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MEAN-GINI ANALYSIS, STOCHASTIC EFFICIENCY AND WEAK RISK AVERSION

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Stochastic dominance methods lately have been used to derive efficient strategies for given risk aversion intervals. A new decision approach, which makes use of the Gini coefficient, is shown to represent effectively the preferences of weakly risk averse individuals. The approach also has distinct advantages over stochastic dominance analysis. An application is provided of farmers' choices among alternative co-operative pooling rules.

Applied research in the economics of risky decisions relies heavily on procedures for identifying stochastically efficient strategies. Chief among methods employed are mean-variance and stochastic dominance analysis, including Meyer's (1977) popular generalisation of the latter. Recently stochastic dominance has been favoured because it may be used to identify any strategy that individuals in a given risk aversion class unanimously would find inferior to another strategy. Such a property is especially desirable in view of the strict validity of mean-variance techniques only for quadratic utilities, normally distributed prospects or vanishingly small risks (Samuelson 1970).

Yet, application of stochastic dominance techniques has proven troublesome. Second-degree dominance, which represents the interests of all risk averters, appears typically to eliminate only one-half to four-fifths of the strategies proposed, often leaving a large number of undominated ones from which to choose (Levy and Hanoch 1970, p. 71; Porter and Gaumnitz 1972, p. 444; Anderson 1974, p. 164; Anderson 1975, p. 101). Although additional strategies can be eliminated by further restricting the absolute risk aversion interval, it sometimes is difficult to know what sub-interval to employ. Further, stochastic dominance methods are not convenient for analysing portfolio problems where an optimal weighted combination of strategies is desired (Anderson 1975, p. 105).

Yitzhaki (1982) recently proposed an alternative decision model in which risk is reflected by a function of the mean absolute difference. The technique has some of the convenience of mean-variance analysis and is robust, like stochastic dominance, with respect to utility and probability functional form. In this paper the basis for the technique is described. It is then applied to a series of risky decision problems and the results are

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compared with those from mean-variance and stochastic dominance analyses. It is argued that Yitzhaki's method is appropriately employed for weakly risk averse decision makers and that the method has distinct advantages over stochastic dominance when dealing with weakly risk averse situations.

Mean-Gini Analysis

The variability of a random variable x can be described, among other ways, by its 'Gini coefficient' (Kendall and Stuart 1969, p. 46)

$$(1) \qquad \Gamma_F = \triangle/2 = \int_a^b \int_a^b |x-y| \, dF(x) \, dF(y)/2$$

where $F(\bullet)$ is a cumulative density function defined on the range $[a, b]$ and x, y is a pair of values of x.[1] The Gini coefficient is one-half the expected absolute difference between a randomly selected pair of values of x. It is to be distinguished from the mean absolute deviation of x from its mean. Gastwirth (1972) and others show that (1) can be rewritten as[2]

$$(2) \qquad \Gamma_F = \int_a^b [1-F(x)] \, dx - \int_a^b [1-F(x)]^2 \, dx$$

$$= \mu_F - a - \int_a^b [1-F(x)]^2 \, dx$$

where μ_F is the mean of x when it has distribution F.

Mean-Gini dominance

The usefulness of (2) in decision analysis arises from the following statement proven by Yitzhaki (1982, pp. 179-80). Let $F(x)$ and $G(x)$ be the cumulative density functions of two risky prospects such that $F(x)$ stochastically dominates $G(x)$ in the second degree (SSD), such that

$$(3) \qquad \int_a^t F(x) \, dx \leq \int_a^t G(x) \, dx$$

for all t in $[a, b]$, with strict inequality for at least one t. Then it is necessary that

$$(4) \qquad \int_a^b [1-F(x)]^n \, dx \geq \int_a^b [1-G(x)]^n \, dx$$

for all positive values of n, with strict inequality for at least one n. Provided $F(x)$ and $G(x)$ cross at most once, (4) is sufficient as well as necessary for equation (3).

