Fair Value of Whole-Farm and Crop-Specific Revenue Insurance

David A. Hennessy
Professor
Iowa State University
Dept. of Economics
& CARD
578C Heady Hall
Ames, IA 50011-1070
U.S.A.
Phone: 515 294-6171
Fax: 515 294-6336
email: hennessy@iastate.edu

Alexander E. Saak
Research Scientist
Iowa State University
Dept. of Economics
& CARD
565 Heady Hall
Ames, IA 50011-1070
U.S.A.
Phone: 515 294-0696
Fax: 515 294-6336
email: asaak@iastate.edu

Bruce A. Babcock
Professor
Iowa State University
Dept. of Economics
& CARD
578F Heady Hall
Ames, IA 50011-1070
U.S.A.
Phone: 515 294-5764
Fax: 515 294-6336
email: babcock@iastate.edu

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Abstract

The U.S. market in subsidized commodity revenue insurance contracts has expanded rapidly since 1996. By far the most prevalent contract forms are crop-specific, rather than the whole-farm design which has a better claim to being optimal. For an arbitrary acre allocation vector, this paper inquires into absolute and relative determinants of the actuarial costs of these forms. The actuarial value of whole-farm insurance increases under a particular characterization of 'more systematic' risk, whereas the actuarial value of insurance through crop-specific contracts need not change. Upon fixing stochastic revenue interactions, we identify conditions such that the expected cost of revenue insurance via crop-specific contracts is increasing under a more dispersed vector of location-and-scale adjusted revenue guarantee parameters. Then we identify two precise requirements such that the actuarial values of the two contract forms converge. To give insights on the difference, fair premiums for whole-farm and crop-specific contracts are compared for typical farms in two states.

Keywords: acreage allocation, efficiency, insurance subsidies, rate setting, systematic risk

JEL Classification: Q1, D8

Fair Value of Whole-Farm and Crop-Specific Revenue Insurance

Federal involvement in U.S. agricultural output insurance markets has grown to become a major source of subsidies to U.S. farmers. The Agricultural Risk Protection Act of 2000 increased premium subsidy levels and removed the requirement that subsidies be no greater than that available to individuals who purchased traditional yield insurance (APH). This change has increased the demand for non-traditional forms of crop insurance, such as subsidized revenue insurance (RI) contracts. The most popular RI product, Crop Revenue Coverage (CRC), typically has an higher premium than APH. In 2001 and 2002, 47% and 49% of total U.S. crop insurance premiums were spent on the four available forms of RI contract (U.S. Department of Agriculture).

These four RI products were developed in response to two acts of Congress that were passed during the 1990s. Among the reforms introduced in the Federal Insurance Reform Act of 1994 was a provision that authorized the Federal Crop Insurance Corporation (FCIC) to subsidize and re-insure innovative risk management products. The 1996 Farm Bill went further by requiring the FCIC to implement a pilot revenue insurance program. In 1996 the Federal government provided a RI product called Income Protection (IP), while a private insurance company brought CRC to market. In 1997 the Revenue Assurance (RA) product became available, while Group Risk Income Protection (GRIP) made its debut in 1999. Since their introduction, these contracts have expanded to cover almost all states and the more important crops, including, corn, soybeans, wheat, canola, barley, cotton, rice, and grain sorghum.[1]

The vast majority of RI contracts are crop-specific rather than whole-farm, in the sense of Turvey. RA is the only product that offers whole-farm coverage. Premiums covered under RA contracts amounted to 10% of all premium revenue under RI contracts in 2001, and reached 38% in 2002. Whole-farm insurance constituted a tiny share of the RI market as premiums paid for whole-farm contracts comprised 0.8% of premiums under RA contracts issued in 2001. In 2002, premium revenue under whole-farm contracts accounted for 0.2% of that under RA contracts (U.S. Department of Agriculture).

The choice of crop-specific contracts over whole-farm contracts is somewhat surprising because rotation effects make it efficient for many farms to produce a plural number of crops. Hennessy, Babcock, and Hayes (HBH), and also Mahul and Wright (MW), have demonstrated the superiority of whole-farm RI relative to crop-specific RI contracts.[2] As has become standard since early research on the valuation of farm-level revenue insurance contracts (e.g., Turvey and Amanor-Boadu; Stokes, Nayda, and English; Stokes), neither work considered problems of moral hazard and adverse selection arising from information asymmetry.

HBH, in a state-contingent framework, demonstrated superiority by studying the expected costs of different portfolios of contracts. MW, using a principal-agent framework, went further by demonstrating the optimality of the whole-farm insurance contract studied in HBH. Sufficient conditions for the superiority of whole-farm RI relative to crop-specific RI are that each grower is risk-averse, the insurer is risk-neutral, the transactions cost of administering the contract is an increasing function of the contract's actuarially fair value (i.e., expected value which we may write as 'cost'), and the indemnity depends on producer-specific variables. MW also established that the whole-farm option may no longer dominate if the indemnity is based on imperfect estimators of producer-specific variables (e.g., if futures prices and aggregate yields are used in place of farm-gate prices and individual yields to determine the indemnity payment.)

The intent of this paper is to establish the factors that determine the difference in actuarially fair values between the whole-farm and crop-specific contract designs. The issue is important for at least two reasons. It should assist rate setters in pricing contracts, and so reduce the risks that private companies face. It should also facilitate policymakers in determining which products to promote when seeking to assist growers in managing risk.

The analysis proceeds by first considering the actuarially fair value of whole-farm RI. Here, we decompose the problem into a) the effects of interactions between revenue risks, controlling for marginals, and b) the effects of shifts in the marginals, controlling for interactions between marginals. We then turn to valuing crop-specific insurance for the same farm. For these contract forms, the contract specifications ensure that interactions between the marginals do not matter. We identify conditions under which the cost of insuring the farm resolves to a weighted majorization of location-and-scale adjusted guarantee levels.

The third main section integrates the analysis of the earlier sections. In it we study what determines the magnitude of the difference between the cost of whole-farm insurance and the cost of crop-specific insurance for the same farm, having controlled for the magnitude of the farm-level revenue guarantee. The difference can be decomposed into two effects. One concerns how dispersed the crop-specific guarantee levels are. The other concerns the systematic component of crop revenue risks. After showing that our analysis also applies to area yield insurance, the paper concludes with numerical applications to farms typical in North Dakota and in Arkansas.

Whole-Farm Insurance

A farm has available $A \in \bar{\mathbb{R}}_+ = [0,\infty)$ acres on which to grow *n* crops, where $a_i \in \bar{\mathbb{R}}_+$ acres are allocated to the i^{th} crop and $A = \sum_{i=1}^n a_i$. Here, the crops are denoted by the index set $i \in \{1,2,\dots,n\} = \Omega_n$. Viewed from time point 0 (planting), the per acre revenue on the i^{th} crop at time point 1 (harvest) is stochastic with random value x_i. An insurer agrees to guarantee whole-farm revenue of $\bar{r} \in \bar{\mathbb{R}}_+$ to the grower, so that the actuarially fair value is

$$(1) \qquad C^{wf}(\cdot) = E\{\max[\bar{r} - \textstyle\sum_{i=1}^n a_i x_i,\ 0]\},$$

where $E\{\cdot\}$ is the expectation operator over the multivariate distribution $F(x_1,x_2,\dots,x_n)$. In this section, we seek to understand the determinants of $C^{wf}(\cdot)$.[3] Our inquiry will decompose the effects of the multivariate distribution in Eqn. (1) into impacts across the marginals, or systematic risk impacts, and impacts along the marginals, which we label 'dispersion impacts'.

Increase in Systematic Risk

Clearly the function $T(\vec{x};\vec{a},\bar{r}) = \max[\bar{r} - \sum_{i=1}^n a_i x_i, 0]$ is (weakly) monotone decreasing and convex in the argument vector $\vec{x}$. A less obvious attribute of the function is that it is supermodular. That is, for any pair $\{x_j, x_k\}$ with $j,k \in \Omega_n$, and for evaluations $x_{j,2} \ge x_{j,1}$, $x_{k,2} \ge$

$x_{k,1}$, we have $T(x_1,x_2,\ldots,x_{j,1},\ldots,x_{k,1},\ldots,x_n;\vec{a},\bar{r}) + T(x_1,x_2,\ldots,x_{j,2},\ldots,x_{k,2},\ldots,x_n;\vec{a},\bar{r}) \geq T(x_1,x_2,\ldots,x_{j,1},\ldots,x_{k,2},\ldots,x_n;\vec{a},\bar{r}) + T(x_1,x_2,\ldots,x_{j,2},\ldots,x_{k,1},\ldots,x_n;\vec{a},\bar{r})$. The concept of supermodularity plays an important role in the theory of comparative statics, in part due to its connection with the economic notion of complementarity (see, e.g., Topkis or Athey). Shaked and Shanthikumar, and also Hennessy and Lapan in economic contexts, have studied a related stochastic order. We tailor their order to apply to the present context.

