When to Get In and Out of Dairy Farming: A Real Option Analysis

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Abstract

The Dixit entry/exit real option model was applied to the entry/exit decisions of New York dairy farmers. For the cost structure of a 500-cow farm the entry milk price is $17.52 per hundredweight (cwt.) and the exit milk price is $10.84. For the 50-cow farm cost structure the entry price is higher at $23.71 per cwt., and the exit price is also higher at $13.48. If infinite numbers of representative farms enter and exit at these prices, the price of milk should range between $13.48 and $17.52 per cwt.

Keywords: Dairy farming, entry-exit, investment, real options

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Introduction

Over forty years ago Glenn Johnson discussed how supply response in agriculture was nonsymmetrical such that supply elasticity empirically often appeared to be lower for a price decrease than for a price increase. He postulated this was due to fixed investment in land and labor, such that the opportunity costs were too great for exit except at very low prices. At that time the economic theory and mathematics to model this asymmetric response had not been developed, except for ad hoc approaches estimating separate output responses to price increases and decreases (Tweeten and Quance). Beginning with McDonald and Siegel (1985, 1986) among others, and developed and popularized by Dixit and Pindyck, entry and exit into an industry can now be modeled using real option concepts. Essentially, this approach uses financial option theory applied to physical assets rather than financial assets, with the realization that the entry decision can be modeled as a call option and the exit decision can be modeled as a put option.

This article uses the model developed by Dixit to model the entry and exit decision of the dairy farmer. This model can result in a spread between the milk price which would encourage a dairy farm to exit the industry and which would encourage a new entrant. This is the case even while losses are incurred or profits are foregone. These losses or profits occur without exit or entry because the farmer holds unexercised options to exit or enter the industry. These exit and entry options have value and will not be exercised until the discounted losses or discounted profits exceed the value of the exit and entry option values, encouraging their exercise.

We determine what milk prices should encourage farmers to exit and enter the industry given the investment and cost structure of different types of New York dairy farms. What we find is that there are lower and upper prices such that exit does not occur until milk price moves below the lower price bound, and entry does not occur until milk price moves above the upper price bound, producing hysteresis between the price bounds. Since dairy producers have different costs of production, these price bounds vary by type of farm, although all may have the same milk price movement expectation.

There have been applications of real option concepts to agricultural investment decisions, including Richards and Patterson, and Carey and Zilberman, among many others. For dairy investment decisions, Purvis, Boggess, Moss and Holt modeled the freestall housing investment as a real option problem, and found that the present value of the investment would have to be much greater than the investment cost before the investment would be made. Engel and Hyde found the same for the adoption of robotic milking systems.

The Farming Entry and Exit Decisions as Options

Why a farmer may not get out of farming, even when he is currently experiencing losses, is easily expressed by any farmer. Next year might be better, and he is keeping his options open. Why someone may also hesitate to get into farming can also be expressed in option terminology. There may be profit today but it might be wise to see if profitability continues before making the investment. The exit decision is viewed as a put option and the entry as a call option, with the farmer as a holder (buyer) of these options. These options have value.

The standard economic operating decision, given perfect information and no adjustment costs, is to invest when the product price is above the sum of fixed and variable cost. In a multi-period setting, that would be when NPV is positive. The decision to shut down is when the product price is below variable cost. Given positive fixed and variable costs, this would generate both a lower and upper milk price band such that new investment would not occur until the upper milk price is reached, and exit would not happen until the lower milk price is reached. These options further increase the upper price and decrease the lower price. That is because if the upper price band is reached and you make the investment, you kill your option value to wait. Thus, it takes an even higher milk price than the sum of fixed and variable costs before you make the investment. In contrast, when you exit you kill the option to continue operating, and this takes a lower milk price than the variable cost alone.

