Food Desert and Weight Outcome: Disentangling Confounding Mechanisms

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Abstract

Food deserts are increasingly considered as a potential cause of overweight and obesity, yet existing literature is largely inconclusive in part due to the infeasibility of sorting out multiple confounding mechanisms from a purely empirical perspective. This article investigates the hypothesized causality in a rational-choice framework, where the individual chooses how much to patronize a distant supermarket and/or a nearby convenience store, broadly defined, and the weight outcome depends on this choice. Results suggest that neither limited supermarket access nor low income, the key features of food deserts, would determine the weight outcome, which is also affected by individual preferences as well as time and monetary costs associated with grocery shopping. Parametric conditions under which varying effects on weight occur are further derived to elicit policy implications.

\textbf{Key words}: Food deserts, food choice, weight, food environment, obesity

\textbf{JEL classification}: D13, I12, Q18

Debbie has lived in Urbansburg for twenty years. Like any other college town in the Southeast, Urbansburg has a fairly good collection of shops consisting of three supermarkets plus a mid-sized shopping center that provides durable goods to 53,421 townspeople and about 8,000 college

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students that flock in and out over the seasons. Debbie’s house is located about a mile from the
town center. She stays at home and her husband Duncan works as a procurement clerk and makes
36,500 dollars a year. While their financial situation is comparable to other households in the
neighborhood, the family has four school-aged children and still struggles to make ends meet.

As Duncan drives their only family car to work, Debbie usually walks to the supermarket. The
nearest supermarket is roughly two miles from home, which is reachable within one half hour at a
brisk pace. Debbie enjoys this exercise as she thinks it could help her lose weight, especially after
learning about the risk of type two diabetes at a recent community health fair, a condition which
now afflicts her older sister Mary. This knowledge has also led Debbie to increase the servings of
fresh fruits and vegetables on her family’s dining table, yet this differs from many of her neighbors
who often buy foods from a nearby convenience store.

While Debbie is not apparently obese, her older sister Mary is. Mary and their youngest sister
Katie live on small, divided family farms in Ruralsville, an unincorporated area where they grew
up. Unlike Debbie, Mary and Katie each have to drive more than ten miles to the nearest
supermarket to buy food. Mary has a compact car that gets good gas mileage, but Katie has an old
pickup truck and each trip to the supermarket costs her several dollars in gas. To save gas, Katie
carefully plans grocery shopping trips, which she limits to once a week. She is sometimes reluctant
to stock up on fresh perishables during these trips out of concern that foods may spoil during the
week, and occasionally mixes in processed foods with longer shelf life. Mary, also on a limited
budget, thinks differently and believes that time, rather than money, is the real catch. Grocery
shopping leaves her little time to cook, and so she always makes sure to get something fast like
frozen pizzas and chips to feed her family on shopping days. In fact, these processed items are
common her household’s menu since her children like them and they are inexpensive compared to fresh produce.

Debbie, Mary and Katie are just fictional characters abstracted from millions of Americans living in “food deserts”, the low-income urban and rural areas with limited access to healthy foods (U.S. Department of Agriculture, 2009; Dutko, Ver Ploeg, and Farrigan 2012). Supermarkets, the primary source of healthy foods, are difficult to access from these areas, and so residents may presumably obtain and consume more energy-dense, unhealthy foods from other food retailers such as convenience stores and fast food restaurants. The limited access to supermarkets could therefore adversely affect the diet quality of food desert residents and could result in weight gain.

Despite this appealing logic, empirical analyses of food deserts as an obesity argument have produced only mixed findings (e.g. Courtemanche and Carden 2011; Alviola, Nayga and Thomsen 2013; Thomsen et al., 2015). Such inconclusiveness is partly due to the failure to differentiate multiple confounding mechanisms from a purely empirical perspective. Among the characters described above, Debbie is trying to be healthy and is able to procure healthy foods despite lengthy walks. A moderate change in distance from the supermarket would affect Debbie primarily through exercise incurred on walks to the supermarket. A closer store for Katie may allow her to make more frequent trips and facilitate her buying fresh items. The gas money she saves would also be available for additional food items, some of which may be less healthy. In contrast, a nearer store for Mary might simply mean that unhealthy food items are available at lower time and monetary costs. Heterogeneities regarding individual preferences and travel costs, however, are generally not considered in existing studies given data limitations. Important features are also missing on the supply side. For instance, supermarkets generally provide foods, including unhealthy foods, at lower prices (Drewnowski 2004), which could inadvertently increase food consumption and
therefore result in weight gain (Courtemanche and Carden 2011). These conflicting forces have not been formally analyzed, yet such information is needed to effectively assist policy designs from both corrective and preventive perspectives in face of the high overweight and obesity rates in the U.S.

This article helps disentangle these confounding mechanisms within a rational-choice framework that associates food access with weight outcomes through the mediation of food choice. It shows that a longer distance to the supermarket does not increase weight in general, while a shorter distance does in certain cases. The effect of income on weight is also ambiguous. Parametric conditions are further explored under which the weight of an individual is positively or negatively affected by these food desert features. The results jointly suggest a necessity to re-evaluate policy priorities to combat obesity.

A rational-choice model

To appropriately describe food deserts, consider two food retail outlets around an individual’s residence: a supermarket that sells both healthy foods and junk foods, and a convenience store that sells only junk foods. These stores reasonably represent the food environment faced by many people, which is comprised of large grocery stores with many kinds of foods and small retailers with limited selections of energy-dense processed foods. In the real world, the convenience store can also represent another food outlet such as a fast food restaurant that provides similar types of foods. The supermarket is farther away from the individual’s residence than the convenience store. Otherwise, the latter would no longer be “convenient”. Geographical access to the supermarket is captured by travel distance.
Consider an overweight individual for whom weight increase may generate disutility. The following utility function is to be maximized:

\[ V = U(F, l) - \rho W \]

where \( F \) is food consumption, \( l \) is leisure, \( W \) is weight, and \( \rho \) is a weight sensitivity parameter which equals zero if the individual is weight-insensitive but takes a positive value if the individual is weight-sensitive. Conventional assumptions are satisfied: \( U_F > 0, U_l > 0, U_{FF} < 0, U_{ll} < 0, \) and \( U_{Fl} \geq 0 \). The last assumption implies that both food and leisure are normal goods.