Condition (4) can be expressed in more intuitive form by defining, after Shalit and Yitzhaki (1982, p. 25), the 'extended Gini coefficient'

[1] For simplicity, the discussion is restricted to continuous, bounded variates. However, the results may be derived for discrete and unbounded cases as well (Kendall and Stuart 1969; Shalit and Yitzhaki 1982). Equation (1) differs from Gini's 'coefficient of concentration', often used in income distribution studies, in that in the latter, (1) is divided by the mean of x (Gastwirth 1972, p. 307).

[2] The bottom version of equation (2) can be obtained from the top version by noting that $\int [1-F(x)] \, dx = b - a - \int F(x) \, dx$ and that $b - \int F(x) \, dx = \int x F(x) \, dx = \mu$.

$$\Gamma_F(n) = \int_a^b [1 - F(x)]\, dx - \int_a^b [1 - F(x)]^n\, dx \tag{5}$$

$$= \mu_F - a - \int_a^b [1 - F(x)]^n\, dx$$

Definitions (5) and (2) differ only in that the exponent in the integral of (5) is generalised: Γ_F in equation (2), the 'simple' Gini coefficient, is equivalent to $\Gamma_F(2)$. Since

$$\mu_F - \Gamma_F(n) = a + \int_a^b [1 - F(x)]^n\, dx \tag{6}$$

condition (4) can be expressed in the 'extended mean-Gini' form as

$$\mu_F - \Gamma_F(n) \geq \mu_G - \Gamma_G(n) \qquad \text{all } n > 0 \tag{7}$$

The latter says that if F dominates G in the second degree, then F's mean minus extended Gini is at least as large as G's mean minus extended Gini, with strict inequality holding for at least one n. Condition (7) is difficult to apply because it may involve an infinite number of numerical comparisons between F and G.[3] For practical purposes, Yitzhaki suggests implementing the comparisons for $n = 1$ and $n = 2$ only. When $n = 1$, $\Gamma(n) = 0$ and (7) becomes

$$\mu_F \geq \mu_G \tag{8}$$

When $n = 2$, $\Gamma(n)$ reduces to simple Gini coefficient (2); so (7) becomes

$$\mu_F - \Gamma_F \geq \mu_G - \Gamma_G \tag{9}$$

that is,

$$\int_a^b [1 - F(x)]^2\, dx \geq \int_a^b [1 - G(x)]^2\, dx \tag{9}$$

Taken together, equations (8) and (9) will be called the mean-Gini (MG) criterion. F dominates G by MG if both (8) and (9) hold, providing that at least one is a strict inequality.

Discriminating power of alternative criteria

Because (8) and (9) are necessary conditions for (7), and (7) is necessary for (3), relations (8) and (9) are necessary for second-degree stochastic dominance. Being generally necessary only, (8) and (9) are easier (or at least no more difficult) to satisfy than (3). Consider two SSD-efficient strategies, A and B, and a third strategy, C, that is SSD-dominated by A but not by B.

(a) Since all necessary conditions exist for C to be SSD-dominated by A, necessary conditions (8) and (9) (the MG criterion) also will indicate that C is dominated by A. That is, *every SSD-inefficient strategy also will be MG-dominated.*

(b) Although sufficient conditions are not present for B to be SSD-dominated by A (or *vice versa*), necessary conditions (8)-(9) for

[3] If F and G cross at most once, it is sufficient to compare them for $n = 1$ and $n \to \infty$ only (Yitzhaki, p. 183).

either dominance may be present. Hence, *a strategy may be MG-dominated even if it is SSD-efficient.*

(c) Suppose A is undominated by MG criterion (8)-(9). Then A also must be SSD-efficient because at least two conditions necessary for it to be SSD-inefficient are lacking. In other words, *every strategy that is MG-undominated is SSD-efficient.*

A principal conclusion is that while all MG-undominated strategies are SSD-efficient, some SSD-efficient strategies may be MG-dominated.

