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INCOMPLETE DEMAND SYSTEMS AND SEMILOGARITHMIC DEMAND MODELS

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The methodology of LaFrance and Hanemann for analysing the structure of incomplete demand systems is applied to models that are linear or logarithmic in quantities, prices and/or income. The structure of each model is presented when the implications of consumer choice theory are satisfied. The usefulness of the approach is illustrated. It is shown that considerable prior information is obtained from the theory of consumer choice when it is applied to this set of functional forms for demand equations.

In applied demand analysis, an incomplete information set is the rule and not the exception. We are always concerned with a subset of the total number of commodities that are purchased by consumers. Data limitations, finite computer memory, and the increased complexity and time required for numerical computations in large models make it necessary to abstract from a completely specified system of consumer demands with a different equation for each of the countless goods available in the market.

Basically only three practical solutions have been proposed to deal with this dimensionality problem. One approach is to aggregate across commodities and estimate a complete system of demand equations with the commodity aggregates (for example, food, clothing, housing, transportation, entertainment and all other goods) as functions of the corresponding set of aggregated price indices and total consumer expenditure (income, for short). This approach has at least two drawbacks. First, the conditions are quite restrictive for consumer preferences to be consistent with such a high degree of price and quantity aggregation. Second, considerable information is lost concerning the demands for individual commodities.

The second approach appeals to separability properties of consumer preferences. A common empirical practice is to assume that preferences are separable and estimate a complete system of conditional demands for the goods of interest as functions of that subset of prices and total expenditure on those goods. This approach is based on the fact that weak separability of a subset of goods from all other goods in the consumer's utility function is necessary and sufficient for the existence of conditional demand equations for the separable goods (Primont 1970; Gorman 1971; Blackorby, Primont and Russell 1978).

However, there is potential for simultaneous equations bias in conditional demand models. Due to the joint determination of the

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quantities demanded and group expenditure, bias will occur unless either the joint probability density function of the residuals or the class of conditional demand functions is restricted (Pollak 1971; Theil 1971, 1975, 1976; Deaton 1975; LaFrance 1989). Moreover, conditional demand functions only reveal the structure of the subutility function. This results in three problems: (1) the estimated effects of changes in prices and group expenditures on quantities omit the impacts of changes in group expenditure arising from the changes in prices and in income; (2) welfare estimates calculated from the conditional demand model are biased, even if the exact compensating or equivalent variation measures are employed; and (3) because group expenditure is treated as an exogenous variable, the model may not be consistent with the overall maximisation of utility (LaFrance 1989).

The third approach specifies an incomplete system of demand equations as functions of the prices of the goods of interest, the prices of related goods, and income. This approach has been challenged by Richardson (1976, p. 80) in his review of previous studies of the domestic demand for agricultural products in Australia:

> 'Only limited violence is done to the authors of such papers, if the broad approach they adopt is described as 'Ad Hoc'. Econometric studies of demand and prices for individual commodities do not exploit a rigorous basis for analysis in terms of consumer behaviour or factor demand theories. The approach is rather one of ad hoc selection of variables to enter estimating equations. Consistency with the arguments of underlying utility or production functions of demanders is largely ignored.'

There are good reasons to take Richardson's critique seriously in empirical work. The economic theory of consumer choice results in useful parameter restrictions that can be incorporated during model estimation, for example, symmetry of the Slutsky substitution terms and zero degree homogeneity of the demands in prices and income. Furthermore, recovering the underlying structure identifies the flexibility of preferences implied by the functional form of the model. Rejection of the empirical implications of the theory is more likely to be due to a restrictive maintained hypothesis embedded in the chosen functional form of the demand system than to irrational consumer behaviour. Thus, the *a priori* flexibility of preferences is a valid criterion at the model selection stage of the analysis. Finally, a common use of empirical estimates of demand relationships is for the measurement of the economic welfare effects of various policies and programmes. The implications of the theory are both necessary and sufficient for the existence of the exact money metrics compensating and equivalent variation. Furthermore, even the approximating arguments for the use of consumer's surplus (Currie, Murphy and Schmitz 1971; Harberger 1971; Willig 1976) are based on the assumption that an underlying preference function exists, and therefore that the implications of the theory are satisfied by the demand equations. Therefore, in almost every aspect of applied economic analysis, the concerns raised by Richardson are relevant.

This *ad hoc* empirical approach can be reconciled with the theory through an application of the recent results on the dual structure of incomplete demand models obtained by LaFrance and Hanemann (1989). These results rely on the fact that incomplete demand models

that are the result of utility maximisation subject to a linear budget constraint have four properties: (a) the demands are positive valued; (b) the demands are homogeneous of degree zero in all prices and income; (c) the matrix of substitution effects for each subset of goods is symmetric, negative semidefinite; and (d) income is greater than the total expenditure on any proper subset of the goods consumed. If the demand model satisfies these four conditions, then each of the following is also true: (1) the conditional preference structure for the goods under study can be recovered from the demand equations; (2) the dual structures for the recoverable parts of the utility, expenditure and indirect utility functions are analogous to the dual structures for complete demand systems and the structure is embodied in the properties of the demand equations; and (3) exact welfare measures can be derived from the incomplete demand system. Consequently, a coherently specified incomplete demand model contains all of the necessary information to complete any of the usual tasks of applied economic analysis (LaFrance and Hanemann 1989).