\( F \) consists of two types of foods: junk foods \((J)\) and healthy foods \((H)\). Junk foods, which do not require preparation and are ready to eat, can be obtained from either the supermarket \((J^S)\) or the convenience store \((J^C)\), and so the total junk food consumption can be represented by \( J = J^S + J^C \). The individual discriminates between the two types of junk foods as well as healthy foods with nonnegative taste parameters \( \mu \) and \( \lambda \) (where \( J^S \) is weighted by the unit):

\[ F = f^S + \mu f^C + \lambda H \]

In reality, each food category would consist of numerous types of foods and inclusion of all of them is neither feasible nor interesting in the current study. Still, \( J^S, J^C \) and \( H \) can be plausibly interpreted as typical (e.g., most frequently purchased) food bundles and thus are treated as scalar variables throughout the analysis (along with their scalar prices defined below).

Following Yaniv, Rosin and Tobol (2009), assume that healthy foods require preparation at home with both ingredients \((G)\) and cooking time \((k)\):

\[ H = \min \left( \frac{G}{\alpha}, \frac{k}{\beta} \right) \]

where \( \alpha \) and \( \beta \) are technology parameters \((\alpha > 1)\). \( G \) is only available in the supermarket with travel distance \( D \) from the individual’s residence. On the other hand, the convenience store is
readily accessible at zero distance. The individual’s non-working time (normalized to 1) is therefore allocated among cooking \( k \), travel to the supermarket \( t \), and leisure \( l \). \(^6\)

\[
(4) \quad k + t + l = 1
\]

Assume that the individual is a “routine grocery shopper” who conducts a fixed number of supermarket trips in a given period (e.g. once per week, twice per month). \(^7\) In this case, travel time is specified as a simple linear function of the supermarket distance (with coefficient \( \theta \)):

\[
(5) \quad t = \theta D
\]

On the other hand, the individual’s budget is applied to the purchase of junk foods \( J^S \) at the price of \( p^S_j \) from the supermarket and/or \( J^C \) at the price of \( p^C_j \) from the convenience store), ingredients of healthy foods \( G \) at the price of \( p_g \) from the supermarket), and supermarket travel (gas, vehicle maintenance, public transportation fare, etc.) which is a linear function of \( D \) (with coefficient \( \eta \)):

\[
(6) \quad p^S_j J^S + p^C_j J^C + p_g G + \eta D = I
\]

Real world observations further suggest that the price of healthy food ingredients is higher than that of junk foods (Drewnowski 2004; Drewnowski and Specter 2004), i.e. \( p^S_j < p_g \), \( p^C_j < p_g \), and that convenience stores charge more than grocery stores for the same goods (Broda, Leibtag and Weinstein 2009), i.e. \( p^S_j < p^C_j \). \(^8\)

The individual’s weight \( W \) is affected by calorie intake from the consumption of both healthy foods \( H \) and junk foods \( J \), physical activity \( E \) and baseline calorie consumption as measured by the basal metabolic rate (BMR):

\[
(7) \quad W = \omega H + \psi J - E - BMR
\]

where \( \psi \) and \( \omega \) are calorie densities of each type of foods: \( \psi > \omega \). Following existing literature (e.g. Auld and Powell 2009; Dragone and Savorelli 2012), full consideration of physical activity as an individual choice is abstracted as it is not the focus of the current study. Reviews of the public
health literature also generally suggest that physical activity alone can lead to only modest weight loss or not at all (e.g. Glenny et al. 1997; Miller, Koceja and Hamilton 1997; Reilly and McDowell 2003). Therefore, $E$ is assumed to include only a fixed set of exercises ($E_0$) that does not vary with grocery shopping and shopping-related exercises ($e$):

$E = E_0 + e$

where $e$ is a function of supermarket access ($D$). Further assume that if the individual drives/takes public transportation to the supermarket, $e_D \equiv \partial e / \partial D = 0$; if he/she walks, $e_D > 0$.

General results

As a start of analysis, first consider the case where $H \neq 0$ and $J \neq 0$ ($J^S$ and $J^C$ do not simultaneously equal zero). Substituting equation (3) in equation (6) and rearranging, it becomes:

$H = \frac{I - p_J^S J^S - p_J^C J^C - \eta D}{\alpha p_G}$

Further substituting equation (9) in equation (7) and rearranging provides:

$W = \gamma J^S + \delta J^C - e + \Gamma$

where $\gamma \equiv \psi - \omega p_J^S / \alpha p_G$, $\delta \equiv \psi - \omega p_J^C / \alpha p_G$, and $\Gamma \equiv \omega (I - \eta D) / \alpha p_G - E_0 - BMR$. Note that both $\gamma$ and $\delta$ are positive given assumptions above, suggesting that consumption of junk foods from either source increases weight.

Since weight is now expressed as a function of $J^S$ and $J^C$, it is necessary to express $U(F, l)$ also in terms of $J^S$ and $J^C$ for comparative static analysis. Substituting equation (9) into (2) provides:

$F = J^S + \mu J^C + \lambda \frac{I - p_J^S J^S - p_J^C J^C - \eta D}{\alpha p_G}$

On the other hand, by substituting equations (5) and (9) in equation (4) and rearranging, leisure can be expressed as:
(12) \[ l = 1 - t - k = 1 - \theta D - \beta \frac{l - p_j f^{S} - p_j f^{C} - \eta D}{\alpha p_g} \]

Now the individual’s (unconstrained) problem is to maximize the following utility function expressed in terms of only \( J^S \) and \( J^C \):

(13) \[ V = U \left( J^S + \mu J^C + \lambda \frac{l - p_j f^{S} - p_j f^{C} - \eta D}{\alpha p_g}, 1 - \theta D - \beta \frac{l - p_j f^{S} - p_j f^{C} - \eta D}{\alpha p_g} \right) - \rho (\gamma J^S + \delta J^C - e + \Gamma) \]

The first-order conditions are:

(14) \[ V_{J^S} J^S = \left[ U_F \left( 1 - \frac{\lambda p_j ^S}{\alpha p_g} \right) + U_l \frac{\beta p_j ^S}{\alpha p_g} - \rho \gamma \right] J^S = 0 \]

(15) \[ V_{J^C} J^C = \left[ U_F \left( \mu - \frac{\lambda p_j ^C}{\alpha p_g} \right) + U_l \frac{\beta p_j ^C}{\alpha p_g} - \rho \delta \right] J^C = 0 \]

Each condition individually satisfies complementary slackness: in equation (14), either \( J^S > 0 \) and \( V_{J^S} = 0 \), or \( J^S = 0 \) and \( V_{J^S} < 0 \); and in equation (15), either \( J^C > 0 \) and \( V_{J^C} = 0 \), or \( J^C = 0 \) and \( V_{J^C} < 0 \). The second-order conditions are satisfied providing that \( V_{J^S J^S} < 0 \), \( V_{J^C J^C} < 0 \) and \( U_F U_{ll} - U_{F l} ^2 > 0 \).

