Farm-level Acreage Allocation under Risk

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Abstract

We model the area allocation decision problem for a fixed size crop farm under random yields and prices for a risk-averse farmer. We assume that in the short run, the variable input expenses are fixed per hectare and per crop (an assumption that is motivated by our data). Therefore the cost function depends only on the non-stochastic area allocation. The first order conditions of the model involve integrals across functions of random variables that do not in general have closed form solutions. Numerical simulation techniques are used to calibrate the parameters of the cost function.

The two sources of randomness, price and yield, are combined into a single random variable, the yield-in-value. Based on examination of panels of yield-in-value data, we assume independence across the yield-in-value distributions and that the farmers know these distributions.

We have modeled the sugar quota constraint, the Common Agricultural Policy subsidies and set-aside, and one Agri-Environmental Measure called “buffer zone”.
Farm-level Acreage Allocation under Risk

1. Introduction

In this paper, we are interested in constructing a model for Northwestern European crop farms, exploiting two EU databases: The Farm Accounting Data Network (FADN) and the agricultural land covers census. We model the short-run area allocation decision problem for a fixed size crop farm under random yields and prices. The model is currently in an intermediate phase.

We assume that farmers intend to maximize the expected utility of their net stochastic income. The utility function reflects risk-aversion. Income consists of stochastic revenues (prices times quantities) minus the cost of production. We assume constant per hectare variable input expenses. Even though that assumption may appear strong, farmers in our sample present fairly stable input expenses and most variable input expenses are committed at the time of area allocation. Therefore, the cost of production is a function of only the vector of decision variables (the area allocations) and not of any random element. The cost of production can then be captured through a non-stochastic cost function. That is an important advantage of the model (see e.g. Moschini, 2001).

At the time of allocation, neither prices nor yields are known for sure. These two sources of randomness are combined in the model into a single one: the random yield-in-value. Examination of panels of yield-in-value data for individual farmers from the FADN sample for the Walloon region of Belgium for the period 1995-2004 have led us to assume independence across the distributions of yield-in-value for different crops based on fixed effects panel data model residuals. This is important because the first order conditions of the model involve integrals across functions of (possibly non-normal) random yields-in-
value. Those integrals do not in general have closed form solutions, but since there is independence of the yield-in-value residual distributions, we can easily make use of simulation techniques to find numerical solutions for the parameters of the cost function. The distributions are not always normal; in particular, skewed-to-left distributions may fit the wheat yield-in-value best, possibly reflecting the price floor (“intervention”) policy of the EU. The model assumes that the farmers know these distributions, an assumption that we regard as a generalization of the adaptive expectation hypothesis.

Theoretically, our land allocation model is similar to Chavas and Holt’s (1990), however they consider an aggregate (representative) farmer who is not subject to land constraint; it is therefore not an allocation problem. Guyomard, Baudry and Carpentier (1996) consider land as a fixed and allocatable factor, but they use a sector level model that is not stochastic. Moro and Sckokai (1999) develop a farm-level non-stochastic land allocation model. Sckokai and Moro (2006) present a generalization of Moro and Sckokai (1999) to the case of stochastic prices together with a mean-variance utility function. In some respects, such as random yields, Constant Relative Risk Aversion utility function and the modeling of the beet sugar quota, our model is more general. However, Sckokai and Moro (2006) go much further in the modeling of the farmer’s choice of crops and inputs.

Currently, we apply the model to single farms in the FADN sample by means of a calibration approach. We assume that the cost function has a quadratic form and impose that the off-diagonal terms are zero. We calibrate it on the basis of the last year of fully complete available data (2003). We adjust the variable cost taken from the FADN for the opportunity cost of land implied by the total area constraint. We assume that the utility function has a Constant Relative Risk Aversion form.

Using this framework, numerous policies can be simulated. In particular, we argue that in a stochastic framework with risk aversion every policy change has to be considered not only from the point of view of its effect on the expected value of the random variables of the
model, but also on their variance. For a subsample of 24 FADN crop farms, we have simulated the following scenarios.

A reduction of the price of wheat under the form of a production “tax” (the negative of an output subsidy) equal to 10% of the average price of wheat induces a reduction in the total wheat area of about 3.5%; a decrease of 10% of the price of wheat induces roughly the same reduction of the wheat area. This is surprising since the former type of price decrease has practically no effect on the variance of the yield in value while the later type induces nearly a 20% decrease in variance. This is however consistent with the result that the introduction of crop insurance for wheat has negligible impacts on the area allocations. This is due to the fact that both yield and price of wheat are already quite stable in that region; therefore changes in variance are relatively unimportant compared to changes in expectation.

Sugar beet production is very important for Walloon farmers. It is also a heavily regulated sector, in which beet growers own delivery rights to sugar refineries at a guaranteed high price. We model two scenarios. In one, we let the world price for sugar drop to zero; that scenario is intended to represent a ban on EU sugar exports. On average that scenario leads to a reduction of the beet sugar hectarage of nearly 9%. In the second scenario, the beet growers’ delivery rights are decreased by 10%. The result is a reduction of sugar beet area of only 2.5%. That suggests that at least a few Walloon beet growers are able to sell sugar profitably on the world market.