DEFINITION 1. *Distribution $G(\vec{x})$ is larger than $F(\vec{x})$ in the decreasing, convex, supermodular order if $\int v(\vec{x})\, dG(\vec{x}) \geq \int v(\vec{x})\, dF(\vec{x})$ for all decreasing, convex, and supermodular functions $v(\vec{x})$. The ordering is written as $G(\vec{x}) \overset{dcs}{\geq} F(\vec{x})$.*

The stochastic order possesses a number of agreeable properties. In particular, it is closed under monotone increasing transformations of the random variables. Thus, suppose that $\vec{z} = (g_1(x_1), g_2(x_2), \ldots, g_n(x_n))$ where the $g_i(x_i)$ are all increasing functions. Then $G(\vec{x}) \overset{dcs}{\geq} F(\vec{x})$ implies $G(\vec{z}) \overset{dcs}{\geq} F(\vec{z})$. Also, the order continues to adhere upon marginalization, e.g., upon integrating out x_n.

To appreciate why the definition captures aspects of the notion of 'more systematic risk', observe that the function $-U(a_1x_1 + a_2x_2)$ is decreasing and supermodular in the x_i for positive a_i whenever $U(\cdot)$ is increasing and concave. So with $(a_1,a_2) \in \bar{\mathbb{R}}^2_+ = [0,\infty)\times[0,\infty)$, then $G(x_1,x_2) \overset{dcs}{\geq} F(x_1,x_2)$ implies $\int U(a_1x_1 + a_2x_2)\, dG(x_1,x_2) \leq \int U(a_1x_1 + a_2x_2)\, dF(x_1,x_2)$ for all $U(\cdot)$ increasing and concave. If $U(\cdot)$ is a utility function, then we see that risk averters prefer distributions that are smaller in the order because marginals are shifted rightward and they are also, in some sense, less well-aligned. To see the relation between utility function $U(a_1x_1 + a_2x_2)$ and $T(x_1,x_2;a_1,a_2,\bar{r})$, note that an integration by parts establishes the equivalence between $U(a_1x_1 + a_2x_2)$ and $-\int T(x_1,x_2;a_1,a_2,\bar{r})\, dH(\bar{r})$ for some positive measure $H(\bar{r})$. Applying Definition 1 to $T(\vec{x};\vec{a},\bar{r})$ establishes[4]

PROPOSITION 1. *The fair value of whole-farm insurance is increasing in the* $\overset{dcs}{\geq}$ *order.*

Notice that Definition 1 does not require a change in any marginal distribution. We could fix all marginals, and then increase the covariability in their interactions to arrive at an ordering satisfying $G(\vec{x}) \overset{dcs}{\geq} F(\vec{x})$ such that the fair value increases strictly upon $F(\vec{x}) \rightarrow G(\vec{x})$. It should also be noted that the related concept of zone-based group insurance that are risk profile conditioned has already received attention in Wang. That applied study is concerned with grower assignment to areas for an area yield insurance that purges moral hazard from grower incentives. The intent of the assignment is to align grower yield risks within an area so that within area systematic risks are high and grower indemnities correlate strongly with grower crop losses.

Having considered the effects of stochastic inter-dimensional interactions, in the next section we control for these interactions and deal with how the way the marginals themselves relate to each other affects the whole-farm contract fair value.

Dispersion of Marginals

Whole-farm insurance may be viewed as the assumption of a portfolio position by the insurer. Because the insurer's loss function, $T(\vec{x};\vec{a},\bar{r})$, is convex it is intuitive that the risk-neutral insurer would have a preference for increased diversification.[5] In this section we will clarify how one might measure, and assign preferences over, the amount of risk faced by the insurer. To do this we make an assumption on the multivariate distribution:

ASSUMPTION 1. *The crop revenues are such that* $x_i = \mu_i + \sigma_i \epsilon_i$, $\mu_i > 0$, $\sigma_i > 0$, $E\{\epsilon_i\} = 0$, *where the* $\epsilon_i \in [\underline{\epsilon}, \bar{\epsilon}] \subseteq \mathbb{R}$ *are exchangeable. By exchangeable it is meant that if* $\vec{\epsilon}$ *follows distribution* $H(\vec{\epsilon})$ *then* $H(\vec{\epsilon}) = H(\vec{\epsilon}_\tau)$ *where the subscripted* τ *transposes any two of the arguments of* $\vec{\epsilon}$.

Exchangeable random variables $\vec{\epsilon}$ do not need to independent, and so the specification $x_i =$

$\mu_i + \sigma_i \epsilon_i$ allows for the crop revenues to be correlated.[6] Because the μ_i merge with $\bar{r}$ in equation (1) under fixed acre parameters are fixed, the location parameters are of no interest in assessing whole-farm liability when acres are fixed. Vector $\vec{\sigma}$ is more interesting, however. Upon accepting Assumption 1, we may write the fair value of whole-farm insurance as $C^{wf}(\vec{\sigma};\vec{a},\vec{\mu}) = E\{\max[\bar{r} - \sum_{i=1}^{n} a_i \mu_i + \sum_{i=1}^{n} a_i \sigma_i \epsilon_i,\ 0]\}$. The curve in set $\vec{\sigma} \in \bar{\mathbb{R}}_+^n$ that is given by $C^{wf}(\vec{\sigma};\vec{a},\vec{\mu}) = \kappa$, $\kappa \in \bar{\mathbb{R}}_+$, is called the whole-farm iso-value curve. To ascertain the nature of $C^{wf}(\vec{\sigma};\vec{a},\vec{\mu})$, a definition is required.

DEFINITION 2. (See Marshall and Olkin, p. 7) For vectors $\vec{u} \in \mathbb{R}^n$ and $\vec{v} \in \mathbb{R}^n$, denote the respective k^{th} largest components as $u_{[k]}$ and $v_{[k]}$. Write $\vec{u} \prec \vec{v}$ if a) $\sum_{k=1}^{i} u_{[k]} \le \sum_{k=1}^{i} v_{[k]}$ $\forall\ k \in \{1,2,\ldots,n-1\}$, and b) $\sum_{k=1}^{n} u_{[k]} = \sum_{k=1}^{n} v_{[k]}$. Then vector $\vec{v}$ is said to *majorize* vector $\vec{u}$.

If a vector, $\vec{\epsilon}$, of random variables is exchangeable, then it is readily shown (see Proposition 11.B.2 in Marshall and Olkin, pp. 287–288) that functions of the specification $V(\vec{p}) = E\{\max[K - \sum_{i=1}^{n} p_i \epsilon_i,\ 0]\}$ are both symmetric and convex. Consequently, they are Schur-Convex in $\vec{p}$. By Schur-Convex, it is meant that $V(\vec{p}^{\,\prime}) \le V(\vec{p}^{\,\prime\prime})$ whenever $\vec{p}^{\,\prime} \prec \vec{p}^{\,\prime\prime}$. We see then that $V(\vec{p}) = C^{wf}(\vec{\sigma};\vec{a},\vec{\mu})$ is Schur-Convex in $\vec{p}$ where $p_i = a_i \sigma_i$ indicates the risk position associated with crop i. And so, focusing on the riskiness of crop revenues, $p_i = a_i \sigma_i$, rather than on the scale parameters, the fair value of the whole-farm insurance increases as the risk positions become more 'spread out' in the majorization sense. This observation can be used to establish

PROPOSITION 2. *Under Assumption 1, in the scale parameter space, the whole-farm iso-value curve is concave to the origin, and the fair value is smaller for iso-value curves closer to the origin.*

The proof is provided in the appendix. Figure 1 presents an illustration in two dimensions, and where the shaded area represents smaller levels of expected cost. Let point t_1 be such that

$\sigma_1 < a_2\sigma_2/a_1$. The matrix

$$(2) \qquad \begin{pmatrix} 0 & a_2/a_1 \\ a_1/a_2 & 0 \end{pmatrix},$$

which is its own inverse, is a linear map of points on the half-orthant $\sigma_1 \leq a_2\sigma_2/a_1$, $\sigma_1 \in \bar{\mathbb{R}}_+$, onto points on the half-orthant $\sigma_1 \geq a_2\sigma_2/a_1$, $\sigma_2 \in \bar{\mathbb{R}}_+$. In particular, it maps some point t_1 onto t_2 as shown in figure 1 so that the iso-value points on either sides of the line $\sigma_1 = a_2\sigma_2/a_1$ must conform. In the proof it is demonstrated that the fair values at these two points are the same, and that the fair value on any $t_3 = \lambda t_1 + (1-\lambda)t_2$, $\lambda \in [0,1]$ is smaller than either. Further, it is shown that the expected cost on any $\theta \times t_1$, $\theta \in [0,1)$ is smaller than at t_1. These observations suffice to demonstrate the result because any point in the shaded area of figure 1 may be obtained from a point on the iso-cost curve through one of these two operations. Proposition 2 reveals the nature of the trade-offs across the crop revenue risk scaling parameters, σ_i, that leave the fair value of the whole-farm insurance contract unchanged.