A common empirical practice is to estimate demand equations that are linear or logarithmic in the variables. A small sample from an extensive literature includes Alston and Chalfant (1987), Bewley (1987), Burt and Brewer (1971), Chavas (1983), Cheng and Capps (1988), Chicchetti, Fisher and Smith (1976), Fisher (1979), Freebairn and Rausser (1975), George and King (1971), Gruen and McLaren (1967), Havrila (1989), Heien (1977), Huang and Haidacher (1983), Kaiser, Streeter and Liu (1988), Labys (1976), LaFrance and DeGorter (1985), Leser (1960), Main, Reynolds and White (1976), Marceau (1967), Martin and Porter (1985), Safyurtlu, Johnson and Hassan (1986), Taylor (1961, 1963) and Tsolakis *et al.* (1983). It is the author's opinion that analysts need to understand the implications of the functional forms they choose for empirical demand analysis. In an effort to provide a mechanism to further this understanding, this paper uses the methodology developed by LaFrance and Hanemann (1989) to analyse the structure of semilogarithmic demand models. The models considered are linear or logarithmic in quantities, prices and income, but are neither linear nor logarithmic in all three sets of variables. Each of these demand models also can be represented as linear functions of the parameters when the theoretical parameter restrictions are not imposed, or are imposed at a single point such as the sample means of the data. Models linear in quantities, prices and income and models logarithmic in all three sets of variables are discussed in LaFrance (1985) and LaFrance (1986), respectively. Thus, the results of this paper characterise all of the remaining structures for incomplete demand models that are either linear or logarithmic in the variables of interest.

A complete set of results on the parameter restrictions and the dual conditional preference structures for each of these models is presented. This information will be useful to applied researchers interested in semilogarithmic demand models that can be rationalised by the theory of consumer behaviour. The results demonstrate the usefulness of these techniques when they are employed in analyses of the implications of economic theory for incomplete systems of demand equations. They also illustrate the informative nature of the theory when it is applied to

these functional forms for demand models. In the paper, the main results of LaFrance and Hanemann (1989) on the dual structure of incomplete demand models are summarised. These concepts are then applied to the problem of recovering a full complement of information regarding the dual structure of semilogarithmic demand models. The last section summarises and concludes.

The Structure of Incomplete Demand Systems

The structural implications of utility maximisation on an incomplete system of demand equations, a methodology for identification of this structure and recovery of the conditional preference functions, and proofs of the results summarised below are contained in LaFrance and Hanemann (1989). A brief overview of the main ideas is presented here.

Let $\mathbf{x}=[x_1, \ldots, x_n]'$ be the vector of consumption levels for the commodities of interest and $\mathbf{p}=[p_1, \ldots, p_n]'$ be the corresponding price vector; let $\mathbf{z}=[z_1, \ldots, z_m]'$ be the vector of consumption levels of all other commodities, with $m \geq 2$, and $\mathbf{q}=[q_1, \ldots, q_m]'$ be the corresponding price vector; and let income be y. Suppose that we observe or estimate the n demands for $\mathbf{x}$,

$$(1) \qquad \mathbf{x}=\mathbf{h}^x(\mathbf{p}, \mathbf{q}, y)$$

but we neither observe nor estimate the demands for $\mathbf{z}$. The question that we are immediately faced with is, How much information about the consumer's preferences can we glean from the demand functions in equation (1)? The answer depends on whether the implications of utility maximisation are satisfied for a range of values of the price and income variables, at a point, or not at all. If economic theory is only a guide to the choice of the right-hand-side variables in the demand functions, then the demand equations relay little information about the structure of consumer preferences. This interpretation is justified by differences between individual and market behaviour and other aggregation issues. At the opposite extreme, the implications of the theory apply to a range of values for the price and income variables. An intermediate view is that the restrictions of the theory apply at a given base point. A Taylor's series expansion then permits the calculation of the slopes and curvature of preferences at the point of approximation.

Throughout this paper it is assumed that the theoretical restrictions on the subset of demands for $\mathbf{x}$ apply to an open neighbourhood of each observed data point. This is the approach taken by LaFrance and Hanemann (1989). This approach permits the recovery of the implied preference structure and the calculation of the exact welfare effects of changes in the prices of the goods under study. The other interpretations of empirical demand models do not share these desirable properties.

As noted above, there are four properties of demand functions which result from constrained utility maximisation:

(a) the demands are positive valued, $\mathbf{h}^x(\mathbf{p}, \mathbf{q}, y) \geq \mathbf{0}$;

(b) the demands are zero degree homogeneous in all prices and income, $\mathbf{h}^x(\mathbf{p}, \mathbf{q}, y) \equiv \mathbf{h}^x(t\mathbf{p}, t\mathbf{q}, ty)$ for all $t \geq 0$;

(c) the $n \times n$ matrix of compensated substitution effects for $\mathbf{x}$, $\mathbf{S}_x \equiv \partial \mathbf{h}^x/\partial \mathbf{p}' + \partial \mathbf{h}^x/\partial y \mathbf{h}^{x\prime}$, is symmetric, negative semidefinite;

(d) income is greater than total expenditures on a proper subset of the goods consumed, $\mathbf{p}'\mathbf{h}^x(\mathbf{p}, \mathbf{q}, y) < y$.

The first three properties are the same if the system of demand equations is complete or incomplete. The last property is the essence of an incomplete demand model; only part of the consumer's budget is allocated to the consumption of $\mathbf{x}$. Thus, the main source of information loss in an incomplete demand model is due to the fact that the adding up condition does not apply to a subset of the goods consumed.