Finally, we have modeled one Agri-Environmental Measure (AEM) from the EU Common Agricultural Policy accompanying measures. This AEM is called buffer zones; farmers who chose to uptake it have to maintain large strips of the local flora on the perimeter of their fields. We treat the buffer zones AEM as if it was an additional crop, without an output, but with an area subsidy. The model can be used to show that a 10% increase of the buffer zones subsidy leads to a 5 to 6% increase of the buffer zones area in our sample.
The remainder of the paper is divided in four parts. In the next section, we present the model for crops farms, including subsidies, mandatory set aside, sugar quota and buffer zones. In section three, we give a description of a sample of purely crops farms extracted from the FADN sample for the Walloon region. Section four presents several policy simulations and their effects on the area allocation choices. Section five discusses the results.

2. The model

This section is divided in four parts. We first motivate the model, then we introduce a core model designed to capture the short-run behavior of a generic multi-output crop farm in a hypothetical environment without agricultural policy distortions. Next we introduce subsidies per hectare and mandatory set aside, reflecting EU agricultural policies. In the third part, we introduce the modeling of the sugar sector. In our sample nearly all the crop farms produce sugar beet. Finally, we present the modeling of the buffer zones.

2.1 Motivation

Following a large body of applied literature, we assume a Constant Relative Risk Aversion (CRRA) utility function, that is, of the form \( U = \frac{1}{1-\rho} income^{1-\rho} \). Following OECD (2001), several authors have shown that there exist agents presenting decreasing absolute risk aversion (Arrow, 1965; Binswanger, 1981; Saha, Shumway and Talpaz, 1994 and Chavas and Holt, 1990). While Saha, Shumway and Talpaz (1994) have found empirical evidence supporting increasing relative risk aversion, many authors have assumed constant relative risk aversion and have tried to estimate \( \rho \). Following OECD (2001) results using Italian FADN data and presented in Table 1, relative risk aversion would show considerable variations according to farm size. In Belgium, small farmers often have an additional job
and may therefore be expected to present smaller risk aversion than their Italian counterparts.

Table 1. Relative risk aversion coefficients ($\rho$)

<table>
<thead>
<tr>
<th>Farm size</th>
<th>$\rho$</th>
<th>Asymptotic std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small &lt; 20 ha</td>
<td>3.29</td>
<td>0.88</td>
</tr>
<tr>
<td>Medium 20-40 ha</td>
<td>0.72</td>
<td>0.20</td>
</tr>
<tr>
<td>Large &gt; 40 ha</td>
<td>0.06</td>
<td>0.43</td>
</tr>
</tbody>
</table>


In a famous experiment conducted in India, Binswanger (1980) shows that farmers’ choices are consistent with expected utility (while not with the “safety first” model); that they respond to fluctuations of their income rather than their wealth; and that their attitude towards risk is rather properly approximated by a CRRA utility function defined over their income with an aversion coefficient $\rho$ ranging from 0.3 for small income changes and 1.7 for larger changes. The mean variance approach is a restrictive particular case of the expected utility model (Moschini and Hennessy, 2001).

A difficulty with stochastic farm models when they are defined in terms of the dual cost function is that the cost function has a stochastic argument (farm output). Even though Pope and Just (1996) and Moschini (2001) have shown that an ex-ante cost function is still well defined when yields and prices are stochastic, its specification becomes much more complicated. A simple solution to avoid that problem is to define a cost function on the surface allocations. The implicit hypotheses underlying such definition is that farmers intend to reach the same yield across all the areas planted to one crop and that input decisions are not stochastic. If that is true, then surface allocations can be regarded as a measure of output and the yield in value as a stochastic price.

In this way, the problem of correlated price and yield (see e.g. Moschini, 2001) also disappears from the model. Since both prices and yields are observed per farm and per year for most crops in the FADN sample, it is possible to infer the distribution of the yield in value. Additionally, when an area of the farm is under contract with a buyer for the
production of some crops, typically potatoes, flax and colza, only the yield in value appears in the FADN data, not the price and the yield separately.

Input decisions can be stochastic if they are primarily a reaction to random events such as irrigation can be a reaction to drought. In Northwestern European agriculture however, water is usually not a limiting factor and it can be argued that most input decisions are committed at the same time as the farmer chooses the surface allocation.

We assume that the input mix is fixed per hectare of each crop. There are five types of variable input expenses recorded in the RICA for crops (there are more for livestock): fertilizers (chemical and organic, pooled until 2001), treatments or pesticides, hired services, seeds and other inputs (unspecified). Only input expenses are observed in the FADN sample, not the prices or any measure of quantity.\(^1\) Those inputs are rather complementary, supporting the hypothesis that input expenses are fixed per ha.

Nearly all the variable inputs are crop-specific. That is self evident for seeds, but that is also the case for the other variable inputs to a great extent. Consequently, in a multi-output variable cost function, second-order (cross-products) terms can reasonably be assumed equal to zero. Another consequence is that across crops, the price per unit of a certain input does not refer to the same products, therefore, in terms of input intensity of an output, only the expense is relevant.

Given these facts and the hypotheses that we have made, the variable cost can be written as a function of the surface allocations only.

Fixed factors in the FADN belong to the “non-allocable” inputs (that is, for which the allocation per crop is not given in the FADN data). The data concern primarily labor (family and on farm) and capital (machines and other). Each crop requires the services of those fixed factors during definite periods in the course of one year (Just and Pope, 2003), but that allocation is not known in the FADN data. We assume that both capital and land

\(^1\) Considering the hundreds of products and services that belong to those inputs, it does not seem possible to consider actual quantity measures. That is why our modeling approach focuses on expenses.
amounts and crop mix are chosen to match family labor, in other words, such that family labor is occupied more or less equally across the whole year.