It is rather obvious that contract specification (1) generates a jointness across marginals. When some x_j increases, then the upper bound on an x_i such that the insurer is not liable decreases as a consequence. Crop-specific insurance contracts are not possessed of this attribute, and the implications of removing this jointness are quite marked.

Crop-Specific Insurance of a Farm

As an alternative to a whole-farm contract, the insurer might offer the grower a portfolio of crop-specific insurance contracts to cover losses beyond the crop-specific guarantee, $\bar{r}_i$, on the i^{th} crop. Then, for the same farm, the farm-level indemnity is

$$(3) \qquad C^{cs}(\cdot) = \sum_{i=1}^{n} a_i E\{\max[\bar{r}_i - x_i,\ 0]\}.$$

The intent of this section is to ascertain the determinants of expected liability $C^{cs}(\cdot)$.

If we invoke Assumption 1 then we may, without further loss of generality, hold that each

ϵ_i follows distribution $G(\epsilon_i|\omega)$, where ω is an external common source of risk, i.e., the multivariate distribution $J(\vec{\epsilon})$ satisfies $J(\vec{\epsilon}) = \int\prod_{i=1}^{n} G(\epsilon_i|\omega)\, dH(\omega)$ for some probability measure $H(\omega)$ on the environmental conditioner ω.[7] We will write $G(\epsilon_i) = \int G(\epsilon_i|\omega)\, dH(\omega)$, and we will assume that the usual derivative exists. Notice that we place no other restrictions on the univariate distribution $G(\epsilon_i)$. Integration by parts then demonstrates that (3) may be re-written as

$$(4) \qquad C^{cs}(a_i\sigma_i,\phi_i) = \sum_{i=1}^{n} a_i\sigma_i G^2(\phi_i), \quad \phi_i = (\bar{r}_i - \mu_i)/\sigma_i, \quad G^2(\phi_i) = \int_{\underline{\epsilon}}^{\phi_i} G(\epsilon)\, d\epsilon .$$

Expression $\phi_i = (\bar{r}_i - \mu_i)/\sigma_i$, often called the (ex-ante) Sharpe index, has been regularly encountered in the financial portfolio literature, and measures the expected differential return per unit of risk (Sharpe; Campbell, Huisman, and Koedijk). Here $\bar{r}_i - \mu_i$ can be thought of as a differential return and σ_i as the standard deviation of the differential return relative to a riskless security with return $\bar{r}_i$. However, the portfolio of insurance contracts for individual revenues differs from an investment portfolio composed of long positions in riskless securities (revenue guarantees) and short positions in risky assets (crop revenues) because the former portfolio curbs the downside risk (on the part of the insured). Expression (4) illustrates the connection between the Sharpe index and the expected value of the portfolio of crop-specific contracts. Clearly, the expected value of the crop-specific insurance is increasing in each Sharpe index ϕ_i. In contrast to the investment portfolio management literature, and due to the nature of insurance contracts, this relationship is not linear because $C^{cs}(\vec{\phi})$ is convex in ϕ_i.[8] To better understand this feature of expression (4), a definition is warranted.[9]

DEFINITION 3. *(Cheng) Let* $\vec{p} \in \bar{\mathbb{R}}_{+}^{n}$. *For* $\vec{z}^{a} \in \mathbb{R}^n$ *and* $\vec{z}^{b} \in \mathbb{R}^n$, *assign* $z_1^a \geq z_2^a \geq \ldots \geq z_n^a$ *and require the vectors be similarly ordered, i.e.,* $(z_i^a - z_j^a)(z_i^b - z_j^b) \geq 0 \;\forall\, i \in \Omega_n, \forall\, j \in \Omega_n$. *Then vector* $\vec{z}^{a} \in \mathbb{R}^n$ *is said to be p-majorized by vector* $\vec{z}^{b} \in \mathbb{R}^n$ $(\vec{z}^{a} \prec_p \vec{z}^{b})$ *if*

(5) $$\sum_{i=1}^{n} p_i h(z_i^a) \le \sum_{i=1}^{n} p_i h(z_i^b)$$

whenever $h(z): \mathbb{R} \to \mathbb{R}$ *is convex. Equivalently,* $\vec{z}^a \prec_p \vec{z}^b$ *if*

(6) $$\sum_{i=1}^{k} p_i z_i^a \le \sum_{i=1}^{k} p_i z_i^b \;\; \forall \;\; k \in \Omega_{n-1}, \qquad \sum_{i=1}^{n} p_i z_i^a = \sum_{i=1}^{n} p_i z_i^b .$$

For a coordinate-wise strictly positive weighting vector $(p_1, p_2, \ldots, p_n) \in \mathbb{R}_+^n$, a generalized Schur-Convex function is one that is increasing in the *p*-majorization order, i.e., function $S(\vec{z})$ is generalized Schur-Convex if $S(\vec{z}^a) \le S(\vec{z}^b)$ whenever $\vec{z}^a \prec_p \vec{z}^b$.[10] Expression $C^{cs}(\vec{\phi})$ is generalized Schur-Convex with $(p_1, p_2, \ldots, p_n) = (a_1\sigma_1, a_2\sigma_2, \ldots, a_n\sigma_n) \in \mathbb{R}_+^n$.[11] And so, the *p*-majorization order is defined in terms of the expected differential returns given by the products of the risk position times the Sharpe index for each crop. Furthermore, Cheng shows the following property of continuously differentiable generalized Schur-convex functions

(7) $$(\phi_i - \phi_j)\left(\frac{1}{p_i} \frac{\partial C^{cs}(\vec{\phi})}{\partial \phi_i} - \frac{1}{p_j} \frac{\partial C^{cs}(\vec{\phi})}{\partial \phi_j} \right) \ge 0 \quad \forall \; \vec{\phi} \in \Upsilon$$

where Υ is the domain of definition. In other words, for any $\vec{\phi}$ such that $\phi_i \ge \phi_j$, an increase in Sharpe index ϕ_i by marginal amount $\partial(p_i \phi_i)$ accompanied by a decrease in Sharpe index ϕ_j by marginal amount $\partial(p_j \phi_j)$ increases the fair value of the portfolio of crop-specific contracts. This is another way of saying that the fair value of crop-specific insurance is positively related to the variability among the Sharpe indices ϕ_i adjusted for the riskiness of the crop, $a_i \sigma_i$.

Summarizing, Definition 3 together with Equation (4) reveal:[12]

PROPOSITION 3. *Under Assumption 1, let* $p_i = a_i \sigma_i > 0$, $i \in \Omega_n$ *be fixed numbers. Then the fair value of crop-specific insurance, at the farm-level, is smaller under Sharpe index vector* $\vec{\phi}^a \in \mathbb{R}^n$ *than under Sharpe index vector* $\vec{\phi}^b \in \mathbb{R}^n$ *whenever* $\vec{\phi}^a \prec_p \vec{\phi}^b$.

The proposition's inference lends itself to a graphical interpretation, as provided in figure 2. Since function $C^{cs}(\cdot)$ is convex, its level sets are convex. In the two-crop context one might fix on a level set, or crop-specific iso-value curve, as specified by $C^{cs}(\cdot) = \kappa$. The terms of trade are then given by $\partial\phi_2/\partial\phi_1 = -p_1G(\phi_1)/p_2G(\phi_2)$ so that $\partial\phi_2/\partial\phi_1 = -p_1/p_2$ on the bisector, $\phi_1 = \phi_2$. In (ϕ_1,ϕ_2) space, points satisfying $p_1\phi_1 + p_2\phi_2 \geq (p_1+p_2)\hat{\phi}$ (it can be seen that the pair (ϕ_1,ϕ_2) is more 'spread out' than $(\hat{\phi},\hat{\phi})$) incur higher expected cost than points on the crop-specific iso-value curve that passes through $(\hat{\phi},\hat{\phi})$.[13]

At this point we turn to another feature of the equation (4), namely that it is possessed of the arrangement increasing property (Boland, Proschan and Tong).[14] In particular, suppose we consider the values of p_i, p_j, ϕ_i, and ϕ_j. If it so happens that $(p_i - p_j)(\phi_i - \phi_j) \leq 0$, then the value of $C^{cs}(\cdot)$ increases upon making the transposition $\phi_i \leftrightarrow \phi_j$. That is,

$$(8) \qquad p_iG^2(\phi_i) + p_jG^2(\phi_j) \geq p_iG^2(\phi_j) + p_jG^2(\phi_i) \quad \forall\, i \in \Omega_n,\ \forall\, j \in \Omega_n,$$

whenever (p_i,p_j) and (ϕ_i,ϕ_j) are well-ordered, i.e., whenever $(p_i - p_j)(\phi_i - \phi_j) \geq 0$. It can, therefore, be concluded

PROPOSITION 4. *Under Assumption 1, let $p_i = a_i\sigma_i > 0$, $i \in \Omega_n$. For any pair of crops, the fair value of crop-specific insurance, at the farm-level, is larger when the given Sharpe index values are well-ordered with the p weightings.*

The fair value of a portfolio of crop-specific revenue insurance contracts will be smaller when the Sharpe indices match inversely with the scale-adjusted acre weights. The proposition may be applied in a number of ways. If $a_i = A/n\ \forall\, i \in \Omega_n$ then the insurer will prefer that a fixed scale vector, $\vec{\sigma}$, is arranged with a fixed (up to permutations) Sharpe index vector, $\vec{\phi}$, so that the sum, $\sum_{i=1}^{n}\sigma_i\phi_i$, is minimized. This will occur when the two vectors, the vector of revenue risks and the vector of Sharpe indices associated with each revenue, are ordered inversely; largest value with smallest value and so on. Alternatively, if the values of the σ_i are invariant across crops then the insurer will prefer the arrangement of given vectors $\vec{a}$ and $\vec{\phi}$ that minimizes the value of $\sum_{i=1}^{n} a_i(\bar{r}_i - \mu_i)$.

