ABSTRACT

VaR gives a prediction of potential portfolio losses, with a certain level of confidence, that may be encountered over a specified time period due to adverse price movements in the portfolio’s assets. For example, a VaR of 1 million dollars at the 95% level of confidence implies that overall portfolio losses should not exceed 1 million dollars more than 5% of the time over a given holding period. This research examines the effectiveness of VaR measures, developed using alternative estimation techniques, in predicting large losses in the cattle feeding margin. Results show that several estimation techniques, both parametric and non-parametric, provide well calibrated VaR estimates such that violations (losses exceed the VaR estimate) are commensurate with the desired level of confidence. In particular, estimates developed using JP Morgan’s Risk Metrics methodology seem promising.
INTRODUCTION

The topic of Value-at-Risk (VaR) has recently come to the forefront of financial risk management and is considered by many as the “state-of-the-art” in risk measurement. VaR gives a prediction of potential portfolio losses, with a certain level of confidence, that may be encountered over a specified time period due to adverse price movements in the portfolio’s assets. For example, a Value-at-Risk estimate of 1 million dollars at the 95% level of confidence implies that overall portfolio losses should not exceed 1 million dollars more than 5% of the time over the given holding period (Jorion). VaR is usually estimated using either parametric or full-valuation procedures. Parametric procedures rely on estimates of variances and covariances (correlations) in creating portfolio standard deviation measures which are scaled by a factor corresponding to the desired confidence level (e.g., 1.65 for the 95% confidence level). Full-valuation procedures model the entire return distribution with the VaR measure being the quantile associated with the desired confidence level (e.g., 5% quantile for the 95% confidence level). Studies to date (Mahoney; Jackson, Maude, and Perraudin) find that the performance of either parametric or full-valuation procedures is sensitive to the data and portfolio examined as well as the predetermined factors of the VaR model itself (e.g., confidence level and time horizon).

The major market risks to cattle feeding are the variability of fed cattle prices (output price) and the variability of corn and feeder cattle prices (input prices). The difference between fed cattle prices and the prices of corn and feeder cattle, under assumed production technology, is referred to as the cattle feeding margin (Leuthold and Mokler; Peterson and Leuthold). In the context of livestock risk management, these prices have often been examined in a portfolio framework (Peterson and Leuthold). In addition, there has been renewed interest in the variability of fed cattle, feeder cattle, and corn prices and their effect on the profitability of cattle feeding operations (Schroeder et al.; Jones et al.).

Considering the interest in VaR and the variability of the market risk factors of the cattle feeding margin, the overall objective of this paper is to examine VaR measures in the context of the cattle feeding margin. In particular, this paper develops and tests VaR measures incorporating alternative estimation
techniques (both parametric and full-valuation), in predicting large losses in the cattle feeding margin (e.g., the number of times the VaR is exceeded relative to its pre-determined confidence level).

DATA

Return series are constructed from Wednesday cash prices of fed cattle, feeder cattle, and corn. Fed cattle prices ($/cwt) are for the Texas-Oklahoma direct market (1100 to 1300 lb steers), feeder cattle ($/cwt) are for the Oklahoma City terminal market (650 to 700 lbs), and corn prices ($/bu) are for Central Illinois (#2 yellow). Returns are defined as $R_{t,i} = \ln(p_{t,i}) - \ln(p_{t-1,i})$ where $R_{t,i}$ is the weekly return of commodity $i$, $\ln$ is the natural logarithm, $p_{t,i}$ is the current Wednesday price of commodity $i$ and $p_{t-1,i}$ is the previous Wednesday price. Weekly price data are used since fed cattle and feeder cattle are actively traded only one day per week, with that day typically in mid week (Rob). If a Wednesday price is not available, then a Tuesday price is used. The three data series span from January 1984 through December 1997 providing 14 years (729 observations) of returns for estimation and out-of-sample testing.

METHODS

Leuthold and Mokler, Peterson and Leuthold, and Schroeder and Hayenga describe similar cattle feeding scenarios that incorporate fixed feeding technology. It is assumed that cattle are placed on feed at 650 pounds and fed to 1100 pounds, consuming 45 bushels of corn in the process.\(^1\) Based on this technology, the cattle feeding margin is defined as:

\[
(1) \quad \text{margin ($/cwt)} = \frac{(\text{fed cattle price})_{11} - (\text{feeder cattle price})_{6.5} - (\text{corn price})_{45}}{11}. \quad 2
\]

On large feedlots, cattle are continually marketed and placed on feed. As well, feedlots with a capacity of 30,000 head or more typically do not maintain corn inventories for more than two weeks (Davies and Widawsky). Therefore, it is assumed that cattle feeding is a continuous process with decision makers routinely evaluating the variability of fed cattle, feeder cattle, and corn prices in a portfolio framework.