Before investigating special cases where either \( J^S \) or \( J^C \) is zero, consider a “mixed eater” who consumes all types of foods (i.e., none of \( J^S \), \( J^C \) or \( H \) is equal to zero). He/she is considered representative of the majority of the population who mainly consume healthy foods but still occasionally consume some junk foods from both types of stores. In this case, the first-order conditions become:

(14') \[ V_{J^S} = U_F \left( 1 - \frac{\lambda p_j ^S}{\alpha p_g} \right) + U_l \frac{\beta p_j ^S}{\alpha p_g} - \rho \gamma = 0 \]

(15') \[ V_{J^C} = U_F \left( \mu - \frac{\lambda p_j ^C}{\alpha p_g} \right) + U_l \frac{\beta p_j ^C}{\alpha p_g} - \rho \delta = 0 \]

Now categorize the mixed eater by weight sensitivity. For a weight-insensitive individual (\( \rho = 0 \)), equations (14') and (15') imply that \( 1 - \lambda p_j ^S / \alpha p_g < 0 \) and \( \mu - \lambda p_j ^C / \alpha p_g < 0 \) given the signs
of the partial derivatives of $U(F, l)$ (or the constraints are no longer negatively sloped and the individual would not consume $H$ at all). Yet for a weight-sensitive individual ($\rho > 0$), the sign of either $1 - \lambda p^*_f / \alpha p_G$ or $\mu - \lambda p^*_c / \alpha p_G$ is indeterminate given that the last two terms in both equations possess different signs. For analytical purposes, assume that the taste parameters ($1, \mu$ and $\lambda$) are bounded, i.e. none of them are large enough to rule out the existence of an interior solution. This assumption is relaxed later in the examination of extreme preferences.

**Proposition 1.** *The effect of supermarket access on the weight of a mixed eater is ambiguous. For a weight-insensitive mixed eater, if the time cost of supermarket travel is sufficiently smaller than the monetary cost, longer supermarket distance decreases weight; while if the monetary cost of supermarket travel is sufficiently smaller than the time cost, longer supermarket distance increases weight. For a weight-sensitive mixed eater, such effect is ambiguous and depends on complicated relationships among the taste and cost parameters.*

**Proof.** See Appendix A.

These results are straightforward. For a weight-insensitive individual, if $\theta$ is sufficiently smaller than $\eta$, the income effect dominates as $D$ increases and total food consumption ($H + J$) would decrease. However, if $\eta$ is sufficiently smaller than $\theta$, the substitution effect (towards $J$) dominates as $D$ increases, in which case $k$ shrinks quickly, resulting in a significant drop of $H$ but an even greater increase of $J$ given the lower prices of the latter ($p^*_f$ and/or $p^*_c$), and total food consumption would increase on balance. For a weight-sensitive individual, these results still hold if his/her preference for healthy foods is strong enough, while the overall effect on weight would depend on two conflicting effects if his/her preference for healthy foods is relatively weak. On one hand, the substitution effect towards $J$ could result in the increase of $F$, which increases utility. On the other
hand, the increase of $J$ would also increase weight, which decreases utility. As is shown in Appendix A, the effect on weight is jointly determined by all the taste and cost parameters.

Although it is widely hypothesized in the food desert literature that longer distance (thus more difficult access) to the supermarket would result in weight gain, it is not generally true when individual preferences and travel costs are considered. This may partly explain the mixed empirical findings in the literature and implies that *a priori* speculation of the effect of supermarket access on weight without consideration of individual heterogeneity may be less informative for policy making. To further establish the results, it is necessary to extend the baseline analysis above to different cases, including special food environments, extreme preferences, random supermarket travel, and income changes.

*Special food environments*

Consider two special food environments. In the first case, there is only a supermarket but no convenience store. This may reflect some rural areas where food retailers are sparse and the nearest store is a supermarket. Such a case is important because, as compared to urban areas, rural areas with low income are more likely to be food deserts (Dutko, Ver Ploeg, and Farrigan 2012). In the second case, the convenience store exists, while the supermarket is replaced by a farmers market where junk foods are not available. This may describe some inner city areas where convenience stores are readily available, fresh produce vendors are available, but supermarkets are not. Such a situation is similar to that analyzed by Raja, Ma and Yadav (2008). Appendix A shows that Proposition 1 continues to hold in both of these cases.

*Extreme preferences*
Equation (2) suggests that the individual may have different tastes among $J^S$, $J^C$ and $H$, where the taste parameters ($1$, $\mu$ and $\lambda$) are assumed to be bounded to allow for an interior solution. This assumption is relaxed for now. In one extreme case, the individual may strongly prefer healthy foods and consume no junk foods at all (the “healthy eater” hereafter). At the other extreme, it is also possible that the individual consumes only junk foods but no healthy foods at all (the “unhealthy eater” hereafter). Two subcases are further considered under the unhealthy eater category: where the individual consumes junk foods from only one of the two sources (and thus either $J^S$ or $J^C$ is zero). These cases may jointly represent only a limited portion of the population, yet they still deserve upfront discussion regarding the possible effect of supermarket access on weight. For simplicity, assume that changes in $D$ do not result in switches between a corner solution and an interior solution. This allows the focus to be on the gradual behavioral change resulting from the change in $D$, which is of primary interest.

**Proposition 2.** The increase of supermarket distance unambiguously decreases the weight of a healthy eater, yet the effect on weight is ambiguous for an unhealthy eater.

**Proof.** See Appendix B.

These results are again straightforward. For the healthy eater who only purchases $G$ to prepare $H$, the increase of $D$ shrinks both $G$ and $k$, reducing the consumption of $H$ and therefore decreasing $W$. The increase of shopping-related exercises further reduces weight if he/she walks to the supermarket. For the unhealthy eater, such effect would only occur if both $\eta$ (capturing the income effect) and $\mu$ (capturing the preference of $J^C$) are sufficiently large to cause $J$ to decrease with the increase of $D$. If both $\eta$ and $\mu$ are sufficiently small, the substitution effect towards $J^S$ can be larger than income effect, and $J$ may increase on balance with the increase of $D$ (given that $p_J^S < p_J^C$). Appendix B derives the parametric conditions under which these opposite effects would occur.
The effects on weight in the two subcases mentioned above are also worth some discussion, yet in neither case is such effect positive. For the unhealthy eater who only consumes \( J^C \), his/her weight would remain intact with the increase of \( D \) as he/she never travels to the supermarket. If he/she consumes only \( J^S \), the increase of \( D \) would unambiguously reduce weight due to the income effect and possible increase of \( e \).