Consequently, surface allocation implicitly reflects fixed factors allocation along the year. With that simplification in mind, we see the farmer’s short-term economic problem as one purely of allocation in which deviation from the present allocation is increasingly costly.

We borrow the argument from Howitt (1995): in the short run, if the farmer wants to increase the allocation of one crop (short of increasing the total farm surface), he has to allocate areas that are not as appropriate, for example because they fall out of rotation, but also because he will have to use fixed factors (including his own labor) more intensively or hire them. Because crops are requiring fixed factors services at different moment across the year, changing one crop allocation will not affect the marginal cost of producing another crop on the farm other than through the total surface constraint. In other words, a short-term agricultural multi-output cost function should not have cross-products (second-order) terms.

In modeling term, we can therefore write a short-term cost function simply by using the surface, without cross-products terms. It is a simplification, but it is a convenient way to represent technical relations on which there is no data since fixed factors allocation across crops during the year is not given in the FADN.

In the following subsection, we present the formal crop model without agricultural policy. Then, in the next three subsections, we introduce the three main EU policies: subsidies and set-asides, sugar quota, agri-environmental measures.

### 2.2 Formal crop model

Let

- $\mathbb{E}$ the mathematical expectation operator
- $\theta$ the vector of yields in value calculated as output prices $P$ (usually in euro per T) times physical yields $\Psi$ (usually in T/ha)
- $h$ the vector of area allocations (in hectares)
- $C(h)$ the variable (or short-run) cost function
- $j$ the agricultural land use domain, including crops, $j = 1\ldots J$
The farmer has a Constant Relative Risk Aversion (CRRA) utility function and has the following maximizing behavior:

$$\max_{h} \left[ \left( \sum_{j} \tilde{\theta}_j h_j - C_j(h_j) \right)^{1-\rho} \right] \quad \text{s.t.} \quad \sum_{j} h_j = \bar{h} \quad [\omega] \quad [1]$$

with

$$\sum_{j} \tilde{\theta}_j h_j - C_j(h_j) \quad \text{net income,}$$

$$U = \frac{1}{1-\rho} (\text{net income})^{1-\rho} \quad \text{CRRA utility,} \quad [2]$$

$$C_j(h_j) = \alpha_j h_j + \frac{1}{2} \beta_j h_j^2 \quad \text{variable cost function.}$$

Since the average variable cost per crop $\bar{C}_j = \alpha_j + \frac{1}{2} \beta_j h_j$ is a data, we can write $C_j' = \bar{C}_j + \frac{1}{2} \beta_j h_j$. We define the following shorthand notations: $\Omega = EU'$ and $\Omega_j = E[\tilde{\theta}_j U']$ where $U'$ indicates the first derivative of the utility function. The First Order Conditions (FOC) can then be written:

$$\Omega_j = (\bar{C}_j + \frac{1}{2} \beta_j h_j) \Omega + \omega \quad \text{marginal revenue=marginal cost,}$$

$$\sum_{j} h_j = \bar{h} \quad \text{total area constraint.} \quad [3]$$

From the farmer’s point of view, the FOC constitute a system of J+1 non-linear equations in J+1 unknowns: $h_j$ – the land allocations – and $\omega$ – the opportunity cost of land. The parameters of the cost function $C_j(h)$ and of risk aversion $\rho$ are known to him while the yields in value $\tilde{\theta}$ are random but of known distribution.

From the investigator’s point of view, we observe supposedly optimal hectare allocations and realizations of the yields in value that are used to infer their distributions. In the system
of J+1 equations, there remain J+2 unknowns: the risk aversion coefficient $\rho$, the opportunity cost of land $\omega$ and the $J$ $\beta$ coefficients from the cost function.

In general, there is therefore infinity of solutions; we impose a value of one for the coefficient $\rho$ based on Table 1. System [3] can then be used to solve for $\beta$ and $\omega$ by means of a numerical non-linear solver\(^2\) in the following way:

\[
\Omega_j = \left( \overline{C}_j + \frac{1}{2} \beta_j h_j \right) \Omega + \omega \quad \text{as in the FOC}[3],
\]

\[
\sum_j \frac{2}{\Omega \beta_j} \left( \Omega_j - \overline{C}_j \Omega - \omega \right) = \overline{h} \quad \text{substituting } h_j \text{ in the total area constraint.}
\]

A key operational difficulty of that calibrating approach is that $\Omega = EU'$ and $\Omega_j = E[\hat{\theta}_j U']$ are integrals over non-linear function of random variables and therefore cannot in general be solved. However, provided that the distribution of those random variables is known, they can be numerically calculated by computer generating realizations of those variables. To this effect, we have used the FADN sample to test what distribution best fitted the yield in value of each crop. We have used a fixed effect panel data model to filter out farm specific changes in yields. It turned out that, although most crops did not follow the normal distribution, their distribution tended to be uncorrelated. To simplify the computer simulation of those random variables, we have made the hypothesis that they were independent; that is, we assume that each $\hat{\theta}_j$ is independent of the other elements of the vector $\hat{\theta}$. Numerically solving the system [4] therefore only implies generating $J$ series of $S$ simulated values of the estimated distributions of the vector $\hat{\theta}$. That is much easier than generating one series of simulations from a vector of correlated random variables. Therefore, the operational version of the calibrating system [4] is:

\(^2\) We use CONOPT3 as provided with the GAMS software.
\[
\Omega_j = \frac{1}{3} \sum_s \left[ \tilde{\theta}_j \sum_j \left( \tilde{\theta}_j h_j - C_j(h_j) \right)^{-\rho} \right] s = 1 \ldots S
\]
\[
\Omega = \frac{1}{3} \sum_s \left[ \sum_j \left( \tilde{\theta}_j h_j - C_j(h_j) \right)^{-\rho} \right] s = 1 \ldots S
\] [5]

Using [5], the system [4] can be solved farm per farm for every year for which there are data. The solutions will be different each time, implying a lack of robustness of the results. A potential solution would be to use econometric techniques to estimate the parameters of a more general cost function, but in the meantime, we calibrate the model on the year 2003 on the basis that it is the last year with fully complete data.

