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THE DEMAND FOR MILK IN AUSTRALIA ESTIMATION OF PRICE AND INCOME EFFECTS FROM THE 1984 HOUSEHOLD EXPENDITURE SURVEY*

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Cross-sectional data are used to estimate a three-equation generalised addilog demand system (GADS); two equations are used to express the demand for milk by method of sale and a residual equation is used to close the system. It is shown that, as the average budget share of the residual equation approaches unity, the GADS equations for the incomplete system are approximately equivalent to double logarithmic equations. It is found that aggregate milk demand is relatively insensitive to both price and income, but the degree of substitution between delivered and non-delivered milk is both large and highly significant. A new test for influential data in the system context is developed and it suggests that the reported results are robust to variations in the sample space.

Grouped data from the 1984 Australian Household Expenditure Survey have been made available in two separate one-way classifications. Household expenditure on a vast array of goods and services has been classified by household income for Australia as a whole, and by state of residence of the household. This paper combines these two one-way classifications of grouped data to estimate both price and income effects for milk demand at this single point in time. The variation in the sample that is necessary to estimate price responses at a single point exists because milk prices have been subject to controls determined separately in each state.

The aim of the paper is to describe the demand for fresh milk and cream, both delivered to the home and purchased from other outlets. When an incomplete demand system such as this is of interest, a number of modelling approaches are possible. At one extreme, a residual item 'all other expenditure' can be defined to close the system or, at the other end of the spectrum, separability assumptions can be invoked to enable the subsystem to be estimated in isolation. As Fisher (1979) discussed, it is common practice in agricultural economics to describe demand allocation with an incomplete system and to utilise a double logarithmic specification. This approach occupies the middle ground; the relationship between certain commodity demands and total expenditure is estimated with the excluded category being implicitly defined. In this paper, it is demonstrated that the incomplete double logarithmic demand model is the limiting case of the generalised addilog demand system (GADS) as the proportion of the household budget spent on the residual approaches unity.[1]

The twin problems of autocorrelation and multicollinearity are particularly prevalent in time-series demand applications. The effects of

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[1] The GADS was developed in Bewley (1982*a*, 1986) and Bewley and Young (1987).

both of these econometric problems are minimised in this study by considering a single cross section and commodities that exhibit substantial spatial price variation.[2] Although only one-way data classifications are available, an assumption of independence in the formation of the classifications renders the generalised least squares (GLS) estimator feasible and, indeed, Bewley (1987) showed that two one-way classifications can be efficiently estimated in such circumstances by the modified weighted least squares procedure described below.

The Milk Market

Although the price of milk is controlled in each state, the price level differs widely between states. The interstate range in 1984 was 64c/L to 77c/L in supermarkets and 63.3c/L to 73.3c/L for home-delivered milk.[3] Moreover, the differential between the price of milk delivered to the home and that available at supermarkets, for example, varies widely across the country.

The year of 1984 marked the beginning of a new era in the milk industry, particularly in Victoria and New South Wales. A dispute arose in April of that year between these two states that led to a reorganisation of milk control in New South Wales. This relaxation in the degree of control manifested itself in two forms. First, there was a change from a fixed milk price in New South Wales prior to 6 July 1984 to a minimum-maximum price range that allowed retail outlets much greater flexibility in pricing policies. This pricing decision was accompanied by changes in the conditions of distribution. Second, supermarket chains won certain rights that had the effect of introducing a 2 L plastic container of milk at a reduction of 5c/L over the standard 1 L price. It was this reorganisation of the milk market that allowed supermarket chains in New South Wales to compete effectively on price and contribute to the ensuing major shifts in the demand for milk by product type.[4] However, owing to consumer inertia, the composition of milk demand in New South Wales did not alter greatly during 1984 but after 12 to 18 months, a substantial switch to 2 L containers had been established. Thus, an analysis of 1984 data collected over the year is likely to reveal the

[2] Of course, spatial autocorrelation can exist in a cross-sectional sample and it can be argued that dependence in the residuals is a result of using an inappropriate functional form. See Prais and Houthakker (1955).

[3] Interestingly, the price of milk in 600 mL bottles in New South Wales in March 1984 (just prior to the change in pricing policy) was 40c but 43c for the same sized carton. The former container was essentially the domain of home-delivered milk. The prices of milk published for the same point in time by the Australian Bureau of Statistics (1984) were 82c for 2×600 mL bottles delivered and 70c for 1 L carton in supermarkets. Since the latter survey price was indeed the set price, the former discrepancy could be thought of as a 1c per bottle delivery charge. The seven pairs of survey prices (six states plus the Australian Capital Territory) indicate that supermarket milk was cheaper on a cents per litre basis in only two cases.

[4] For example, prior to the Victorian milk dispute of April 1984, the 600 mL bottle, the dominant form in home-delivered milk, constituted 24 per cent of the New South Wales market and 2 L cartons held 4 per cent. After the introduction of the 2 L plastic container and the accompanying change in pricing policies, the combined market share of 2 L containers (cartons and plastic) rose to 38 per cent by the end of 1986 while 600 mL bottle sales only held 15 per cent of the market at the same point in time (NSW Dairy Corporation, personal communication).

existing patterns of demand rather than the direction of future demand.