The basic Dixit model assumes that the original investment is lost and there may be additional costs to exit. Yet for many dairy farms a significant amount of the initial investment can be recovered upon exit. Cows are liquid and land always has value. If that

is the case, the Dixit model can be modified with a negative exit cost, reflecting what the farmer may recover of the original investment. A current farmer may then find it optimal to exit while the milk price is even greater than variable cost. Although the farmer may be covering variable cost, he may not be covering total cost, and the stochastic price may go even lower than the current price. It might be best to "get out while you are ahead – if you can get back in at little cost". If you can recover all investment and re-enter at no cost, you will exit when price falls below total cost and re-enter when price moves above total cost.

It is interesting that the uncertainty of the milk price is what determines these costs, and it is not necessary for the producer to be risk averse. In fact, most analysis is done assuming that the farmer is risk neutral. Simply the existence of price variability and entry and/or exit costs produce option value.

The model to be used specifies that milk prices evolve as a Geometric Brownian Motion, which generates a lognormal price distribution. Alternative price movements include a mean-reverting process, where price reverts to the long-run marginal cost of production. However, evidence shows that there is a large range of production costs on U.S. dairy farms, ranging almost uniformly between $10 and $20 per cwt. of milk produced (Short). Dixit and Pindyck also model price floors with a point mass at that truncation point. This could pertain to milk with a support price, although in recent years the milk support price has rarely been binding.

An important assumption of the Dixit model is that delayed investments remain available in the future. That is easily the case for proprietary investment, but might not be the case in a competitive industry where a competitor might make the investment instead and kill the option. Leahy addressed this issue with firm homogeneity and competition (a competitive industry comprised of a large number of identical firms), and showed that the investment strategy using real options models is still optimal in competitive equilibrium. With homogeneity of firms, the introduction of competition reduces the value of investment options but does so by reducing the value of the invested capital. Since competition reduces the value of actual and potential capital at the same time, the trade-off between the two is unaffected. Decision makers may treat the price process as an exogenous diffusion process whose mean and variance are a fixed function of the price

level. His proof, however, assumes that firms are homogeneous. Caballero and Pindyck also take up this issue and look at both firm and industry uncertainty, but again by assuming firm homogeneity. It is clear that dairy farms are heterogeneous, as Short reveals with U.S. dairy farms with different fixed and variable costs of production. Yet, individual farms probably do not know the cost structure of potential competitors, so in our analysis we will assume that each firm with a specific cost structure believes that the industry is populated with an infinite number of similar firms. Individual farmers have various cost structures and may enter and leave the industry at different milk prices, as our later analysis demonstrates. Entry and exit also depends upon expected price dynamics, which will be modeled identically across farms.

Mathematics of the Entry and Exit Option Model

The Dixit model requires assumptions concerning the characteristics of the investment. First is that the investment has an infinite life and is nondepreciating. Land has an infinite life and buildings have long lives. It is clear that components of the dairy farm do depreciate, although land does not, and buildings depreciate slowly. Depreciation can be included into the model by one of two methods. If the investment depreciates and that depreciation is not restored, then depreciation can be modeled like a stock dividend by adjusting the discount rate. If depreciation is restored by replacement, then the depreciation necessary to maintain the investment is added to the constant operating cost. We elect to add depreciation to the operating cost presuming most farmers replace depreciated equipment.

Assume that the price of milk follows a Geometric Brownian Motion specified as:

$$dP = \mu P dt + \sigma P dz \qquad (1)$$

where:

P is the price of milk,

μP is the expected drift rate of P,

$\sigma^2 P^2$ is the variance rate of P, and

dz follows a Wiener process, i.e., $dz = \varepsilon\sqrt{dt}$, with ε being a random draw from a standardized normal distribution ($E(\varepsilon) = 0$ and standard deviation of ε is 1).

Using the square root of time allows the process to be Markovian. Note that in keeping with conventional notation the variable P is used to represent the stochastic market price of the product.

If the cost of production is assumed to be constant, or at least not extremely variable over time, then the value of the farm is strictly a function of the milk price and a stochastic component represented by time, expressed as $V(P,t)$. If cost is expected to vary significantly over time, then it can be entered as an additional stochastic variable which makes the mathematics more complex; or alternatively, the price variable P can be altered to represent an annual net operating return variable (NR). The modeling approach of price variable and cost constant is used since the price of milk is a transparent and published statistic while net return is not.