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\(^1\)Studies examining the hedging of the cattle feeding margin typically assume the consumption of corn to be in the range of 42 to 49 bushels.

\(^2\) Typically, other variable costs are also subtracted from the revenues and expenses presented in (1). Since this study focuses on market risk, other variable costs (e.g., vet costs) are held constant.
VaR measures are developed and evaluated for weekly horizons consistent with the periodicity of the three return series presented earlier. The variance of the cattle feeding margin ($\sigma_{fm}^2$) is defined as:

\[
\sigma_{fm}^2 = w_{fc}^2 \sigma_{fc}^2 + w_{fdr}^2 \sigma_{fdr}^2 + w_c^2 \sigma_c^2 + 2 w_{fc} w_{fdr} \rho_{fc,fdr} \sigma_{fc} \sigma_{fdr} + 2 w_{fc} w_c \rho_{fc,c} \sigma_{fc} \sigma_c + 2 w_{fdr} w_c \rho_{fdr,c} \sigma_{fdr} \sigma_c
\]

where $\sigma_{fc}^2$, $\sigma_{fdr}^2$, and $\sigma_c^2$ are the variances of fed cattle, feeder cattle, and corn returns and $\rho_{fc,fdr}$, $\rho_{fc,c}$, and $\rho_{fdr,c}$ are the respective correlation coefficients between returns. The portfolio weights ($w_{fc}$, $w_{fdr}$, and $w_c$) are defined as $P_i Q_i$ where $P_i$ is the price of commodity $i$ and $Q_i$ is quantity of commodity $i$ based on the assumed production technology allowing equation (2) to be expressed in dollar terms (Jorion, p. 156).

Considering equation (2) in a forecasting framework such that the individual variances and correlation coefficients are forecasts, VaR at any given week $t$ is:

\[
VaR_{t, fm} = \alpha \hat{\sigma}_{t+1, fm}
\]

where $\hat{\sigma}_{t+1, fm}$ is the portfolio volatility forecast of the cattle feeding margin ($$/cwt) and $\alpha$ is the scaling factor corresponding to the desired confidence level.

Several variance forecasts are employed in estimating (3). Appendix A and B explicitly present these procedures. The methods include a long-run historical average model of variances and covariances where the sample size is anchored to the first return observation in the series, a 150 week historical moving average, a GARCH (1,1) $\sim t$, implied volatility from nearby futures options contracts, and an exponentially weighted moving average variance forecast advocated by JP Morgan’s Risk Metrics. For the Risk Metrics forecasts, three fixed decay factors are used including $\lambda=.94$ and $\lambda=.97$ which are recommended by Risk Metrics for weekly and monthly data respectively as well as a $\lambda$ optimized for weekly data over the respective sample period of the return series ($\lambda=.96$). Some of the volatility models described above (e.g., implied volatility) do not have corresponding methods to explicitly develop covariances needed for correlation estimation. Because of this, correlations are developed using three

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3 In each of the variance and covariance forecasts, mean returns are constrained to zero which is a common practice in the volatility forecasting literature. In addition, Figlewski provides empirical evidence showing that setting the mean of the return series to zero can provide more accurate volatility estimates.
methods including the long-run historical average, 150 week moving average, and *Risk Metrics* correlations (Appendix A and B). Since the use of *Risk Metrics* volatilities and correlations is advocated as a simple alternative to multivariate GARCH procedures, a constant correlation MGARCH procedure is also used for creating a time-varying covariance matrix in creating VaR estimates (Appendix C).  

Using these parametric methods, VaR measures are estimated from January, 1987 through October, 1997 providing 564 weekly forecasts of volatility and correlations among the three return series. VaR (equation 3) is calculated for the 90% ($\alpha=1.28$), 95% ($\alpha=1.65$), and 99% ($\alpha=2.33$) levels of confidence. In addition to the parametric VaR estimates, a simple full-valuation procedure (historical simulation) is also developed for the 90%, 95%, and 99% confidence levels. The historical simulation method models the entire return distribution with the VaR designated as the quantile associated with the desired level of confidence (Linsmeier and Pearson). The historical simulation procedure is outlined in Appendix D and follows the methods of Linsmeier and Pearson. The specific parametric and historical simulation VaR measures developed and tested are described and outlined in table 1.

