = \$1$, conservation and cultivation are each measured in terms of real expenditures, $w/p = v/p = 1$, and the crop is measured in terms of revenue, $p = \$1$. With these normalizations, the equilibrium relationship between f_x and g_x is $f_x = 1 - g_x/r$, and the equilibrium relationship between f_y and g_y is $f_y = 1 - g_y/r$. For simplicity, assume that $g_x = -g_y$ at the steady state, so that \$1 of additional conservation expenditure saves as much soil as \$1 of additional cultivation expenditure loses. Combining the first-order conditions then implies $f_x = 2 - f_y$ in the steady state. Combining this with $f_x > 0$ and $f_y < 0$ requires $f_x > 2$; therefore let $f_x = 2.25$ and $f_y = -0.25$. A four percent discount rate, $r = 0.04$, is then equivalent to $g_x = -0.05$ and $g_y = 0.05$ at the long-run steady state solution.

TABLE 1
Qualitative Short- and Long-run Price Responses

	Cultivation Strongly Dominates Conservation								
	Short-run Response to a Change in					Long-run Response to a Change in			
Variable	p	w	v	r	s_o	p	w	v	r
Cultivation	+	?	?	+	+	+	–	–	?
Conservation	?	?	–	–	–	+	–	–	?
Crop Production	+	?	?	+	+	?	–	–	?
Soil Resource	–	–	–	–	0	–	–	–	–

	Cultivation Weakly Dominates Conservation								
	Short-run Response to a Change in					Long-run Response to a Change in			
Variable	p	w	v	r	s_o	p	w	v	r
Cultivation	+	–	?	+	+	+	–	–	?
Conservation	?	?	–	–	–	+	–	–	?
Crop Production	+	–	?	+	+	?	?	–	?
Soil Resource	–	+	–	–	0	–	+	–	–

	Conservation Weakly Dominates Conservation								
	Short-run Response to a Change in					Long-run Response to a Change in			
Variable	p	w	v	r	s_o	p	w	v	r
Cultivation	?	–	?	+	+	+	–	–	?
Conservation	+	?	–	–	–	+	–	–	?
Crop Production	?	–	?	+	+	+	?	–	?
Soil Resource	+	+	–	–	0	+	+	–	–

	Conservation Strongly Dominates Cultivation								
	Short-run Response to a Change in					Long-run Response to a Change in			
Variable	p	w	v	r	s_o	p	w	v	r
Cultivation	?	–	?	+	+	+	–	–	?
Conservation	+	?	–	–	–	+	–	–	?
Crop Production	?	–	?	+	+	+	?	?	?
Soil Resource	+	+	+	–	0	+	+	+	–