These are different ways of illustrating that the insurer prefers if the weight is shifted from the revenues with high Sharpe indices to the revenues with low Sharpe indices in the insurance portfolio. Were the risk-neutral insurer paying out the plain differences between the guarantee levels and the realized revenues this would have been clearly true because $-\sum_{i=1}^{n} p_i \phi_i$ is exactly the expected differential return on such a portfolio. Proposition 4 demonstrates the relevance of the expected differential return statistic in the case of crop-specific insurance contracts.

Rather than re-arrange given values of Sharpe indices among the farm's acre allocation vector, one may wonder whether something can be said about shifts in stochastic revenue parameters. The answer is in the affirmative.

PROPOSITION 5. *Make Assumption 1.*

a) *The fair value of the farm-level portfolio of crop-specific insurance contracts decreases with an increase in the expected values of the crop revenues (i.e., in the location parameters μ_i $\forall\, i \in \Omega_n$).*

b) *If $\bar{r}_i - \mu_i = \kappa$ $\forall\, i \in \Omega_n$, then the fair value increases under more dispersion in the p-majorization sense among the crop revenues risks σ_i where the p weightings are given by the crop acres a_i.*

This proposition should be viewed as the crop-specific counterpart to Proposition 2. Rather than trading off across dimensions, it exploits the effectively univariate nature of the crop-specific portfolio problem. The preferences expressed in part a) is a first-order stochastic dominance result, where acres provide the positive discrete measure. Part b) is effectively an acre-weighted mean-preserving spread on the scale parameters.[15] This part complements the Sharpe index dispersion result in Proposition 3 by establishing that an increase in the dispersion of acre-weighted scale parameters also increases the farm-level expected liability upon insuring crop-specific contracts. In light of the convexity in scale parameters that is present in both whole-farm and crop-specific revenue insurance contracts, it is not surprising that the fair values of both types of insurance rise with an increase in the diversity among crop revenue risk levels.

Now that we have a sense of what determines the fair values of both a portfolio of crop-specific RI contracts and a whole-farm contract, we are well-positioned to inquire into how they compare.

The Difference

It has been shown by Hennessy, Babcock, and Hayes, and also by Mahul and Wright, that crop-specific insurance will be more costly in all states of nature (and so in expectation) than whole-farm insurance. That is, $\mathfrak{L}_{wf}^{cs} \geq 0$ where

$$\text{(9)} \qquad \mathfrak{L}_{wf}^{cs} = C^{cs}(\cdot) - C^{wf}(\cdot) = \sum_{i=1}^{n} a_i E\{\max[\bar{r}_i - x_i,\ 0]\} - E\{\max[\bar{r} - \sum_{i=1}^{n} a_i x_i,\ 0]\},$$

and $\bar{r} = \sum_{i=1}^{n} a_i \bar{r}_i$. The magnitude of this difference reflects the extent of inefficiency in risk management because the difference arises from the increased likelihood of poorly targeted cash inflows to the farm. The inflows are poorly targeted in that the crop-specific contracts are designed in an uncoordinated manner so that inflows often occur when marginal utility is low.

The constraint $\bar{r} = \sum_{i=1}^{n} a_i \bar{r}_i$ ensures that the revenue guarantee at the farm-level is invariant to the contract design, crop-specific or whole-farm, by which the farm revenue is insured. It is not immediately clear, however, what determines the magnitude of the difference. In this section, we will identify factors that determine the magnitude of $\mathfrak{L}_{wf}^{cs}$. Our analysis will proceed in two steps. First, we will ascertain the nature of an insurer's preferences over guarantee vectors. It will be shown that dispersion in the guarantee levels is responsible for some of the value of $\mathfrak{L}_{wf}^{cs}$. Then we will demonstrate that the residual unexplained component of quantity $\mathfrak{L}_{wf}^{cs}$ is due to systematic risk. As to dispersion in the guarantee levels,[16]

PROPOSITION 6. *Under Assumption 1, let* $p_i = a_i \sigma_i > 0$, $i \in \Omega_n$ *where the* σ_i *are fixed.*

Then, for the set of Sharpe index vectors such that $\sum_{i=1}^{n} p_i \phi_i$ *is constant, the fair value of crop-specific insurance (at the farm-level) is smallest under Sharpe index vector* $\hat{\phi} \times \vec{1}$ *where* $\hat{\phi}$ = $\sum_{i=1}^{n} p_i \phi_i / \sum_{i=1}^{n} p_i$. *The fair value of whole-farm insurance is unaffected by location on the set of Sharpe index vectors determined by fixing the values of* $\sum_{i=1}^{n} p_i \phi_i$ *and* $\vec{\sigma}$.

Proposition 6 establishes that dispersion, in the p-majorization sense, away from the p_i-weighted mean of the location-and-scale adjusted crop-specific revenue guarantees, ϕ_i, is responsible for some of the value of $\mathfrak{L}_{wf}^{cs}$. If we decompose

$$\text{(10A)} \qquad \Delta_p^{disp} = \sum_{i=1}^{n} p_i E\{\max[\phi_i - \epsilon_i, 0]\} - \sum_{i=1}^{n} p_i E\{\max[\hat{\phi} - \epsilon_i, 0]\},$$

$$\text{(10B)} \qquad \Delta_p^{sys} = \sum_{i=1}^{n} p_i E\{\max[\hat{\phi} - \epsilon_i, 0]\} - E\{\max[\bar{r} - \sum_{i=1}^{n} a_i \mu_i - \sum_{i=1}^{n} a_i \sigma_i \epsilon_i, 0]\},$$

so that

$$\text{(11)} \qquad \mathfrak{L}_{wf}^{cs} = \Delta_p^{disp} + \Delta_p^{sys},$$

then Proposition 6 identifies the condition under which the first right-hand term in (11) vanishes.[17] It remains to establish the determinants of Δ_p^{sys}.

We will presently demonstrate that what remains after accounting for the dispersion, i.e., Δ_p^{sys}, is due entirely to systematic risk. To verify our claim, we seek a condition such that the value of expression Δ_p^{sys} recedes to zero. To this end, we require a result similar to Proposition 1. However, (10B) differs from the expression in (1) because each random variable enters the expression in two ways. Expression Δ_p^{sys} is not necessarily monotone or of uniform curvature in the random variables, and so the $\overset{dcs}{\geq}$ order no longer suffices. It is necessary to fix the marginals so that function $\sum_{i=1}^{n} p_i E\{\max[\hat{\phi} - \epsilon_i, 0]\}$ is invariant in value.[18]

DEFINITION 4. *Distribution* $G(\vec{x})$ *is larger than* $F(\vec{x})$ *in the supermodular order if* $\int v(\vec{x})\, dG(\vec{x}) \geq \int v(\vec{x})\, dF(\vec{x})$ *for all supermodular functions* $v(\vec{x})$. *The ordering is written as*

$G(\vec{x}) \overset{sm}{\geq} F(\vec{x})$.

Because the class of functions covered is larger than in Definition 1, this order cannot be more discriminating than that in Definition 1, i.e., if $G(\vec{x}) \overset{sm}{\geq} F(\vec{x})$ then $G(\vec{x}) \overset{dcs}{\geq} F(\vec{x})$.[19] An application of the $\overset{sm}{\geq}$ order provides:

PROPOSITION 7. *Suppose that* $G(\vec{x}) \overset{sm}{\geq} F(\vec{x})$. *Then the positive difference between the fair value of the farm-level portfolio of crop-specific insurance contracts and the fair value of the whole-farm insurance contract is smaller under distribution* $G(\vec{x})$ *than under distribution* $F(\vec{x})$. *Equivalently, the value of* Δ_p^{sys} *is smaller under distribution* $G(\vec{x})$ *than under distribution* $F(\vec{x})$.

By contrast with propositions 2 through 6, an inspection of the proof in the appendix shows why Proposition 7 need not assume exchangeability up to location-and-scale parameters.[20] The dominated and dominating multivariate probability measures need not be symmetric in any way, and the changes in probability weightings that leads to the $\overset{sm}{\geq}$ relation are not required to conform to any symmetry constraints either.