These results suggest the importance of individual preferences in addition to supermarket travel costs in weight production. Given heterogeneous individual preferences, the effect of supermarket access on weight is difficult to predict. Even for an unhealthy eater, the increase of supermarket distance can reduce weight in certain situations.

**Random supermarket travel**

The above analysis considers a routine grocery shopper who conducts a fixed number of supermarket trips during the study period. Although this suitably describes most, especially low-income people (Wilde and Ranney 2000; Yoo et al. 2006), alternative patterns of supermarket travel are also worth investigation in search of generality. Now consider a “random grocery shopper” whose number of supermarket trips is not fixed but depends on the purchase of \( J^S \) and \( G \). For analytical purposes, assume his/her supermarket travel is infinitely divisible. In this case, the time cost of supermarket travel is positively affected by \( J^S \) and \( G \) (with coefficient \( \kappa \)) but not \( J^C \) (as the purchase from a convenience store does not require travel). Time cost is therefore assumed a linear function of \( D(J^S + G) \):

\[
(5') \quad t = \kappa D(J^S + G)
\]

On the other hand, the monetary cost of supermarket travel is also affected by \( J^S \) and \( G \) (with coefficient \( \phi \)):
(6') \( p_j^S J^S + p_j^C J^C + p_G G + \phi D (J^S + G) = I \)

Now use equations (5') and (6') to respectively replace equations (5) and (6) in the original model. To solve the problem, \( H \) is expressed as:

\[
H = \frac{I - (p_j^S + \phi H) J^S - p_j^C J^C}{\alpha (p_G + \phi D)}
\]

Substituting equation (16) in equation (7) and rearranging, the weight function can be written as:

\[
W = \gamma' J^S + \delta' J^C - e' + \Gamma'
\]

where \( \gamma' \equiv \psi - \omega (p_j^S + \phi D) / \alpha (p_G + \phi D) \), \( \delta' \equiv \psi - \omega p_j^C / \alpha (p_G + \phi D) \), and \( \Gamma \equiv \omega I / \alpha (p_G + \phi D) - E_0 - BMR \). \( \gamma' \) and \( \delta' \) are both positive given assumptions outlined above. The effect of supermarket access on weight therefore can be derived as:

\[
\frac{dW}{dD} = \frac{d\gamma'}{dD} J^S + \gamma' \frac{dJ^S}{dD} + \frac{d\delta'}{dD} J^C + \delta' \frac{dJ^C}{dD} - \frac{de'}{dD} - \frac{\omega \phi I}{\alpha (p_G + \phi D)^2}
\]

where \( e' \) is the updated version of shopping-related exercises, which is a function of \( D, J^S \) and \( G \), as supermarket travel also increases with either \( J^S \) or \( G \) (or both). It is obvious that \( d\gamma' / dD = \phi \omega (p_j^S - p_G) / \alpha (p_G + \phi D)^2 < 0 \), yet \( d\delta' / dD = \phi \omega p_j^C / \alpha (p_G + \phi D)^2 > 0 \). The first and third terms on the right hand side of equation (18) therefore differ in sign, and the overall effect is ambiguous.

With more complicated derivations, it can be shown that the signs of both \( dJ^{S*} / dD \) and \( dJ^{C*} / dD \) are ambiguous. Consequently, the ambiguity of the effect also exists in either of the two special food environments considered above (i.e. rural area without convenience store and city center without supermarket but with farmers market). Moreover, the results above regarding extreme preferences still hold: longer supermarket distance unambiguously reduces the weight of a healthy eater but is ambiguous for an unhealthy eater. In sum, the main results apply to both
routine and random grocery shoppers, suggesting that supermarket access alone does not determine the effect on weight regardless of the supermarket travel pattern.

Income effects

So far the analysis has been performed without explicit consideration of the individual’s income status, which has been treated as given. In other words, the above results would hold at any given income level. Still, possible changes in income need to be formally addressed. The U.S. Department of Agriculture defines food deserts as communities with both low income population and low access to supermarket (Dutko, Ver Ploeg, and Farrigan 2012). It is therefore necessary to understand how weight changes with income in the food environment context, which could assist policy interventions specifically targeted at low-income population. For analytical purposes, assume that the income change is small (so that the individual remains a low-income food desert resident).10

Proposition 3. The effect of income on weight is ambiguous in general. For a mixed eater, income reduction unambiguously decreases weight if he/she is weight-insensitive, yet increases weight if he/she is weight-sensitive but his/her preference for healthy foods is sufficiently small. For a healthy eater, income reduction unambiguously decreases weight. For an unhealthy eater, the effect is ambiguous and depends on his/her relative tastes for junk foods from the two sources.

Proof. See Appendix C.

The ambiguity of the income-weight relationship may at first appear counterintuitive, given the fact that higher obesity rates are indeed observed in food deserts (Cummins and Macintyre 2006; Ghosh-Dastidar et al. 2014). However, there is consistent evidence that the effect of income on weight is generally unclear (see Ball and Crawford 2005; McLaren 2007; Shrewsbury and Wardle
2008 for recent literature reviews), and could be the net effect of numerous conflicting mechanisms. In the food environment context, individual preferences that affect food choices may play a more important role in weight gain. However, the effects of food preferences usually confound with income effects since it is consistently found that economically disadvantaged and minority population have less healthy food preferences (e.g. Drewnowski 1997; Turrell 1998; Granner et al. 2004), which could in part explain the observed obesity disparities by food desert status. Regarding income per se, the above results suggest that the hypothesized inverse income-obesity relationship would not be observed in a mixed eater if his/her preference for healthy foods is strong, and it may not necessarily occur even to an unhealthy eater.

Discussion

The above results suggest that neither limited supermarket access nor low income has a clear effect on weight, which depends on individual preferences as well as grocery shopping costs. This may help explain the mixed findings in existing studies (e.g. Courtemanche and Carden 2011; Alviola, Nayga and Thomsen 2013; Thomsen et al., 2015), as the net effect of multiple conflicting mechanisms may not be of either a clear sign or statistical significance. Hence, there is a need to extend the current literature with consideration of the possible determinants discussed above.