VaR estimates are evaluated on their ability to predict large losses in the cattle feeding margin resulting from fluctuations in fed cattle, feeder cattle, and corn prices. If actual portfolio losses over the desired horizon (e.g., 1 week) exceed the VaR estimate, a violation occurs. Hence, if violations are in excess to that implied by a particular confidence level, the VaR measure is considered inadequate in measuring large losses of the cattle feeding margin. To determine if violations are commensurate with the designated confidence level of VaR, a likelihood ratio test is developed following the procedures of Lopez. The null hypothesis is $H_0: \delta = \delta^*$ where $\delta$ is the desired coverage level (e.g., 5%) corresponding to the given confidence level (e.g., 95%), $\delta^*$ is $X/N$ where $X$ is the number of realized violations and $N$ is the number of out-of-sample observations. The probability of realizing $X$ violations of VaR for a sample of $N$ and the likelihood ratio test statistic is (Lopez, p. 7):

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4 Manfredo, Leuthold, and Irwin examine the univariate forecasting performance of these various volatility forecasts. It is important to note that the forecasting performance of these procedures in a univariate context are different than their performance as VaR measures due to the composition of the portfolio and correlation effects.
which has an asymptotic $\chi^2$ distribution with 1 degree of freedom. Similarly, a test of bias in VaR estimates is conducted consistent with the procedures of Mahoney (p. 206 and 207) which is based on a binomial probability distribution. The expectation of the number of violations of a VaR estimate is $N(1-c)$ where $N$ is the number of out-of-sample observations and $c$ is the confidence level expressed in decimal form (e.g., 0.95 for 95% level of confidence). The variance of this estimate is $Nc(1-c)$. Thus, the test for bias is defined as a $Z$ test, which in large samples is distributed normally, such that:

$$Z_c = \frac{L_{\text{realized}} - N(1-c)}{\sqrt{Nc(1-c)}}$$

where $L_{\text{realized}}$ = the number of observed violations of VaR at a given confidence level $c$ (Mahoney).

Hence, if the $Z$ statistic is significantly positive (negative) then VaR regularly underestimates (overestimates) actual downside risks. Summary statistics of the VaR violations are also used to evaluate the VaR models including the number of violations realized ($X$), the percentage of violations ($X/N$)*100, the average size violation (“sum of violations”)/$X$), the maximum violation, and the minimum violation. Therefore, if several VaR measures are determined to be “well calibrated” (e.g., $\delta = \delta^*$) the preferred VaR model among alternatives is the one with the smallest of these summary statistics.

**EMPIRICAL RESULTS**

The results of the likelihood ratio test (equation 4) and $Z$ test (equation 5) for each VaR confidence level are presented in tables 2 through 4. The Risk Metrics models ($\lambda=.97$, $\lambda=.94$, $\lambda=.96$), the historical moving average model (H150-VaR), and the full-valuation historical simulation (HISTSIM-VaR) provide coverage consistent with all three confidence levels (90%, 95%, and 99%). However, the Risk Metrics specification using the $\lambda=.97$ (RM97-VaR) appears to provide the best VaR estimates for

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5 The likelihood ratio statistic and $Z$ statistic are commonly used in the literature as well as by industry professionals, however, Lopez notes that these tests are designed to determine unconditional coverage, and do not take into consideration potential serial dependence of violations. The development of VaR evaluation measures that examine conditional coverage is a topic of current research (see Lopez; Crnkovic and Drachman).
each confidence level using a wide array of evaluation criteria. It is important to note though that the improvement provided by RM97-VaR relative to the other well calibrated VaR models is fairly minimal. Furthermore, evaluation based on the summary statistics presented, beyond the results of the LR test and Z test, is somewhat subjective.⁶

Since it is well known that composite forecasting techniques often yield superior forecasts (Clemen), a composite VaR estimate is also created and examined versus the individual models. The composite measure is constructed as a simple average of RM97-VaR and HISTSIM-VaR (COMP-VaR). These two methods are chosen for combining since they are both constructed using different methods (parametric vs. full-valuation). COMP-VaR is found to provide coverage consistent with all three confidence levels and provides improvement over HISTSIM-VaR.

