STORAGE AND PRICING OF APPLES: SOME EMPIRICAL EVIDENCE ON THE STRUCTURE OF THE VICTORIAN WHOLESALE MARKET

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The problems which orchardists face in marketing fresh fruit are often viewed as arising from a lack of control over their markets, particularly lack of control over prices or the quantities offered for sale, and the efficiency of the spatial, temporal and form distribution of the product. These problems and concerns have given rise to a considerable number of economic studies. Some typical studies are those by Piggott (1975) and Rae (1978) reported in this Journal and those by Fuchs, Farish and Bohall (1974), and O’Rourke and Masud (1980).

A feature of such studies is the restricted range of theoretical market structures within which marketing processes take place. Markets for fruit are traditionally thought to be competitive thus competitive theory is used to characterise the initial or given market structure. Intervention to improve competitive market outcomes for orchardists generally has involved marketing boards, marketing orders (in the US) or through generating an informed reallocation of stocks indirectly by improving the information state of the markets. The benefits of intervention involving monopoly control are then estimated using the principles of price discrimination illustrated by Waugh, Burtis and Wolf (1936).

Deficiencies of this conventional methodology are at least two-fold: the market structure under which the allocation takes place may not approximate competition, and the use of monopolistic price discrimination probably overstates the degree of control over the market which can be exercised. These deficiencies are not commonly acknowledged but they can be important.

In this paper an unconventional approach to modelling market structure is described which draws on the theory of ‘offer variation’ described by Scitovsky (1952) and is termed the ‘mark-up’ model because of its similarity to the setting of optimal price margins in a retailing context. Linear Programming (LP) is used to solve the optimising problem. While the LP technique was demonstrated by Piggott (1975), the model developed here employs Lagrange Multipliers as a dummy activity, and leads to a different conclusion to that reached by Piggott as to the value of the resultant shadow prices.

*Julian Alston, John Kennedy, Rover Piggott and two anonymous referees helped with comment on an earlier version of this paper. ‘Storage and Pricing of the Victorian Apple Crop’, presented to the 1982 AAES conference.
Apple Prices and Market Structure

During the 1970s the prices of stored apples sold at the Melbourne Wholesale Fruit and Vegetable Market exhibited seemingly inconsistent patterns of price variation. The pattern of prices is shown in Table 1.

<table>
<thead>
<tr>
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<th></th>
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<tbody>
<tr>
<td>June</td>
<td>231</td>
<td>392</td>
<td>245</td>
<td>623</td>
<td>304</td>
<td>684</td>
<td>546</td>
<td>671</td>
<td>680</td>
<td>1,051</td>
</tr>
<tr>
<td>July</td>
<td>243</td>
<td>446</td>
<td>266</td>
<td>624</td>
<td>306</td>
<td>747</td>
<td>655</td>
<td>730</td>
<td>635</td>
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<tr>
<td>August</td>
<td>264</td>
<td>490</td>
<td>312</td>
<td>676</td>
<td>414</td>
<td>778</td>
<td>621</td>
<td>744</td>
<td>706</td>
<td>1,101</td>
</tr>
<tr>
<td>September</td>
<td>300</td>
<td>565</td>
<td>302</td>
<td>649</td>
<td>449</td>
<td>813</td>
<td>658</td>
<td>830</td>
<td>696</td>
<td>1,179</td>
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<tr>
<td>October</td>
<td>358</td>
<td>615</td>
<td>380</td>
<td>690</td>
<td>503</td>
<td>998</td>
<td>710</td>
<td>959</td>
<td>723</td>
<td>1,204</td>
</tr>
<tr>
<td>November</td>
<td>360</td>
<td>608</td>
<td>415</td>
<td>696</td>
<td>528</td>
<td>1,083</td>
<td>729</td>
<td>1,028</td>
<td>749</td>
<td>1,214</td>
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<tr>
<td>Price increase</td>
<td>129</td>
<td>216</td>
<td>170</td>
<td>73</td>
<td>224</td>
<td>399</td>
<td>183</td>
<td>357</td>
<td>69</td>
<td>163</td>
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<tr>
<td>Percentage increase</td>
<td>56</td>
<td>55</td>
<td>69</td>
<td>12</td>
<td>74</td>
<td>58</td>
<td>33</td>
<td>53</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

*Increase in price from June to November for each year and the corresponding percentage.