A Taylor expansion of the function $V(P,t)$ around the variables P and t produces:

$$dV = \partial V/\partial P\, dP + \partial V/\partial t\, dt + 1/2\, \partial^2 V/\partial P^2\, (dP)^2 + \ldots , \qquad (2)$$

where terms $(dt)^2$, $(dP)^3$ and higher vanish in the limit. In ordinary calculus the term $(dP)^2$ would also vanish but not in this case since dP follows a Brownian Motion.

Inserting equation (1) for dP and the square of equation (1) for $(dP)^2$ into equation (2) produces the following Ito process:

$$dV = \left(\frac{\partial V}{\partial P} \mu P + \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial P^2} \sigma^2 P^2 \right) dt + \frac{\partial V}{\partial P} \sigma P dz \qquad (3)$$

Since this is an infinite horizon problem, the variable t is not a decision variable and the derivative $\frac{\partial V}{\partial t}$ can be deleted.

Denote $\frac{\partial V}{\partial P} = V'(P)$, $\frac{\partial^2 V}{\partial P^2} = V''(P)$ for notational simplification, then (3) can be re-written as:

$$dV = \left(V'(P) \mu P + \frac{1}{2} V''(P\ )\sigma^2 P^2 \right) dt + V'(P) \sigma P dz$$

Taking the expected value of both sides of the equation yields

$$E\,(dV) = \left(V'(P) \mu P + \frac{1}{2} V''(P) \sigma^2 P^2 \right) dt \qquad (4)$$

since the expectation of dz, a normal standard deviate, is zero.

Deriving the Functional Form of the Value of an Idle Project

In equilibrium the expected capital gain of an idle project (denoted by $E(dV_0(P))$) should equal the normal return from the value of the investment ($= \rho V_0(P)dt$), where ρ is the discount (interest) rate. We use a risk-adjusted interest rate rather than the risk-free rate appropriate under contingent valuation (Dixit and Pindyck). There are short-term milk futures but these would not completely span the future, so long-term risk could not be hedged to justify contingent valuation. Equating produces:

$$[V_0'(P)\ \mu P + \frac{1}{2} V_0''(P)\ \sigma^2 P^2]dt - \rho V_0(P)dt = 0$$

Dividing the above equation by dt, produces the differential equation:

$$V_0'(P)\ \mu P + \frac{1}{2} V_0''(P)\ \sigma^2 P^2 - \rho V_0(P) = 0$$

As shown by Dixit, the general solution for this differential equation is of the form: $V_0(P) = AP^{\alpha} + BP^{\beta}$

where:[1]

$$\alpha = \frac{\sigma^2 - 2\mu - ((\sigma^2 - 2\mu)^2 + 8\rho\sigma^2)^{1/2}}{2\sigma^2} < 0 \qquad (5)$$

$$\beta = \frac{\sigma^2 - 2\mu + ((\sigma^2 - 2\mu)^2 + 8\rho\sigma^2)^{1/2}}{2\sigma^2} > 1 \qquad (6)$$

assuming $\rho > \mu$, and A and B are constants to be determined.

For an idle project, such that no capital investment has been made, the option to make the capital investment should go to zero as the price P goes to zero. Since $\alpha < 0$ and $\beta > 1$, the option value of $V_0(P) = AP^{\alpha} + BP^{\beta}$ goes to zero when P goes to zero only if $A = 0$. So the functional form of the value of an idle project (denoted by V_0) becomes

$$V_0(P) = BP^{\beta} \qquad (7)$$

Deriving the Functional Form of the Value of an Active Project

[1] In Dixit, equation (5) is defined and used as $-\alpha$ in further derivations since it has a negative value. In Dixit and Pindyck, they keep $\alpha < 0$ but use β_2 for notation.

In equilibrium the following condition holds for an active project:

Normal return = expected capital gain + net revenue flow. This is stated as:

$$\rho V_1(P)\, dt = E[dV_1] + (P - C)dt$$

where C is variable cost above the milk price per hundredweight of milk produced since P is the milk price.