Undominated sets resulting from application of alternative efficiency criteria are illustrated in Figure 1. The fewer or weaker the conditions required for dominance, the more discriminating is the criterion and the smaller (generally) is the undominated set. Sets D, C, B, and A result from successive removal of dominance conditions and therefore successively must enclose one another. Without knowing the distribution families involved, we cannot safely identify the mean-variance (MV) un-

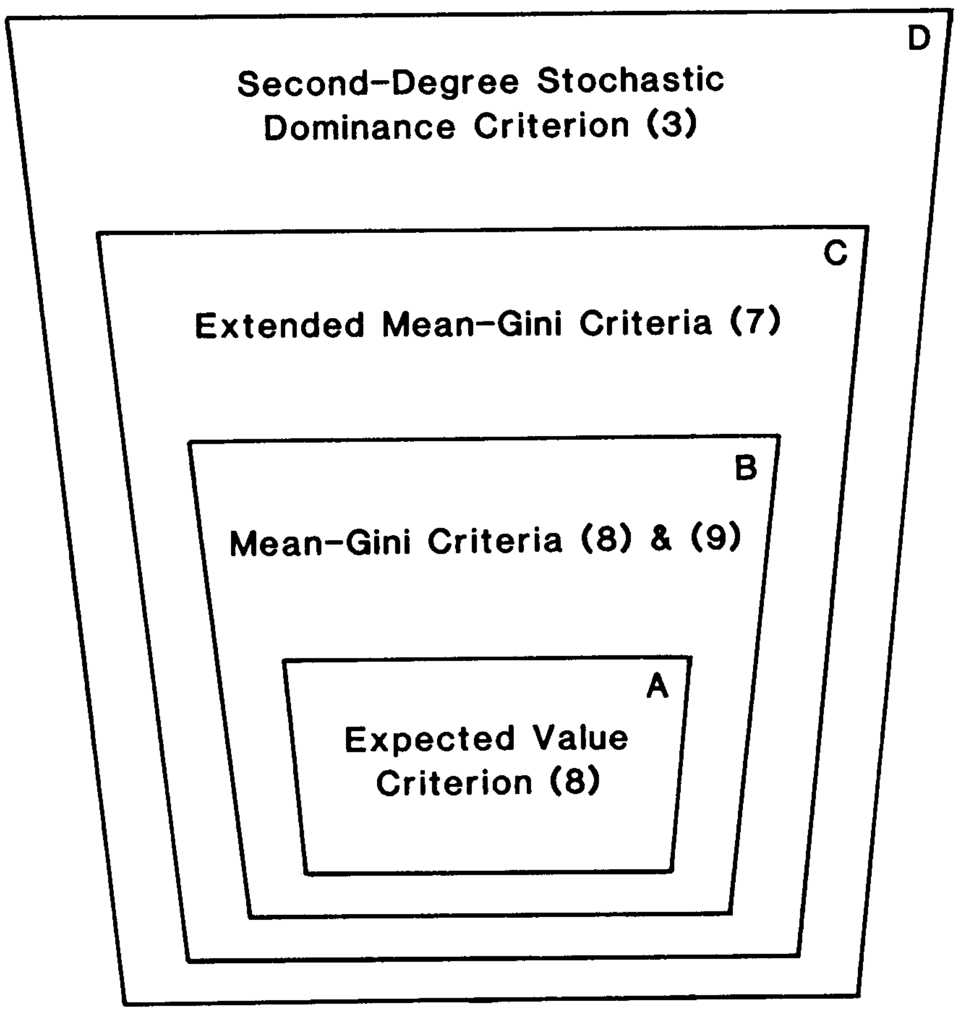

FIGURE 1—Undominated Sets under Alternative Dominance Criteria.

dominated set in Figure 1. But the latter is coterminous with D if the distributions are normal, and it contains D if the distributions are lognormal (Levy and Hanoch 1970; Levy 1973, pp. 610-11).