In a competitive market the price increase over the storage period in any year should be a crude measure of the premium for the costs of storage and the acceptance of intertemporal market risk. But in most years the premium appeared to be far more than these costs. Shortfalls in quantity did not appear to be a prime factor since releases from storage followed quite consistent patterns from year to year. Given other pertinent information on the marketing environment, a further plausible hypothesis was that storage premiums were not always competitively determined.

A characteristic of the market which added support to this hypothesis was the behaviour and stated policy of the local growers’ association the Orchardists and Fruit Cool Stores Association (OFCSA), which disseminated price information to its members ‘... not merely reporting sales made but the level of values (prices) which could be made during the following week on known information and trends’ (OFCSA 1979). Thus the marketing strategy of apple storers who followed this advice involves both price and quantity fixing. Price fixing in the context of the wholesale market involved adopting a strong negotiating or bargaining stance over the marketing margin attributable to storage. The empirical implication of this activity was that apple storers were in fact negotiating the position of the demand function at the wholesale level from week to week and month to month.
The Demand Model

Several demand models were evaluated to examine the empirical strength of this view of the demand function. These models are described in Tunstall (1982). The model evaluation procedure involved comparing various logarithmic and semi-logarithmic transformations of a linear model. Further different specifications of annual and seasonal demand variations were tested. The statistical evaluation procedure amounted to the 'occasional sin' of pre-testing discussed in Wallace (1977). Thus the empirical results reported here (Table 2) may reflect reduced variance achieved at the expense of increased bias.

Although logarithmic specifications produced estimates with lower variances the linear model was preferred on other grounds. Generally the linear model produced lower elasticity estimates with smoother seasonal gradation, broadly similar in pattern to the findings of other researchers. Linearity was also favoured on grounds of simplicity and convenience in mathematical programming applications as described by Takayama and Judge (1971). Possible reservations based on misspecified functional form were regarded as minor since the linear form approximates well to other forms within limited data ranges and the model was not used to extrapolate beyond the range of the data.

The selected linear demand model can be written as follows:

\[ PA_{im} = a + bQ_1A_{im} + cQ_2P_{im} + dY_{im} + e_mD_m + f_mS_m + g_lX_l + h_lZ_l + V_{im} \]

where \( PA \) is the monthly average price of apples deflated by the CPI Melbourne All Groups Index 1971 = 100 (cents/kg), \( QA \) is the monthly disappearance of apples from storage (kg/head), \( QP \) is the monthly disappearance of pears from storage (kg/head), \( Y \) is the monthly

TABLE 2

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>35.770</td>
<td>(31.245)</td>
<td>( D_1 )</td>
<td>-5.903</td>
<td>(7.497)</td>
</tr>
<tr>
<td>( QA )</td>
<td>4.829</td>
<td>*</td>
<td>( Z_1 )</td>
<td>-3.085</td>
<td>*</td>
</tr>
<tr>
<td>( Y )</td>
<td>-0.0224</td>
<td>(0.0294)</td>
<td>( Z_2 )</td>
<td>-8.058</td>
<td>*</td>
</tr>
<tr>
<td>( QP )</td>
<td>-0.343</td>
<td>(1.019)</td>
<td>( Z_3 )</td>
<td>-5.045</td>
<td>*</td>
</tr>
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<td>( X_1 )</td>
<td>-5.903</td>
<td>(7.497)</td>
<td>( Z_4 )</td>
<td>11.872</td>
<td>*</td>
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<tr>
<td>( X_2 )</td>
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<td>(5.898)</td>
<td>( Z_5 )</td>
<td>-5.216</td>
<td>*</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>-2.532</td>
<td>(4.655)</td>
<td>( Z_6 )</td>
<td>6.328</td>
<td>*</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>-8.701</td>
<td>(6.509)</td>
<td>( Z_7 )</td>
<td>0.448</td>
<td>(2.384)</td>
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<tr>
<td>( X_5 )</td>
<td>-1.189</td>
<td>(3.343)</td>
<td>( Z_8 )</td>
<td>-5.754</td>
<td>*</td>
</tr>
<tr>
<td>( X_6 )</td>
<td>10.742</td>
<td>*</td>
<td>( Z_9 )</td>
<td>2.264</td>
<td>(2.384)</td>
</tr>
<tr>
<td>( X_7 )</td>
<td>-7.329</td>
<td>(4.3980)</td>
<td>( Z_{10} )</td>
<td>3.157</td>
<td>(1.863)</td>
</tr>
<tr>
<td>( X_8 )</td>
<td>5.094</td>
<td>(4.383)</td>
<td>( S_1 )</td>
<td>2.017</td>
<td>(1.742)</td>
</tr>
<tr>
<td>( X_9 )</td>
<td>-12.552</td>
<td>*</td>
<td>( S_2 )</td>
<td>-4.196</td>
<td>*</td>
</tr>
<tr>
<td>( X_{10} )</td>
<td>3.042</td>
<td>(3.178)</td>
<td>( S_3 )</td>
<td>-2.812</td>
<td>(2.117)</td>
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<tr>
<td>( X_{11} )</td>
<td>6.271</td>
<td>*</td>
<td>( S_4 )</td>
<td>-4.374</td>
<td>(3.056)</td>
</tr>
</tbody>
</table>