Although the demands for $\mathbf{z}$ are not observed, the budget identity continues to apply to the full vector of goods consumed,

$$(2) \qquad \mathbf{p}'\mathbf{h}^x(\mathbf{p}, \mathbf{q}, y) + \mathbf{q}'\mathbf{h}^z(\mathbf{p}, \mathbf{q}, y) \equiv y$$

where $\mathbf{h}^z(\mathbf{p}, \mathbf{q}, y)$ is the vector of demands for the unobserved goods. Since $\mathbf{x}$ is observed, the budget identity (2) implies that the total expenditure on $\mathbf{z}$,

$$(3) \qquad s \equiv \mathbf{q}'\mathbf{z} \equiv y - \mathbf{p}'\mathbf{x}$$

is also observed. As a result, the set of $n+1$ choices for $(\mathbf{x}, s)$ is a complete system, where the composite commodity s is a *numeraire* good, the prices $\mathbf{q}$ are shift parameters, and the 'demand' for s is completely determined by the n demands for $\mathbf{x}$ and the budget identity (3).

This relationship between an incomplete demand system with n goods and a complete system with $n+1$ goods and a *numeraire* composite commodity for the last good greatly simplifies the analysis of incomplete demand systems. If the n demands for $\mathbf{x}$ satisfy properties (a) to (d) then there is a function $\omega(\mathbf{x}, s, \mathbf{q})$, called the *quasi-utility function*, defined over the goods $(\mathbf{x}, s)$ and the prices $\mathbf{q}$, with all of the properties of a utility function for $(\mathbf{x}, s)$. That is, for fixed $\mathbf{q}$, ω is increasing and quasiconcave in $(\mathbf{x}, s)$. Also, if the utility function for all goods is $u(\mathbf{x}, \mathbf{z})$, then the conditional preference structure of u with respect to $\mathbf{x}$ for given $\mathbf{z}$ is completely determined by the structure of ω with respect to $\mathbf{x}$ for given $(s, \mathbf{q})$. The demands $\mathbf{x} = \mathbf{h}^x(\mathbf{p}, \mathbf{q}, y)$ and $s = \sigma(\mathbf{p}, \mathbf{q}, y) \equiv y - \mathbf{p}'\mathbf{h}^x(\mathbf{p}, \mathbf{q}, y)$ solve the utility maximisation problem

$$(4) \qquad \phi(\mathbf{p}, \mathbf{q}, y) \equiv \max_{\mathbf{x}, s} \{\omega(\mathbf{x}, s, \mathbf{q}) \colon \mathbf{p}'\mathbf{x} + s \leq y,\ \mathbf{x} \geq 0,\ s \geq 0\}$$

The function $\phi(\mathbf{p}, \mathbf{q}, y)$ is called the *quasi-indirect utility function*. It is related to the true indirect utility function,

$$(5) \qquad v(\mathbf{p}, \mathbf{q}, y) \equiv \max_{\mathbf{x}, \mathbf{z}} \{u(\mathbf{x}, \mathbf{z}) \colon \mathbf{p}'\mathbf{x} + \mathbf{q}'\mathbf{z} \leq y,\ \mathbf{x} \geq \mathbf{0},\ \mathbf{z} \geq \mathbf{0}\}$$

by the identity

$$(6) \qquad v(\mathbf{p}, \mathbf{q}, y) \equiv \psi[\mathbf{q}, \phi(\mathbf{p}, \mathbf{q}, y)]$$

The function $\psi[\mathbf{q}, \omega(\mathbf{x}, s, \mathbf{q})]$ is called the *variable indirect utility function* (Diewert 1978; Epstein 1975). It is dual to the utility function with respect to $(\mathbf{q}, s)$ and $\mathbf{z}$ (Epstein 1975; LaFrance and Hanemann 1989). That is, the variable indirect utility function is obtained by maximising utility with respect to $\mathbf{z}$, with $\mathbf{x}$ fixed and total expenditure on $\mathbf{z}$ equal to s,

$$\psi[\mathbf{q}, \omega(\mathbf{x}, s, \mathbf{q})] \equiv \max_{\mathbf{z}}\{u(\mathbf{x}, \mathbf{z}): \mathbf{q}'\mathbf{z} \leq s, \mathbf{z} \geq \mathbf{0}\} \tag{7}$$

As with complete demand systems, the quasi-utility function is dual to the quasi-indirect utility function,

$$\omega(\mathbf{x}, s, \mathbf{q}) \equiv \min_{\mathbf{p}, y}\{\phi(\mathbf{p}, \mathbf{q}, y): \mathbf{p}'\mathbf{x} + s \leq y, \mathbf{p} \geq \mathbf{0}\} \tag{8}$$

Thus, all of the information that is lost in a coherently specified incomplete demand model is associated with the structure of ψ with respect to $\mathbf{q}$.

The strength of these results lies in the fact that they are completely general; nothing has been assumed *a priori* about the structure of the utility function, the indirect utility function or the expenditure function beyond their usual properties. The approach is very straightforward and can be applied to any incomplete demand model. The incomplete system is artificially augmented by a *numeraire* composite commodity equal to the total expenditure on all other goods and the structure of this augmented system is analysed. This procedure exhausts the information contained in the incomplete system arising from constrained utility maximisation, reveals the conditional preference map for the goods of interest and permits the calculation of exact welfare measures due to changes in the prices of the goods of interest. The structure of preferences with respect to the individual elements of the composite commodity cannot be recovered, but this does not affect any of the results.