Mean-Gini and weak risk aversion

The fact that some SSD-efficient strategies may be eliminated under the MG criterion is not necessarily bad. The researcher may not be interested in the class of risk averters whose preferences solely account for the SSD-efficiency of a given group of prospects. Structural similarity between a prospect's certainty equivalent, $(\mu-\pi)$, and its $(\mu-\Gamma)$ value suggest that strategies characteristically eliminated by MG also would be rejected by a broadly identifiable subset of risk averters. Pratt (1964, pp. 125-6) has shown that the risk premium, π, of a small-risk prospect is approximately

$$\pi \approx r(\mu)\sigma^2/2 \tag{10}$$

where σ^2 is the variance of the prospect and $r(\mu)$ is absolute risk aversion evaluated at μ. For fixed μ, Γ and σ^2, $\Gamma \approx \pi$ if and only if $r(\mu) \approx 2\Gamma/\sigma^2$. Hence, at the latter value of $r(\mu)$, Γ is approximately the risk premium and $(\mu-\Gamma)$ is approximately the certainty equivalent. Thus also, criterion (9) approximately involves comparing the certainty equivalents of F and G for a particular neighbourhood of absolute risk aversion.

There are several reasons to believe that (9) best represents the weakly risk averse portion of the utility spectrum. One may view $\Gamma = \Gamma\,(2)$ in (9) as a penalty deducted from μ on account of the riskiness of x. Intuitively, a risk penalty should rise with increasing risk aversion as well as with increasing dispersion of x. Yet for a given dispersion, the risk penalty represented by $\Gamma\,(2)$ is relatively small. Successively larger penalties could be produced by utilising extended Gini coefficient (5) with successively larger n. The subclass represented by $n=2$, therefore, is relatively weakly risk averse in the interval $n=1\ldots,\infty$ admitted by extended mean-Gini criterion (7).

Another way of viewing this is to note that under SSD criterion (3), F cannot dominate G if the minimum value of F lies to the left of the minimum value of G. That is equivalent to satisfying the requirement of minimaxers that the least desirable outcome of F be at least as good as the least desirable outcome of G. By contrast, mean-Gini criterion (9) places only 'mild' emphasis on lower distribution tails because squaring a cumulative density only 'slightly' de-emphasises upper tails relative to lower ones. Analysts could further de-emphasise upper tails by raising the exponents in the integrals of (9) to higher powers. In fact $[\mu-\Gamma(n)]$ approaches a, the minimum variate value, as $n \to \infty$. The implication, again, is that use of the mean-Gini criterion would be appropriate for relatively weakly risk averse individuals.

Empirical Application

Meyer's stochastic dominance with respect to a function (MSD), in which efficient strategies may be identified for an arbitrary risk aversion interval, already has been utilised by agricultural economists to limit attention to the weakly risk averse case (Kramer and Pope 1981; King and Oamek 1983). It is natural to ask whether strategies undominated under the mean-Gini criterion would be the same as the stochastically efficient set for some 'low' risk aversion interval. The correspondence between MG- and MSD-undominated sets was tested by applying each method to the same series of risky decision problems. Mean-variance dominance (MV) also was applied for purposes of comparison.

Setting

The problem involved selecting a pooling rule that would be used to allocate net proceeds of a horticultural processing co-operative among its grower-members. The co-operative processes and sells a wide variety of fruits and vegetables. Three pool structures were considered: (a) *multiple pools*, where net revenues of each fruit and vegetable are accounted for separately and allocated in proportion to the volume of each product each member delivers; (b) a *single pool*, where net revenues of all products are combined and allocated in proportion to the total value of raw product each member delivers; and (c) *grouped pools*, where combined fruit net revenues are grouped separately from combined vegetable net revenues, each allocated in proportion to the value of raw product each member delivers to each group. Raw products in (b) and (c) alternately were valued at farm market prices ('farm-price basis') and at an index of expected processing profitability ('profitability basis'). Thus, five pooling rules were examined in all.

Depending upon the mix of products a member contributes to the co-operative, the member could participate in many pools under structure (a), in at most one pool under structure (b), and in at most two pools under structure (c). Differences between structures in the probability distribution of a member's total payment depend upon his product mix. For example, although payment variability for some members may be relatively lower in more diversified pools, those supplying products with especially stable prices may experience lower risk in multiple (less diversified) pools. Members contributing products with especially high (low) mean returns would receive higher (lower) mean payments in multiple pools than in single or grouped ones. Hence, it is desirable to represent separately the decision problem faced by any member having a distinctive mix of raw products. In the present study, separate dominance analyses for ten characteristic raw product mix classes were conducted.