### 2.3 Subsidies and Set-asides

The EU Common Agricultural Policy is characterized by area subsidies denoted \( S_j \) that are received for the so-called “COP” crops. The subsidies actually received per farm and per year are available per crop in the FADN dataset. Farmers are entitled to receive those subsidies only if they set-aside a certain percentage denoted \( \rho_j \) of their COP area each year. In the formal model below, \( \rho_j = 0 \) for non-COP crops. Farmers receive a subsidy \( S_f \) on the set aside area. The set aside area also has a cost \( C_f \) since it must be seeded, sowed and generally maintained. We assume that because the farmer does not intend to maximize output on the set aside area, the cost function is simply linear, that is

\[
C_f(h_f) = \alpha_j \sum \rho_j h_j.
\] Formally, model \([1]\) becomes

\[
\max_{\bar{h}} \left[ \left( \sum_j \left( \tilde{\theta}_j + S_j + \rho_j S_f - \alpha_j - \frac{1}{2} \beta_j h_j - \alpha_j \rho_j h_j \right) h_j \right)^{1-\rho} \right]^{1-\rho}
\]

\[
\sum_j \left( 1 + \rho_j \right) h_j = \bar{h}
\] [\( \omega \)]

The FOC are:

\[3\] Cereals, oleaginous crops, oilseed crops called “protéagineux” in French, hence the name.
\[
\Omega_j + (S_j + \rho_j S_f) \Omega = (C_j + \frac{1}{2} \beta_j h_j + \alpha_j \rho_j) \Omega + \omega \quad \text{marg.revenue=marg.cost},
\]
\[
\sum_j (1 + \rho_j) h_j = \bar{h} \quad \text{total area constraint}.
\]

There are still J+1 equations in the FOC (set-aside is not included in the \( j \) domain), but there are now J+1 average cost equations since the FADN dataset provides the cost of the set-aside area. Since we have assumed that the set-aside cost function was linear, there is only one additional parameter \( \alpha_f \) and it is identified by the additional average cost equation.

### 2.4 Sugar Beet Quota

In the Walloon region, sugar is exclusively produced by means of sugar beet. Nearly all crop farms in that region produce sugar beets and it is deemed the most profitable crop. The sector is heavily regulated. The EU allocates sugar production quotas to each member state that allocates them to the sugar refineries. In Belgium, the sugar refineries themselves are regulated: each plant is assigned a zone for collecting the beet and allocates delivery rights \( Q_a \) to beet growers; no outsider may enter the industry. Farmers receive a fixed price \( P_a \) per ton of beet delivered to the refinery up to the level of his delivery right; they may deliver more, but then receive only the uncertain world price \( \bar{P}_c \). The problem of the beet growers is then to meet at least their delivery rights. The penalty for not reaching \( Q_a \) is not receiving the high price \( P_a \) while the penalty for supplying too much is the

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4 In many EU countries, there is a quota called “A”, for which the EU pays the highest price (\( P_a \)), and a “B” quota, for which the price is lower, yet quite higher than the world price. Any amount of sugar produced above the quota must be exported (indeed “dumped”) outside the EU at world price; by analogy with the A and B quotas, the exported sugar is called “C quota” while there is no quantity restriction and the world price is called C price (\( P_c \)). In Belgium, the A and B quotas have been pooled, we call the pooled quota “A” and its price \( P_a \) for simplicity.

5 Farmers have the possibility of delivering up to 10% more than their assigned quota but whatever excess they have will be discounted from their quota the next year. In the formal model, the delivery right \( Q_a \) is the actual right including the additional 10% minus the previous year excess supply.
difference between $\hat{P}_c$ and the cost of the sugar beet produced above $Q_a$. Writing the index $sb$ for sugar beet and $\psi$ for the random physical yield, formally, model [1] becomes:

$$\max E_h \left[ \left( \hat{R} + \sum_j \left( \hat{\theta}_j - \alpha_j - \frac{1}{2} \beta_j h_j \right) h_j \right)^{1-\rho} \right]^{1/\rho} \quad j \neq sb$$

$$\sum_j h_j + h_{sb} = \bar{h} \quad [\omega]$$

with

$$\hat{R} = P_a Q_a + \left[ \bar{z} \hat{P}_c + (1 - \bar{z}) P_a \right] (\psi_{sb} h_{sb} - Q_a) \quad \text{sugar revenue}$$

$$\bar{z} = 1 \text{ if } \psi_{sb} h_{sb} > Q_a; \quad \bar{z} = 0 \text{ else} \quad \text{sugar indicator}$$