The Household Expenditure Survey

For Australia as a whole, data on individual households were classified into ten income groups and the average weekly household expenditure was recorded, as was the expenditure on a number of specific goods and services. The two expenditure items of interest are 'fresh milk and cream expenditure' classified by whether or not the goods were delivered. Unfortunately, it is not possible to isolate the fresh milk component, for which there is good price data, from the milk plus cream expenditure. However, milk consumption dominates the aggregate data on the combined component and, therefore, the results can be interpreted broadly as being representative of the demand for fresh milk alone.[5] Certain related information, such as household size, is also recorded.

The ten pieces of data grouped by income present information in only one direction and no price effects can be ascertained from this set. However, these data can be supplemented by a further eight observations that are grouped not by income but by the state (or territory) of residence of the household. Although there is little income variation in this direction, there are large differences between the general level of milk prices in the various states. Moreover, the price differential between home-delivered milk and milk purchased from other outlets varies between states. It is this interstate price variability that enables price responses from survey data to be measured.

The complete data set consists of 18 observations and is made up from two one-way classifications. All 18 observations are analysed within a single model but it is clear that the spatial classification amounts to grouping by price and this section of the data is the driving force behind the estimates of price responsiveness. On the other hand, the income distribution classification component naturally dominates the data set in terms of income variability and these data provide the necessary information for the income response estimates. Thus, there are data groupings for the two standard determinants of demand.

The price data are taken from the Australian Bureau of Statistics (1984). In that publication, the prices for two 600 mL bottles (delivered) and a 1 L carton in supermarkets are given. Appropriate data for cream are not available. It should be noted that the 'delivered' price in Darwin has never been made available.

It is usual in Engel curve analysis to assume that all consumers face the same set of prices. In the current context, good price data are available for milk but not for 'other goods and services'. Thus, the standard practice of assuming no price variation is followed for the 'other goods' category. Using the consumer price index for the residual

[5] In 1984–85 in New South Wales, total market milk sales amounted to 528·5 ML while the sales of sweet cream in the same period amounted to only 5·4 ML or about 1 per cent. Flavoured milk accounted for 5 per cent of market milk (NSW Dairy Corporation, personal communication).

category was considered but that index is a temporal and not a spatial measure of price variability.[6]

The Model

The GADS is a development of Theil's (1969) multinomial logit model and is founded on two important principles. First, the model ensures that the sum of the component demands is identical to total expenditure, the so-called adding-up property. Second, the nature of the functional forms ensures that the implied demands for all goods are strictly positive and this feature can be of particular importance when narrowly defined and variable components are considered.[7]

The specific form of the GADS is, from Bewley (1986) and Bewley and Young (1987):

$$(1)\quad \bar{w}_i \ln(q_{it}/w_t^\dagger) = \alpha_i + \theta_i \ln(y_t/P_t) + \sum_{j=1}^{n} \Pi_{ij} \ln(p_{jt}) + v_{it} \qquad \begin{matrix} i=1,\ldots,n \\ t=1,\ldots,N \end{matrix}$$

where $\ln(P_t) = \Sigma \bar{w}_j \ln(p_{jt})$ is an overall price index; p_{it}, q_{it}, and $\bar{w}_i$ represent the price, quantity demanded and average budget share (at the mean) of the i-th good for household (group) t ($t=1, \ldots, N$); $y_t = \Sigma_i p_{it} q_{it}$ is total expenditure; θ_i and Π_{ij} are marginal budget shares and Slutsky parameters evaluated at $\bar{w}_i$ ($i=1, \ldots, n$); $w_t^\dagger$ is the weighted geometric mean of the average budget shares also evaluated at $\bar{w}_i$ ($i=1, \ldots, n$); and v_{it} is a disturbance term with the usual properties. Providing that the n goods in (1) exhaust total expenditure, adding-up implies that $\Sigma_i \theta_i = 1$; $\Sigma_i \alpha_i = 0$; and $\Sigma_i \Pi_{ij} = 0$ ($j=1, \ldots, n$). Homogeneity implies that $\Sigma_j \Pi_{ij} = 0$ ($i=1, \ldots, n$) and Slutsky symmetry implies that $\Pi_{ij} = \Pi_{ji}$ ($i>j=1, \ldots, n-1$).