Procedures used

Probability data were generated by simulating the annual net returns per acre that would have been allocated to each product mix class by each alternative pooling rule during the periods 1960-70 and 1972-80. (The year 1971 was deleted due to accounting anomalies.) Net returns included final product sales revenues less processing and farm production costs. Data were deflated and detrended, and means were adjusted to reflect future revenue and cost expectations. Cumulative probability values F then were found by dividing the rank of each observation in each series by the sample size of 20. A Gini coefficient was calculated by taking twice the covariance between x and its cumulative density F. This avoids the necessity of integrating to find Γ (Trajtenberg and Yitzhaki 1982).

For each of the ten product mix classes, MSD, MV, and MG were used alternatively to model members' preferences among the five alternative pooling rules. MSD-efficient sets for selected risk aversion intervals were obtained with a program written by Meyer (1977) and modified by James Richardson. SSD-efficient sets were simply those for which the MSD risk aversion interval was $0 \leq r \leq +\infty$. MV-efficient sets were found by applying the rule: F dominates G if and only if $\mu_F \geq \mu_G$ and $\sigma_F^2 \leq \sigma_G^2$, with at least one strict inequality (Tobin 1958). To obtain MG-efficient sets, μ, Γ, and $(\mu - \Gamma)$ first were estimated for every pooling rule. Rules' μ estimates then were arrayed from highest to lowest, with $(\mu - \Gamma)$ estimates listed next to the corresponding means. Any rule was eliminated if its $(\mu - \Gamma)$ estimate was not higher than that of an alternative with higher mean.[4] An example of this procedure for product mix V is given in Table 1. Rules 3 and 4 were eliminated first because 295 exceeds 284 and 287. Rule 2 was eliminated next because 338 exceeds 309.

Results

Results of the dominance analyses are shown in Table 2. Initially the MG-dominant sets were derived. The MSD-efficient sets were then derived for alternative risk aversion intervals to see how the latter would compare with the MG-dominant sets. In one series of comparisons, the lower-r bound was held at zero and the positive upper-r bound gradually increased. The upper bound nearest zero providing results different from expected net revenue maximisation was $0 \leq r \leq 0.0005$; in this interval, only members in product mix class V acted differently from risk-neutral individuals. The next-higher upper bound providing a solution change was $0 \leq r \leq 0.0015$, in which class III members also departed from risk neutrality. The latter solution, shown in Table 2, was identical to that generated by the mean-Gini criterion. Results listed under $0 \leq r \leq 0.0045$

TABLE 1

Example of Strategy Elimination by Mean-Gini (MG) Dominance[a]

				Eliminations	
Rule	μ	Γ	μ-Γ	First round	Second round
5	547	252	295		
1	543	205	338		
2	529	220	309		x
3	511	227	284	x	
4	471	184	287	x	
Mean-Gini undominated pooling rules: 1,5					

[a] Pooling rule numbers are as follows: 1 — multiple; 2 — single (farm-price basis); 3 — single (profitability basis); 4 — grouped (farm-price basis); 5 — grouped (profitability basis).

[4] Yitzhaki (p. 183) suggests initially eliminating any strategy G if there is an F such that $\mu_F \geq \mu_G$ and $\Gamma_F \leq \Gamma_G$, with at least one strict inequality. Such conditions, which Yitzhaki collectively calls the 'MΓ criterion', are sufficient but not necessary for MG criterion (8)-(9). Hence, application of the MΓ criterion must be followed by application of the MG criterion as just described in the text. Initial elimination by MΓ, which is completely analogous to mean-variance analysis, would perhaps be useful if a large number of strategies had to be compared.

and $0 \leq r \leq \infty$ in the table suggest how stochastically efficient sets began to diverge from the mean-Gini-dominant set as more risk averse utilities were included.