The sugar revenue equation can be interpreted as follows. The farmer is entitled to a sure price $P_a$ for a known delivery right $Q_a$. If he produces at least $Q_a$, $\bar{z}=1$, he receives the world price for the quantity delivered above the right, that is $\hat{P}_c (\psi_{sb} h_{sb} - Q_a)$. Conversely if he does not meet the right, $\bar{z}=0$, his entitlement is reduced by the missing quantity valued at the high price, $P_a (\psi_{sb} h_{sb} - Q_a)$. The indicator variable $\bar{z}$ is stochastic because the beet yield is uncertain; the more hectares the farmers plants with beet, the surer he is to meet his quota. The marginal stochastic revenue for sugar is $m\hat{R} = \left[ \bar{z} \hat{P}_c + (1 - \bar{z}) P_a \right] \psi_{sb}$: for a given area of beet, for an additional hectare of beet, the farmer will receive $\hat{P}_c \psi_{sb}$ if he meets his quota but $P_a \psi_{sb}$ if he does not.

The FOC are:

$$E(m\hat{R}U') = \left( \bar{C}_{sb} + \frac{1}{2} \beta_{sb} h_{sb} \right) \Omega + \omega \quad \text{sugar: marg.revenue=marg.cost},$$

$$\Omega_j = \left( \bar{C}_j + \frac{1}{2} \beta_j h_j \right) \Omega + \omega \quad \text{other crops: marg.revenue=marg.cost},$$

$$\sum_j h_j + h_{sb} = \bar{h} \quad \text{total area constraint}.$$
The system of FOC can be used to calibrate the parameters of the variable cost function in the same way as in model [1]. It is remarkable that in a stochastic model with risk aversion, the sugar beet FOC becomes smooth because the quota is reached ex ante in probability rather than with certainty. This is illustrated in Figure 1 below. The left panel represents the beet sugar FOC in a non-stochastic model in which the marginal revenue drops discreetly from the supported price to the world price. In the right panel on the other hand, the marginal revenue becomes probabilistic and the quota is reached ex ante with increasing probability as the area planted with beet increases.

Figure 1. Stochastic sugar beet marginal revenue
2.5 Buffer Zones

EU farmers have the possibility to engage in Agri-Environmental Measures (AEM). AEM are outlined in a EU directive, but the detailed implementation is designed at the regional (“NUTS2”) level. AEM are, broadly speaking, non-strictly productive activities that are of local environmental or cultural interest. In the Walloon region, AEM include buffer zones, soil cover during winter, maintenance of hedges and ponds, rearing of local threatened domestic species and extensive (low input) management of grassland. Each AEM is defined by a detailed plight of conditions. The farmer must declare his intention to uptake an AEM and commit to it for five years; in exchange, he receives a subsidy.

We modify model [1] to include the most successful (in terms of total subsidies paid) AEM in the Walloon region: buffer zones.\(^7\) Buffer zones are strips of grassland at most 20m wide that must be located along the borders of a tiled crop field. That limits the maximum possible total surface of buffer zones to 20m times the sum of the perimeters of the fields on the farms. The total buffer zones surface also cannot exceed 8% of the total tiled crop surface on the farm. We call \(h_{bf}\) the minimum of these two maximums.\(^8\) Computation of \(h_{bf}\) requires access to the agricultural land cover census at the field level. Such a dataset exists in EU countries because farmers must declare their land covers to the administration for the computation of the CAP subsidies they are entitled to. We call \(h_{bf}\) the total area of buffer zones that the farmer has committed to. This data is also not present in the FADN dataset but there exists a census of all the AEM per farm and per year in every EU country.

Writing \(S_{bf}\) as the buffer zone subsidy, model [1] becomes:

\(^7\) It is the only AEM that competes with conventional crops for the allocation of surface. The other AEM do not occupy cropland, although they do require some fixed factors services, in particular, the farmer’s time.\(^8\) Buffer zones must also comply with a set of other conditions. They must be at least 4m wide on one or several sides of a tiled crops field. Each buffer zone must be at least 0.08ha but must be no larger than the crop area in that field. They must be seeded with local flora. No fertilizer or treatment can be used except for localized treatments against certain plagues. They must be sowed at least once no earlier that late summer and generally kept free of any use.
\[
\max_h E \left[ \left( \left( S_{bf} - \alpha_{bf} - \frac{1}{2} \beta_{bf} h_{bf} \right) h_{bf} + \sum_j \left( \theta_j + S_j + \rho_j S_f - \alpha_j - \frac{1}{2} \beta_j h_j - \alpha_f \rho_j \right) h_j \right)^{1-\rho} \right]^{1-\rho} \\
\sum_j (1 + \rho_j) h_j + h_{bf} = \bar{h} \\
h_{bf} \leq \bar{h}_{bf} \\
[a_j] \\
\lambda_{bf} \\
\text{The FOC are:}
\]

\[
S_{bf} \Omega = \left( \bar{C}_{bf} + \frac{1}{2} \beta_{bf} h_{bf} \right) \Omega + \omega + \lambda_{bf} \\
\Omega_j + \left( S_j + \rho_j S_f \right) \Omega = \left( \bar{C}_j + \frac{1}{2} \beta_j h_j + \alpha_f \rho_j \right) \Omega + \omega \\
\sum_j (1 + \rho_j) h_j + h_{bf} = \bar{h} \\
\left( \bar{h}_{bf} - h_{bf} \right) \lambda_{bf} = 0
\]

The model with buffer zones is quite similar to the model without. The key difference is that there is an upper quantity constraint on buffer. In spite of the general belief that buffers are highly profitable, most farms are not at the upper limit. We deduce that there must be hidden costs that may be caused by the fact that buffer operations take place at roughly the same time during the year as cereals. To capture such an interior point solution we resort to a complementary slackness constraint for buffer: either the upper limit is reached, and then the shadow cost of buffer may be positive, or the upper limit is not reached. Either way, the system of FOC allows us to calibrate the coefficients of the cost and the shadow values of the constraints.