FIGURE 3
Constrained Optimal Paths for Conservation, Output, and Soil

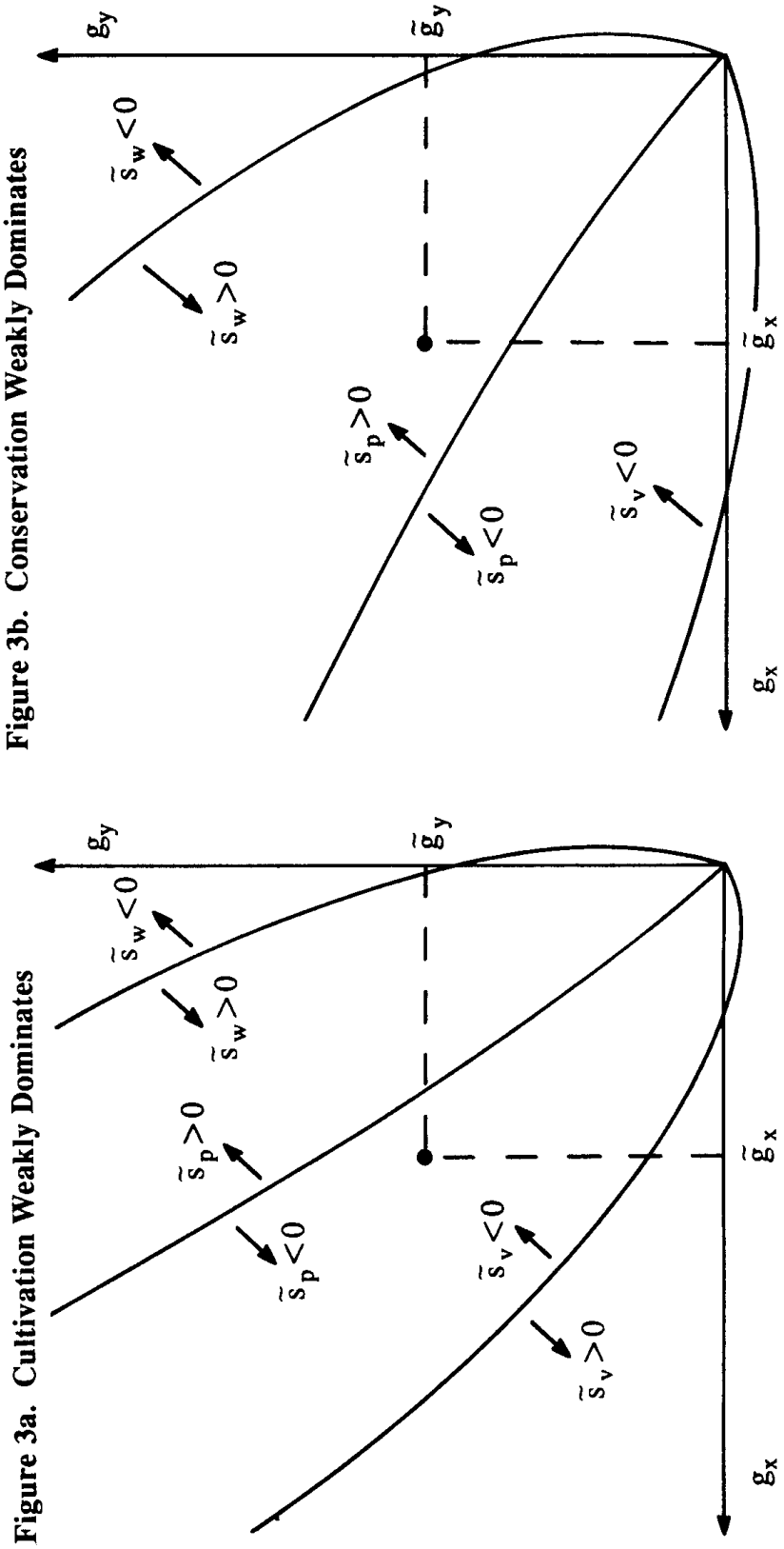


Figure 3b. Conservation Weakly Dominates

Figure 3a. Cultivation Weakly Dominates

The interpretation of these hypothetical choices is as follows. At the steady state, \$1 in additional cultivation expenditure results in an increase of \$2.25 in current crop revenues and a reduction in future crop revenues of \$0.05 per period for all future periods due to additional soil losses, which gives a present value of foregone future revenues in the amount of \$1.25 at a four percent rate of discount. Similarly, \$1 of marginal conservation expenditure results in a decrease of \$0.25 in current crop revenues and an increase in future crop revenues of \$0.05 per period for all future periods due to additional soil savings, which gives a present value of additional future revenues of \$1.25. These choices put conservation and cultivation on essentially equal footing at the margin in the long-run equilibrium.