The mildness of risk aversion implicit in mean-Gini is emphasised by the fact that, in all cases but III and V, MG-dominant strategies also maximised expected net revenue. For product mix III, mean-Gini eliminated only one of five strategies, while for product mix V it eliminated three of the five strategies. In the case of every product mix, each MG-dominated strategy had a lower mean than all undominated ones. Nothing in the MG criterion explicitly requires this.

As the risk aversion interval in MSD is widened, an increasingly heterogeneous group of decision makers is asked to agree on strategies to be stochastically dominated. Hence, the efficient set generally enlarges. Such was decidedly the case in the present study. The average number of SSD-efficient strategies was just over three, similar to the average number of MV-efficient strategies. SSD- and MV-efficient sets differed in six of the ten product mix classes shown. Despite this, all MG-dominant strategies were both SSD-efficient and MV-efficient. MG-dominant strategies need not, of course, be MV-efficient in all research applications.

Conclusions

Whether the weak risk aversion inherent in mean-Gini is an asset or liability depends upon the class of risks or decision makers one wishes to represent. Clearly it would not be an appropriate model if catastrophic risks are involved or if the decision makers are very risk-fearing or heterogeneous in risk attitude. In the absence of more definitive information, however, the assumption of weak risk aversion is not a bad idea. It probably best describes the inclinations of the typical farm decision maker faced with a moderate-sized gamble (Young 1979, p. 1067;

TABLE 2

Undominated Pools by Members' Product Mix and Dominance Criterion[a]

Members' product mix	Stochastic dominance (MSD)[b]			Mean-variance (MV)	Mean-Gini (MG)
	0-0.0015	0-0.0045	0-∞ (SSD)		
I	4	4	1,4	1,2,4,5	4
II	4	1,3,4,5	1,3,4,5	1,2,3,4,5	4
III	1,2,3,5	1,2,3,5	1,2,3,5	1,2,3,5	1,2,3,5
IV	4	4	2,4	2,4	4
V	1,5	1,3,5	1,2,3,4,5	1,4,5	1,5
VI	4	4	4,5	2,4,5	4
VII	5	3,5	1,2,3,4,5	2,4,5	5
VIII	4	4	2,4	2,4	4
IX	1	1	1,3,5	1	1
X	2	1,2,4	1,2,4	1,2,4	2

[a] Pooling rule numbers are described in footnote a of Table 1.
[b] Ranges listed are absolute risk aversion intervals. Net returns used in the study were expressed in whole dollars and on a per-acre basis.

Kramer and Pope 1981, p. 124; Bond and Wonder 1980, pp. 25-8). The assumption also avoids the unrealistic emphasis sometimes placed on third and fourth probability moments by more risk averse model solutions.

For representing weakly risk averse individuals, mean-Gini has several natural advantages over Meyer's stochastic dominance. First, stochastic dominance analysis of portfolio problems requires initially generating a series of portfolios with alternative weightings of the individual options (Porter and Gaumnitz 1972; Anderson 1975). Such sample portfolios, even though numerous, could fail to include others that are preferable. By contrast, a piecewise linear program can be used to identify minimum-Γ portfolios at selected mean profit levels (Yitzhaki 1982, p. 184). MG-dominant portfolios then can be found by applying to this μ, Γ frontier the procedure illustrated in Table 1 (see footnote 4).

Second, because mean-Gini analysis relies on parameter estimates rather than on sequential inspection of entire cumulative density functions, its sampling properties may easily be found analytically. The sample variance of an estimate of Γ has been derived by Lomnicki (1952). For normal distributions it reduces approximately to $0.1627\sigma^2N^{-1}$, where N is sample size.[5] Since in the normal family the expected value of a Γ estimate is $\sigma\pi^{-1}$, the coefficient of variation of the estimate is approximately $1.2673N^{-0.5}$ under normally distributed returns. With uniformly distributed returns, the coefficient of variation of sample Γ is only $0.4472N^{-0.5}$, suggesting that mean-Gini is relatively more reliable when dealing with shorter-tailed distributions. Sampling properties have not been derived analytically for second-degree or Meyer's stochastic dominance (Stein and Pfaffenberger 1983).