Semilogarithmic Demand Models

The above methodology is applied to these semilogarithmic demand models:

$$\text{(M1)} \quad x_i = \alpha_i(\mathbf{q}) + \sum_{j=1}^{n} \beta_{ij}\log(p_j) + \gamma_i y$$

$$\text{(M2)} \quad x_i = \alpha_i(\mathbf{q}) + \sum_{j=1}^{n} \beta_{ij}\log(p_j) + \gamma_i \log(y)$$

$$\text{(M3)} \quad x_i = \alpha_i(\mathbf{q}) + \sum_{j=1}^{n} \beta_{ij} p_j + \gamma_i \log(y)$$

$$\text{(M4)} \quad x_i = \alpha_i(\mathbf{q})\exp\left\{\sum_{j=1}^{n} \beta_{ij} p_j + \gamma_i y\right\}$$

$$\text{(M5)} \quad x_i = \alpha_i(\mathbf{q})\exp\left\{\sum_{j=1}^{n} \beta_{ij} p_j\right\} y^{\gamma_i}$$

$$\text{(M6)} \quad x_i = \alpha_i(\mathbf{q})\left\{\prod_{j=1}^{n} p_j^{\beta_{ij}}\right\} e^{\gamma_i y}$$

It is assumed that the functional form of the demands for $\mathbf{x}$ are the same for all $i = 1, \ldots, n$ in each of these models. To conserve space, the detailed derivations of the results are omitted, but they are contained in a separate Appendix that is available from the author upon request.

All prices and income are assumed to have been deflated by a linear homogeneous function of the prices for **z**. Thus, $(\mathbf{p}, \mathbf{q}, y)$ should be interpreted as 'real' prices and income. All additional influences of the prices for **z** on the demands for **x** are assumed to be captured by the functions, $\alpha_i(\mathbf{q})$, which serve as demand shifters. This is mainly a convenience that does not affect the qualitative results. The price deflator can be any positive valued, increasing, and linear homogeneous function of a non-empty subset of the prices for **z**, for example, the price of gold or the consumer price index for non-food items. The most important property of the price deflator is that it does not depend on the prices for **x**. This hypothesis simplifies the analysis considerably.

The intuition for this assumption is straightforward. In many empirical studies an aggregate price index is included among the right-hand-side variables to measure the 'costs' of consuming other goods. The prices included separately are assumed to have only infinitesimal effects on the aggregate price index, and demand slopes and elasticities are calculated as if these effects are in fact zero. Including an index of the prices of other goods as an explanatory variable in an incomplete system of demand equations is justified if preferences are homothetically separable (Blackorby, Primont and Russell 1978), although this is not a necessary condition (LaFrance and Hanemann 1989). When the prices of the goods of interest have no measurable effect on the price index the aggregate price index is formally equivalent to an index that is not a function of the prices for **x**. Our assumption simply makes this aspect of the demand model precise.

When prices and income are deflated by a price index, the demands are always homogeneous of degree zero in prices and income. Therefore, the symmetry conditions are the only equality restrictions arising from the theory. All other restrictions (the demands are positive valued, the $n \times n$ Slutsky matrix is negative semidefinite and income exceeds total expenditure on **x**) are inequalities. Inequality restrictions provide information about the signs and magnitudes of the demand parameters and the region of regular behaviour for the demand equations, but do not reduce the number of parameters *per se*. Therefore, the main source of information concerning the conditional preference structure for incomplete demand models that are specified with deflated prices and income is symmetry of the Slutsky substitution terms. For the semilogarithmic demand models, for all $i, j = 1, \ldots, n$, the symmetry conditions are

(S1) $\beta_{ij}/p_j + \gamma_i x_j = \beta_{ji}/p_i + \gamma_j x_i$

(S2) $\beta_{ij}/p_j + \gamma_i x_j/y = \beta_{ji}/p_i + \gamma_j x_i/y$

(S3) $\beta_{ij} + \gamma_i x_j/y = \beta_{ji} + \gamma_j x_i/y$

(S4) $\beta_{ij} x_i + \gamma_i x_i x_j = \beta_{ji} x_j + \gamma_j x_i x_j$

(S5) $\beta_{ij} x_i + \gamma_i x_i x_j/y = \beta_{ji} x_j + \gamma_j x_i x_j/y$

(S6) $\beta_{ij} x_i/p_j + \gamma_i x_i x_j = \beta_{ji} x_j/p_j + \gamma_j x_i x_j$

Symmetry as the main determinant of the structure of incomplete demand models contrasts with complete demand systems, where the

budget constraint often generates many of the restrictions on the parameters in the demand equations (Lau 1976). Indeed, if they were complete systems, models M2, M3, M4 and M6 would satisfy the adding up condition at most at a single point, while models M1 and M5 would satisfy the budget constraint at more than one point if and only if consumer preferences are generated by Leontief (fixed proportions) indifference curves. As will be seen in the results below, each semilogarithmic demand model can be rationalised by a conditional preference map that is not necessarily Leontief. Thus, for these functional forms, incomplete demand systems are more general than complete demand systems.