Substituting $E\,[dV] = \left(V'(P)\mu P + \frac{1}{2}V''(P\ )\sigma^2 P^2\right)dt$ from (4) into the equation above, dividing both sides by dt, and rearranging the equation produces:

$$V_1'(P)\ \mu P + \frac{1}{2}\ V_1''(P)\ \sigma^2 P^2 - \rho V_1(P) + P - C = 0.$$

The general solution for this differential equation is:

$$V_1(P) = P/(\rho - \mu) - C/\rho + AP^{\alpha} + BP^{\beta}$$

where:

$P/(\rho - \mu) - C/\rho$ is the present value of the net revenue.

$AP^{\alpha} + BP^{\beta}$ is the value of the option to abandon the project.

Clearly, as the price P goes to infinity, this option value of abandonment goes to zero. Since $\alpha < 0$ and $\beta > 1$, the option value $AP^{\alpha} + BP^{\beta}$ goes to zero when P goes to infinity only if $B = 0$.

Therefore, the functional form of the value of an active investment project becomes

$$V_1(P) = P/(\rho - \mu) - C/\rho + AP^{\alpha} \tag{8}$$

Note that the solution value of an active project does not include the $AP\alpha$ term, whereas previously the solution value of an idle project did not include the $BP\beta$ term.

Deriving the Investment Trigger Point and Abandonment Point

At the investment trigger point H, the value of the option (the value of the idle project) must equal the net value obtained by exercising it (value of the active project minus sunk cost of investment, represented by K). So we must have:

$V_0\,(H) = V_1(H) - K$ or

$$V_1(H) - V_0\,(H) = K \tag{9}$$

This is the value-matching condition.

The smooth-pasting condition requires that the two value functions meet tangentially:

$$V_1'(H) - V_0'(H) = 0 \qquad (10)$$

Similarly, at the abandonment point L we have:

$$V_1(L) - V_0(L) = -X \qquad \text{(value-matching condition)} \qquad (11)$$

$$V_1'(L) - V_0'(L) = 0 \qquad \text{(smooth-pasting condition)} \qquad (12)$$

where X is the cost of abandoning the investment K, which is assumed worthless. If some of the original investment K is recovered, such that value remains after liquidation costs, then those net proceeds are entered as a positive X value.

Substituting the functional form of V_0 and V_1 from equations (7) and (8) into equations (9), (10), (11), (12) produces the following system of equations:

$$H/(\rho - \mu) - C/\rho + AH^{\alpha} - BH^{\beta} = K \qquad (13)$$

$$1/(\rho - \mu) + \alpha AH^{\alpha-1} - \beta BH^{\beta-1} = 0 \qquad (14)$$

$$L/(\rho - \mu) - C/\rho + AL^{\alpha} - BL^{\beta} = -X \qquad (15)$$

$$1/(\rho - \mu) + \alpha AL^{\alpha-1} - \beta BL^{\beta-1} = 0 \qquad (16)$$

In this system of equations, ρ, μ , σ^2 are parameters which can be estimated directly from empirical data. Then α, β can be calculated by applying formulae (5) and (6). Finally, the four unknowns A, B, L, H can be obtained numerically as a simultaneous numerical solution to the four equation system (13-16). This is done with the use of Mathcad software.

Equation (13) can be rearranged into $H/(\rho - \mu) - C/\rho$ - K, which is the discounted net return form the farm investment plus AH^{α}, the option to exit (after entry), equated to (BH^{β}), the value of the option to enter (i.e. to continue being idle) all valued at the milk price of H. Any milk price about the solution value of H encourages the farm investment and creation of the option to exit. Equation (14) is simply the derivative of equation (13) with respect to the critical milk price of H. This is the "smooth pasting" condition where the farm investment value function is tangent to the options value function, and provides an additional equation for solution. Equation (15) can be rearranged into $L/(\rho - \mu) - C/\rho$, which is the expected net return if you never exit when the milk price is currently L, plus

AL^{α}, the value of the option to exit, equated to BL^{β}, the option value of being idle at milk price L, minus –X, the cost of exiting (salvage value). A milk price lower than the solution value of L encourages exit. Equation (16) then is the derivative of equation (15) with respect to the milk price of L and provides another "smooth pasting" condition and equation.