While robust in this sense, Proposition 7 does not establish a path of dominance relations such that Δ_p^{sys} vanishes in a finite number of steps. Therefore, it is not forthcoming with insights on when one might expect relation $\overset{sm}{\geq}$ to pertain. Our final result will be more constructive in that it provides such a path. The result will show how systematic risk may be viewed as a concentration in the sources of independent risk that contribute to farm revenue risk.

In (10B), suppose that each of the ϵ_i may be decomposed into *m* conditionally independent and identically distributed mean-zero shocks of the form $(\eta_{i,1}, \eta_{i,2}, \cdots, \eta_{i,m})$, where the conditioning is on ω. Then, with the substitution $K = \bar{r} - \sum_{i=1}^{n} a_i \mu_i$, we may write

$$(12) \qquad \Delta_p^{sys} = \sum_{i=1}^{n} a_i \sigma_i E\{\max[\hat{\phi} - \textstyle\sum_{j=1}^{m} \eta_{i,j}, 0]\} - E\{\max[K - \textstyle\sum_{i=1}^{n} a_i \sigma_i \sum_{j=1}^{m} \eta_{i,j}, 0]\},$$

The path adopted such that Δ_p^{sys} vanishes will increase the value of the subtracted expectation on the right in equation (12) to meet the value of the sum of expectations.

Commence by writing the vector of risk positions

$$\vec{e}^{\,1} = (\underbrace{a_1\sigma_1, a_1\sigma_1, \ldots, a_1\sigma_1}_{m\ times}, \underbrace{a_2\sigma_2, a_2\sigma_2, \ldots, a_2\sigma_2}_{m\ times}, \ldots, \underbrace{a_n\sigma_n, a_n\sigma_n, \ldots, a_n\sigma_n}_{m\ times}) \in \mathbb{R}^{nm}. \tag{13}$$

Now suppose that independence is no longer adhered to because some of the $\eta_{i,j}$ are replications, i.e., copies. For example, $\eta_{2,1}$ might be replaced by a copy of another $\eta_{i,j}$. Then more systematic risk has been introduced into the portfolio because there is now perfect correlation between two among the *nm* sources of risk. If, for example, $\eta_{2,1}$ becomes a copy of $\eta_{1,1}$ then $\vec{e}^{\,1}$ becomes

$$\vec{e}^{\,1} = (a_1\sigma_1 + a_2\sigma_2, \underbrace{a_1\sigma_1, \ldots, a_1\sigma_1}_{m-1\ times}, 0, \underbrace{a_2\sigma_2, \ldots, a_2\sigma_2}_{m-1\ times}, \ldots, \underbrace{a_n\sigma_n, a_n\sigma_n, \ldots, a_n\sigma_n}_{m\ times}) \in \mathbb{R}^{nm}. \tag{14}$$

When identifying the path, if no $\eta_{i,j}$ can be replicated by an $\eta_{i,k}$ then the summation $\sum_{i=1}^{n} a_i\sigma_i E\{\max[\hat{\phi} - \sum_{j=1}^{m}\eta_{i,j}, 0]\}$ is unaffected by the replacement operation. Remember that, as in Proposition 2, $E\{\max[K - \sum_{i=1}^{n} p_i \sum_{j=1}^{m}\eta_{i,j}, 0]\}$ is Schur-Convex in the random variable coefficients whenever the identified random variables are exchangeable. Independent and identically distributed random variables are exchangeable. This allows us to see that expression Δ_p^{sys} in equation (12) will increase upon $\vec{e} \to \vec{e}^{\,\prime}$ if $\vec{e}^{\,\prime} \prec \vec{e}$ and Assumption 2 below holds

ASSUMPTION 2. *No $\eta_{i,j}$ can be replicated by an $\eta_{i,k}$.*

Upon replacing all $\eta_{i,j}$, $i \neq 1$, with $\eta_{1,j}$ we have

$$\vec{e} = \vec{e}^* = (\underbrace{\textstyle\sum_{i=1}^n a_i\sigma_i, \sum_{i=1}^n a_i\sigma_i, \ldots, \sum_{i=1}^n a_i\sigma_i}_{m\ times}, \underbrace{0, 0, \ldots, 0, \ldots, 0}_{mn-m\ times}) \in \mathbb{R}^{nm}. \tag{15}$$

It is only then, or for one of the $m - 1$ permutations where all $\eta_{i,j}$ are replaced with some $\eta_{t,j}$, $t \in \{2, 3, \ldots, m\}$, that the value of (12) declines to zero.

To see that (12) does decline to zero, write

$$\Delta_p^{sys}(\vec{e}^*) = \sum_{i=1}^n a_i\sigma_i E\{\max[\hat{\phi} - \textstyle\sum_{j=1}^m \eta_{1,j}, 0]\} - E\{\max[K - \sum_{j=1}^m \eta_{1,j}\sum_{i=1}^n a_i\sigma_i, 0]\}. \tag{16}$$

Equality $\Delta_p^{sys}(\vec{e}^*) = 0$ adheres due to the fact that the points of non-differentiability of each integrand, i.e., $\hat{\phi}$ for the crop-specific integrations and $K/\sum_{i=1}^n a_i\sigma_i = \sum_{i=1}^n a_i\sigma_i\hat{\phi}/\sum_{i=1}^n a_i\sigma_i$ for the whole-farm integration, have the same value. Consequently, we have

PROPOSITION 8. *If $\vec{e}^{\,\prime} \prec \vec{e}$ and Assumption 2 adheres, then $\Delta_p^{sys}(\vec{e}^{\,\prime}) \geq \Delta_p^{sys}(\vec{e})$. The expression assumes the value 0 when $\vec{e} = \vec{e}^*$, as in (15) above, or when $\vec{e}$ is one of the $m - 1$ permutations of $\vec{e}^*$.*

Propositions 6-8 establish the following. Suppose that each revenue is subject to the same sources of risk, and the variation in crop revenue guarantees is due to the variation in crop revenue's location and scale parameters: $\bar{r}_i = \mu_i + \sigma_i(K/\sum_{i=1}^n a_i\sigma_i)$, so that $\bar{r} = \sum_{i=1}^n a_i\bar{r}_i$. These are the conditions such that the difference in the indemnities under the two types of insurance contracts completely disappears. This happens because there are no idiosyncratic components in revenue risks, and all (ex-post) revenues are exact replicas of each other up to the location and scale parameters. Then the sum of crop revenues is also a replica of each individual revenue after the adjustment for location and scale. Hence, the payments under the whole-farm RI and the portfolio of crop-specific RI are always identical. In this case, there is no 'natural' risk diversification achieved by allocating farm acreage to multiple crops because all crop revenues are perfectly correlated.

In summary, when revenue risks are exchangeable up to location and scale parameters then the fair values of the alternative approaches to crop insurance converge under two conditions. The revenue guarantees, when adjusted for location and scale parameters, must be common. Further, all risk must be systematic.

Simulation Analysis

To give us some idea of the practical magnitude of the results developed in this paper, we turn to an examination of an hypothetical North Dakota farm in Cass County. This farm grows corn, soybeans and spring wheat in rotation. The farm's proven yields for crop insurance purposes for the three crops equal county trend yields: 100 bu/acre for corn, 34 bu/acre for soybeans, and 40 bu/acre for wheat. The crop insurance scenarios will be based on the rates and rating assumptions of the RA product, where the main rating methods are as given in Babcock and Hennessy.[21] Without going into valuation details, the valuation procedure assumes a log-normal price marginal distribution, and a beta yield distribution. The correlation structure is imposed by repeated use of a procedure due to Johnson and Tenenbein.

Equation (11) reveals that if all risk is systematic, then the difference in actuarial value between crop-specific and whole-farm coverage is due to dispersion in effective insurance guarantees. But clearly, corn, soybean, and wheat crop revenues are not perfectly correlated. Weather events affect farm yields differently and crop prices do not move together. So at least part of the difference in value pertains to nonsystematic risk.

Table 1 presents the covariance matrix for crop yields and prices used to rate the RA product in North Dakota. The resulting average degree of correlation across revenue for the three crops is approximately 0.45. Table 2 presents actuarially fair crop-specific and whole-farm RA premiums for all available coverage levels. The RA product requires a projected price input, and the projected prices used in the calculations are \$2.35/bu for corn, \$4.50/bu for soybeans, with

\$3.00/bu for wheat. The price volatilities for the log-normal distribution are 0.21, 0.18, and 0.17 respectively. Equal acreage is assumed planted to each crop.

The value of $\mathfrak{L}_{wf}^{cs}$ equals the difference between average crop-specific premiums and the whole-farm premium. It ranges from \$5.01/acre at 65% coverage (56% of the average crop-specific premium) to \$7.05/acre at 85% coverage (36% of the average crop-specific premium). These large differences indicate that the choice between crop-specific coverage and whole-farm coverage involves fairly large financial decisions.

A decomposition of $\mathfrak{L}_{wf}^{cs}$ in the manner of (11) requires additional programming. Revenue draws were obtained with correlations derived from the Table 1 correlation matrix, while means and variances were taken from the Cass County hypothetical farm with log-normal prices and beta yields. These draws were used to numerically integrate the expressions in (10A) and (10B), and the decomposition in (11) was accomplished.