Theoretical predictions regarding supermarket access and weight are in line with very recent empirical findings in the food desert literature. Boone-Heinonen et al. (2011) employ panel data of U.S. adults 18-30 years old and conclude that better supermarket access is associated with neither diet outcomes nor fruit and vegetable intake levels. Rahkovsky and Snyder (2015) investigate the correlation between households that live in food deserts and their purchases of major groups of foods with varying diet quality, and find a modest negative effect which is only
slightly alleviated with the increase of supermarket distance. Fitzpatrick, Greenhalgh-Stanley and Ver Ploeg (2015) focus on the elderly and see little evidence that living in a food desert affects food-related distress, yet transportation difficulties are more likely to cause food insufficiency than limited supermarket access. These findings consistently speak to the importance of individual preferences regarding food source/healthfulness that factor into food choices as well as travel costs that occur with grocery shopping.

Due to the complicated relationship between supermarket access and weight outcomes, policy designs that simply assume a homogeneous consumer response could be less effective. There is quasi-experimental evidence that simply adding neighborhood supermarkets may have little benefit to diet quality (Cummins et al. 2005; Elbel et al. 2015), which is consistent with the above result that supermarket access alone does not have a clear effect on weight. On the other hand, it is not effective either to regulate unhealthy food providers that are prevalent in food deserts (Sturm and Cohen 2009), which also resembles with the above observation that the effect of supermarket access on weight remains ambiguous even when the neighborhood convenience store (or other retailer of unhealthy foods) is not present.

The ambiguous effect of income on weight is also increasingly supported by empirical evidence (see Ball and Crawford 2005; McLaren 2007; Shrewsbury and Wardle 2008 for literature reviews). Therefore, efforts in providing affordable foods to low-income populations may not always lead to desirable weight results. For instance, it is found that Supplemental Nutrition Assistance Program (SNAP) benefits could result in unintended weight gain among recipients (Chen, Yen and Eastwood 2005; Meyerhoefer and Pylypchuk 2008).

In the above analysis, individual preference and travel cost are two factors that may be important in the realm of public policy. In all cases, it is implied that a preference shift towards
food healthfulness would lead to lower weight. Hence, enhancing healthful food preferences could be especially meaningful to food desert residents by helping to reduce purchases from unhealthy food retailers. Nutrition education could be a potentially effective tool in this regard. Nutrition education to combat obesity has been commonly practiced (Flodmark, Marcus and Britton 2006; Pérez-Escamilla et al. 2008; Shaya et al. 2008), yet interventions specifically targeted at food desert residents have not been called for until very recently (Theuri 2015). The merits of nutrition education to address challenges in food deserts, especially its cost-effectiveness in this context, require further investigation.

Travel cost is increasingly discussed as a determinant of food access, which could also be a more accurate measure of access than the supermarket distance (Ver Ploeg, Dutko and Breneman 2014; Fitzpatrick, Greenhalgh-Stanley and Ver Ploeg 2015). Vehicle ownership is considered in these recent studies to capture travel cost. Vehicle ownership, however, may need to be examined further regarding fuel economy, maintenance cost, road condition, traffic congestion, etc., as the relative importance of time and monetary costs could lead to opposite effects on weight (see Proposition 1). Moreover, since the majority of the food desert population reside in urban areas, especially those with higher poverty rates (Dutko, Ver Ploeg and Farrigan 2012), the role of alternative travel methods in grocery shopping, such as public transportation and walking, needs to be better understood. Advancement in this knowledge could assist the design of interventions that help the food desert population overcome transportation barriers.

**Concluding remarks**

Food deserts are increasingly hypothesized as a causal factor of overweight and obesity in the U.S., yet empirical findings are rather mixed and largely inconclusive in part due to the infeasibility of
sorting out the multiple confounding mechanisms from a purely empirical perspective. This article considers this hypothesized relationship from the perspective of food choice in a rational-choice framework. It is found that longer distance to the supermarket does not generally increase weight, yet in certain cases a shorter distance does. The effect of income on weight is also ambiguous. In fact, the causality is far more complicated when mediated by food choice, which is collectively affected by not only supermarket access or income but also individual preferences and travel costs associated with grocery shopping. Case studies of special food environments, extreme preferences, random supermarket travel, and income changes reinforce the conclusion that the effect of food desert on weight is often ambiguous when food choice is taken into consideration.

The results imply that simply adding neighborhood supermarkets or restricting unhealthy food outlets may not be cost effective if obesity reduction and/or prevention is the policy goal. Also, food-purchasing assistance without addressing individual food choices may not result in desirable weight outcomes. Rather, the enhancement of healthful food preferences could play a more beneficial role. Moreover, efforts in removing the transportation barrier faced by certain food desert residents could be important. There may be a need to reevaluate policy priorities given these findings. Possible interventions in line with the preference- and cost-shifting strategies need further examination in a variety of settings.

As a first attempt at disentangling the potential confounding mechanisms behind the hypothesized relationship between food deserts and obesity, the current analysis observes several limitations. For instance, the fact that many convenience stores do provide a limited number of healthy foods is not formally incorporated into the model. Although this is deemed reasonable as major healthy foods such as fresh fruits and vegetables are rarely available in convenience stores, the dichotomy of supermarket with healthy foods and convenience stores without results could be
an oversimplification and the results may not apply to all individuals. In addition, the current framework considers only a single individual and ignores possible interactions among individuals that could affect food choices. Social learning and peer effects, which are beyond the scope of the current analysis, could be further addressed from both theoretical and empirical perspectives.
Notes

1 In reality, many convenience stores also provide a limited number of healthy foods (e.g. juice, yogurt), yet these provide a limited number of calories to most people on a daily basis. The primary source of healthy foods, such as fresh fruits and vegetables, are still large grocery stores and especially supermarkets (U.S. Department of Agriculture 2009). The assumption that convenience stores sell only junk foods helps simplify the analysis while still captures the essential features of the problem.

2 It can be speculated that food consumption at apparently “healthier” food outlets, such as full service restaurants, may not necessarily lead to weight gain. However, this is not true as meals from both full service and fast food restaurants have been shown to result in similar calorie increases (Binkley 2008; An 2015), and food away from home generally increases weight regardless of food outlet (Mancino, Todd and Lin 2009). For simplicity, full service restaurants are abstracted as their food prices are not very attractive to low-income food desert residents, which limits their policy relevance in context.

3 Overweightness is a theoretical result. Levy (2002) shows in a lifetime-utility maximizing framework why a weight-sensitive individual would choose a weight above the physiologically optimal level. In the U.S., 69% of adults (20 years and above) are overweight in 2011-2012 (Center of Disease Control and Prevention, see www.cdc.gov/nchs/fastats/obesity-overweight.htm). Therefore, the individual considered here represents the majority of population. In fact, he/she may further describe any normal-weight individual for whom weight loss generates a nonnegative utility.