Performance of any VaR measure is greatly affected by the correlation structure incorporated. Those VaR models that combine univariate volatilities (variances) in conjunction with alternative correlation estimates (mixed models) consistently underestimate the true downside risk of the cattle feeding margin. However, performance of these mixed VaR models improve as the correlations go from being long-run historical to conditional *a la Risk Metrics* (e.g., IVHIST-VaR to IVRM97-VaR). The parametric VaR measures found to be well calibrated across confidence levels use variances and correlations that are estimated from the same underlying method (i.e., *Risk Metrics*). This observation calls into question the computational validity of using implied volatility or another univariate volatility estimation procedure for VaR that does not have a corresponding way of defining covariances and subsequently correlations. Furthermore, the weak performance of both multivariate GARCH specifications is likely due to the constant correlation assumption incorporated which was necessary to provide a positive definite covariance matrix as well as model convergence.

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⁶ These are interesting results given that in a univariate volatility forecasting context (Manfredo, Leuthold, and Irwin), *Risk Metrics* volatility forecasts do not perform well relative to competing forecasts. However, it is important to remember that VaR relies on portfolio volatility when dealing with more than one asset and that the *Risk Metrics* methodology provides a time-varying covariance matrix such that variances and covariances (correlations) are estimated with a matching procedure.
Seasonality is also found to be a factor in the performance of the tested Value-at-Risk measures. The majority of the violations from RM97-VaR were realized during the period of April through October. For instance, at the 90% level of confidence, 41 out of the 62 violations (66%) occur from April to October. Similarly, at the 95% the April through October time periods saw 22 out of the 32 violations. At the 99% confidence level, 5 out of the 6 violations were realized during the May to October time span. These observations are consistent with known increased price variability of fed cattle and corn prices during these months.

SUMMARY AND CONCLUSIONS

The methods recommended by JP Morgan’s Risk Metrics, in particular using $\lambda=.97$, provide a robust specification for a covariance matrix in calculating weekly parametric VaR for the examined portfolio (the cattle feeding margin). Other specifications, including a simple historical simulation (full-valuation procedure), also produce well calibrated VaR measures for all three confidence levels examined. The fact that both parametric procedures (e.g. Risk Metrics) and full-valuation (e.g., historical simulation) were found to provide well calibrated VaR measures is most likely due to the fact that the cattle feeding margin, as defined in this study, is a portfolio of linear instruments (cash prices). Overall, it is concluded that at least for this portfolio, correlations are more important to the overall performance of a particular VaR measure than volatilities (variances). Furthermore, the majority of violations of VaR occur during times of observed seasonal increases in the volatility of fed cattle and corn returns.

This study is the first known attempt at empirically examining the performance of various VaR measures in the context of an agricultural enterprise (e.g., cattle feeding). To date, all known empirical studies examining the performance of alternative VaR measures have been conducted in the context of portfolios containing currency, interest rate, or equity data with portfolios often developed randomly (Mahoney; Hendricks). The cattle feeding margin provides a realistic alternative setting, as well as new data, for studying existing techniques of VaR estimation.
Table 1. Value-at-Risk Measures Key.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGARCH</td>
<td>Multivariate GARCH (constant correlation)</td>
</tr>
<tr>
<td>MGARCH-t</td>
<td>Multivariate GARCH using the student’s t-distribution in the estimation</td>
</tr>
<tr>
<td></td>
<td>(constant correlation)</td>
</tr>
<tr>
<td>H150-VaR</td>
<td>150-day moving average variances and correlations</td>
</tr>
<tr>
<td>HISTAVG-VaR</td>
<td>Long-run historical average variances and correlations</td>
</tr>
<tr>
<td>RM97-VaR</td>
<td><em>Risk Metrics</em> variances and correlations using $\lambda=.97$</td>
</tr>
<tr>
<td>RM94-VaR</td>
<td><em>Risk Metrics</em> variances and correlations using $\lambda=.94$</td>
</tr>
<tr>
<td>RM96-VaR</td>
<td><em>Risk Metrics</em> variances and correlations using optimized $\lambda=.96$</td>
</tr>
<tr>
<td>GRM97-VaR</td>
<td>GARCH-t variances and RM97 correlations</td>
</tr>
<tr>
<td>GH150-VaR</td>
<td>GARCH-t variances and H150 correlations</td>
</tr>
<tr>
<td>GHIST-VaR</td>
<td>GARCH-t variances and HISTAVG correlations</td>
</tr>
<tr>
<td>IVRM97-VaR</td>
<td>Implied volatility (IV) and RM97 correlations</td>
</tr>
<tr>
<td>IVH150-VaR</td>
<td>Implied volatility (IV) and H150 correlations</td>
</tr>
<tr>
<td>IVHIST-VaR</td>
<td>Implied volatility (IV) and HISTAVG correlations</td>
</tr>
<tr>
<td>HISTSIM-VAR</td>
<td>Historical simulation</td>
</tr>
<tr>
<td>COMP-VaR</td>
<td>Simple average composite of RM97-VaR and HISTSIM-VaR</td>
</tr>
</tbody>
</table>