Estimating the Entry and Exit Price of Milk

Parameter Estimates

The Department of Applied Economics and Management at Cornell University collects annual farm business data on a group of cooperating farms, which provides information on investment and cost of production (Knoblauch, Putnam, and Karszes). The average annual price of milk received by Dairy Farm Business Summary (DFBS) participants and their annual operating costs per hundredweight of milk produced over 10 years are shown in Table 1. It would be possible to use these annual milk prices to estimate both μ and σ^2 for the option model since prices that are a random walk at the monthly level would also display a random walk at the annual level. However, monthly prices are available from USDA surveys and provide many more observations to estimate milk price volatility. Monthly cost data are not available from any source, so annual DFBS investment and cost of production data are used.

The premise underlying option pricing is that the stochastic price variable follows a random walk. The option model developed further assumes that milk prices are log normally distributed. Monthly milk prices from farm gate milk prices received by New York farmers over the period 1993 through the end of 2003 produces 120 observations (New York State Agricultural Statistics). To calculate mean and variance of the milk price, the statistic $d_t = ln(P_t/P_{t-1})$ was calculated and used to calculate the monthly mean and variance (Hull). These were annualized by multiplying by 12, which resulted in an annually adjusted mean of 0.00 and variance of 0.0278.[2]

[2] In financial options, daily prices are often used such that the variance is much larger than the mean estimate, with the mean often ignored or set to zero, especially since it is not required in the Black-Sholes Formula because of risk-free arbitrage. Mean returns are also difficult to accurately estimate (Luenberger).

Table 1. Milk Price and Operating Costs for New York DFBS Farms

Year	*Milk price $/cwt.*	*Operating Costs Reported $/cwt.*
1993	13.14	10.18
1994	13.44	10.47
1995	13.03	10.40
1996	14.98	12.00
1997	13.65	11.76
1998	15.60	11.50
1999	14.91	11.22
2000	13.38	11.31
2001	15.98	11.87
2002	12.98	11.01

Whether New York milk price is a random walk was tested with a Dickey-Fuller test. Regressions were estimated for a unit root with a drift (intercept) and trend, with a drift and no trend, and with no drift and no trend. The Durbin-Watson statistic initially indicated that errors in each equation were not strictly white noise, so lagged differences were included beginning with five terms, and deleting those not statistically significant. The Durbin-Watson statistics of the final models imply the remaining errors are white noise. In only the random walk equation with no drift and no trend could the null hypothesis of a unit root not be rejected, allowing the weak conclusion of the existence of a unit root and a random walk. It appears, however, that there is drift in the monthly milk price, but no trend; and in that equation the null hypothesis of a unit root was rejected, implying that the milk price does not follow a random walk. However, Tomek concludes that since the data generating processes for commodity prices are complex and difficult to forecast, and given the costs of arbitrage, any systematic behavior of prices cannot be used to make profitable forecasts. It is thus reasonable to assume that farmers act as if prices do follow a random walk.

Even if the mean is not zero, most financial options have short lives such that the volatility overwhelms any modeled mean price increase. In our application there is no option expiration.

A trend line fitted through the DFBS annual operating cost data leads to the rejection of any trend in costs.[3] The fact that operating costs per cwt. essentially have not increased might surprise some since the cost of inputs has increased, but offsetting input price increases is the continuous increase in milk production per cow. Thus, the mean percentage change of operating costs and variance were assumed zero in the following analysis. Assuming a constant operating cost allowed formulating the model in terms referenced to the price of milk. Later the stochastic variable in the model is redefined as the net operating return per hundredweight of milk rather than milk price per hundredweight of milk.