4 The additivity of weight as a utility argument follows existing literature (e.g. Yaniv, Rosin and Tobol 2009; Dragone and Savorelli 2012), and reduces model complexity because weight per se is not a choice but an outcome, and it does not affect the utility of a weight-insensitive individual.

5 This assumption helps avoid some mathematical complexity in comparative static analysis. In general, convenience store shopping also requires travel (for a shorter distance). In that sense, $D$ may be interpreted as the difference between the supermarket and the convenience store distances with the abstraction of
convenience store travel costs (both time and monetary). The main results are the same with or without introducing the convenience store distance.

6 The shopping time (filling the cart, checking out, loading, unloading, etc.) is abstracted to avoid unnecessary complexity. This makes sense for food desert residents as the variation in shopping time should be comparatively small as compared with that of supermarket travel. It is therefore implicitly assumed that the individual shops for a period of time that is representative for the majority of population.

7 Admittedly, the number of grocery shopping trips is endogenously determined, but this assumption is still reasonable given that household grocery shopping behavior is characterized by a certain number of big trips to the supermarket on a weekly, biweekly or monthly basis (Yoo et al. 2006). The authors’ own calculation using Nielsen Homescan data (2009) also shows that an average household only conducts roughly 1.1 grocery shopping trips per week. Low-income people, however, travel even less frequently to the supermarket. For example, Wilde and Ranney (2000) find that 43.2% of the food stamp recipients conduct major grocery shopping trips only once per month or less. Therefore, the “routine grocery shopper” is representative in a food desert setting. This assumption is later relaxed to allow for random grocery trips.

8 Prices may depend on competition and local food market structure. Yet this is beyond the focus of the model presented here. Food prices for any given individual are reasonably exogenous.

9 From equations (14) and (15) and assuming interior solutions, it is straightforward to show that \( V_{jsjs} = U_{FF} (1 - \lambda p_j^F / \alpha_G)^2 + U_{Fl} (1 - \lambda p_j^F / \alpha_G) (\beta p_j^C / \alpha_G) + U_{fl} (\beta p_j^F / \alpha_G)^2 \), and \( V_{jcjc} = U_{FF} (\mu - \lambda p_j^C / \alpha_G)^2 + U_{Fl} (\mu - \lambda p_j^C / \alpha_G) (\beta p_j^C / \alpha_G) + U_{fl} (\beta p_j^C / \alpha_G)^2 \). For the weight-insensitive individual \((\rho = 0)\), \( V_{jsjs} < 0 \) and \( V_{jcjc} < 0 \) are automatically satisfied given that \( 1 - \lambda p_j^F / \alpha_G < 0 \) and \( \mu - \lambda p_j^C / \alpha_G < 0 \) are required for equations (14') and (15') to hold. The existence of a unique optimum also requires that \( V_{jsjs} V_{jcjc} - V_{jsjc}^2 > 0 \), in which case it can be shown that \( U_{FF} U_{fl} - U_{FF}^2 > 0 \) must hold.

10 Higher income people have broader choice sets, and there is more scope to substitute expenditure-intensive food procurement practices for time-intensive ones. These possibilities, however, are less relevant in the food desert context, and thus are assumed away with a small income change.
References


Appendix A: Proof of Proposition 1

Totally differentiate equation (14') with respect to $J_S$ and $D$ and rearrange:

(A1) \[ \frac{dJ_S^*}{dD} = \frac{1}{V_{JS}} \left\{ \left[ U_{FF} \frac{\lambda \eta}{\alpha_p G} + U_{FL} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \left( 1 - \frac{\lambda p_J^S}{\alpha_p G} \right) + \left[ U_{FL} \frac{\lambda \eta}{\alpha_p G} + U_{ll} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \frac{\beta p_J^S}{\alpha_p G} \right\} \]

where $V_{JS} = U_{FF} (1 - \lambda p_J^S / \alpha_p G)^2 + 2U_{FL}(1 - \lambda p_J^S / \alpha_p G) (\beta p_J^S / \alpha_p G) + U_{ll} (\beta p_J^S / \alpha_p G)^2$. Totally differentiate equation (15') with respect to $J_C$ and $D$ and rearrange:

(A2) \[ \frac{dJ_C^*}{dD} = \frac{1}{V_{JC}} \left\{ \left[ U_{FF} \frac{\lambda \eta}{\alpha_p G} + U_{FL} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \left( \mu - \frac{\lambda p_J^C}{\alpha_p G} \right) + \left[ U_{FL} \frac{\lambda \eta}{\alpha_p G} + U_{ll} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \frac{\beta p_J^C}{\alpha_p G} \right\} \]

where $V_{JC} = U_{FF} (\mu - \lambda p_J^C / \alpha_p G)^2 + 2U_{FL}(\mu - \lambda p_J^C / \alpha_p G) (\beta p_J^C / \alpha_p G) + U_{ll} (\beta p_J^C / \alpha_p G)^2$. Now totally differentiate equation (10) with respect to $D$, and substitute in equations (A1) and (A2):

(A3) \[ \frac{dW}{dD} = \frac{\gamma}{V_{JS}} \left\{ \left[ U_{FF} \frac{\lambda \eta}{\alpha_p G} + U_{FL} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \left( 1 - \frac{\lambda p_J^S}{\alpha_p G} \right) + \left[ U_{FL} \frac{\lambda \eta}{\alpha_p G} + U_{ll} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \frac{\beta p_J^S}{\alpha_p G} \right\} \]

\[ + \frac{\delta}{V_{JC}} \left\{ \left[ U_{FF} \frac{\lambda \eta}{\alpha_p G} + U_{FL} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \left( \mu - \frac{\lambda p_J^C}{\alpha_p G} \right) + \left[ U_{FL} \frac{\lambda \eta}{\alpha_p G} + U_{ll} \left( \theta - \frac{\beta \eta}{\alpha_p G} \right) \right] \frac{\beta p_J^C}{\alpha_p G} \right\} - \frac{\omega \eta}{\alpha_p G} \]