The remaining factors that influence $\tilde{s}_p(\alpha)$, $\tilde{s}_w(\alpha)$, and $\tilde{s}_v(\alpha)$ are the second-order own- and cross-partial derivatives of the Hamiltonian, H_{xx}, H_{yy}, H_{ss}, H_{xy}, H_{xs}, and H_{ys}, evaluated at the steady state values. It is evident from the definition of weak and strong domination that each of these concepts is homogeneous of degree zero in the second-order cross partial derivatives. Thus, only their relative (rather than absolute) magnitudes matter. The example will be most enlightening if it does not include any *a priori* assumptions that favour one type of input over the other. In light of this consideration, the values $H_{xx} = H_{yy} = H_{ss} = -3$, $H_{xy} = -1$, and $H_{xs} = 1$ are chosen. These values imply that the slope of the hyperbola $Q(g_x, g_y) = 0$ at the origin is minus one, i.e., $\hat{g}'_y(0) = -1$, and the relationship between cultivation and conservation is symmetric at this point.

The hypothetical values for the unknown equilibrium parameters also must be consistent with strict concavity of the Hamiltonian. Given the other values of the equilibrium parameters, this requires $H_{ys} > -2\frac{1}{3}$. Therefore, two cases are considered: (a) In the first case set $H_{ys} = -2$ so that the negative effect of additional conservation expenditures on the marginal productivity of soil is twice as strong as the positive effect of additional soil on the marginal productivity of cultivation expenditures. It follows that $d^2g_y/dg_x^2 > 0$ along the $\tilde{s}_p(\alpha) = 0$ curve in this case, which is depicted in figure 3a. (b) In the second case, set $H_{ys} = -\frac{1}{2}$ so that the negative effect of additional conservation expenditures on the marginal productivity of soil is only half as strong as the positive effect of additional soil on the marginal productivity of cultivation expenditures. It follows that $d^2g_y/dg_x^2 < 0$ along the $\tilde{s}_p(\alpha) = 0$ curve in this case, which is depicted in figure 3b.

There is nothing particularly special about these specific hypothetical values for the equilibrium parameters *per se* except that they are carefully chosen so that, to the extent possible, cultivation and conservation are on equal footing in this illustration. However, it can be shown that the two cases considered in this numerical example reflect the generic properties and ambiguities of the long-run equilibrium for the soil degradation problem. That is, regardless of the true equi-

librium values for g_x, g_y, etc., the shapes, relative positions, and regions of the possible signs and values for $\tilde{s}_p(\alpha)$, $\tilde{s}_w(\alpha)$, and $\tilde{s}_v(\alpha)$ are as depicted in either figure 3a or figure 3b. Thus, in this example the essence of the problem and any ambiguities that result from it are captured completely.

Proposition 1, table 1, and figure 3 show that it is possible for higher crop prices to result in sufficient additional soil conservation activity relative to additional cultivation intensity to cause the long-run soil stock to increase. Although there is some evidence suggesting that this is unlikely and the opposite conclusion has been a long-standing proposition held by economists writing on the economics of soil conservation (Bunce 1942; Burt 1983; and CiriacyWantrup 1952), it is important to realize that this outcome is a logical possibility. But it is equally important, if not more so, to recognize that higher crop prices may lead to greater land degradation in both the short and long run.

In Figure 3 is shown an important point regarding effective economic methods to influence the rate of change or the long-run level of a natural resource such as soil. That is, regardless whether higher output prices lead to a larger or smaller equilibrium stock of the soil resource, subsidizing or taxing the price of the crop is less likely to be effective for achieving long-run resource improvements than either taxing the level of cultivation intensity or subsidizing the level of conservation activity. In general, whether higher crop prices increase or decrease the long-run soil stock, a much larger set of values for the relevant parameters implies an increase in the soil stock when cultivation is taxed or conservation is subsidized.

For example, under the conditions in figure 3a, $g_x = -0.05$and $\tilde{s}_w < 0$ require that the steady state level of g_y must be greater than 0.175; i.e., the marginal soil savings from conservation expenditures must be more than 3½ times the marginal soil losses from cultivation. Conversely, $g_y = 0.05$ and $\tilde{s}_w < 0$ require that the steady state level of g_x must be greater than –0.00045; a marginal dollar's worth of conservation must save more than 100 times the soil losses due to a marginal dollar's worth of cultivation. Conditions for $\tilde{s}_w < 0$ in figure 3b as well as for $\tilde{s}_v > 0$ in figures 3a and 3b also are strong. In figure 3a, $g_x = -0.05$ and $\tilde{s}_v > 0$ requires conservation expenditures to be productive for growing crops, while in figure 3b this requires conservation to degrade the soil.