The proportion of $\mathfrak{L}_{wf}^{cs}$ accounted for by Δ_p^{disp} under various coverage level scenarios was calculated. If all single-crop guarantees are originally 75% of projected revenue, then Δ_p^{disp} accounts for about two percent of $\mathfrak{L}_{wf}^{cs}$. This share rises to 12% when corn single-crop coverage is 85%, soybeans is 75% and wheat is 65% and to 14% when corn coverage is 75%, soybean coverage is 65% and wheat coverage is 85%. As Proposition 3 suggests, we conclude that increasing coverage dispersion increases the importance of Δ_p^{disp}. But, when coverage of corn (the crop with the most per-acre projected value) is 65%, soybean is 85%, and wheat coverage is 75%, then Δ_p^{disp} accounts for only three percent of $\mathfrak{L}_{wf}^{cs}$. When compared with the (75%, 75%, 75%) scenario, the latter scenario increases percent coverage dispersion as well as the value of Δ_p^{disp}. But it actually decreases dispersion in per-acre coverage, thus accounting for the small share of Δ_p^{disp}.

These results suggest that the reduction in the fair premium for whole-farm revenue insurance relative to single-crop insurance is primarily driven by risk pooling across crops and not by a reduction in coverage dispersion, particularly if a farmer compares single-crop and whole-farm insurance at the same coverage level for all crops. The example included here is a fairly diversified farm with significant amounts of non-systematic risk because a relatively uncorrelated crop, wheat, was added to two highly correlated crops, corn and soybeans.

Another example will highlight the effect of crop diversification on the magnitude of $\mathfrak{L}_{wf}^{cs}$. Table 3 presents $\mathfrak{L}_{wf}^{cs}$ as a percentage of the average single-crop premium for all non-monoculture combinations of four crops in Arkansas Co., Arkansas.[22] To show how the addition of crops affects the magnitude of $\mathfrak{L}_{wf}^{cs}$, each of the four crops is treated as a "base" crop, to which additional crops are introduced. For each rotation, each crop is grown just once. The combinations are provided as a diversification lattice in figure 3. Under Assumption 1, and if the values of the a_i, the μ_i, and the σ_i are common across crops, then the higher (i.e., more diversified tiers) on the lattice would represent a more diversified portfolio. From Proposition 8, we could then infer that the higher tiers on the lattice have larger $\mathfrak{L}_{wf}^{cs}$ values even if lines do not connect them.[23]

From table 3 or figure 3, it can be seen that the marginal effect on $\mathfrak{L}_{wf}^{cs}$ upon adding a crop to a rotation is always positive. For example, adding cotton to a rice-corn combination increases $\mathfrak{L}_{wf}^{cs}$ from 0.44 to 0.53. And adding soybeans to this extended rotation increases $\mathfrak{L}_{wf}^{cs}$ to 0.55. But $\mathfrak{L}_{wf}^{cs}$ may be lower under three crop combinations than two crop combinations. For example, $\mathfrak{L}_{wf}^{cs}$, under a soybean-rice combination equals 0.48. But $\mathfrak{L}_{wf}^{cs}$ equals 0.40 under a soybean-corn-cotton combination. This illustrates that it is not only the number of crops that affect the magnitude of $\mathfrak{L}_{wf}^{cs}$, but also the stochastic interactions between the crops. There is a failure in exchangeability among the revenue random variables, and so Proposition 8 does not apply.

Table 4 presents the correlation matrix for the Arkansas crop example. Notice that the corn-soybean correlations tend to be larger than the corn-cotton correlations. Notice too that the normalized value of $\mathfrak{L}_{wf}^{cs}$ for the corn-cotton rotation, as given in table 3, is 0.36 while the value for the corn-soybean rotation is 0.23. We see that the diversification impact of adding cotton to the corn-soybean rotation exceeds the impact of adding soybean to the corn-cotton rotation. The inclusion of an additional uncorrelated farming enterprise, such as livestock returns would significantly decrease insurance premiums further than the levels suggested by table 3.[24]

Discussion

This paper has taken a very close look at systematic risk as it relates to crop revenue insurance.[25] Our empirical analysis establishes that the whole-farm contracts are, due to the extent of idiosynchratic risk, considerably more efficient as risk management tools than are portfolios of well-designed crop-specific contracts.

Because the benefits from the whole-farm contract design can be so substantial for those risk-averse growers who cannot adequately protect themselves without using insurance markets, one wonders why almost all RI contracts are crop-specific. The answer may be in the nature of the federal insurance subsidy. Insurance salespeople are paid on commission, and their remunerations are in proportion to the premiums on sales made. The marketer has the incentive to market a portfolio of crop-specific contracts rather than a single whole-farm contract. A grower that is not strongly risk averse may be receptive to the marketer's sales pitch because subsidies are provided as a percent of the actuarially fair premium and the subsidy is presently well in excess of the load to cover administration costs. To circumvent these agency problems and to guard against the moral hazard problem that arises from political pressure to provide ex-post disaster assistance packages, the government might consider issuing fixed-sum crop/revenue insurance vouchers to growers.

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Appendix

PROOF OF PROPOSITION 2. The proof proceeds in two steps.

Step 1: Write

$$\text{(A.1)} \qquad C^{wf}(\cdot) \;=\; E\{\max[\bar{r} - \textstyle\sum_{i=1}^{n} a_i\mu_i - \sum_{i=1}^{n} a_i\sigma_i\epsilon_i,\; 0]\} \;=\; V(\vec{p}),$$

where $p_i = a_i\sigma_i \;\forall\, i \in \Omega_n$. Function $V(\vec{p})$ is Schur-Convex. We will find two vectors, $\vec{p}^{\,\alpha}$ and $\vec{p}^{\,\beta}$, such that $V(\vec{p}^{\,\alpha}) = V(\vec{p}^{\,\beta})$. From convexity, we then know that $V(\lambda\vec{p}^{\,\alpha} + (1-\lambda)\vec{p}^{\,\beta}) \le V(\vec{p}^{\,\alpha}) \;=\; V(\vec{p}^{\,\beta}) \;\forall\, \lambda \in [0,1]$. As in the study by Lapan and Hennessy of optimal portfolio allocation vectors, let $\vec{p}^{\,\alpha} = (a_1\sigma_1, \ldots, a_i\sigma_i, \ldots, a_j\sigma_j, \ldots, a_n\sigma_n)$ and $\vec{p}^{\,\beta} = (a_1\sigma_1, \ldots, a_i\hat{\sigma}_i, \ldots, a_j\hat{\sigma}_j, \ldots, a_n\sigma_n)$ where $\hat{\sigma}_i = a_j\sigma_j/a_i$ and $\hat{\sigma}_j = a_i\sigma_i/a_j$. Then, upon substitution into (A.1), cancellations and exchangeability establish that $V(\vec{p}^{\,\alpha}) = V(\vec{p}^{\,\beta})$.

Step 2: For $\theta \times \vec{p}^{\,\alpha} = \theta \times (a_1\sigma_1, \ldots, a_i\sigma_i, \ldots, a_j\sigma_j, \ldots, a_n\sigma_n) = (a_1\theta\,\sigma_1, \ldots, a_i\theta\,\sigma_i, \ldots, a_j\theta\,\sigma_j, \ldots, a_n\theta\,\sigma_n)$, it will be shown that $V(\theta \times \vec{p}^{\,\alpha})$ is monotone increasing in $\theta \in \bar{\mathbb{R}}_+$. Differentiating

$$\text{(A.2)} \qquad V(\theta \times \vec{p}) \;=\; E\{\max[\bar{r} - \textstyle\sum_{i=1}^{n} a_i\mu_i - \theta\sum_{i=1}^{n} a_i\sigma_i\epsilon_i,\; 0]\}$$

with respect to θ provides

$$\text{(A.3)} \qquad \frac{dV(\theta \times \vec{p})}{d\theta} \;=\; -\,E\{I_{[\theta\psi \le \bar{r} - \sum_{i=1}^{n}\mu_i a_i]}\,\psi\,]\}, \qquad \psi \;=\; \textstyle\sum_{i=1}^{n} a_i\sigma_i\epsilon_i,$$

where $I_{[B]} = 1$ if event B is realized and $I_{[B]} = 0$ otherwise. Note, that $\mathrm{cov}[g(\psi), h(\psi)] \ge 0$ whenever $g(\psi)$ and $h(\psi)$ are both monotone in the same direction. Because $E\{\epsilon_i\} = 0 \;\forall\, i \in \Omega_n$ and $-\,I_{[\theta\psi \le \bar{r} - \sum_{i=1}^{n}\mu_i a_i]}$ is monotone increasing in ψ, we have that $V(\theta \times \vec{p}^{\,\alpha})$ is monotone increasing in θ. Therefore, and for any $\vec{p}^{\,\alpha} \in \bar{\mathbb{R}}_+^n$, expected liability decreases as θ contracts toward the origin.