Data from the year 2002 of the New York DFBS were used to estimate costs and investments (Knoblauch, Putnam, and Karszes). The reported and plotted operating costs in that annual publication include interest paid and exclude depreciation and the value of operators' labor, so interest paid was subtracted, depreciation on buildings, machinery and equipment was added, and the values of operators' labor were added to operating costs. All depreciation is assumed reinvested into the farm to maintain the investment. Operators' labor is treated as an operating cost rather than modeled as an investment. There is an active market for dairy workers and managers so little human capital would be lost. Cull cows and other receipts besides milk are produced by these farms and the cost of producing those receipts is reflected in operating cost. Thus, the value of those receipts per cwt. of milk is also subtracted from operating cost to produce the variable cost of producing milk only. On average, these farms paid an interest rate of 5 percent. An additional 300 basis points provides a discount rate of 8 percent.

The other variables necessary to make the model operational are an estimate of the investment per cwt. of milk and the cost of liquidating that investment. Dividing total farm assets of various farm sizes by the total annual milk production of that farm size produced investment cost per hundredweight as shown in Table 2. Investment per cwt. of milk decreases by farm size, from a maximum of $46.65 for the 50-cow farm (data from size class 50 to 74 cows), to a low of $27.04 for the 500-cow farm (data from size class 400 to 599 cows). Liquidation costs were estimated at 50 percent of real estate value, 40 percent of machinery and equipment value, and 10 percent of cows, feed, and other

[3] The change in annual operating cost was 0.0104 with a variance of 0.004.

assets, all of which are more liquid. Sensitivity analyses on these liquidation costs are reported. Parameters used in the option model are summarized in Table 2.

Table 2. Option Model Parameters for Various Size Dairy Operations in New York

Number of cows (range of size)	*Investment per cwt. of milk*	*Liquidation value per cwt. of milk*	*Variable cost per cwt. of milk*	*Total cost per cwt. of milk*
50 (50-74)	$46.65	$30.37	$14.66	$18.39
100 (100-149)	43.46	28.84	14.01	17.49
150 (150-199)	37.59	25.59	13.44	16.45
250 (200-299)	29.77	20.50	12.05	14.43
500 (400-599)	27.04	19.00	11.85	14.01

Data generated from Year 2002 NY Dairy Farm Business Summary Report. Future growth in the price of milk was 0.00. Variance of the percentage change in milk price was 0.0278. No projected change in operating cost, and variance of that cost is zero. Discount rate of 8 percent.

Results

Milk prices by farm size that would encourage exit and entry into milk production are shown in Table 3. It is important to remember that entry prices reflect turnkey entry of a similar type farm and not incremental investment of current farms. Also, if farmers were able to recover all investment costs upon exit, then entry and exit prices would be equal to the total cost of production – operating and fixed. There would be no cost of entering and exiting the industry and the options would have no value.

The 250-cow farm has a variable cost of $12.05, and with an investment of $29.77 at 8 percent interest would have a fixed cost of $2.38, for a total cost of $14.43, producing an operating price range from $12.05 to $14.43. With the addition of exit and entry options, the operating range expands to the range of $11.03 to $18.18. Exit prices range from a low of $10.84 for the 500-cow dairy to a high of $13.48 for the 50-cow dairy. The lowest milk price in New York during the last 10 years has been $11.80, which is below the exit price of all but the 250-cow and 500-cow farms, and thus would trigger exit of the smaller farms. During those same 10 years, new entry of larger farms would have been justified because the highest milk price during that period was $18.20,

whereas the entry price of the 250-cow and 500-cow farms were both lower at $18.18 and $17.52, respectively.[4]

Table 3. Exit and Entry Milk Price for Various Size Dairy Operations in New York

Number of cows	*Total cost per cwt. of milk*	*Variable cost per cwt.*	*Milk price to exit dairy*	*Milk price to enter dairy*	*Entry-exit price range*
50	$18.39	$14.66	$13.48	$23.71	$10.23
100	17.49	14.01	12.92	22.45	9.53
150	16.45	13.44	12.37	20.90	8.53
250	14.43	12.05	11.03	18.18	7.15
500	14.01	11.85	10.84	17.52	6.68

Solution by real option model. Model parameters from Table 2.

There may be other financial and non-financial factors impacting entry and exit decisions, and exit causes have recently been estimated for Maine dairy producers (Bragg and Dalton). Many farmers simply are at an age to retire, and beginning farmers will enter as long as the price is reasonable.