\[ R^2 = 0.97 \quad \widetilde{R}^2 = 0.94 \quad \text{Durbin-Watson} = 1.90 \]

Sum of Squared Residuals = 42.26

\[ F_{(31,36)} = 37.32 \]

* = significant at 0.05 (one-tail test)
average weekly earnings expressed in real 1971 dollars, \( D \) is a monthly dummy intercept variable defined such that \( D_m = 1 \) in month \( m \), and zero otherwise, \( S \) is a monthly dummy slope variable defined such that \( S_m = QA \) in month \( m \) and zero otherwise, \( X \) is an annual dummy intercept variable defined such that \( X_i = 1 \) in year \( i \) and zero otherwise, \( Z \) is an annual dummy slope variable defined such that \( Z_i = QA \) in year \( i \) and zero otherwise, \( V \) is an error term about which the usual requirements for OLS are assumed to hold, \( m \) is a month subscript = 1 in June, 2 in July, \ldots 6 in November, and \( i \) is a year subscript = 1 in 1971, 2 in 1972, \ldots 10 in 1980, and \( a, b, c, \ldots \) are coefficients to be estimated.

Pears were used as a substitute fruit because they were the only comparable fruit with equivalent data. This model incorporates year and month dummy intercept and slope variables to capture demand shifts envisaged in the hypothesis. The annual dummy variables contain demand variation due to the heterogeneity in the apple crop which arises from biennial bearing, and an average shift attributable to the bargaining strength of a particular year. Similarly the monthly dummy variables capture seasonal shifts and an average shift attributable to bargaining for those months. The Durbin–Watson statistic \((D–W)\) in this case is not a meaningful indicator of first-order serial correlation. The statistical procedure used to calculate \(D–W\) accounted for the discontinuous data.

**Intertemporal Pricing Rules**

Monthly pricing rules can be developed using competitive and monopoly theory. The competitive rule is that prices should vary from month to month by a premium for the costs of storage including market risk. It can be inferred from a number of studies of apple storage that storage costs are mostly fixed in nature (see, for example, Mathia and Pasour 1968, Lee and Jack 1974, Kenyon 1971, Mathia and Beals 1977, Beatie 1972, 1981, and Valentine and Mullen 1979). Monthly variable costs are small.

The June price \((PA_1)\) or in the case of the monopoly model the June marginal revenue \((MRA_1)\) was treated as a base opportunity cost, incorporating fixed storage costs, of supplying apples for storage in subsequent months. A characteristic of the mark-up model outlined below was that the previous period's price \((PA_{m-1})\) embodied all of the relevant costs for the current month.

Where appropriate a monthly cumulative cost was included in the optimising algorithms. This cost \((C_m)\), represented the costs of operating a coolstore for a month, this was estimated by Tunstall (1982) to be $0.003 per kilogram of fruit (expressed in real 1971 dollars). The impact of such a small charge on the optimum storage release strategies was negligible.