For models M4, M5 and M6, there are several structures possible for the matrix of price coefficients $\mathbf{B} \equiv [\beta_{ij}]$. It is nearly impossible to present all of the possible combinations, but it is relatively simple to characterise the nature of each particular structure for each model with some simplifying assumptions and a little notation. Towards this end let J, K, L and N denote index sets satisfying $\emptyset \subset J \subset K \subset L \subset N \equiv \{1, 2, \ldots, n\}$, let $\sim$ denote set differences, that is, $N \sim J \equiv \{i \in N\colon i \notin J\}$, and assume that if $J \neq \emptyset$ then $1 \in J$, while if $N \sim L \neq \emptyset$ then $n \in N \sim L$ to fix the location of one element in each of the sets J and $N \sim L$. Nesting the index sets J, K, L and N is a convenient way to reduce the notational burden, while avoiding the need to reorder the demand equations. With these conventions, the parameter restrictions that characterise the basic structures of the semilogarithmic demand models are in Table 1, while the quasi-indirect

TABLE 1

*Symmetry Restrictions for Semilogarithmic Demand Models**

Model		
M1 & M2	A.	$\beta_{ij} = 0 \,\forall\, i, j \in N, i \neq j \qquad \gamma_i = 0 \,\forall\, i \in N$
	B.	$\alpha_i(\mathbf{q}) \equiv \left(\frac{\gamma_i}{\gamma_1}\right) \alpha_1(\mathbf{q}) \,\forall\, i \in N \qquad \beta_{ij} = 0 \,\forall\, i, j \subset N$
M3	A.	$\beta_{ij} = \beta_{ji} \,\forall\, i, j \in N \qquad \gamma_i = 0 \,\forall\, i \in N$
	B.	$\alpha_i(\mathbf{q}) \equiv \left(\frac{\gamma_i}{\gamma_1}\right) \alpha_1(\mathbf{q}) \,\forall\, i \in N \qquad \beta_{ij} = \left(\frac{\gamma_i \gamma_j}{\gamma_1^2}\right) \beta_{11} \,\forall\, i, j \in N$
M4 & M5		$\alpha_i(\mathbf{q}) \equiv \left(\frac{\beta_{ii}}{\beta_{11}}\right) \alpha_1(\mathbf{q}) \,\forall\, i \in N \qquad \beta_{ik} = \beta_{jk} \,\forall\, i, j \in J, k \in N, i \neq j$
		$\beta_{ij} = \beta_{ji} = 0 \,\forall\, i \in N, j \in N \sim J, i \neq j \qquad \gamma_i = \gamma_1 \,\forall\, i \in N$
M6		$\alpha_i(\mathbf{q}) \equiv \left(\frac{1+\beta_{ii}}{1+\beta_{11}}\right) \alpha_1(\mathbf{q}) \,\forall\, i \subset J \qquad \beta_{ij} = 1 + \beta_{ji} \,\forall\, i, j \subset J, i \neq j$
		$\beta_{ij} = \beta_{ji} = 0 \,\forall\, i \in K, j \in K \sim J, i \neq j$
		$\beta_{ij} = \beta_{1j} \,\forall\, i \in N, j \in N \sim K, i \neq j \qquad \beta_{ii} = -1 \,\forall\, i \in N \sim K$
		$\beta_{ij} = 0 \,\forall\, i \in N \sim L, j \in L \qquad \gamma_i = \gamma_1 \,\forall\, i \in L \qquad \gamma_i = 0 \,\forall\, i \in N \sim L$

*The index sets J, K, L and N satisfy $\emptyset \subset J \subset K \subset L \subset N \equiv \{1, 2, \ldots, n\}$, if $J \neq \emptyset$ then $1 \subset J$, and if $N \sim L \neq \emptyset$ then $n \in N \sim L$, where $\sim$ denotes set differences. Each model is assumed to have at most one set of price and income coefficients in each of the sets J, $K \sim J$, $L \sim K$ and $N \sim L$. However, each of these sets may be empty. For models M1, M2 and M3 there are two separate, non-nested ways that the symmetry conditions can be satisfied. They are denoted as cases A and B, respectively.

utility and the quasi-utility functions that can be expressed in closed form are in Tables 2 and 3, respectively.

From Tables 1 to 3, it is clear that all of the semilogarithmic demand models are quite restrictive in one way or another. With one exception, each of these demand models has at most one independent price coefficient for each good. Also, the conditional preference maps for subgroups of goods are often homothetic, and homotheticity results in only one independent intercept term for each subgroup of homothetic