The presentation of the model in (1) gives the GADS strong similarities to more standard demand systems such as Barten (1964) and Theil's (1965) Rotterdam system or Deaton and Muellbauer's (1980) Almost Ideal Demand System (AIDS). However, the complex nature of the regressands in (1) ensures that the average budget shares are strictly positive. The model underlying equation (1) is Theil's (1969) multinomial extension of the linear logit model:

$$(2)\quad w_{it} = \exp[a_i + \Sigma_j b_{ij} \ln(p_{jt}) + c_i \ln(y_t) + u_{it}] / \Sigma_k \exp[a_k + \Sigma_j b_{kj} \ln(p_{jt}) + c_k \ln(y_t) + u_{kt}]$$

where a_i, b_{ij} and c_i are fixed parameters and u_{kt} is a disturbance term. The Slutsky parameters and marginal budget shares for equation (2) are given by the expressions

$$(3)\quad \Pi_{ijt} = w_{it}\varepsilon_{ijt} + w_{it}w_{jt}\mu_{it}$$

and

[6] I would like to thank a referee for having pointed that out the consumer price index is inappropriate in this context. The Australian Bureau of Statistics does not produce a spatial price index for all goods but it does give the prices of selected grocery items. Given the restricted nature of this selection, and problems of defining quality across states, it is preferable to assume that there is no price variability in 'other goods' across states.

[7] Zero expenditures are not permitted in the GADS or multinomial logit model. Although this can hamper the analysis of raw data, zero expenditures are uncommon in grouped data with frequently purchased goods or broad categories.

(4) $\theta_{it} = w_{it}\mu_{it}$

where

(5) $\varepsilon_{ijt} = b_{ij} - \delta_{ij} - \Sigma_k w_{kt} b_{kj}$

and

(6) $\mu_{it} = 1 + c_i - \Sigma_k w_{kt} c_k$

and $\delta_{ij} = 1$ if $i = j$ and 0 otherwise. Clearly the expressions in (3) to (6) depend on the budget shares w_{it} $\{i = 1, \ldots, n\}$. For the purpose of estimation and the presentation in (1), the budget shares in equations (3) to (6) are replaced by some appropriate values, in this case the mean values. The mean elasticity estimates are denoted by Π_{ij}, ε_{ij}, θ_i, and μ_i, that is, with the 't' subscript removed. Note, however, that the underlying estimates of a_i, b_{ij}, and c_i do not depend on the values chosen for w_{it}. A major purpose of using (1) is to be able to estimate directly the elasticities rather than the fixed parameters, a_i, b_{ij} and c_i.

Somermeyer and Langhout (1972) showed implicitly that the multinomial logit elasticities have special properties. Notably, the total expenditure elasticities decline monotonically with increasing income and at the same rate for each good. Similar results flow for the price elasticities.

It follows from equations (3) to (6) that there is a one-to-one relationship between equations (1) and (2) when mean average budget shares are utilised. Equation (1) offers a more convenient framework for estimation, particularly with regard to the imposition of Slutsky symmetry, but it must be stressed that, as in equation (2), Slutsky symmetry can hold only at a single point in the sample space. Therefore, the underlying estimates for a_i, b_{ij} and c_i depend on the values chosen for $\bar{w}_i$ if symmetry is imposed; unconstrained or homogeneity-constrained estimates of a_i, b_{ij} and c_i are independent of the choice of values for $\bar{w}_i$.

The GADS is not a flexible functional form in the sense of the AIDS or Christensen, Jorgenson and Lau's (1975) translog model. That is, it does not provide an arbitrary second-order approximation to either a utility or a cost function. Nevertheless, empirical comparisons of functional forms conducted in Bewley (1982*a*, 1982*b*, 1986) suggest that the GADS performs well in an empirical sense and can outperform the more conventionally based models.

The two milk expenditure components clearly do not exhaust total expenditure but a very broad category, 'all other goods and services', which is the difference between total household expenditure and the sum of the two milk components, can be defined to close the system. The definition of such a dominant component leads to an interesting interpretation.

As w_{nt} approaches unity for all t, $\ln(w_t^{\dagger})$ approaches zero and $\ln(P_t)$ approaches $\ln(p_{nt})$. Thus, equation (1) can be approximated by:

$$(7) \quad \ln(q_{it}) = \tau_i + \mu_i \ln(y_t) + \Sigma\,\varepsilon_{ij} \ln(p_{jt}) + \mu_{it} \qquad i = 1, \ldots n$$

where μ_i is the total expenditure elasticity of the i-th good and ε_{ij} is a price elasticity. The important difference between the double logarithmic model (7) and the GADS (1) is in the *interpretation* of the estimated coefficients. In the case of (7), the elasticities are assumed constant over the entire sample but in (1) the elasticities evolve with total expenditure and price. If the GADS is estimated at the mean, that is w_{it} in (3) to (6) are replaced by the mean average budget shares, the results of the two specifications are liable to be very close. However, in the extreme, major differences can occur if the residual component is not dominant. For example, all of the Engel curves from the double logarithmic model are monotonic functions of total expenditure but asymptotes and turning points can exist for different goods within a single estimated GADS.[8]

Estimation

The three-equation GADS was estimated using two one-way classifications from the 1984 Australian Household Expenditure Survey. Houthakker (undated), Haitovsky (1973), and Maddala (1977) discussed the problems of estimating equations from multiple one-way classifications of grouped data. Because each individual record is included in each one-way classification, there is a double counting problem and, consequently, the disturbance covariance matrix for a single equation is singular. This problem is distinct from the cross-equation singularity that always exists in a demand system as a result of adding up.