Using competitive theory and the data shown in Table 1 it is difficult to support a hypothesis of competitive pricing throughout the data period. The inconsistency between competitive theory and the observed data became apparent when an attempt was made to simulate the competitive market using Linear Programming. However, this study was limited to those seasons where statistically acceptable demand functions were estimated. These were seasons of high price
variability which was inconsistent with the hypothesis of competitive storage. Given the shortcomings of this study it is impossible to rule out competitive pricing in some years where prices were comparatively stable.

But the observed patterns of intertemporal prices reported in Table 1 could be due to monopolised storage of apples because month to month price differences under monopoly depend on the ratio of price to elasticity as well as storage costs as shown in equation (2).

\[
P_{m} - P_{m-1} = (P_{m} - 1/E_{m-1}) - (P_{m} E_{m}) + C_{m}
\]

where \(E_{m}\) is the price elasticity of demand for month \(m\).

Depending on the relative prices and elasticities and their seasonal trends, prices under a monopolist could be less variable or more variable than under competitive storage. However, for the observed pattern of prices to be due solely to monopolised storage improbable dramatic swings in elasticity would need to be assumed.

In this paper a third option is considered, namely that of a mark-up type of pricing pattern. Scitovsky (1952) developed a theoretical construct for determining optimal profit margins for a price-maker which is applicable to many dimensions of the price-maker’s offer. Scitovsky (1952, p. 259) has commented that ‘the behaviour of businessmen in many markets is known to conform to this pattern’.

Following Scitovsky (1952, p. 251), the monthly price/quantity variation cost \((V_{m}C)\), defined to be the loss of receipts due to the price or quantity variation made in order to raise current sales by one unit, can be expressed as follows:

\[
V_{m}C = -P_{m}E_{m}
\]

In an intertemporal context optimal profit margins are obtained where the price difference (net of marginal storage costs) is equated with the variation cost:

\[
P_{m} - P_{m-1} = -P_{m}E_{m} + C_{m}
\]

This intertemporal pricing rule differs from the monopolist’s rule (equation 2) in that the margin or mark-up for the current month depends only on conditions (price, elasticity and cost) for the current month and is determined independently of conditions for other months.

The monopoly and mark-up pricing rules can be applied to the apple storage problem. The question of interest is which theoretical pricing pattern best fits the pattern of actual prices. The intertemporal pricing rules can be compared with actual prices using an optimising algorithm similar to that used by Piggott (1975). The optimising algorithms are formulated using the estimated demand functions as data. This process and the results are described in the next section.

**Optimal Allocations using Linear Programming**

The LP method used by Piggott (1975) was adapted by first casting the problems in a Lagrange Multiplier format. This is possible because in the simple model outlined here only equality constraints are involved.

The Lagrangian expression for the monopolist storer can be written:
(5) \[ TP = \sum_{m=2}^{6} d_m QA_m (PA_m - C_m) + L \left( TS - \sum_{m=2}^{6} QA_m \right), \]

where \( TP \) is the Present Value \((m = 2)\) of Total Profit, \( d \) is the monthly real discount factor, \( L \) is the Lagrangian Multiplier, and \( TS \) = Total Sales (constrained to equal actual sales).

The summation is from month 2 to 6 \((i.e. July to November)\) because predicted marginal revenue from the June market \( MRA_1 \) is used as a marginal opportunity cost of sales in the optimising algorithm \([\text{see (6)}\] below). For the purpose of this study, considering the real interest rates for the period, \( d_m \) is assumed to be unity. The consequences of using other estimates of \( d_m \) within realistic ranges were negligible.

Assuming \( d_m \) to be unity, substituting for \((PA_m - C_m)\) in (5), and taking first partial derivatives yields the following necessary conditions for a maximum,

(6) \[ \frac{\partial TP}{\partial QA_m} = A + 2b_m QA_m - (MRA_1 + 0.003(m - 1)) - L = 0 \]

for \( m = 2 \ldots 6 \), and

(7) \[ \frac{\partial TP}{\partial L} = TS - \sum_{m=2}^{6} QA_m = 0. \]

This yields a set of \( n + 1 \) equations with \( n + 1 \) unknowns, where \( n \) = the number of storage markets \((i.e. five excluding June)\).

The mathematical determination of the second order (sufficient) conditions is complex \((Bresler 1975, p. 316)\). But as discussed by Waugh et al. \( (1936)\), it can be shown that the conditions will be met where the returns curves from all markets are convex, as were the linear demand curves used in this study.