TABLE 2

*Quasi-Indirect Utility Functions for Semilogarithmic Demand Models**

Model	$\gamma_i = 0 \; \forall i \subset N$
M1 & M2	$\phi(\mathbf{p}, \mathbf{q}, y) = y - \boldsymbol{\alpha}(\mathbf{q})'\mathbf{p} - \sum_{i \in N} \beta_{ii} p_i [\log(p_i) - 1]$
M3	$\phi(\mathbf{p}, \mathbf{q}, y) = y - \boldsymbol{\alpha}(\mathbf{q})'\mathbf{p} - \frac{1}{2}\mathbf{p}'\mathbf{B}\mathbf{p}$
M4 & M5	$\phi(\mathbf{p}, \mathbf{q}, y) = y - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \exp\left\{\sum_{i \in J} \beta_{ii} p_i\right\} - \sum_{i \in N \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) \exp\{\beta_{ii} p_i\}$
M6	$\phi(\mathbf{p}, \mathbf{q}, y) = y - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \prod_{i \in J} p_i^{1-\beta_{ii}} - \sum_{i \in K \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) p_i^{1-\beta_{ii}} - \sum_{i \in N \sim K} \alpha_i(\mathbf{q}) \log(p_i)$
	$\gamma_1 \neq 0$
M1	$\phi(\mathbf{p}, \mathbf{q}, y) = \left[y + \left(\frac{\alpha_1(\mathbf{q})}{\gamma_1}\right)\right] \exp\left\{\sum_{i \in N} \gamma_i p_i\right\}$
M4	$\gamma_1 > 0$
	$\phi(\mathbf{p}, \mathbf{q}, y) = \left(\frac{-1}{\gamma_1}\right) e^{-\gamma_1 y} - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \exp\left\{\sum_{i \in J} \beta_{ii} p_i\right\} - \sum_{i \in N \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) \exp\left\{\beta_{ii} p_i\right\}$
	$\gamma_1 < 0$
	$\phi(\mathbf{p}, \mathbf{q}, y) = \left(\frac{-1}{\gamma_1}\right) e^{-\gamma_1 y} - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \exp\left\{\sum_{i \in J} \beta_{ii} p_i\right\} - \sum_{i \in N \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) \exp\left\{\beta_{ii} p_i\right\} - \sum_{i \in N \sim K} \alpha_i(\mathbf{q}) p_i$
M5	$\gamma_1 = 1$
	$\phi(\mathbf{p}, \mathbf{q}, y) = \log(y) - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \exp\left\{\sum_{i \in J} \beta_{ii} p_i\right\} - \sum_{i \in N \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) \exp\left\{\beta_{ii} p_i\right\}$
	$0 < \gamma_1 \neq 1$
	$\phi(\mathbf{p}, \mathbf{q}, y) = \left(\frac{y^{1-\gamma_1}}{\gamma_1}\right) - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \exp\left\{\sum_{i \in J} \beta_{ii} p_i\right\} - \sum_{i \in N \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) \exp\left\{\beta_{ii} p_i\right\}$
	$\gamma_1 < 0$
	$\phi(\mathbf{p}, \mathbf{q}, y) = \left(\frac{y^{1-\gamma_1}}{1-\gamma_1}\right) - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \exp\left\{\sum_{i \in J} \beta_{ii} p_i\right\} - \sum_{i \in K \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) \exp\left\{\beta_{ii} p_i\right\} - \sum_{i \in N \sim K} \alpha_i(\mathbf{q}) p_i$
M6	$\phi(\mathbf{p}, \mathbf{q}, y) = \left(\frac{-1}{\gamma_1}\right) e^{-\gamma_1 y} \prod_{i \in N \sim L} p_i^{-\beta_{ii}} - \left(\frac{\alpha_1(\mathbf{q})}{1+\beta_{11}}\right) \prod_{i \in J} p_i^{1-\beta_{ii}} - \sum_{i \in K \sim J} \left(\frac{\alpha_i(\mathbf{q})}{1+\beta_{ii}}\right) p_i^{1-\beta_{ii}} - \sum_{i \in L \sim K} \alpha_i(\mathbf{q}) \log(p_i)$

*No closed form expressions exist for any quasi functions in models M2 and M3 if the income coefficients are not all zero.