Table 2. Evaluation of VaR Measures for the 90% Confidence Level.

<table>
<thead>
<tr>
<th>VaR Measure</th>
<th>Number of Violations (N=564)</th>
<th>Percent Violations</th>
<th>Avg. Size Violation</th>
<th>Maximum Violation</th>
<th>Minimum Violation</th>
<th>LR Statistic</th>
<th>Z Statistic</th>
<th>Average VaR Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGARCH-t</td>
<td>84</td>
<td>14.890</td>
<td>$9.737</td>
<td>$36.105</td>
<td>$0.063</td>
<td>13.251</td>
<td>3.874 **</td>
<td>$17.862</td>
</tr>
<tr>
<td>MGARCH</td>
<td>73</td>
<td>12.940</td>
<td>$9.013</td>
<td>$34.426</td>
<td>$0.226</td>
<td>5.015</td>
<td>2.330 **</td>
<td>$20.107</td>
</tr>
<tr>
<td>H150-VaR</td>
<td>61</td>
<td>10.820</td>
<td>$8.206</td>
<td>$32.349</td>
<td>$0.044</td>
<td>0.407</td>
<td>0.646</td>
<td>$21.632</td>
</tr>
<tr>
<td>HISTAVG-VaR</td>
<td>72</td>
<td>12.770</td>
<td>$8.425</td>
<td>$32.027</td>
<td>$0.009</td>
<td>4.449</td>
<td>2.190 **</td>
<td>$20.280</td>
</tr>
<tr>
<td>RMOPT-VaR</td>
<td>59</td>
<td>10.460</td>
<td>$8.621</td>
<td>$32.119</td>
<td>$0.451</td>
<td>0.131</td>
<td>0.365</td>
<td>$21.959</td>
</tr>
<tr>
<td>RM94-VaR</td>
<td>60</td>
<td>10.640</td>
<td>$8.726</td>
<td>$32.597</td>
<td>$0.023</td>
<td>0.251</td>
<td>0.505</td>
<td>$21.931</td>
</tr>
<tr>
<td>RM97-VaR</td>
<td>62</td>
<td>10.990</td>
<td>$8.081</td>
<td>$31.887</td>
<td>$0.002</td>
<td>0.600</td>
<td>0.786</td>
<td>$21.925</td>
</tr>
<tr>
<td>GRM97-VaR</td>
<td>75</td>
<td>13.300</td>
<td>$8.898</td>
<td>$32.280</td>
<td>$0.038</td>
<td>6.243</td>
<td>2.611 **</td>
<td>$19.983</td>
</tr>
<tr>
<td>GH150-VaR</td>
<td>75</td>
<td>13.300</td>
<td>$9.534</td>
<td>$34.572</td>
<td>$0.279</td>
<td>6.243</td>
<td>2.611 **</td>
<td>$19.444</td>
</tr>
<tr>
<td>GHIST-VaR</td>
<td>87</td>
<td>15.430</td>
<td>$9.530</td>
<td>$36.649</td>
<td>$0.062</td>
<td>16.101</td>
<td>4.295 **</td>
<td>$17.871</td>
</tr>
<tr>
<td>IVRM97-VaR</td>
<td>69</td>
<td>12.230</td>
<td>$9.726</td>
<td>$35.815</td>
<td>$0.306</td>
<td>2.941</td>
<td>1.769</td>
<td>$19.351</td>
</tr>
<tr>
<td>IVH150-VaR</td>
<td>78</td>
<td>13.830</td>
<td>$9.041</td>
<td>$37.424</td>
<td>$0.070</td>
<td>8.314</td>
<td>3.032 **</td>
<td>$18.880</td>
</tr>
<tr>
<td>IVHIST-VaR</td>
<td>83</td>
<td>14.720</td>
<td>$9.879</td>
<td>$39.120</td>
<td>$0.261</td>
<td>12.357</td>
<td>3.734 **</td>
<td>$17.515</td>
</tr>
<tr>
<td>HISTSIM-VaR</td>
<td>69</td>
<td>12.230</td>
<td>$8.367</td>
<td>$33.283</td>
<td>$0.052</td>
<td>2.941</td>
<td>1.769</td>
<td>$20.854</td>
</tr>
<tr>
<td>COMP-VaR</td>
<td>60</td>
<td>10.640</td>
<td>$8.738</td>
<td>$33.429</td>
<td>$0.126</td>
<td>0.251</td>
<td>0.505</td>
<td>$21.515</td>
</tr>
</tbody>
</table>