### 3. Description of the Sample

Table 2 shows summary statistics for our sample of 24 crops farms without significant livestock in the FADN sample from 1995 to 2004 in the Walloon region. Those farms are a subsample of the complete sample of crop farms. We have also removed farms that are not observe for at least five years.
Table 2. Sample description

<table>
<thead>
<tr>
<th>Farms descriptive statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of crops observations excluding fall</td>
<td>924</td>
</tr>
<tr>
<td>Average number of crops</td>
<td>4.2</td>
</tr>
<tr>
<td>Average ha</td>
<td>78.2</td>
</tr>
<tr>
<td>Average gross margin (Euro/ha)</td>
<td>1021.0</td>
</tr>
<tr>
<td>Average area subsidy (Euro/ha)</td>
<td>162.1</td>
</tr>
</tbody>
</table>

Source: based on FADN 1995-2004

Table 3 presents the profile of agricultural choices. Sixteen crops (excluding set asides) are present in the sample, but the most important ones in term of area are winter wheat and sugar beet. Chicory (a vegetable used for the production of inuline, a sugar substitute) and potato are the second most important crops, but their area is already quite small compared to wheat and beet. Buffer zone occupies only a small area on the farms on which it is present (6 in our sample).

Table 3. Main crops of farms without significant livestock in the Walloon region

<table>
<thead>
<tr>
<th></th>
<th>Winter wheat</th>
<th>Sugar beet</th>
<th>Chicory</th>
<th>Potato</th>
<th>Buffer zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average ha of crop per farm</td>
<td>30.0</td>
<td>27.7</td>
<td>9.7</td>
<td>12.7</td>
<td>1.8</td>
</tr>
<tr>
<td>Average ha per year</td>
<td>887.4</td>
<td>828.5</td>
<td>135.6</td>
<td>132.7</td>
<td>7.2</td>
</tr>
<tr>
<td>Average yield (T)</td>
<td>8.2</td>
<td>35.7</td>
<td>22.4</td>
<td>11.4</td>
<td>n.a.</td>
</tr>
<tr>
<td>Average gross margin including area subsidy (Euro/ha)</td>
<td>970.9</td>
<td>1177.9</td>
<td>1205.1</td>
<td>1769.3</td>
<td>896.2</td>
</tr>
<tr>
<td>Relative average std dev gross margin per ha across years (%)</td>
<td>23.6</td>
<td>30.4</td>
<td>28.1</td>
<td>39.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Average area subsidy (Euro/ha)</td>
<td>359.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>914.7</td>
</tr>
</tbody>
</table>

Source: based on FADN 1995-2004

4. Simulations

In this section, we introduce various scenarios for simulation. The modeling of each scenario is presented in subsections and followed by the results for the Walloon sample.
In simulations, we use the results of the calibration to evaluate the reactions of the farm to of a change of its environment. The simulation model is the same as the calibration model, except that in the simulations, we use the results of the calibration regarding the parameters $\alpha$ and $\beta$ of the cost function and we let the model optimize on the surfaces $h$. The simulation model, complete with subsidies, set-aside, sugar quota and buffer zones is the following. The index $j$ represents any “crop” or land cover except sugar beet ($sb$), set aside (fall $f$) or buffer ($bf$).

$$\text{Max}_h \left( \bar{R} + (S_{bf} - \alpha_{bf} - \frac{1}{2} \beta_{bf} h_{bf}) h_{bf} + \sum_j \left( \bar{\theta}_j + S_j + \rho_j S_f - \alpha_j - \frac{1}{2} \beta_j h_j - \alpha_j \rho_j \right) h_j \right)^{1-\rho}$$

$$\sum_j \left(1 + \rho_j \right) h_j + h_{sb} = \bar{h}$$

$$h_{bf} \leq \bar{h}_{bf}$$

$$[\omega]$$

$$[\lambda_{bf}]$$

with

$$\bar{R} + (S_{bf} - \alpha_{bf} - \frac{1}{2} \beta_{bf} h_{bf}) h_{bf} + \sum_j \left( \bar{\theta}_j + S_j + \rho_j S_f - \alpha_j - \frac{1}{2} \beta_j h_j - \alpha_j \rho_j \right) h_j$$

net income

$$U = \frac{1}{1-\rho} \left( \text{net income} \right)^{1-\rho}$$

CRRA utility

$$\bar{R} = P_a Q_a + \left[ \bar{\psi}_c + (1-\bar{\gamma}) P_a \right] (\bar{\psi}_{sb} h_{sb} - Q_a)$$

sugar revenue

$$\bar{\gamma} = 1 \text{ if } \bar{\psi}_{sb} h_{sb} > Q_a; \quad \bar{\gamma} = 0 \text{ else}$$

sugar indicator

$$C_j (h_j) = \alpha_j h_j + \frac{1}{2} \beta_j h_j^2$$

cost function

$$\bar{C}_j = \alpha_j + \frac{1}{2} \beta_j h_j$$

average cost

As before, we define shorthand notations: $\Omega = EU'$, $\Omega_j = E[\bar{\theta}_j U']$ and