For a weight-insensitive individual, it is known that both $1 - \lambda p_J^S / \alpha_p G$ and $\mu - \lambda p_J^C / \alpha_p G$ are negative so that an internal solution for $H$ exists. If $\theta$ is sufficiently smaller than $\eta$ (e.g. $\theta < \beta \eta / \alpha_p G$), $U_{FF}(\lambda \eta / \alpha_p G) + U_{FL}(\theta - \beta \eta / \alpha_p G) < 0$ and $U_{Fl}(\lambda \eta / \alpha_p G) + U_{ll}(\theta - \beta \eta / \alpha_p G) > 0$, the sums in both braces of equation (A3) are positive and so the overall effect is negative. However, if $\eta$ is sufficiently smaller than $\theta$ (e.g. $\eta \equiv 0$), $U_{FF}(\lambda \eta / \alpha_p G) + U_{Fl}(\theta - \beta \eta / \alpha_p G) > 0$ and $U_{Fl}(\lambda \eta / \alpha_p G) + U_{ll}(\theta - \beta \eta / \alpha_p G) < 0$, in which case the sums in both braces are negative and so the first two terms on the right hand side of equation (A3) are positive. The overall effect on weight is also positive given the smallness of $\eta$ and therefore the smallness of the last term: $\omega \eta / \alpha_p G$.

For a weight-sensitive individual, the signs of $1 - \lambda p_J^S / \alpha_p G$ and $\mu - \lambda p_J^C / \alpha_p G$ are undetermined. The above results still holds if both are negative. However, if either or both terms are positive, the first and second terms in the respective brace(s) of equation (A3) have opposite
signs, and the overall effect on weight is ambiguous, and the sign of the effect depends on the parameter values of $\mu$, $\lambda$, $\theta$ and $\eta$ together with $U_{FF}$, $U_{Fl}$ and $U_{ll}$ evaluated at the optimum.

Now consider two special food environments discussed in the main text: 1) rural areas without a convenience store; and 2) city centers without a supermarket but with a farmers market. In the first case, $J^C$ drops from equations (2) and (6). Following the above procedure, the individual’s (unconstrained) problem can be presented as:

\begin{equation}
V = U \left( J^S + \lambda \frac{l-p_j^S}{\alpha p_G}, 1 - \theta D - \beta \frac{l-p_j^S}{\alpha p_G} \right) - \rho (\gamma J^S - e + \Gamma )
\end{equation}

where $\gamma$ and $\Gamma$ are defined in equation (10). The first-order condition is:

\begin{equation}
V_{J^S} = U_F \left( 1 - \frac{\lambda p_j^S}{\alpha p_G} \right) + U_l \frac{\beta p_j^S}{\alpha p_G} - \rho \gamma = 0
\end{equation}

Totally differentiating equation (A5) with respect to $J^S$ and $D$, equation (A1) is derived. The final effect of supermarket access on weight can be expressed as:

\begin{equation}
\frac{dW}{dD} = \frac{\gamma}{V_{J^S}} \left\{ \left[ U_{FF} \frac{\lambda \eta}{\alpha p_G} + U_{Fl} \left( \theta - \frac{\beta \eta}{\alpha p_G} \right) \right] \left( 1 - \frac{\lambda p_j^S}{\alpha p_G} \right) + \left[ U_{Fl} \frac{\lambda \eta}{\alpha p_G} + U_{ll} \left( \theta - \frac{\beta \eta}{\alpha p_G} \right) \right] \frac{\beta p_j^S}{\alpha p_G} \right\} - \frac{\omega \eta}{\alpha p_G}
\end{equation}

Repeat this procedure for the second case (city center without supermarket but with farmers market), and the effect of supermarket access on weight can be similarly expressed as:

\begin{equation}
\frac{dW}{dD} = \frac{\delta}{V_{J^C}} \left\{ \left[ U_{FF} \frac{\lambda \eta}{\alpha p_G} + U_{Fl} \left( \theta - \frac{\beta \eta}{\alpha p_G} \right) \right] \left( \mu - \frac{\lambda p_j^C}{\alpha p_G} \right) + \left[ U_{Fl} \frac{\lambda \eta}{\alpha p_G} + U_{ll} \left( \theta - \frac{\beta \eta}{\alpha p_G} \right) \right] \frac{\beta p_j^C}{\alpha p_G} \right\} - \frac{\omega \eta}{\alpha p_G}
\end{equation}

Both equations (A6) and (A7) are special cases of equation (A3). It is straightforward to verify that Proposition 1 still holds in both cases. Q.E.D.

**Appendix B: Proof of Proposition 2**

For the healthy eater, both $J^S$ and $J^C$ drop from equations (2) and (6). Consequently, $J$ drops from equation (7). From equations (3) and (6), $H = (I - \eta D)/\alpha p_G$, in which case equation (7) suggests
that $dW/dD = -\omega \eta / \alpha p_G - e_D$, which is unambiguously negative. Intuitively, the income effect shrinks $G$ (and therefore $H$) as $D$ increases, while for the walking individual the increase of $e$ further reduces $W$.

For the unhealthy eater, $H = 0$ and so it drops from equations (2) and (7), and $G$ and $k$, respectively, drop from equations (6) and (4). Substituting $f^S$ for $f^C$ (and assuming the existence of an interior solution), equation (7) can therefore be rewritten as:

$$W = \psi \left( 1 - \frac{p_j^S}{p_j^C} \right) f^S + \frac{\psi (l-\eta D)}{p_j^C} - e - E_0 - \text{BMR}$$

The individual’s (unconstrained) problem becomes:

$$V = U \left( f^S + \mu \frac{l-t_j^S - \eta D}{p_j^C}, 1 - \theta D \right) - \rho \left[ \psi \left( 1 - \frac{p_j^S}{p_j^C} \right) f^S + \frac{\psi (l-\eta D)}{p_j^C} - e - E_0 - \text{BMR} \right]$$

The first-order condition is:

$$V_{f^S} = U_F \left( 1 - \frac{\mu p_j^S}{p_j^C} \right) - \rho \psi \left( 1 - \frac{p_j^S}{p_j^C} \right) = 0$$

Equation (A10) implies that $1 - \mu p_j^S / p_j^C > 0$, or $p_j^C > \mu p_j^S$, otherwise the individual would not consume $f^S$ at all as $V_{f^S} < 0$. Totally differentiating equation (A10) with respect to $f^S$ and $D$ and rearranging, it can be shown that:

$$\frac{d f^S}{dD} = \frac{\mu \eta u_{FF} + \theta p_j^C u_{Fl}}{u_{FF} (p_j^C - \mu p_j^S)}$$

Totally differentiating equation (A8) with respect to $D$:

$$\frac{dW}{dD} = \psi \left( 1 - \frac{p_j^S}{p_j^C} \right) \frac{df^S}{dD} - \frac{\psi \eta}{p_j^C} - e_D$$