In the case of a single one-way classification, it is appropriate to use weighted least squares to remove the heteroscedasticity that flows from the different number of households in each group of the classification. When a two-way classification is available, the joint frequency distribution can be used in a GLS framework to produce an efficient estimator.[9] However, the current case amounts to two marginal distributions of a two-way classification and this presents two problems. First, the covariance between the (grouped) observations is unknown and second, every piece of raw data is counted twice.

In the current situation it is not unreasonable to assume that the manner in which the data are grouped, that is by geographic location and by income, are independent drawings from the population. Therefore, the unobserved joint frequency distribution has non-zero cells only to the extent that there are common randomly drawn records from each classification in the one cell. As previously noted, the double counting of observations renders the within-equation disturbance covariance matrix of the grouped data singular and this degeneracy is distinct from the typical across-equation singularity in demand analysis.

If the underlying disturbances of the model for the N pieces of raw data are homoscedastic and the $r\times 1$ and $s\times 1$ diagonal matrixes F and G, respectively, contain the marginal frequency distributions for the two classifications, the within-equation covariance matrix of the $(r+s)$ grouped observations is

$$Q=\begin{bmatrix} F^{-1} & N^{-1}ij' \\ N^{-1}ji' & G^{-1} \end{bmatrix}$$

[8] See Somermeyer and Langhout (1972) and Bewley (1986).

[9] Note also that efficient in this context relates to other grouped estimators; it is always more efficient to use the raw data when they are available.

where i and j are $r \times 1$ and $s \times 1$ vectors of unit elements respectively. Q is singular owing to the double counting of the raw data and it is necessary to delete one grouped observation before GLS estimation.

Bewley (1987) proves the GLS estimator that takes full account of the non-zero off-diagonal elements in Q after deleting one observation is identical to performing weighted least squares on the full $(r+s)$ observations providing that one of the classifications is expressed in deviations from mean form. That is, weighted least squares using

$$\Phi = \begin{bmatrix} F^{1/2} & 0 \\ 0 & G^{1/2} \end{bmatrix}$$

as the weighting matrix is identical to using Q with one row and one column deleted in a GLS procedure. It is interesting to note that Maddala (1977) conjectured that such a procedure might have useful properties on the basis of an empirical comparison with other estimators.

Because data on the price of home-delivered milk are not available for the Northern Territory, a dummy variable that takes the value one for the Northern Territory observation and zero elsewhere is also included; this effectively removes the observation from the sample leaving 17 data points. It should also be noted that all expenditures are expressed in per person terms and that no discernible household size effects could be found. Quadratic functions of both the number of adults and the number of children were included in the initial phase of the analysis but all terms were separately and jointly insignificant. The lack of household size effects could, of course, be attributed to the relatively small variations in these variables at the aggregate level and the fact that a third classification based on household size could not be included in the sample because the expenditure data are not collected at the same level of aggregation in that direction.

Because the price of 'other goods' is assumed to be constant across states (and set to unity), this price term degenerates into the constant and homogeneity is not a testable assertion. However, the third price effect is identified under Slutsky symmetry as a result of the adding-up constraint.

In order to form a basis for comparison, the GADS was first estimated without Slutsky symmetry being imposed. It is important to note that the estimates for Π_{ij} and θ_i are estimates that depend on the values chosen for $\bar{w}_i$ $(i=1, \ldots n)$; in the current situation, the Slutsky parameters and marginal budget shares are estimated at the sample means of the average budget shares.

Results

The unconstrained estimate of model (1), based on 18 grouped observations, is presented in Table 1. The values of the mean average budget shares are 0·002952 for delivered milk, 0·008420 for non-delivered milk and 0·988628 for the residual category.

Since a demand model is a complete system, single equation diagnostic statistics such as R^2 and DW have little relevance. Objection to these single equation measures is strengthened in cross-sectional studies. Given that there is no natural ordering to two one-way

TABLE 1

The Demand for Milk: Unconstrained Weighted Least Squares Estimation of the Generalised Addilog Demand System[a]

	Good		
	Delivered milk	Non-delivered milk	Other
α_i	−0·0017	0·0069	−0·0052
	(0·30)	(0·98)	(1·55)
θ_i	0·0012	−0·0018	1·0006
	(1·01)	(1·22)	(1453)
Π_{i1}	−0·0403	0·0306	0·0097
	(5·46)	(3·34)	(2·27)
Π_{i2}	0·0376	−0·0304	−0·0071
	(5·56)	(3·68)	(1·81)
ϕ_i	−0·0060	0·0032	0·0028
	(5·17)	(2·26)	(4·16)
R^2	0·83	0·61	0·99
DW	2·23	1·84	2·42

[a] ϕ_i is the coefficient on the Northern Territory dummy variable. t-ratios are given in parentheses.

classifications, it is difficult to even use the DW test as a test for non-linearity of functional form.[10]

A system R^2 is far more appropriate as a measure of goodness-of-fit than its single equation counterparts but this too presents problems in a demand system. Because there are the same regressors in each equation, the standard R^2 is a function of an asymptotic Wald statistic and this results in a value of R^2 that is biased toward one. More importantly, the standard system R^2 is not invariant with respect to which equation is deleted (see Bewley 1985, 1986 for details). Bewley (1985) discusses three variants of R^2 that are appropriate for a demand system: the Wald-based R^2, the Deaton-based statistic and the likelihood ratio based statistic. The estimates for these statistics are 0·79, 0·57 and 0·57, respectively. Since even the equation-invariant Wald-based statistic tends to produce values much less than those commonly employed in single equations, these R^2 values are thought to be quite respectable.