In Table 1 is shown another important aspect of the choice of environmental policies to influence short-run rates of change in or long-run equilibrium levels of a natural resource. Any policy that directly affects the real discount rate or the marginal opportunity cost of the soil resource is without question an effective mechanism for influencing the resource stock in both the short and the long run. That this is true for the discount rate is shown in Table 1. An increase in the discount rate leads to accelerated soil losses in the short run and lower

levels of the equilibrium soil stock for all values of the parameters that influence whether or not input and/or output price changes have this effect.

This result applies with equal weight to a policy that targets the shadow price of the soil resource. Let $\sigma > 0$ denote a per unit tax (subsidy) on negative (positive) rates of change in the soil stock. Suppose a regulatory authority can monitor costlessly the rate of change in the soil per period and has the power to charge the farmer $\sigma g(x,y)$ if $g(x,y) < 0$ and to pay $\sigma g(x,y)$ if $g(x,y) > 0$. Profits from crop production now are

$$\pi = pq - wx - vy + \sigma g(x,y).$$

The farmer is assumed to maximize the discounted present value of profits subject to (1) and (2). The current value Hamiltonian for this modified problem is

$$H = pf(x,y,s) - wx - vy + (\lambda + \sigma)g(x,y). \tag{20}$$

The first-order conditions for an interior solution to this optimal control problem are

$$pf_x - w + (\lambda + \sigma)g_x = 0, \tag{21}$$

$$pf_y - v + (\lambda + \sigma)g_y = 0, \tag{22}$$

$$pf_s = r\lambda - \dot{\lambda}, \lim_{t \to \infty} e^{-rt}\lambda = 0, \tag{23}$$

$$\dot{s} = g(x,y), s(0) = s_0. \tag{24}$$

Partially differentiating (21) and (22) with respect to σ at $t = 0$ and solving for $x^*_\sigma(\alpha,\sigma,0)$ and $y^*_\sigma(\alpha,\sigma,0)$ gives

$$x^*_\sigma(\alpha,\sigma,0) = (1 + \lambda^*_\sigma(\alpha,\sigma,0))(g_y H_{xy} - g_x H_{yy})/\Delta\ , \tag{25}$$

$$y^*_\sigma(\alpha,\sigma,0) = (1 + \lambda^*_\sigma(\alpha,\sigma,0))(g_x H_{xy} - g_y H_{xx})/\Delta\ , \tag{26}$$

where $\Delta \equiv H_{xx}H_{yy} - H^2_{xy} > 0$, as before. The following set of short-run relationships follow immediately from these two expressions:

$$x^*_\sigma(\alpha,\sigma,0) \gtreqless 0 \quad \Leftrightarrow \quad y^*_\sigma(\alpha,\sigma,0) \lesseqgtr 0 \quad \Leftrightarrow \quad q^*_\sigma(\alpha,\sigma,0) \gtreqless 0 \tag{27}$$

$$\Leftrightarrow \quad \dot{\sigma}^*_\sigma(\alpha,\sigma,0) \lesseqgtr 0 \quad \Leftrightarrow \quad \lambda^*_\sigma(\alpha,\sigma,0) \lesseqgtr -1.$$

The long-run steady state for this problem is defined by (21) to (24) and $\dot{s} = \dot{\lambda} = 0$. By precisely the same methods employed to derive the long-run comparative statics with respect to α, we have

$$\tilde{s}_\sigma(\alpha,\sigma) = r(g^2_y H_{yy} + g^2_y H_{xx} - 2g_x g_y H_{xy})/\tilde{\Delta} > 0, \tag{28}$$

where $\tilde{\Lambda}$ is defined as before, except that the second-order derivatives are with respect to the modified Hamiltonian in (20). Also, it is straightforward to show that

$$\tilde{s}_\sigma(\alpha,\sigma) = -r\tilde{s}_r(\alpha,\sigma)/(\tilde{\lambda}(\alpha,\sigma) + \sigma). \tag{29}$$

Thus, an increase in the per unit tax on the rate of soil loss has the opposite qualitative effect in the long run as an increase in the real discount rate.