In this case Linear Programming was used directly to solve the above system of equations developed using the Lagrange multiplier algorithm. This was done by setting the objective function as follows:

Minimize \( Z = L \),

subject to the first order conditions specified in equations (5) and (7) above. Minimizing \( L \) effectively minimizes the difference between the constrained allocation and the time optimum. Interpretation of the Lagrangian multiplier is taken up later in the paper.

The matrix was constructed by substituting the simplified monthly demand functions \((Table 3)\), in this example using data for 1976, and taking the given level of stocks \((TS)\) to be 6·70 kg/head, the actual sales for 1976. \( QA_1 \) is set at a predetermined level of \((1·49 \text{ kg/head})\), the actual storage release for June 1976, in order to establish a temporal opportunity cost \((marginal \text{ revenue forgone})\) at the appropriate level for June 1976. The problem matrix is shown in Table 4.

\begin{table}[h]
\centering
\caption{Estimated Simplified Demand Functions (1976)}
\begin{tabular}{llll}
June  & \( PA_1 = 23.4 - 1.5QA_1 \) & September & \( PA_4 = 33.5 - 5.7QA_4 \) \\
July  & \( PA_2 = 26.4 - 2.9QA_2 \) & October    & \( PA_5 = 34.1 - 4.3QA_5 \) \\
August & \( PA_3 = 29.7 - 3.6QA_3 \) & November   & \( PA_6 = 36.4 - 5.9QA_6 \) \\
\end{tabular}
\end{table}
TABLE 4

Problem Matrix

<table>
<thead>
<tr>
<th>Row name</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Column names</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
<th>L</th>
<th>RHS</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.00</td>
<td>-6.33</td>
</tr>
<tr>
<td>September</td>
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<td></td>
<td>-11.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.00</td>
<td>-10.08</td>
</tr>
<tr>
<td>October</td>
<td>3.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.00</td>
<td>-10.73</td>
</tr>
<tr>
<td>November</td>
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<td>-8.62</td>
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<td>-1.00</td>
<td></td>
<td></td>
<td>-1.00</td>
<td>-12.99</td>
</tr>
<tr>
<td>TS</td>
<td>1.00</td>
<td>1.00</td>
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<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
<td>1.49</td>
<td>1.00</td>
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<td>QA_1</td>
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<td></td>
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</tr>
</tbody>
</table>

The optimal solution for the monopolist storer is compared with actual sales in Table 5. In this table the solution with the total sales constraint relaxed is also presented.

Of interest is the result that the monopolist storer would have reallocated sales from the early and mid part of the season to the later part of the season. The monopolist storer could have increased profits by selling some 2.15 kg/head more apples over the storage season than were actually sold. It should be pointed out, however, that 1976 was a year of abnormally low production of apples. A characteristic of apples produced that year was that they lacked certain keeping qualities which would have made the optimal solution of increasing late season sales difficult to attain. An alternative way of viewing this outcome is that the cost of storage (the deterioration in quality) is understated in the algorithm.

TABLE 5

Actual and Optimal Release from Storage-Monopoly Model

(kg/head)

<table>
<thead>
<tr>
<th>1976 Storage season</th>
<th>Actual sales</th>
<th>Optimal sales (constrained)</th>
<th>Optimal sales (unconstrained)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>1.19</td>
<td>0.70</td>
<td>1.32</td>
</tr>
<tr>
<td>August</td>
<td>1.04</td>
<td>1.01</td>
<td>1.51</td>
</tr>
<tr>
<td>September</td>
<td>1.16</td>
<td>0.97</td>
<td>1.28</td>
</tr>
<tr>
<td>October</td>
<td>0.99</td>
<td>1.35</td>
<td>1.76</td>
</tr>
<tr>
<td>November</td>
<td>0.83</td>
<td>1.19</td>
<td>1.49</td>
</tr>
</tbody>
</table>

|                      | 5.21         | 5.22                         | 7.36                          |

Mark-Up Model

In this model the economic gains or profits from storage, \( GS \), ignoring discounting and the constant marginal cost, are defined as

\[
GS = \sum_{m=2}^{6} QA_m(\bar{P}A_m - \bar{P}A_{m-1}).
\]