$$m\bar{R} = \left[ \bar{\psi}_c + (1-\bar{\gamma}) P_a \right] \bar{\psi}_{sb},$$

the marginal stochastic revenue for sugar. The FOC are:
Marginal revenue = marginal cost constraints

\[ E(\hat{m}RU) = (\bar{C}_{sb} + \frac{1}{2} \beta_{sb} h_{sb}) \Omega + \omega \]

\[ S_{bf} \Omega = (\bar{C}_{bf} + \frac{1}{2} \beta_{bf} h_{bf}) \Omega + \omega + \lambda_{bf} \]

\[ \Omega_j + (S_j + \rho_j S_f) \Omega = (\bar{C}_j + \frac{1}{2} \beta_j h_j + \alpha_j \rho_j) \Omega + \omega \]

Area constraints

\[ \sum_j (1 + \rho_j) h_j + h_{bf} + h_{sb} = \bar{h} \]

\[ (\bar{h}_{bf} - h_{bf}) \lambda_{bf} = 0 \]

4.1 Simple Scenarios: Price, Subsidy and Sugar Quota Changes

In this subsection, we present scenarios that involve only simple changes of exogenous variables. We consider changes of the prices of wheat and “C” sugar (see subsection 2.4), buffer zone subsidy and sugar quota.

In a stochastic model, a change in price has to be considered in two dimensions: expectation and variance. Depending on the change in variance, an increase in expected price may not always be desirable for the farmer. We examine a wheat price change at constant absolute variance (\(Var(P)\) is constant) and at constant relative variance (\(Var(P)/E(P)\) is constant). Recall that the yield in value is the product of the price and the yield \(\tilde{\theta}_j = \tilde{P}_j \tilde{\Psi}_j\) and assume a change in the expected value of the price. The two cases are:

- \(\tilde{\theta}_j^* = (\tilde{P}_j + e) \tilde{\Psi}_j\), when \(e\) is not stochastic, that is a price change at constant absolute variance and decreasing (if \(e > 0\)) relative variance, that could be an output subsidy,

- \(\tilde{\theta}_j^* = (1 + f) \tilde{P}_j \tilde{\Psi}_j\), when \(f\) is not stochastic, that is a change at constant relative variance and increasing (if \(f > 0\)) absolute variance, a demand change could cause such a price change.
We examine two scenarios: a reduction of 10% of the expected price of winter wheat at constant absolute variance (corresponding to a reduction of output subsidy) and at decreasing absolute variance (corresponding to a demand reduction).

The third scenario represents a ban of the EU sugar export. That can be modeled straightforwardly by means of setting the world price of sugar to zero. In the fourth scenario, we present a decrease of the A sugar quota of 10%. Finally, we present a scenario of increase of the buffer zone subsidy of 10%. Table 4 presents the corresponding changes in expected hectareage over the sample of 24 crops farms for winter wheat, sugar beet and buffer zone.

Table 4. Expected Hectarage of Wheat, Sugar and Buffer under Five Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>State</th>
<th>Unit</th>
<th>Winter Wheat</th>
<th>Sugar Beet</th>
<th>Buffer Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Initial situation</td>
<td>Ha</td>
<td>642</td>
<td>650</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>( P_{\text{wheat}} - .1 \bar{P}_{\text{wheat}} )</td>
<td>% difference</td>
<td>-3.4</td>
<td>3.9</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>( .9 P_{\text{wheat}} )</td>
<td>% difference</td>
<td>-3.6</td>
<td>3.8</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>( P_c = 0 )</td>
<td>% difference</td>
<td>6.5</td>
<td>-8.7</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>.9 SugarA quota</td>
<td>% difference</td>
<td>1.6</td>
<td>-2.5</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>( S_{bf} + 10% )</td>
<td>% difference</td>
<td>-0.4</td>
<td>1.2</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Scenarios 1 and 2 correspond to a drop of roughly 10% of the expected yield in value of wheat; however, in scenario 1 the variance remains roughly constant (it decreases in fact by 4%) while in scenarios 2 it decreases by about 20%. That difference in variance does not however cause a sizable difference in area allocation for risk-averse farmers. As discussed in the introduction, that result is consistent with the low variability of the yield in value of the winter wheat. Scenarios 3 and 4 lead to the somewhat surprising result that a ban on EU sugar export \((P_c = 0)\) has a stronger effect (minus nearly 9% sugar beet hectares) than a reduction of 10% of the sugar A quota (minus 2.5% sugar beet hectares). That suggests that Walloon beet growers profitably sell on the world market, thus even if the quota decreases, they continue planting about the same area of beet. Finally, a 10% increase of the buffer zone subsidy leads to a 5.5% increase of the buffer zone area. That result is based on only 6 farmers who do buffer zones in our sample; it can therefore be
quite unreliable but it shows that it is possible to combine geographical information and farm accounting data to analyze scenarios regarding non purely agricultural activities.

4.2 Crop Insurance for Wheat

When a farmer has subscribed crop insurance, if the yield falls below a contractual level (usually the farm average yield minus a deductible), the farmer receives the difference valued at some contractual price (the insurance premium is a function of that price). That is equivalent to truncate the distribution of the physical yield $\Psi_{\text{wheat}}$ at the observed farm average $\Psi_{\text{wheat}}$ minus the deductible. To simplify, we follow the current French insurance system and impose that the farmer can only contract at the farm average price. Thus, assuming that the farmer insures all the area allocated to wheat, if $\Psi_{\text{wheat}} < \phi \Psi_{\text{wheat}}$ (where $\phi = 1$ - deductible), then the distribution of the yield in value $\theta_{\text{wheat}}$ has a spike at $\phi \Psi_{\text{wheat}}$. 