The striking feature of the coefficient estimates in Table 1 is the size and significance of the Slutsky parameters. These coefficients are translated into uncompensated elasticities in Table 2 and it can be seen that the own-price and cross-price effects within the milk market are extremely large. However, the residual good, although significantly affected by individual milk prices in a statistical sense, is relatively insensitive to such effects, particularly when both milk prices alter by the same amount. The marginal budget share of non-delivered milk is negative at the mean and that for the residual is extremely close to one. This suggests that higher income group households substitute

[10] Prais and Houthakker (1955) suggest using the DW statistic to test for the adequacy of a functional form with a one-way classification.

TABLE 2

Implied Elasticities from the Unconstrained Generalised Addilog Demand System[a]

	Good		
	Delivered milk	Non-delivered milk	Other
μ_i	0·41	−0·22	1·01
ε_{i1}	−13·65	3·63	0·01
ε_{i2}	12·74	−3·61	−0·02
ε_{i3}	0·51	0·19	−1·00

[a] Homogeneity restrictions are applied to define the unidentified ε_{i3}.

non-delivered for delivered milk possibly because increased per person income is often the result of an increased number of household members going to work and having less time to shop.

The unusually high significance of the marginal budget share for 'other goods' also deserves some comment. Because the residual item is so dominant (98·9 per cent), the budget share is almost necessarily close to unity. If the t-ratio as a test against zero is replaced by the more relevant test against unity, the t-ratio is only a modest 0·87. In other words, the data suggest that all extra income is spent on 'other goods', but there is an associated substitution between the two milk types.

It was stated in a previous section that, if the residual good dominates total expenditure, the GADS model for the subsystem is closely approximated by double logarithmic equations. In practical terms, this means that dividing the GADS subsystem equations by the mean average budget share produces equations with similar coefficients to the double logarithmic model. Since the mean average budget share for the residual category is 98·9 per cent, the approximation would be expected to hold in this case. The double logarithmic estimates are presented in Table 3 and the correspondence in the estimates is extremely close for the subsystem effects. Indeed, if Engel curves are defined for the average price levels, it is found that the difference between the two models is less than one cent for a range of per person total expenditure well outside the sample. Even the price–quantity relationships are extremely robust to model choice for a wide range of prices.

There is a growing literature on the problems of testing the demand restrictions of homogeneity and symmetry. Laitinen (1978) and Meisner (1979) drew attention to the problems and showed that a major cause of the high rejection rates is the poor small sample performance of asymptotic test statistics. Bewley (1983) showed that a statistic originally proposed by Deaton (1972) greatly improves the situation when testing symmetry but even with this statistic problems can arise. For example, demand restrictions were rejected in Bewley and Young (1987) until the (vector) autocorrelation in the residuals was removed and they conjecture that autocorrelation is often the cause of the rejection of demand restrictions in a time-series context. Unfortunately testing for vector autocorrelation in demand systems is not very well developed. Theil and Shonkwiler (1986) suggested a bootstrapping

TABLE 3

The Demand for Milk: Unconstrained Weighted Least Squares Estimation of the Double Logarithmic Model[a]

	Good		
	Delivered milk	Non-delivered milk	Other
a_i	−0·6639	0·7856	−0·0919
	(0·33)	(0·91)	(14·23)
μ_i	0·4188	−0·2207	1·0161
	(1·00)	(1·22)	(758·0)
ε_{i1}	−14·0250	3·7680	−0·0096
	(5·48)	(3·39)	(1·16)
ε_{i2}	13·0850	−3·7528	0·0091
	(5·55)	(3·67)	(1·20)
δ_i	−2·0788	0·3909	−0·0004
	(5·16)	(2·24)	(0·32)
R^2	0·83	0·61	0·99
DW	2·23	1·84	1·45

[a] δ_i is the coefficient on the Northern Territory dummy variable. t-ratios are given in parentheses.

procedure but even bootstrapping requires some asymptotic justification.

With the use of a single cross-section and, hence, no time-series autocorrelation in the present study, it is refreshing to note the ease with which symmetry is accepted. The Wald statistic, which is the most upwardly biased of all of the standard asymptotic test statistics, including that favoured by Bewley (1983) for the test of symmetry, is 0·97, compared with the critical value of the $\chi^2_{1\text{ d.f.}}$ distribution at a 5 per cent significance level of 3·89. Furthermore, as can be seen from the constrained estimates in Table 4, all of the diagonal Slutsky parameters are indeed negative and the estimated Slutsky matrix is negative semi-definite of rank 2. Therefore, all of the criteria for utility maximisation are fulfilled by these estimates. The elasticities for the constrained model are highly similar to those presented in Table 2 and a selection of these is presented in Table 6.