1 N is the number of weekly VaR estimates and subsequent changes in portfolio value.
2 Avg size violation, maximum violation, minimum violation, and average VaR estimate are in dollars per hundred weight.
*Significant at the 5% level. The Chi-squared critical value is 3.841.
**Significant at the 5% level. The critical Z value is 1.96.
Table 3. Evaluation of VaR Measures for the 95% Confidence Level.

<table>
<thead>
<tr>
<th>VaR Measure</th>
<th>Number of Violations (N=564)</th>
<th>Percent Violations</th>
<th>Avg. Size Violation</th>
<th>Maximum Violation</th>
<th>Minimum Violation</th>
<th>LR Statistic</th>
<th>Z Statistic</th>
<th>Average VaR Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGARCH-t</td>
<td>56</td>
<td>9.930</td>
<td>$8.519</td>
<td>$31.040</td>
<td>$0.539</td>
<td>22.073 *</td>
<td>5.371 **</td>
<td>$23.025</td>
</tr>
<tr>
<td>MGARCH</td>
<td>44</td>
<td>7.800</td>
<td>$7.771</td>
<td>$28.875</td>
<td>$0.394</td>
<td>8.019 *</td>
<td>3.053 **</td>
<td>$25.920</td>
</tr>
<tr>
<td>H150-VaR</td>
<td>34</td>
<td>6.030</td>
<td>$6.580</td>
<td>$26.197</td>
<td>$0.010</td>
<td>1.182</td>
<td>1.121</td>
<td>$27.885</td>
</tr>
<tr>
<td>HISTAVG-VaR</td>
<td>39</td>
<td>6.910</td>
<td>$7.583</td>
<td>$25.796</td>
<td>$0.097</td>
<td>3.910 *</td>
<td>2.087 **</td>
<td>$26.143</td>
</tr>
<tr>
<td>RMOPT-VaR</td>
<td>31</td>
<td>5.500</td>
<td>$7.518</td>
<td>$25.901</td>
<td>$0.388</td>
<td>0.284</td>
<td>0.541</td>
<td>$28.306</td>
</tr>
<tr>
<td>RM94-VaR</td>
<td>32</td>
<td>5.670</td>
<td>$7.913</td>
<td>$26.517</td>
<td>$0.233</td>
<td>0.518</td>
<td>0.734</td>
<td>$28.271</td>
</tr>
<tr>
<td>RM97-VaR</td>
<td>32</td>
<td>5.670</td>
<td>$6.838</td>
<td>$25.061</td>
<td>$0.032</td>
<td>0.518</td>
<td>0.734</td>
<td>$28.262</td>
</tr>
<tr>
<td>GRM97-VaR</td>
<td>46</td>
<td>8.160</td>
<td>$7.541</td>
<td>$26.108</td>
<td>$0.751</td>
<td>10.015 *</td>
<td>3.439 **</td>
<td>$25.759</td>
</tr>
<tr>
<td>GH150-VaR</td>
<td>51</td>
<td>9.040</td>
<td>$7.667</td>
<td>$29.063</td>
<td>$0.033</td>
<td>15.820 *</td>
<td>4.405 **</td>
<td>$25.089</td>
</tr>
<tr>
<td>GHIST-VaR</td>
<td>58</td>
<td>10.280</td>
<td>$8.415</td>
<td>$31.741</td>
<td>$0.206</td>
<td>25.739 *</td>
<td>5.757 **</td>
<td>$23.048</td>
</tr>
<tr>
<td>IVRM97-VaR</td>
<td>44</td>
<td>7.800</td>
<td>$8.339</td>
<td>$30.666</td>
<td>$0.874</td>
<td>8.019 *</td>
<td>3.053 **</td>
<td>$24.945</td>
</tr>
<tr>
<td>IVH150-VaR</td>
<td>45</td>
<td>7.980</td>
<td>$8.712</td>
<td>$32.740</td>
<td>$0.745</td>
<td>8.993 *</td>
<td>3.246 **</td>
<td>$24.338</td>
</tr>
<tr>
<td>IVHIST-VaR</td>
<td>54</td>
<td>9.570</td>
<td>$9.057</td>
<td>$34.926</td>
<td>$0.333</td>
<td>19.826 *</td>
<td>4.985 **</td>
<td>$22.578</td>
</tr>
<tr>
<td>HISTSIM-VaR</td>
<td>36</td>
<td>6.380</td>
<td>$6.705</td>
<td>$26.624</td>
<td>$0.016</td>
<td>2.096</td>
<td>1.507</td>
<td>$27.929</td>
</tr>
<tr>
<td>COMP-VaR</td>
<td>34</td>
<td>6.030</td>
<td>$6.614</td>
<td>$27.200</td>
<td>$0.262</td>
<td>1.182</td>
<td>1.121</td>
<td>$28.258</td>
</tr>
</tbody>
</table>