The effect of insurance on the distribution of the yield in value is not straightforward because the price distribution is correlated with the physical yield distribution. In other words, when the yield is below the insured threshold, the price may be high enough to make the insurance unnecessary, or even unprofitable for the farmer. To estimate the distribution of the yield in value conditional to the physical yield being larger than the insured lower bound (farm average minus deductible), we use the FADN sample in which we delete all the observations such that $\Psi_{\text{wheat}} < \phi \Psi_{\text{wheat}}$ and we use a panel data model to filter out farm-specific effects. Let $\theta_{\text{wheat}}$ be the yield in value conditional to $\Psi_{\text{wheat}} > \phi \Psi_{\text{wheat}}$; after estimating the distribution of $\theta_{\text{wheat}}$, we can computer-generate values $\theta_{\text{wheat}}S$. Thus, in summary:
With probability \( \gamma = \Pr\left\{ \Psi_{\text{wheat}} < \phi \Psi_{\text{wheat}} \right\} , \quad \hat{\theta}_{\text{wheat}} = \phi \Psi_{\text{wheat}} \),

With probability \( 1 - \gamma \), \( \hat{\theta}_{\text{wheat}} = \theta_{\text{wheat}} \), simulated by \( \theta_{\text{wheat}} \).

To choose whether to contract crop insurance for wheat, the farmer solves system [7] modified by replacing \( \hat{\theta}_{\text{wheat}} \) par \( \gamma \phi \Psi_{\text{wheat}} \Psi_{\text{wheat}} + (1 - \gamma) \theta_{\text{wheats}} \) and adding the insurance premium to the cost of producing wheat. These changes also affect \( \Omega = EU' \) and \( \Omega_j = E]\left[ \hat{\theta}_j U' \right] \).

In practice for Walloon farms the distribution of the yield in value does not change much when insurance is contracted. Figure 2 shows the distribution of the residuals of a fixed effects panel data regression on the yields in value for wheat in the FADN Walloon sample in 1996-2003. The top panel shows the distribution for the whole sample while the bottom panel shows all the cases in which physical yields lower than 90% of the farm average over the years 1996-2003 have been removed. In both panels, the rectangles show the observed histogram while the curve represents the best-fitted parametric distribution according to one statistical test.\(^9\) Figure 2 clearly shows that crop insurance barely modifies the distribution of the yield in value: to a low physical yield does not generally correspond a low price, and therefore the crop insurance does not truncate the distribution of the yield in value. That may explain the historical lack of success of crop insurances in Belgium.

\(^9\) Different tests unfortunately lead to different “best-fit” distributions. In the case of wheat, the extreme value, gamma and Weibull distribution are usually among the best distributions for any test. The important feature of those distributions in the present case seems to be that they are skewed to left.
The model imposes the insurance to the farmer at the stated cost, he does not have the choice whether to insure or not. The current typical French premium for winter wheat is about 15 Euro/ha. Table 5 presents the changes in acreage allocations accordingly with the cost of the crop insurance (premium). These changes are very small, reflecting the small yield variability of wheat in the Walloon region. Those results are consistent with the results of scenario 1 and 2 above.

Table 5. Hectareage Allocations for Different Premiums

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>Winter Wheat</th>
<th>Sugar Beet</th>
<th>Buffer Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed (no insurance)</td>
<td>ha</td>
<td>642</td>
<td>650</td>
<td>7</td>
</tr>
<tr>
<td>Simulated with premium = 0</td>
<td>% difference</td>
<td>0.5</td>
<td>-0.1</td>
<td>0.6</td>
</tr>
<tr>
<td>Simulated with premium = 7.5€/ha</td>
<td>% difference</td>
<td>0.3</td>
<td>0.2</td>
<td>0.7</td>
</tr>
<tr>
<td>Simulated with premium = 15€/ha</td>
<td>% difference</td>
<td>0.1</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Simulated with premium = 30€/ha</td>
<td>% difference</td>
<td>-0.3</td>
<td>0.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

5. Discussion

Because the model is still in an intermediate level of development, in this section we will simply discuss some of the directions for future research that we are considering.
We have shown how it is possible to represent the farm-level area allocation behavior of risk-averse farmers in the presence of random yields and prices under fixed total farm surface. We have modeled the sugar quota constraint, the Common Agricultural Policy subsidies and set-aside, and the buffer zone Agri-Environmental Measure. We have applied the approach to a sample of crop farms from Belgian Walloon region.

The model is based solely on ex-post data collected in the FADN and in the agricultural land cover census: yields, prices, area allocations (including for Agri-Environmental Measures) and expenses in variable inputs. These data have a high degree of reliability. Observed panels of prices and yields are used to infer distributions that serve to calibrate the parameters of the cost function. In that sense it is not a calibration model based on a single period. Nevertheless, econometric estimation of the cost function across a panel of farms has proved frustrating, as well as linking input expenses and yields.

Finally, because different statistical tests rank parametric distributions differently regarding their fit to the data, it is difficult to compare the present results, based on the normal distribution, with the results generated by alternative distributions. Therefore, even though the normal distribution is not the one that best-fits our data, alternative distributions are equally questionable.
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