It follows from the results thus far that the two categories of milk demand are extremely price sensitive and very close substitutes. The income effects are small and are indeed of opposite signs. It should be stressed, however, that the high degree of price sensitivity does not carry over to aggregate milk demand. It can be noted that the coefficients on milk prices in the first two equations almost cancel and this, together with the insignificant estimates for marginal budget shares, indicates that aggregate milk demand is largely independent of both the (average) price of milk and income.

Testing for Influential Data

One point that has not yet been addressed is whether or not there are state-specific factors that might have biased the estimates. One way to examine this problem in more detail is in the recently popularised

TABLE 4

The Demand for Milk: Symmetry-Constrained Estimation of the Generalised Addilog Demand System[a]

	Good		
	Delivered milk	Non-delivered milk	Other
α_i	−0·0205	0·0071	−0·0050
	(0·43)	(1·20)	(1·83)
θ_i	0·0011	−0·0016	1·0006
	(1·06)	(1·32)	(1755)
Π_{i1}	−0·0436	0·0364	0·0072
	(8·02)	(6·54)	(2·61)
Π_{i2}	0·0364	−0·0305	−0·0059
	(6·54)	(4·38)	(1·92)
Π_{i3}	0·0072	−0·0059	−0·0013
	(2·61)	(1·92)	—
ϕ_i	−0·0059	0·0032	0·0028
	(6·11)	(2·65)	(4·98)
R^2	0·82	0·58	0·99
DW	2·15	1·66	2·21

[a] The coefficient for Π_{33} is derived and no standard error was calculated. Asymptotic t-ratios (using N as the divisor) are given in parentheses.

literature on testing for outliers and influential data. Belsley, Kuh and Welsch (1980) have provided a framework for analysing single equations and Fiebig (1987*b*) has implemented this methodology in an examination of a cross-country demand system, albeit in a single equation fashion. In this study, this methodology is developed specifically to accommodate and exploit the special features of a demand system.

Fiebig (1987*a*) stresses the difference between disparate data and influential data. Disparate data in simplistic terms are data that do not appear to belong to the same population as the remaining data set and influential data are observations that have a strong influence on coefficient estimates and predictions, for example. Data can be disparate without being influential and *vice versa*.

In a single equation with a matrix of regressors X, the diagonal elements of the 'hat' matrix $X(X'X)^{-1}X'$ represent the distance from the centre of gravity of the data, or leverage, while the residuals can be measured in terms of their magnitude measured in standard errors. The associated L–R plot (leverage versus residuals) is one component in the way in which influential data can be detected. In a demand system such as that presented in Table 1, the adding-up condition implies that it is not theoretically plausible for one observation to be influential in only one equation. Either that same observation is influential for one other good or its influence is spread across a number of goods. This suggests the need for a joint test of significance of the residuals in all equations for a single observation. The appropriate small sample test is Hotelling's T^2 and so it is this statistic that is plotted against leverage which is, of course, the same for each good. The L–R plot is presented in Figure 1.

The T^2 statistics can be expressed as an $F(2,12)$ distribution in this case. The critical values at the 5 per cent, 2·5 per cent and 1 per cent levels are 3·88, 5·09 and 6·93, respectively. Quite clearly, the results for Western Australia cause some concern. To a lesser extent the results for New South Wales and Victoria are disparate but the New South Wales result has small leverage and little bearing on the determination of the coefficients. It is interesting to note that the ten income-grouped data points denoted by D1 to D10 for the ten deciles (in order) are not influential but the lowest income group (D1) is perhaps, not surprisingly, the most disparate of this subset.