PROOF OF PROPOSITION 5. The approach is to treat acres as a positive discrete measure, and then employ standard dominance analysis of the first- and second-order. To show part a) consider the map $\mu_i \to \mu_i + \delta_i$, $\delta_i \geq 0 \ \forall\, i \in \Omega_n$. Clearly, the expected value $C^{cs}(\cdot) = \sum_{i=1}^{n} a_i \sigma_i G^2((\bar{r}_i - \mu_i)/\sigma_i)$ falls under the map because function $G^2((\bar{r}_i - \mu_i)/\sigma_i)$ is monotone decreasing in μ_i. In part b), write $H(\sigma) = \sigma\, G^2(\kappa/\sigma)$ and observe that $d^2H(\sigma)/d\sigma^2 = g(\kappa/\sigma)\kappa^2/\sigma^3 \geq 0$ where $g(\cdot)$ is the density of the absolutely continuous distribution $G(\cdot)$. Part b) then follows by considering the map $\vec{\sigma} \to \vec{\sigma}'$ where $\vec{\sigma} \prec_p \vec{\sigma}'$ with $\vec{p} = \vec{a}$, and applying Definition 3 to $C^{cs}(\cdot) = \sum_{i=1}^{n} a_i H(\sigma_i)$.

PROOF OF PROPOSITION 6. The constancy of $\sum_{i=1}^{n} p_i \phi_i$ implies the constancy of $\sum_{i=1}^{n} a_i(\bar{r}_i - \mu_i)$ (at value $\bar{r} - \sum_{i=1}^{n} a_i \mu_i$). That the σ_i are fixed implies that $\sum_{i=1}^{n} a_i \sigma_i \epsilon_i$ is unaffected in all states. Therefore, the fair value of whole-farm insurance is not affected. Returning to Proposition 3, note that $\vec{\phi}^{\,b} = (\hat{\phi}, \hat{\phi}, \ldots, \hat{\phi})$ is p-majorized by all other vectors on the weighted simplex $\sum_{i=1}^{n} p_i \phi_i = \kappa$, $\kappa \in \mathbb{R}$. Thus, this vector delivers the smallest fair value.

PROOF OF PROPOSITION 7. Note that both $\max[\bar{r}_i - x_i, 0]$ and $-\max[\bar{r}_i - x_i, 0]$ are (weakly) supermodular in $\vec{x}$. Therefore, expressions of the form $\max[\bar{r}_i - x_i, 0]$ must be invariant under any $F(\vec{x}) \to G(\vec{x})$ with $G(\vec{x}) \overset{sm}{\geq} F(\vec{x})$. In ascertaining the impact on Δ_p^{sys}, then, the requirements on $G(\vec{x}) \overset{sm}{\geq} F(\vec{x})$ allow us to confine our attention to the impact on $E\{\max[\bar{r} - \sum_{i=1}^{n} a_i x_i, 0]\}$. The rest follows because functions of the form $\max[\bar{r} - \sum_{i=1}^{n} a_i x_i, 0]$ are supermodular in $\vec{x}$.

Table 1. Correlation Matrix for Yields and Prices

	Yield			Price		
	Corn	Soybeans	Wheat	Corn	Soybeans	Wheat
Corn yield	1.00					
Soybean yield	0.78	1.00				
Wheat yield	0.35	0.30	1.00			
Corn price	0.00	0.00	0.00	1.00		
Soybean price	0.00	0.00	0.00	0.70	1.00	
Wheat price	0.00	0.00	-0.30	0.35	0.25	1.00

Table 2. Revenue Assurance Premiums for a North Dakota Crop Farm

	Crop				
Coverage Level	Corn	Soybeans	Wheat	Average for Crop Specific	Average for Whole-Farm
85%	32.68	14.66	12.05	19.80	12.75
80%	28.16	12.01	9.99	16.72	10.05
75%	24.10	9.62	8.10	13.94	7.71
70%	20.15	7.50	6.37	11.34	5.69
65%	16.63	5.63	4.80	9.02	4.01

Table 3. Diversification Effects on Pricing Revenue Insurance for an Arkansas Crop Farm

Crop Combination[a]	$\mathfrak{L}_{wf}^{cs}$ as a percent of Average Single-Crop Premium	Crop Combination[a]	$\mathfrak{L}_{wf}^{cs}$ as a percent of Average Single-Crop Premium
Rice Combinations		*Soybean combinations*	
RC	0.44	SC	0.23
RS	0.48	SCt	0.28
RCt	0.31	SR	0.48
RSCt	0.50	SCCt	0.40
RCCt	0.53	SCR	0.50
RCS	0.50	SCtR	0.50
RCSCt	0.55	SCCtR	0.55
Cotton Combinations		*Corn Combinations*	
CtC	0.36	CS	0.23
CtS	0.28	CCt	0.36
CtR	0.31	CR	0.44
CtCS	0.40	CSCt	0.40
CtCR	0.53	CSR	0.50
CtSR	0.50	CCtR	0.53
CtCSR	0.55	CSCtR	0.55

[a] The combinations are defined as R for rice, Ct for cotton, S for soybeans, and C for corn.

Table 4. Correlation Matrix Used to Rate Revenue Assurance in Arkansas

	Corn Yield	Soybean Yield	Cotton Yield	RiceYield	Corn Price	Soybean Price	Cotton Price	Rice Price
Corn Yield	1.00							
Soybean Yield	0.51	1.00						
Cotton Yield	0.29	0.51	1.00					
Rice Yield	0.08	0.17	0.26	1.00				
Corn Price	0.00	0.00	0.00	0.00	1.00			
Soybean Price	0.00	0.00	0.00	0.00	0.78	1.00		
Cotton Price	0.00	0.00	0.00	0.00	0.36	0.35	1.00	
Rice Price	0.00	0.00	0.00	0.00	0.50	0.65	0.56	1.00