1 N is the number of weekly VaR estimates and subsequent changes in portfolio value.
2 Avg size violation, maximum violation, minimum violation, and average VaR estimate are in dollars per hundred weight.
*Significant at the 5% level. The Chi-squared critical value is 3.841.
**Significant at the 5% level. The critical Z value is 1.96.
Table 4. Evaluation of VaR Measures for the 99% Confidence Level

<table>
<thead>
<tr>
<th>VaR Measure</th>
<th>Number of Violations (N=564)</th>
<th>Percent Violations</th>
<th>Avg. Size Violation</th>
<th>Maximum Violation</th>
<th>Minimum Violation</th>
<th>LR Statistic</th>
<th>Z Statistic</th>
<th>Average VaR Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGARCH-t</td>
<td>22</td>
<td>3.900</td>
<td>$6.370</td>
<td>$21.730</td>
<td>$0.400</td>
<td>27.655</td>
<td>6.924 **</td>
<td>$32.514</td>
</tr>
<tr>
<td>MGARCH</td>
<td>14</td>
<td>2.480</td>
<td>$5.795</td>
<td>$18.674</td>
<td>$1.301</td>
<td>8.863</td>
<td>3.538 **</td>
<td>$36.602</td>
</tr>
<tr>
<td>H150-VaR</td>
<td>6</td>
<td>1.060</td>
<td>$6.001</td>
<td>$14.891</td>
<td>$0.585</td>
<td>0.023</td>
<td>0.152</td>
<td>$39.377</td>
</tr>
<tr>
<td>HISTAVG-VaR</td>
<td>10</td>
<td>1.770</td>
<td>$6.062</td>
<td>$16.036</td>
<td>$0.607</td>
<td>2.768</td>
<td>1.845</td>
<td>$36.917</td>
</tr>
<tr>
<td>RMOPT-VaR</td>
<td>10</td>
<td>1.770</td>
<td>$4.478</td>
<td>$14.473</td>
<td>$0.427</td>
<td>2.768</td>
<td>1.845</td>
<td>$39.972</td>
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<tr>
<td>RM94-VaR</td>
<td>11</td>
<td>1.950</td>
<td>$5.021</td>
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<td>$0.420</td>
<td>4.028</td>
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<td>$39.922</td>
</tr>
<tr>
<td>RM97-VaR</td>
<td>6</td>
<td>1.060</td>
<td>$5.668</td>
<td>$14.050</td>
<td>$0.596</td>
<td>0.023</td>
<td>0.152</td>
<td>$39.910</td>
</tr>
<tr>
<td>GHIST-VaR</td>
<td>23</td>
<td>4.080</td>
<td>$6.445</td>
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<td>$0.147</td>
<td>30.483</td>
<td>7.347 **</td>
<td>$32.530</td>
</tr>
<tr>
<td>IVRM97-VaR</td>
<td>16</td>
<td>2.840</td>
<td>$4.442</td>
<td>$21.202</td>
<td>$0.533</td>
<td>12.840</td>
<td>4.384 **</td>
<td>$35.225</td>
</tr>
<tr>
<td>IVH150-VaR</td>
<td>17</td>
<td>3.010</td>
<td>$4.858</td>
<td>$24.131</td>
<td>$0.368</td>
<td>15.026</td>
<td>4.808 **</td>
<td>$34.368</td>
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<tr>
<td>IVHIST-VaR</td>
<td>25</td>
<td>4.430</td>
<td>$5.877</td>
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<td>$0.666</td>
<td>36.409</td>
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<td>$31.883</td>
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<tr>
<td>HISTSIM-VaR</td>
<td>10</td>
<td>1.770</td>
<td>$5.381</td>
<td>$17.602</td>
<td>$0.273</td>
<td>2.768</td>
<td>1.845</td>
<td>$37.759</td>
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<tr>
<td>COMP-VaR</td>
<td>6</td>
<td>1.060</td>
<td>$6.168</td>
<td>$17.362</td>
<td>$1.648</td>
<td>0.023</td>
<td>0.152</td>
<td>$39.063</td>
</tr>
</tbody>
</table>