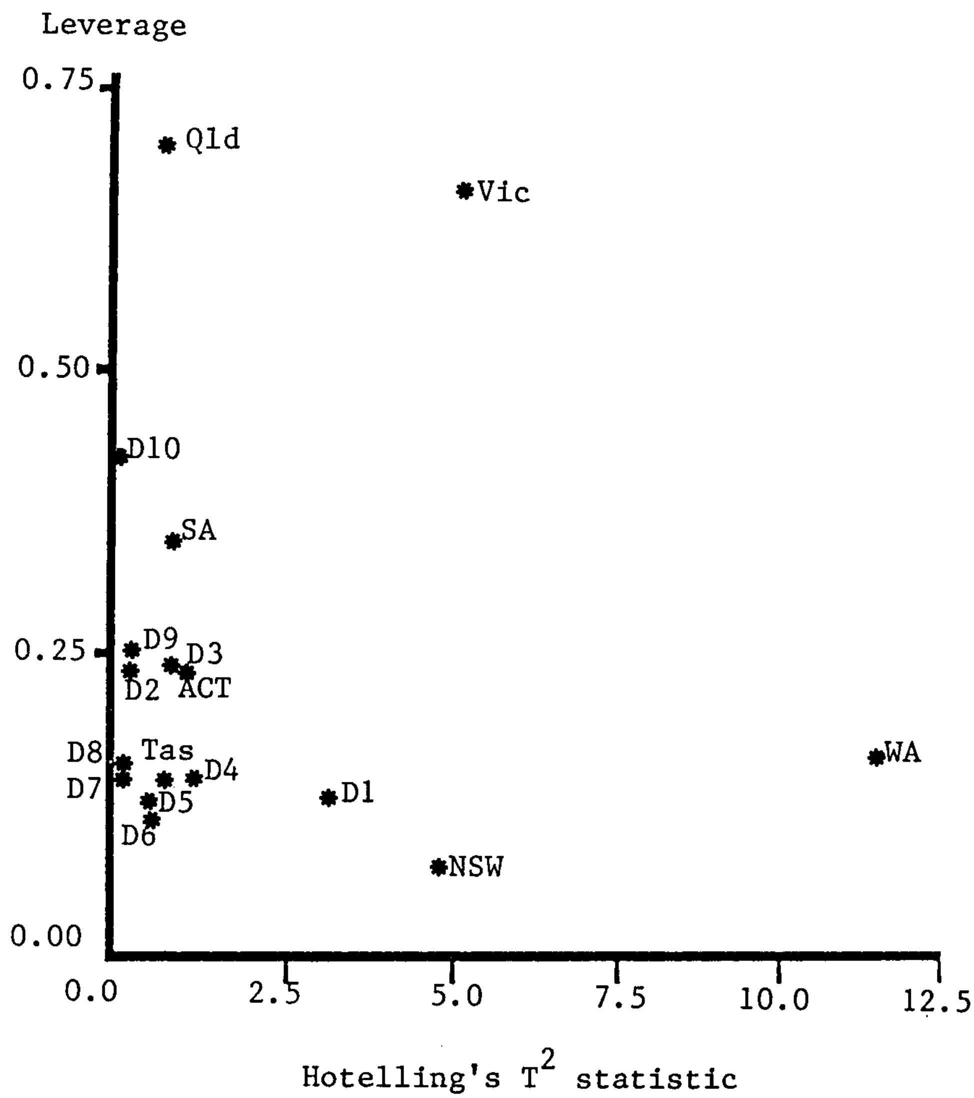

FIGURE 1—Leverage — Residuals Plot.

In order to assess the impact of the two possible offending data points, a subset of the *DBETAS* statistics is presented in Table 5. These statistics are the differences between the coefficients in Table 1 and the corresponding coefficient with the relevant observation being omitted, divided by the standard error of the coefficient from the reduced sample space. The distribution of *DBETAS* is unknown but Belsley et al. (1980) note its 't-like' appearance. They do suggest a critical value of $2/N^{1/2}$ for *DBETAS* but this is for large samples. Given that there are effectively only seven observations on price, it would be unwise to place too much emphasis on the actual values of the tests. It is far more appropriate in this context to examine the relative sizes of the coefficients and the effect on the policy implications of the model.

The *DBETAS* results may be interpreted in the following manner. None of the results for states other than Western Australia and Victoria causes any great concern. The New South Wales observation, although somewhat disparate, has little impact on the value of the coefficients. Perhaps surprisingly, the Western Australian observation does not have a particularly strong effect on the coefficients. On the other hand, one of the coefficients in the delivered milk equation has a *DBETAS* value of 2·18 as a result of the Victorian observation and this suggests some influence from the presence of the Victorian observation.

At this stage of the analysis it is important to distinguish between the statistical and economic implications of influential data on the model. While it is always of interest to consider the statistical properties of an econometric model, the fundamental aim of the analysis is to draw economic conclusions about price and income responsiveness; that is, a loss function based on policy implications is of at least equal importance to the above statistical analysis. The milk price elasticities for the symmetry-constrained data with and without the two disparate observations for Victoria and Western Australia are presented in Table 6.

The results in Table 6 quite clearly demonstrate that the removal of the two most important observations in both a disparate and influential sense has almost no affect on the implications of the model. The fact that the coefficients are very well determined implies that small differences in the estimates appear large in a statistical sense. On the other hand, it matters little for policy purposes whether an elasticity is -12 or -14 since both indicate extreme price sensitivity. Furthermore, the cross-price elasticities are equally robust in an economic sense.