It is apparent that the sign of $df^{S*}/dD$ is critical in determining the final effect. Per equation (A11), if $1 < \mu < p_j^C / p_j^S$ and $\eta > \theta (p_j^S - p_j^C) U_{Fl} / U_{FF} + e_D (p_j^C - \mu p_j^S) / \psi (\mu - 1)$, then $dW/dD < 0$. In this case, longer distance to the supermarket decreases the weight of the unhealthy eater.
Intuitively, if the monetary travel cost is significant, i.e. \( \eta \) is large enough, the income effect associated with the increase in \( D \) reduces total (junk) food consumption (given the individual’s limited willingness to substitute \( J_C \) with \( J_S \)) and therefore decreases weight. However, if \( \mu < 1 \) and \( \eta < \theta(p_f^s - p_f^j)U_{Fl}/U_{FF} + e_D(p_f^C - \mu p_f^S)/\psi(\mu - 1), dW/dD > 0 \). In this case, the income effect is trivial and the substitution effect towards \( J_S \) dominates (given the individual’s strong preference for \( J_S \)), and weight increases as a result. Q.E.D.

Appendix C: Proof of Proposition 3.

Totally differentiate equation (14') with respect to to \( J_S \) and \( I \) and rearrange:

(A13) \[
\frac{dj^S}{dt} = -\frac{1}{V_{jSjS}} \left( U_{FF} \frac{\lambda}{\alpha_p} - U_{Fl} \frac{\beta}{\alpha_p} \right) \left( 1 - \frac{\lambda p_f^S}{\alpha_p} \right) + \left( U_{Fl} \frac{\lambda}{\alpha_p} - U_{ll} \frac{\beta}{\alpha_p} \right) \frac{\beta p_f^S}{\alpha_p}
\]

where \( V_{jSjS} = U_{FF}(1 - \lambda p_f^S/\alpha_p)^2 + 2U_{Fl}(1 - \lambda p_f^S/\alpha_p)(\beta p_f^S/\alpha_p) + U_{ll}(\beta p_f^S/\alpha_p)^2 < 0 \). Totally differentiate equation (15') with respect to to \( J_C \) and \( I \) and rearrange:

(A14) \[
\frac{dj^C}{dt} = -\frac{1}{V_{jCjC}} \left( U_{FF} \frac{\lambda}{\alpha_p} - U_{Fl} \frac{\beta}{\alpha_p} \right) \left( \mu - \frac{\lambda p_f^C}{\alpha_p} \right) + \left( U_{Fl} \frac{\lambda}{\alpha_p} - U_{ll} \frac{\beta}{\alpha_p} \right) \frac{\beta p_f^C}{\alpha_p}
\]

where \( V_{jCjC} = U_{FF}(\mu - \lambda p_f^C/\alpha_p)^2 + 2U_{Fl}(\mu - \lambda p_f^C/\alpha_p)(\beta p_f^C/\alpha_p) + U_{ll}(\beta p_f^C/\alpha_p)^2 < 0 \). Now totally differentiate equation (10) with respect to \( D \), and substitute in equations (A13) and (A14):

(A15) \[
\frac{dW}{dt} = -\frac{\gamma}{V_{jSjS}} \left( U_{FF} \frac{\lambda}{\alpha_p} - U_{Fl} \frac{\beta}{\alpha_p} \right) \left( 1 - \frac{\lambda p_f^S}{\alpha_p} \right) + \left( U_{Fl} \frac{\lambda}{\alpha_p} - U_{ll} \frac{\beta}{\alpha_p} \right) \frac{\beta p_f^S}{\alpha_p} + \frac{\omega}{\alpha_p}
\]

\[
-\frac{\delta}{V_{jCjC}} \left( U_{FF} \frac{\lambda}{\alpha_p} - U_{Fl} \frac{\beta}{\alpha_p} \right) \left( \mu - \frac{\lambda p_f^C}{\alpha_p} \right) + \left( U_{Fl} \frac{\lambda}{\alpha_p} - U_{ll} \frac{\beta}{\alpha_p} \right) \frac{\beta p_f^C}{\alpha_p} + \frac{\omega}{\alpha_p}
\]

It is obvious that \( U_{FF} \lambda/\alpha_p - U_{Fl} \beta/\alpha_p < 0 \) and that \( U_{Fl} \lambda/\alpha_p - U_{ll} \beta/\alpha_p > 0 \). Therefore, the sign of the effect on weight depends on the value of \( 1 - \lambda p_f^S/\alpha_p \) and \( \mu - \lambda p_f^C/\alpha_p \). For a weight-insensitive mixed eater, both terms are negative, and the overall effect on weight is
unambiguously positive, i.e. weight increases with income. In this case, lower income is associated with lower weight. For a weight-sensitive mixed eater, however, the sign of neither $1 - \lambda p_j^S / \alpha p_G$ nor $\mu - \lambda p_j^C / \alpha p_G$ is known, and the overall effect on weight is undetermined.

The positive relationship between income and weight still holds if the individual’s preference for $H$ (captured by $\lambda$) is strong enough as compared to those for both $J^S$ (captured by the unit) and $J^C$ (captured by $\mu$), yet this effect could be negative if his/her preference for $H$ is sufficiently small. In other words, the often observed poverty-obesity nexus would occur only if the individual’s preference for junk foods are sufficiently strong. It is easy to verify that these results hold for the special food environments considered above.

Now consider extreme preferences. For a healthy eater, it is clear from equations (3) and (6) that $H = (I - \eta D) / \alpha p_G$, and consequently from equation (7) that $dW / dI = \omega / \alpha p_G > 0$. Hence, income reduction unambiguously decreases weight. On the other hand, unhealthy eaters as a category may constitute a larger portion of food desert residents. For an unhealthy eater, totally differentiating equation (A10) with respect to $J^S$ and $I$, and rearranging yields:

(A16) $\frac{dj_{j^S}^{n^*}}{di} = -\frac{\mu}{p_j^C - \mu p_j^S}$

It can be further derived from equation (A8) that:

(A17) $\frac{dw}{di} = \psi \left( 1 - \frac{p_j^S}{p_j^C} \right) \frac{dj_{j^S}^{n^*}}{di} + \frac{\psi \mu (1-\mu)}{p_j^C - \mu p_j^S}$

It becomes obvious that if $\mu < 1$, income reduction decreases weight; if $1 < \mu < p_j^C / p_j^S$, income reduction increases weight. Q.E.D.