TABLE 3

*Quasi-Utility Functions for Semilogarithmic Demand Models**

Model	$\gamma_i = 0 \ \forall i \subset N$
M1 & M2	$\omega(\mathbf{x}, s, \mathbf{q}) = \sum_{i \in N} \beta_{ii} \exp\left\{\frac{x_i - \alpha_i(\mathbf{q})}{\beta_{ii}}\right\} + s$
M3	$\omega(\mathbf{x}, s, \mathbf{q}) = \frac{1}{2}[\mathbf{x} - \alpha(\mathbf{q})]'\mathbf{B}^{-1}[\mathbf{x} - \boldsymbol{\alpha}(\mathbf{q})] + s$
M4 & M5	$\omega(\mathbf{x}, s, \mathbf{q}) = \left(\frac{R(\mathbf{x})}{\beta_{11}}\right)\left[\log\left(\frac{R(\mathbf{x})}{\alpha_1(\mathbf{q})}\right) - 1\right] - \sum_{i \in N \sim J}\left(\frac{x_i}{\beta_{ii}}\right)\left[\log\left(\frac{x_i}{\alpha_i(\mathbf{q})}\right) - 1\right] + s$
M6	$\delta = 1$
	$\omega(\mathbf{x}, s, \mathbf{q}) = \begin{cases} -\infty & \text{if } H(\mathbf{x}, \mathbf{q}) < 1 \\ \sum_{i \in K \sim J}\left(\frac{\beta_{ii}}{1+\beta_{ii}}\right)\left(\frac{x_i^{1+\beta_{ii}}}{\alpha_i(\mathbf{q})}\right)^{1/\beta_{ii}} + \sum_{i \in N \sim K} \alpha_i(\mathbf{q}) \log(x_i) + s & \text{if } H(\mathbf{x}, \mathbf{q}) \geq 1 \end{cases}$
	$\delta \neq 1$
	$\omega(\mathbf{x}, s, \mathbf{q}) = (\delta - 1) H(\mathbf{x}, \mathbf{q})^{1/\delta - 1} + \sum_{i \in K \sim J}\left(\frac{\beta_{ii}}{1+\beta_{ii}}\right)\left(\frac{x_i^{1+\beta_{ii}}}{\alpha_i(\mathbf{q})}\right)^{1+\beta_{ii}} + \sum_{i \in N \sim K} \alpha_i(\mathbf{q}) \log(x_i) + s$
	$\gamma_1 \neq 0$
M1	$\omega(\mathbf{x}, s, \mathbf{q}) = \left(\frac{G(\mathbf{x})}{\gamma_1}\right) \exp\left\{\frac{\alpha_1(\mathbf{q}) + \gamma_1 s - G(\mathbf{x})}{G(\mathbf{x})}\right\}$
M4	$\gamma_1 > 0$
	$\omega(\mathbf{x}, s, \mathbf{q}) = -S(\mathbf{x})\left[e^s\left(\frac{R(\mathbf{x})}{\alpha_1(\mathbf{q})}\right)^{R(\mathbf{x})/\beta_{11}} \prod_{i \in N \sim J}\left(\frac{x_i}{\alpha_i(\mathbf{q})}\right)^{x_i/\beta_{ii}}\right]^{1/S(\mathbf{x})}$
	$\gamma_1 < 0$
	$\omega(\mathbf{x}, s, \mathbf{q}) = \left(\frac{\alpha_n(\mathbf{q})}{T(\mathbf{x}, \mathbf{q})}\right)\left\{\left(\frac{1}{\gamma_1}\right)\left[\log\left(\frac{\alpha_n(\mathbf{q})}{T(\mathbf{x}, \mathbf{q})}\right) - 1\right] + \left(\frac{R(\mathbf{x})}{\beta_{11}}\right)\left[\log\left(\frac{\alpha_n(\mathbf{q}) R(\mathbf{x})}{\alpha_1(\mathbf{q}) T(\mathbf{x}, \mathbf{q})}\right) - 1\right] - \sum_{i \in K \sim J}\left(\frac{x_i}{\beta_{ii}}\right)\left[\log\left(\frac{\alpha_n(\mathbf{q}) x_i}{\alpha_i(\mathbf{q}) T(\mathbf{x}, \mathbf{q})}\right) - 1\right] + s\right\}$
M5	$\gamma_1 < 0$
	$\omega(\mathbf{x}, s, \mathbf{q}) = \left(\frac{\gamma_1}{1-\gamma_1}\right) T(\mathbf{x}, \mathbf{q})^{1-\gamma_1/\gamma_1} + \left(\frac{\alpha_n(\mathbf{q})}{T(\mathbf{x}, \mathbf{q})}\right)\left\{\left(\frac{R(\mathbf{x})}{\beta_{11}}\right)\left[\log\left(\frac{\alpha_n(\mathbf{q}) R(\mathbf{x})}{\alpha_1(\mathbf{q}) T(\mathbf{x}, \mathbf{q})}\right) - 1\right] - \sum_{i \in K \sim J}\left(\frac{x_i}{\beta_{ii}}\right)\left[\log\left(\frac{\alpha_n(\mathbf{q}) x_i}{\alpha_i(\mathbf{q}) T(\mathbf{x}, \mathbf{q})}\right) - 1\right] + s\right\}$
M6	$\delta = 1$
	$\omega(\mathbf{x}, s, \mathbf{q}) = H(\mathbf{x}, \mathbf{q})^{-1}\left\{\left(\frac{-1}{\gamma_1}\right)\left[\log\left(\frac{H(\mathbf{x}, \mathbf{q})}{W(x)}\right) + \sum_{i \in N \sim L} \beta_{1i} + 1\right] + \sum_{i \in K \sim J}\left(\frac{\beta_{ii}}{1+\beta_{ii}}\right)\left(\frac{x_i^{1+\beta_{ii}}}{\alpha_i(\mathbf{q}) H(\mathbf{x}, \mathbf{q})}\right)^{1/\beta_{ii}} + s\right\} + \sum_{i \in L \sim K} \alpha_i(\mathbf{q})\left[\log\left(\frac{x_i}{\alpha_i(\mathbf{q}) H(\mathbf{x}, \mathbf{q})}\right) + 1\right]$

*The parameter δ is defined by $\delta \equiv \sum_{i \in J}(1 + \beta_{ii})$. The functions $H(\mathbf{x}, \mathbf{q})$, $R(\mathbf{x})$, $W(\mathbf{x})$, $G(\mathbf{x})$, $S(\mathbf{x})$ and $T(\mathbf{x}, \mathbf{q})$ are defined by:

$$H(\mathbf{x}, \mathbf{q}) \equiv \left(\frac{1+\beta_{11}}{\alpha_1(\mathbf{q})}\right) \prod_{i \in J}\left(\frac{x_i}{1+\beta_{ii}}\right)^{1+\beta_{ii}} \qquad R(\mathbf{x}) \equiv \min_{i \in J}\left\{\left(\frac{\beta_{11}}{\beta_{ii}}\right) x_i\right\} \qquad W(\mathbf{x}) \equiv \prod_{i \in N \sim L}\left(\frac{\gamma_1 x_i}{-\beta_{1i}}\right)^{\beta_{ii}}$$

$$G(\mathbf{x}) \equiv \min_{i \in N}\left\{\left(\frac{\gamma_1}{\gamma_i}\right) x_i\right\} \qquad S(\mathbf{x}) \equiv \left(\frac{1}{\gamma_1}\right) + \left(\frac{R(\mathbf{x})}{\beta_{11}}\right) + \sum_{i \in N \sim J}\left(\frac{x_i}{\beta_{ii}}\right) \qquad T(\mathbf{x}, \mathbf{q}) \equiv \min_{i \in N \sim K}\left\{\left(\frac{\alpha_n(\mathbf{q})}{\alpha_i(\mathbf{q})}\right) x_i\right\}$$

demands. The exception to the limited number of price coefficients is the model that is linear in both quantities and prices and has income coefficients that are all equal to zero. This model has a quadratic conditional preference map, which is relatively general. Because the income effects are zero, however, the conditional preferences for **x** are again homothetic and this model is restrictive in this way.