TABLE 5

Testing for Outliers

Omitted state	*DBETAS* Π_{11}	Π_{12}	Π_{22}	θ_1	θ_2	T^2 residual
New South Wales	0·05	0·17	0·01	0·29	0·14	4·75
Victoria	2·18	1·62	0·88	0·27	0·19	5·24
Queensland	0·56	0·74	0·51	0·15	0·17	0·60
South Australia	0·23	0·07	0·16	0·00	0·96	0·78
Western Australia	1·53	1·64	1·20	0·11	0·07	11·67
Tasmania	0·12	0·01	0·04	0·00	0·06	0·72
Australian Capital Territory	0·23	0·10	0·69	0·01	0·11	1·03

TABLE 6

Implied Milk Price Elasticities from the Symmetry-Constrained Generalised Addilog Demand System and the Effect of Outliers

	Good	
	Delivered milk	Non-delivered milk
Full data set		
ε_{i1}	−14·77	4·32
ε_{i2}	12·33	−3·62
Victoria and Western Australia removed		
ε_{i1}	−11·61	3·79
ε_{i2}	10·82	−3·66

Conclusions

Two important conclusions can be drawn from this study. First, if only a small demand subsystem is under consideration, as is the case in Fisher (1979) and Bewley and Young (1987) *inter alia*, the double logarithmic model for the subsystem can produce estimates and implications at the mean, and the extremes of the sample, that are entirely consistent with the generalised addilog demand system that necessarily fulfils the adding-up constraint. Thus, at the very least, preliminary investigations on such subsystems can be readily justified with the simplistic double logarithmic model.

The extremely well-determined price and cross-price elasticities show quite clearly that although aggregate milk demand is relatively inelastic, the average consumer's demand for milk by method of sale (delivered or non-delivered) is extremely price sensitive. Thus, it can be concluded that if the substitution between milk by point of sale is typical of other disaggregations of milk demand using other criteria (by, say, size of container, package type or type of retail outlet) then severe changes in market shares would result from small price differentials. Given the major changes in pricing policies in late 1984 in New South Wales, it is concluded that at least some of the dramatic changes in New South Wales milk market since that time have resulted from changes in pricing policies.

References

Australian Bureau of Statistics (1984), *Average Retail Prices of Selected Items: Eight Capital Cities*, Cat. No. 6403.0, AGPS, Canberra.

Barten, A. P. (1964), 'Consumer demand functions under conditions of almost additive preferences', *Econometrica* 32(1–2), 1–38.

Belsley, D. A., Kuh, E. and Welsch, R. E. (1980), *Regression Diagnostics*, John Wiley, New York.

Bewley, R. A. (1982*a*), 'The generalized addilog demand system applied to Australian time series and cross-section data', *Australian Economic Papers* 21(37), 177–92.

—— (1982*b*), 'On the functional form of Engel curves: the Australian household expenditure survey 1975–76', *Economic Record* 58(160), 82–91.

—— (1983), 'Tests of restrictions in large demand systems', *European Economic Review* 20(1–3), 257–69.

—— (1985), 'Goodness-of-fit for allocation models', *Economics Letters* 17(3), 227–9.
—— (1986), *Allocation Models: Specification, Estimation and Applications*, Ballinger, Cambridge.
—— (1987), Weighted least squares estimation of grouped data. School of Economics, University of New South Wales, mimeograph.
—— and Young, T. (1987), 'Applying Theil's multinomial extension of the linear logit model to meat expenditure data', *American Journal of Agricultural Economics* 69(1), 151–7.
Christensen, L. R., Jorgenson, D. W. and Lau, L. J. (1975), 'Transcendental logarithmic utility functions', *American Economic Review* 65(3), 367–82.
Deaton, A. S. (1972), 'The estimation and testing of systems of demand equations: a note', *European Economic Review* 3(1–4), 399–411.
—— and Muellbauer, J. (1980), 'An almost ideal demand system', *American Economic Review* 70(3), 312–26.
Fiebig, D. G. (1987*a*), *Diagnostic Checking in Practice: The Case of Outliers and Influential Data*, Econometrics Discussion Paper No. 87-01, University of Sydney.
—— (1987*b*), The identification of outliers and influential data in a cross-country demand system. University of Sydney, mimeograph.
Fisher, B. S. (1979), 'The demand for meat — an example of an incomplete commodity demand system', *Australian Journal of Agricultural Economics* 23(3), 220–30.
Haitovsky, Y. (1973), *Regression Estimation from Grouped Observations*, Hafner, New York.
Houthakker, H. S. (undated), The combined use of differently grouped observations in least squares regression, mimeograph.
Laitinen, K. (1978), 'Why is demand homogeneity so often rejected?', *Economics Letters* 1(2), 187–91.
Maddala, G. S. (1977), *Econometrics*, McGraw Hill, New York.
Meisner, J. F. (1979), 'The sad fate of the asymptotic Slutsky symmetry test for large systems', *Economics Letters* 2(3), 231–3.
Prais, S. J. and Houthakker, H. S. (1955), *The Analysis of Family Budgets*, Cambridge University Press, New York.
Somermeyer, W. H. and Langhout, A. (1972), 'Shapes of Engel curves and demand curves: implications of the expenditure allocation model, applied to Dutch data', *European Economic Review* 3(3), 351–86.
Theil, H. (1965), 'The information approach to demand analysis', *Econometrica* 33(1), 67–87.
—— (1969), 'A multinomial extension of the linear logit model', *International Economic Review* 10(3), 251–9.
—— and Shonkwiler, J. S. (1986), 'Monte Carlo tests of autocorrelation', *Economics Letters* 20(2), 157–60.