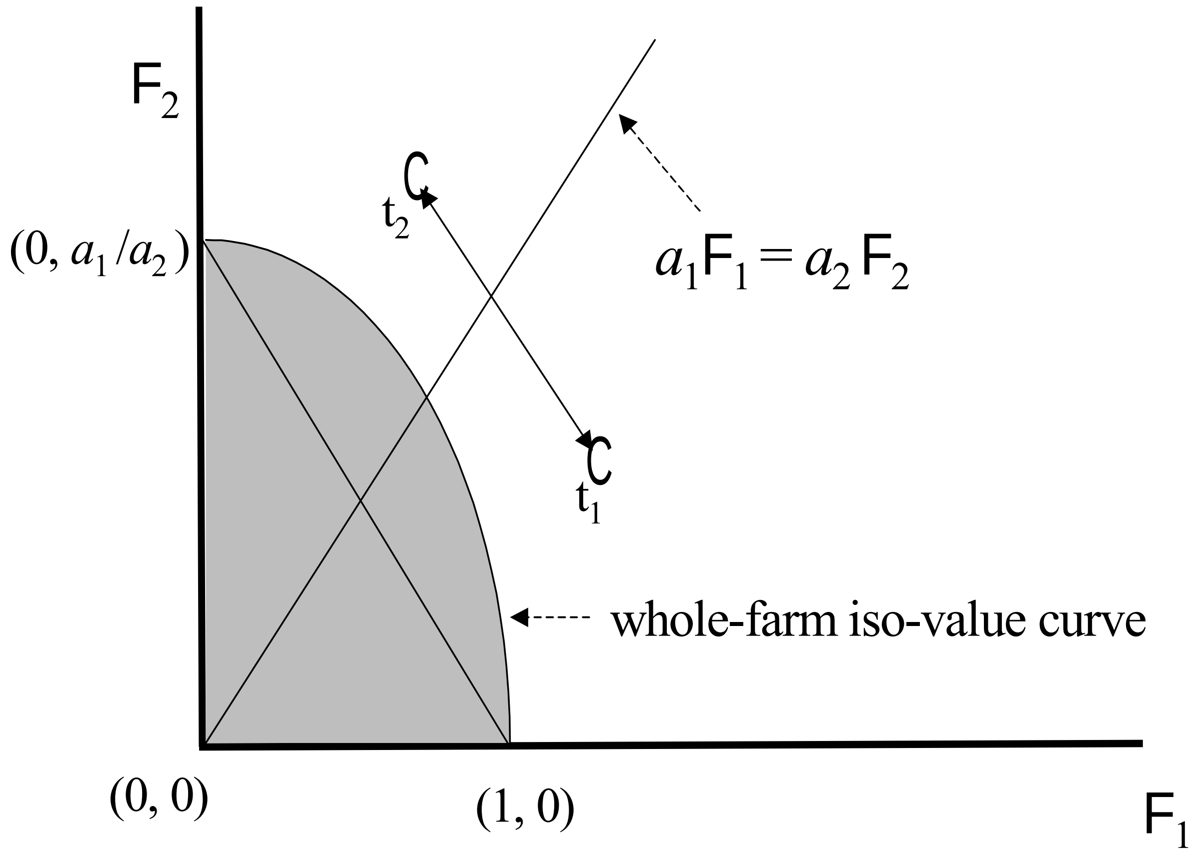

Figure 1. Insurer's preferences over scale parameters in whole-farm insurance

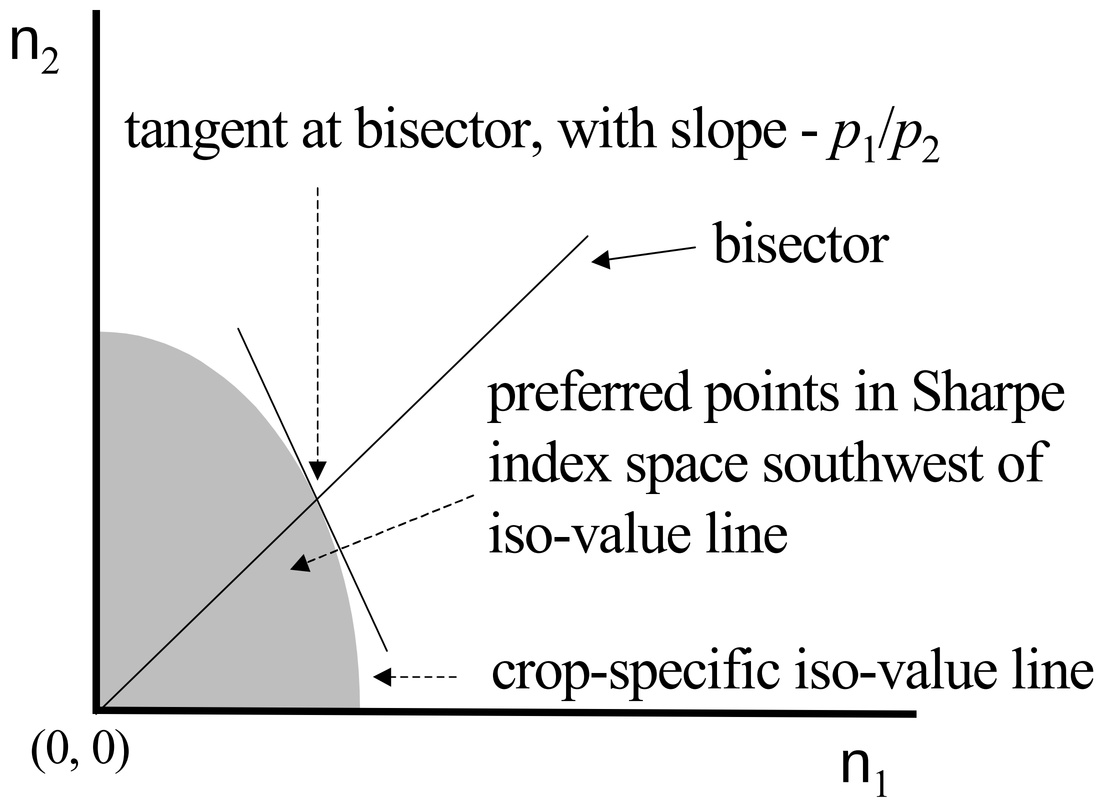

Figure 2. Insurer's preferences over Sharpe index parameters in crop-specific insurance

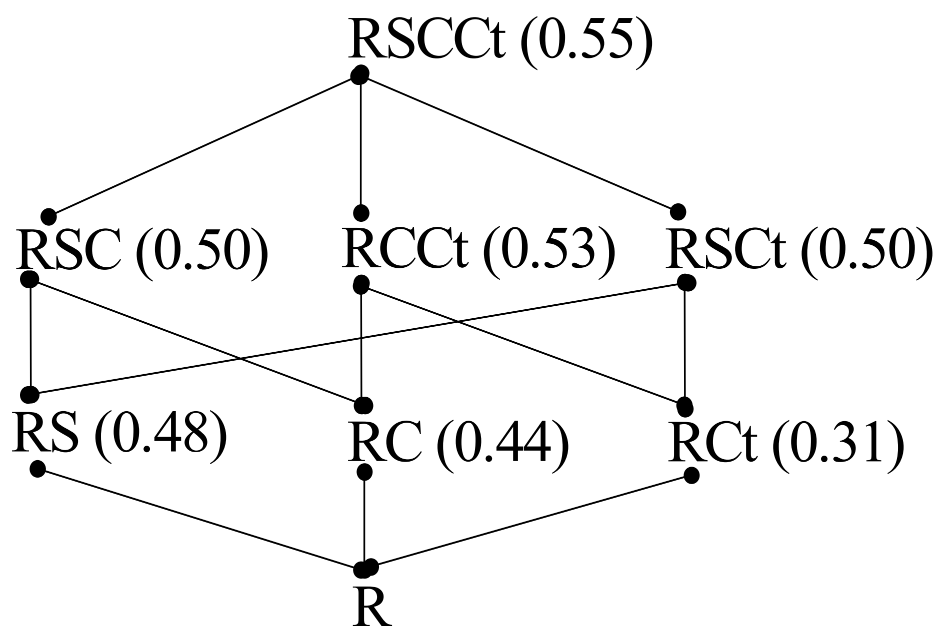

Figure 3. Diversification lattice for Arkansas crop rotations, with rice as the base crop

Endnotes

1. In 2001, CRC was available in all contiguous states except Nevada.

2. Raviv also shows that in the case of multiple losses the optimal indemnity function depends on the aggregate loss rather than individual losses.

3. We do not study partial insurance of losses. All the results to follow can be readily modified to accommodate a variety of approaches, such as coverage levels less than 1, to sharing losses.

4. As an illustration of their notion of systematic risk, some consequences of an increase in such risk on the price of an insurance premium are provided in Hennessy and Lapan.

5. A risk-averse insurer would have even stronger preferences for diversification.

6. A theorem due to de Finetti shows that conditional independence is both necessary and sufficient (Chow and Teicher, p. 232).

7. That is, exchangeable random variables are conditionally independent with common conditional distributions. See Tong, p. 110, or Chow and Teicher as given in footnote 6.

8. In the case of an investment portfolio, the relationships between the expected returns on assets and Sharpe indices (multiplied by the corresponding risk positions) are clearly linear.

9. For alternative but equivalent definitions of p-majorization, see Marshall and Olkin (pp. 417-421).

10. See Chambers and Quiggin for more on generalized Schur-Convex functions.

11. In the portfolio management context, Sharpe refers to quantity $a_i\sigma_i$ as the risk of the position in the zero-investment strategy relative to the total assets.

12. If the equality in (6) is replaced by the weak inequality #, then Proposition 3 could be modified to pertain for the class of increasing and convex functions.

13. See also figure 3.14 in Chambers and Quiggin (p. 117). Our crop-specific fair value function is generalized Schur-Convex whereas their preference function $W(\cdot)$ is generalized Schur-Concave, i.e., $-W(\cdot)$ is generalized Schur-Convex.

14. The concept of arrangement increasing functions is due to Hollander, Proschan, and Sethuraman. But they termed it 'decreasing in transposition', a label that has fallen into disuse.

15. On the relation between majorization and mean-preserving spreads, see pp. 16-17 in Marshall and Olkin.

16. This result is just an application to our context of a finding in Cheng, as reported on p. 421 of Marshall and Olkin.

17. A crude attempt at an analysis of this decomposition was provided in Hennessy. There, the weighting vector $(p_1, \dots, p_n)$ was uniform and the analysis concerned random vectors rather than probability measures.

18. Stochastic order $\overset{sm}{\geq}$ is precisely the order studied by Shaked and Shanthikumar and by Müller and Scarsini.

19. An example of a shift in a multivariate distribution that satisfies Definition 4 is the case where the multivariate normal distribution undergoes an increase in one or more of its correlation coefficients. See Müller and Scarsini.

20. The way (10B) is written may be confusing in this regard. In equation (10B) it is, however, clear that the subtracted term in (10A) has been added back. Equation (9) may be of more assistance in providing an understanding of Proposition 7.

21. All premium calculations in this paper can be closely replicated with the USDA Risk Management Agency's Premium Calculator program available at http://www.rma.usda.gov/.

22. The table was also simulated for the North Dakota example. The contents display findings similar to those in Table 3.

23. RCCt is higher up the lattice than RS, but they are not connected because one cannot obtain RCCt by adding rotation crops to RS.

24. Adding livestock to whole-farm crop insurance is possible now only with the Adjusted Gross

Revenue (AGR) program developed by USDA. AGR is offered only in limited areas. But the Agricultural Risk Protection Act of 2000 expanded the federal crop insurance program by mandating two pilot livestock insurance programs. These pilot programs will be offered to Iowa hog producers in 2003.

25. The relevance of our analysis to risk management goes beyond revenue insurance. U.S. federal subsidies are also available for area yield insurance contracts. Under area yield insurance, indemnities are paid whenever yield in a contract-defined area falls short of a guarantee level. The problem of analyzing the payment schemes where all acres are insured together and separately is structurally identical to the RI problem studied here. See Vercammen or references cited therein for more detail on area yield insurance.