The analysis assumes the farmer can recover only a fraction of the initial investment upon exit. If almost all investment can be recovered, then the entry price falls and the exit price increases. That is because there is little capital loss to exit and re-enter farming. In fact, the exit price can rise above the variable cost of production but not the total cost of production. This is illustrated with the 150-cow dairy, using the parameters for that farm as shown in Table 2 but increasing the liquidation value from $25.59 to $33.83, which represents losing just 10 percent of the initial investment upon exit. The exit and entry prices then become $19.44 and $13.77. The $13.77 exit price is above the $13.44 variable cost. Setting the liquidation value to the investment value generates entry and exit prices of $16.45, which is the total cost of production. In that case K=-X in equations (13) and (14). There is no lost capital to repeatedly exit and enter the industry

[4] Many of these new entrants were current dairy farmers who built new dairy structures, either abandoning or converting their old facilities to other uses.

so that exit will occur whenever the milk price falls below the total cost of production, and entry will occur whenever the milk price moves above the total cost of production.

At the other extreme the farm may become worthless upon exit. That would lower the exit price. For the 50-cow farm the exit price would decrease from $13.48 to $10.71. This is much lower than the variable cost of production of $14.66 for that farm, and produces operating losses, but selling the farm produces no revenue, and there is the chance that prices might get better.

Results Modeling New Return Variability

The analysis so far assumes that any variation in net return is from milk price changes since the cost of production per cwt. of milk was held fixed at C. However, since some variation in costs have occurred on these New York dairy farms, analysis was also completed using net operating return variability from the 10 years of New York DFBS data, which is the difference between the milk price received by participants and operating cost as shown in Table 1. This entailed using only 10 observations to estimate new return variability. The mean annual growth in net return was 0.04520 with a variance of 0.27194. The option model was solved setting operating cost (*C*) equal to zero and specifying P as the net return NR rather than the price per cwt. of milk. Option values are also computed based upon net return rather than the price of milk. Using the investment cost of $37.59 and sales value of $25.59 for the 150-cow farm produces an entry operating net return of $7.18 and an exit net operating return of $0.86. From these amounts would be subtracted a fixed cost of $3.01 (interest rate of 0.08) resulting in entry and exit net profit of $4.17 and negative $2.15. Since the average milk price received by these farm business summary farms over the 10-year period was $14.11, this corresponds to an entry milk price of $18.28 and an exit milk price of $11.96. This compares to entry and exit milk prices of $20.90 and $12.37, respectively, for the 150-cow farm using milk price variability and a constant cost of production.

Conclusions

The entry and exit decisions of the dairy farm were modeled as real options. Conventional economics would dictate that a farmer should exit the industry when the milk price falls below variable cost of production. However, the milk price may recover in the future so a farmer continues to produce, essentially keeping his options open. The value of that option is computed such that the milk price before exit is lower than simply the variable cost of production. At the other end, a farmer should enter the industry when the price of milk is greater than fixed plus variable cost of production. Again, a farmer may not enter immediately since the high milk price may be transient. He wants to see if the high milk price has duration, so essentially keeps his investment option open. The milk entry price must be above not just the sum of fixed and variable cost of production but also the addition of the option value.

Computation of entry prices for various sized dairy farms in New York produced entry prices that were from $3.00 to $5.00 above the total cost to produce a cwt. of milk. Exit prices were around $1.00 lower than variable costs of producing milk. This is a wide price band which milk prices can move without attracting entry or encouraging exit. As farm size increases, the price band moves down. Small dairy farms will exit at a higher price then large farms, and a higher price is needed to induce entry by a small dairy farm. If there are an infinite number of potential large farms, then the price of milk in New York should not exceed $17.52, the entry milk price for the 500-cow farm. Since a large number of small farms still exist in New York, the price of milk should not fall below $13.48, the exit milk price for the 50-cow farm, assuming a sufficient inventory of small farms since their entry price would not be reached. Over 1995 through 2004 the lowest marketing year milk price was $13.10 and the highest was $16.80 (New York Agricultural Statistics).