A shortcoming of mean-Gini is that the absolute risk aversion interval represented by setting $n = 2$ is not (in the absence of a comparative study such as the present one) known exactly. This prevents one from applying utility assessment results to derive efficient strategies for a precisely known class of utility functions. However, in some circumstances the disadvantage may not be great. Estimates of absolute risk aversion coefficients are not only subject to sampling error but also sensitive to the money scale employed and to the wealth level at which they are evaluated. Cross-study comparisons of such coefficients are often awkward and one may sometimes be warranted only in assuming a broad class of risk preferences. In weakly risk averse situations, mean-Gini analysis deserves close consideration.

References

Anderson, J. R. (1974), 'Risk efficiency in the interpretation of agricultural production research', *Review of Marketing and Agricultural Economics* 42(3), 131-84.

—— (1975), 'Programming for efficient planning against non-normal risk', *Australian Journal of Agricultural Economics* 19(2), 94-107.

Bond, G. and Wonder, B. (1980), 'Risk attitudes amongst Australian farmers', *Australian Journal of Agricultural Economics* 24(1), 16-34.

Gastwirth, J. L. (1972), 'The estimation of the Lorenz curve and Gini index', *Review of Economics and Statistics* 54(3), 306-16.

Kendall, M. G. and Stuart, A. (1969), *The Advanced Theory of Statistics*, Vol. I, 3rd edn, Haffner, New York.

[5] Note that Γ as used here is one-half Lomnicki's Δ, and var $(\Delta/2) = $ var $(\Delta)/4$.

King, R. P. and Oamek, G. E. (1983), 'Risk management by Colorado dryland wheat farmers and elimination of the Disaster Assistance Program', *American Journal of Agricultural Economics* 65(2), 247-55.

Kramer, R. A. and Pope, R. D. (1981), 'Participation in farm commodity programs: a stochastic dominance analysis', *American Journal of Agricultural Economics* 63(1), 119-28.

Levy, H. (1973), 'Stochastic dominance among log-normal prospects', *International Economic Review* 14(3), 601-14.

—— and Hanoch, G. (1970), 'Relative effectiveness of efficiency criteria for portfolio selection', *Journal of Financial and Quantitative Analysis* 5(1), 63-76.

Lomnicki, Z. A. (1952), 'The standard error of Gini's mean difference', *Annals of Mathematical Statistics* 23, 635-7.

Meyer, J. (1977), 'Choice among distributions', *Journal of Economic Theory* 14(2), 326-36.

Porter, R. B. and Gaumnitz, J. E. (1972), 'Stochastic dominance vs. mean-variance portfolio analysis: an empirical evaluation', *American Economic Review* 62(3), 438-46.

Pratt, J. W. (1964), 'Risk aversion in the small and in the large', *Econometrica* 32(1), 122-36.

Samuelson, P. A. (1970), 'The fundamental approximation theorem of portfolio analysis in terms of means, variances, and higher moments', *Review of Economic Studies* 37(4), 537-42.

Shalit, H. and Yitzhaki, S. (1982), *Mean-Gini, Portfolio Theory, and the Pricing of Risky Assets*, Paper No. 8204, Center for Agricultural Economic Research, Israel.

Stein, W. and Pfaffenberger, R. (1983), *An Examination of Tests for Stochastic Dominance*, Working Paper No. 3, Department of Decision Science, Texas Christian University.

Tobin, J. (1958), 'Liquidity preference as behaviour towards risk', *Review of Economic Studies* 25(2), 65-86.

Trajtenberg, M. and Yitzhaki, S. (1982), *The Diffusion of Innovations: A Methodological Reappraisal*, Working Paper No. 1008, National Bureau of Economic Research, Cambridge, Massachusetts.

Yitzhaki, S. (1982), 'Stochastic dominance, mean variance, and Gini's mean difference', *American Economic Review* 72(1), 178-85.

Young, D. L. (1979), 'Risk preferences of agricultural producers: their use in extension and research', *American Journal of Agricultural Economics* 61(5), 1063-70.