This is an appropriate objective function given that the previous period's price embodies all of the variable costs associated with
intertemporal variation in the quantity-fixer’s offer. A total sales constraint can be applied and the gain from storage function converted to a Lagrangian Function in much the same way as was done with the monopoly model. In the mark-up model the relationship between the constrained mark-up, the price flexibility of demand \((F)\) and the Lagrangian Multiplier \((L)\) is,

\[
(\bar{P}_A - \bar{P}_A_{-1})\bar{P}_A = \left| F_m \right| + L/\bar{P}_A
\]

In this form the mark-up is expressed as a proportion of the current price and is equal to the price flexibility of demand, corrected by a factor \((L/P_A)\) reflecting the degree of suboptimality imposed by the joint constraints of supply (stocks) and the positions of the demand curves (relative bargaining strength).

The optimal storage release or monthly sales pattern for 1976 is shown in Table 6. This shows a quite different pattern of sales from that shown in Table 5 for the monopoly model. It is acknowledged, however, that both sales patterns are probably not significantly different from actual sales because of the variance which surrounds the demand estimate. Nevertheless a comparison made between the models as to their relative ability to forecast actual sales showed that the mark-up model was marginally superior. The details of the tests are reported in Tunstall (1982).

<table>
<thead>
<tr>
<th>1976 Storage season</th>
<th>Actual sales</th>
<th>Optimal sales (constrained)</th>
<th>Optimal sales (unconstrained)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>1.19</td>
<td>1.32</td>
<td>0.93</td>
</tr>
<tr>
<td>August</td>
<td>1.04</td>
<td>1.29</td>
<td>0.83</td>
</tr>
<tr>
<td>September</td>
<td>1.16</td>
<td>0.93</td>
<td>0.59</td>
</tr>
<tr>
<td>October</td>
<td>0.99</td>
<td>0.95</td>
<td>0.46</td>
</tr>
<tr>
<td>November</td>
<td>0.83</td>
<td>0.73</td>
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</tr>
<tr>
<td></td>
<td>5.21</td>
<td>5.21</td>
<td>3.17</td>
</tr>
</tbody>
</table>

*The Interpretation of the Lagrangian Multiplier and its Relationship to the Marginal Value Product of Limiting Resources*

In the general problem the Lagrangian Multiplier can be interpreted as a shadow price or opportunity cost. It is a measure of the extent to which an imbalance of resources such as an excess or shortage of stocks prevents the attainment of a true maximum. It therefore represents the potential increase in the objective (profit) function obtainable from an appropriate marginal change in certain resources. A more specific interpretation of the Lagrangian, and indeed the Marginal Value Product (MVP) of limiting resources, depends on considering the problem in context.

In the simple problem described above, the Lagrangian Multiplier \((L)\) is constrained to take the same value for all months, thereby marginalist pricing is maintained. The marginal cost of an excess or shortage of stocks is equated across all markets. This interpretation of the Lagrangian Multiplier applies equally to both LP solutions.
There is also an interdependence between the LP problem specification and the interpretation of the MVPs of limiting resources. In the LP problem described above the objective is to the absolute value of $L$. A zero value of $L$ can only be attained if the level of stocks (sales) allows the attainment of a true maximum level of profit, or where the constraint is not binding (i.e. abandonment is allowed). In these cases the MVP of extra units of resource will be zero. However, where the total sales constraint is binding, in the sense that it restricts the attainment of a true maximum (as in this example), then the MVPs will be non-zero.

Consider the situation depicted in Figure 1. $DD$ represents demand in any storage month and $DM$ is the respective marginal revenue curve. The curve $CC$ represents the appropriate level of marginal revenue (marginal cost) for optimal pricing and sales. However, the constraint on sales is such that prescribed sales are at $QQ$ rather than $Q'Q'$ and the Lagrangian Multiplier ($L$) is shown by the vertical distance $C'C$ which results from subtracting $DM$ from $CC$ at the quantity $QQ$ (see equation 10).