1N is the number of weekly VaR estimates and subsequent changes in portfolio value.
2Avg size violation, maximum violation, minimum violation, and average VaR estimate are in dollars per hundred weight.
*Significant at the 5% level. The Chi-squared critical value is 3.841.
**Significant at 5% level. The critical Z value is 1.96.
Appendix A: Volatility and Covariance Forecasts Used in Developing Parametric VaR.

Long-run historical average and 150 week moving average (T=150):
\[ \hat{\sigma}_{t+1,i,j} = \frac{1}{T} \sum_{m=0}^{T-1} R_{t-m,i,j} \]
\[ \hat{\sigma}_{t+1} = \frac{1}{T} \sum_{m=0}^{T-1} R_{t-m} \]

GARCH (1,1) ~ t:
\[ \hat{\sigma}_{t+1,i} = \sqrt{\alpha_0 + \alpha_1 R^2_{t,i} + \beta_1 \sigma^2_{t,i}} \]
Risk Metrics (exponentially weighted moving average where \( \lambda = 0.97, \alpha = 0.94, \beta = \text{optimized} \):
\[ \hat{\sigma}_{t+1,i} = \sqrt{\lambda \sigma^2_{t,i} + (1-\lambda) R^2_{t,i}} \]
\[ \hat{\sigma}_{t+1,j} = \lambda \hat{\sigma}_{t,j} + (1-\lambda) R_{t,j} \]

Implied volatility from nearby, at-the-money, options on corresponding futures contracts:
\[ \hat{IV}_{t+1,i} = \frac{IV_{\text{annual},t,i}}{\sqrt{52}} \]

Appendix B: Correlation Used in Developing Parametric VaR Using Historical or Risk Metrics Covariances.
\[ \hat{\rho}_{t+1,i,j} = \frac{\hat{\sigma}_{t+1,i,j}}{\hat{\sigma}_{t+1,i} \hat{\sigma}_{t+1,j}} \]

Appendix C: Constant Correlation Multivariate GARCH Specification.
\[ \sigma^2_{u,t} = c_{u,t} + \alpha_{u,t} R^2_{u,t-1} + \beta_{u,t} \sigma^2_{u,t-1} \]
\[ \sigma_{y,t} = \rho_{y} \sigma_{u,t} \sigma_{y,t} \]

Appendix D: Historical Simulation Method.
1. At time t, the cattle feeding margin is calculated as in equation (1).
2. Prices of fed cattle, feeder cattle, and corn at time t are exposed to their respective previous 150 weeks of returns such that \( P^* = P(t+1+R_{t-T}) \) for all \( T = 1...150 \).
3. The cattle feeding margin is recalculated using these new prices \( P^* \) creating 150 new values of the cattle feeding margin.
4. Next, each of these new values of the cattle feeding margin is subtracted from the actual feeding margin realized at week t, providing 150 differences between the cattle feeding margin at week t and the simulated values of the feeding margin.
5. From the distribution of these differences, the quantile associated with the desired confidence level (e.g., 5% quantile for 95% level of confidence) becomes the VaR estimate.
REFERENCES