Finally, the saddle point nature of the optimal path and the relations in (27) imply

$$x^*_\sigma(\alpha,\sigma,0) < 0,\ y^*_\sigma(\alpha,\sigma,0) > 0,\ \dot{s}^*_\sigma(\alpha,\sigma,0) > 0,$$

$$q^*_\sigma(\alpha,\sigma,0) < 0 \text{ and } \lambda^*_\sigma(\alpha,\sigma,0) > -1.$$

Thus, a policy that either lowers the real discount rate or places a per unit tax (subsidy) on the rate of soil losses (gains) unequivocally reduces the rate of land degradation in the short run and increases the equilibrium stock of soil in the long run.

Conclusions

Not all increases in farm income increase the long-run stock of soil. Higher profits do not increase the incentives faced by farmers to take better care of the soil simply because soil is a more valuable asset. If the increased profits are due to subsidies on conservation activities, such as the conservation reserve program of the United States of America, then it is likely that the soil stock is exploited less intensively in the short run and the long-run equilibrium stock of soil increases. When soil degrading inputs are subsidized, such as the subsidized cost of irrigation water in the Murray-Darling Basin of south-eastern Australia, then it is likely that the soil is exploited more intensively in the short run and the long-run equilibrium soil stock declines. If commodity prices are subsidized, soil may be improved or further degraded, although the physical nature of soil degradation and past experiences in the United States of America suggest that the latter case is the more likely outcome. However, a per unit tax on the rate of soil loss, or a reduction in the real discount rate, is certain to lead to relative improvements in the soil in both the short and the long run.

It has been known for more than two decades that the only sure way to control pollution emissions is to measure and tax the emissions (Plott 1966). In this light, the results in this analysis should not be too surprising. Of course, the difficulty with this proposition is that pollution emissions (soil degradation) are difficult and costly to monitor and enforcement of this kind of policy often is untenable.

Nevertheless, there are some important implications from the results of this analysis. First, without a substantial body of empirical evidence on the issue, it is unclear whether commodity subsidies contribute to or mitigate land degradation in agriculture. However, to

claim that output price subsidies lead to greater long-run levels of soil requires that the argument also must be made that soil is more akin to produced capital than a natural resource. This is at odds with the available physical and empirical evidence.

Second, it appears likely that subsidies on the use of land degrading inputs contribute to soil degradation, while subsidies on conservation activities have the opposite effect. This seems to be nothing more than common sense. However, the opposite conclusion for one input (*but not both at the same time!*) is also possible. The precise conditions under which such a counter intuitive and perverse result can be expected have been shown in this paper.

Thus, if we are searching for a mechanism that increases farm incomes (due to a social criterion, say, that income transfers to agriculture are good) and creates an incentive for farmers to take better care of their land, then a conservation subsidy is more likely than an output price subsidy to produce the desired result. If the objective is merely to control soil degradation, then a per unit tax on cultivation is as likely to be successful as a per unit subsidy on conservation. Clearly, a tax on cultivation intensity would be preferred by taxpayers and individuals who suffer the off-farm effects of soil degradation.

Finally, even if monitoring and taxing soil losses directly is too costly to be viable, any policy that lowers the real discount rate reduces the incentive to trade-off lower future profits for greater profits in the present. In this paper such policies have been shown to be as effective for reducing the rate of soil degradation as measuring and taxing soil losses directly. Thus, it is possible that a solution to the soil degradation problem can be achieved more effectively with macroeconomic policies than with microeconomic market intervention schemes. Although this now appears somewhat obvious, it seems to have been overlooked in previous discussions of the issue of soil degradation.

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