![Figure 1 - The Lagrangian Multiplier and its relationship to MVP of limiting resources.](image-url)
In this situation profits will not be maximised unless the constraints posed by at least one of the curves $DM(DD)$, $CC$ or $QQ$ are relaxed. In fact profits will be maximised, other things being equal, in any situation where $L$ has a zero value. A direct outcome of relaxing the constraint on stocks or allowing abandonment is to achieve such a result by reducing sales from $QQ$ to $Q'O'$. However, through the use of LP other means are suggested. These are indicated by a shift in the appropriate curves to new positions, $C'C'$ and $D'M' (D'D')$ respectively. The MVPs generated by the LP procedure indicate the worth of achieving those shifts; that is, a parallel increase in demand or a reduction in cost.

Piggott (1975, p. 21) deemed these 'shadow prices' to be 'of little economic interest', since in his model they represented changes in Marginal Net Revenue of a particular period rather than changes in aggregate net revenue. However, it is of interest here that such parallel shifts in demand may result from advertising or promotion, improved bargaining over prices (in the context of wholesale markets), or in improvements to apple quality. Further, the concept of the 'MVP of a shift in demand' can be related to Scitovsky's (1952) 'variation cost concept' and the approach to optimal advertising expenditure taken by Dorfman and Steiner (1954). Hence, in many situations the non-zero MVPs generated from LP solutions to problems such as this could be of economic interest.

The MVPs for the exemplar case of the monopoly model, using 1976 data, were July 0.29, August 0.23, September 0.15, October 0.19, and November 0.14. The MVPs in this case measure the change in the value of $L$ from a unit increase in resource.

When constrained profits are at a maximum

(10) \[ L = MRA_m - MRA_1. \]

That is, $L = \text{Marginal Net Revenue}$, since $MRA_1$ (from equation 5) represents the intertemporal opportunity cost ($CC$ in Figure 1) in the problem as specified. The constrained allocations are therefore suboptimal in the sense that optimisation would require zero marginal net revenue in all markets, giving a true maximum net revenue.

In the constrained optimisation of this study some useful points emerge. The MVPs were generally higher in the early months (when demand curves are further to the left) and hence attention to factors which lead to rightward shifts in demand is relatively more profitable in these months. This infers that the established industry practice of marketing controlled atmosphere apples earlier is likely to be the correct policy since such an activity leads to an improvement in the quality of apples sold in earlier months.

The fact that the MVPs tend to decline over the storage season could be interpreted as an indication of diminishing returns to bargaining over the wholesale/retail margin. Indeed, if orchardists are forced to determine the storage release or intertemporal allocation in advance, say in June, then the contrast between optimal and actual allocations provides only part of the intertemporal marketing picture. The other part is provided by the dual solution (i.e. the MVPs of the limiting resources), reflecting the potential gains from efforts to shift marginal revenue by bargaining during the season, given the limited freedom orchardists may have over the storage release decision.
Summary and Conclusions

In this paper the application of optimising algorithms to the problem of intertemporal pricing and storage of Victorian apples has been illustrated using a simplified example. It must be acknowledged that there is a measure of uncertainty about the results reported and the conclusions which can be drawn from them, but it is possible to draw some conclusions about the formation of apple prices. It was argued that, under reasonable assumptions, the prices and sales of stored apples in those years were not typical of a competitive market.

Two alternative market structures were examined. The monopoly model and the 'mark-up' model. These models performed better as predictors of actual sales than the competitive model. In their simple form both were 'unbiased' predictors. As a rule in four months out of five actual sales and prices tended to fall between the prescription of the two alternative market structures. However, the prediction errors with the monopoly model were larger than those with the mark-up model.

Modelling of market behaviour as a static structure in the storage market for apples is unlikely to capture evolution in the market or the opportunistic behaviour of market participants as market conditions change. Real markets are likely to display a hybrid set of pricing practices rather than exhibit solely the characteristics of a monopoly or competition. In some circumstances one might expect that customary mark-up behaviour will prevail in the market for stored apples. At other times monopolistic behaviour among storers is encouraged by the level of supplies and when the otherwise loose ties of the OFCSA are strengthened by conditions favourable to compliance with Association advice in particular seasons.

Identification of the structure which most closely mirrors the essence of actual market behaviour is further blurred by the level of aggregation in the modelling process which was constrained by available data. The major sources of these aggregation problems relate to the technical conditions of storage and the varietal mix of the crop.

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