While the results for the semilogarithmic demand models are largely negative and somewhat discouraging, it would be a mistake to conclude that applications of these methods always result in restrictive preference structures. For example, suppose that the demands for **x** are linear and quadratic in **p** and linear in y with no interaction terms between **p** and y,

$$(9) \qquad x_i = \alpha_i(\mathbf{q}) + \sum_{j=1}^{n} \beta_{ij} p_j + \tfrac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \theta_{ijk} p_j p_k + \gamma_i y$$

for all $i = 1, \ldots, n$. Without loss in generality, assume that $\theta_{ijk} = \theta_{ikj}$ for all $i, j, k = 1, \ldots, n$. Also, as for the semilogarithmic models, assume that all prices and income are deflated by a linear homogeneous function of the prices of the other goods. Then the quasi-indirect utility function that generates the demands for **x** in equation (9) is

$$(10) \qquad \phi(\mathbf{p}, \mathbf{q}, y) = y - \boldsymbol{\alpha}'\mathbf{p} - \tfrac{1}{2}\mathbf{p}'\mathbf{B}\mathbf{p} - \delta(\mathbf{q})\exp\{-\boldsymbol{\gamma}'\mathbf{p}\}$$

The demands for **x** can therefore be written as

$$(11) \qquad \mathbf{x} = \boldsymbol{\alpha} + \mathbf{B}\mathbf{p} + \boldsymbol{\gamma}[y - \boldsymbol{\alpha}'\mathbf{p} - \tfrac{1}{2}\mathbf{p}'\mathbf{B}\mathbf{p} - \delta(\mathbf{q})]$$

where **B** is a symmetric, negative semidefinite matrix.

The demand model (11) is derived from (10) by Roy's (1947) identity,

$$(12) \qquad \mathbf{h}^x(\mathbf{p}, \mathbf{q}, y) \equiv -[\partial\phi(\mathbf{p}, \mathbf{q}, y)/\partial\mathbf{p}]/[\partial\phi(\mathbf{p}, \mathbf{q}, y)/\partial y]$$

It is clear that model (11) is a member of the class of demand functions that are linear and quadratic in **p**, linear in y, and have no interaction terms between **p** and y. Symmetry of the Slutsky substitution terms implies (11) (details are available from the author upon request). This demand model is quite flexible with respect to the price and income elasticities. For example, individual income coefficients may be positive, negative or zero, and the matrix of price effects, $\partial\mathbf{x}/\partial\mathbf{p}' = \mathbf{B} - \boldsymbol{\gamma}(\boldsymbol{\alpha}' + \mathbf{p}'\mathbf{B})$, is not (necessarily) symmetric, so that there is no requirement that the demands for **x** are homothetic. Nevertheless, considerable information is obtained from the theory. The unrestricted model has $\frac{1}{2}n^3 + \frac{3}{2}n^2 + n$ coefficients, but the model that is consistent with the theory has only $\frac{1}{2}n^2 + \frac{5}{2}n + 1$ coefficients. For example, with five commodities the unrestricted model has 105 parameters, while the restricted model has only 26 parameters, a saving of 79 degrees of freedom in the empirical model.

Conclusions

This paper has outlined a general procedure for discovering the parameter restrictions and the underlying conditional preference structure implied by the theory of utility maximisation subject to a linear budget constraint for an incomplete system of demand

equations. The methodology can be applied to any system of demand equations and the results obtained are general; nothing has to be assumed *a priori* about the structure of the consumer's preferences in addition to the usual properties. As an illustration of its usefulness and practical implications, this methodology was applied to several incomplete semilogarithmic demand models.

The results obtained for the semilogarithmic incomplete demand models are more general than, and cannot be derived from, studies of complete demand systems (for example, Lau 1976). This added generality results from two aspects of incomplete demand models. First, the budget constraint is a strict inequality. Second, the demand function for the composite commodity that is added to the incomplete demand system generally does not have the same functional form as the demand functions for the goods of interest. These conditions result in more flexibility for incomplete demand models relative to complete demand systems. Nevertheless, each semilogarithmic demand model is restrictive even with this general concept of integrability for incomplete demand systems.

The fact that each of these models is quite restrictive is useful information. Most of economics, in one form or another, involves the estimation of costs and benefits due to such things as changes in public policies. Hausman (1981) has shown that when deadweight losses are the main welfare measure of interest, the use of exact welfare measures (compensating or equivalent variation) can reduce measurement errors by as much as 50 per cent relative to consumer's surplus approximations. Calculation of the exact welfare measures requires the structure that results from the theory of consumer choice, that is, integrability of the demand system. Demand models that are flexible representations of consumer preferences and that can be rationalised by utility maximisation clearly are superior choices for this task. The fact that each semilogarithmic demand model considered in this paper fails this test is valuable prior information.

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