These results are estimates and different model parameters would change entry and exit prices. Farmers have different cost structures and may have different future price expectations than modeled, changing the entry and exit prices. The inability to secure financing may also deter farm entry. Behavior and non-financial considerations may easily alter prices farmers are willing to enter and exit the industry. The desire to become and remain a farmer will lower the entry and exit prices. Yet, these results demonstrate

that there can be significant ranges in entry and exit prices for individual farmers. Given the parameters, small diary farmers are estimated to exit at a higher milk price than larger farmers, and the significant exit of dairy farms in the industry have been the smaller farms, although many of these may simply have been ready to retire (Bragg and Dalton).

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This worksheet implements the model in Dixit, Entry and Exit Decisions under Uncertainty, Journal of Political Economy, 1989, Vol 97, No. 3, pp 620-638, using Mathcad Software.

Written by Loren Tauer, Cornell University, February 2004

The model essentially treats the entry decision as a call option, and the exit decision as a put option. These are valued depending upon the variability of the revenue from the investment. The example here is entry and exit of the dairy farm. Investment, exit cost, and operating costs are all entered on the cwt. of produced milk basis. Revenue then is the milk price.

Values $\mu := 0.015($ $\sigma 2 := 0.0256($ $\rho := 0.08($

The variable μ is the growth rate of the revenue stream R. The variable $\sigma 2$ is the variance of R. The model assumes that costs, represented by C does not grow. (As an alternative R can represent NR=R-C, which can have a growth rate. The terms C/ρ would then be dropped from the 2 value matching equations below.)

The variable ρ is the discount rate, which must be greater than the growth rate μ or the solution is to wait forever to invest because the discounted net value would continue to increase.

The variables α and β are computed below using the parameters above

$$\alpha(\mu,\sigma 2,\rho) := \frac{\left[\sigma 2 - 2\cdot\mu - \left[(\sigma 2 - 2\cdot\mu)^2 + 8\rho\cdot\sigma 2\right]^{.5}\right]}{2\cdot\sigma 2}$$

$$\alpha(\mu,\sigma 2,\rho) \rightarrow -2.58741411470305374$$

The quadratic below should be equal to zero

$$-.5\cdot\sigma 2\cdot\alpha(\mu,\sigma 2,\rho)\cdot(-\alpha(\mu,\sigma 2,\rho) - 1) + \mu\cdot -\alpha(\mu,\sigma 2,\rho) - \rho \rightarrow 1.13846221046934364780^{-2}$$

$$\beta(\mu,\sigma 2,\rho) := \frac{\left[\sigma 2 - 2\cdot\mu + \left[(\sigma 2 - 2\mu)^2 + 8\rho\cdot\sigma 2\right]^{.5}\right]}{2\cdot\sigma 2}$$

$$\beta(\mu,\sigma 2,\rho) \rightarrow 2.41553911470305374$$

This is the second solution of the quadratic so value below should also be zero

$$.5\cdot\sigma 2\cdot\beta(\mu,\sigma 2,\rho)\cdot(\beta(\mu,\sigma 2,\rho) - 1) + \mu\cdot\beta(\mu,\sigma 2,\rho) - \rho \rightarrow 0$$

Investment cost K, exit cost of l, and annual (period) cost C

$K := 37.59$ $\quad l := -10$ $\quad C := 14.93$ $\quad FC := K \cdot \rho$ $\quad TC := FC + C$

Equations below solve the low return L for disinvestment and the high return H for new investment. The first two equations are for value matching, the second two equations are for the smooth pasting conditions.

Guess Values:

$A := 8000$ $\quad B := .0008$ $\quad H := 20$ $\quad L := 8.00$

Given

$$\frac{H}{(\rho - \mu)} + A \cdot H^{\alpha(\mu,\sigma2,\rho)} - B \cdot H^{\beta(\mu,\sigma2,\rho)} - \frac{C}{\rho} = K$$

$$\frac{L}{(\rho - \mu)} + A \cdot L^{\alpha(\mu,\sigma2,\rho)} - B \cdot L^{\beta(\mu,\sigma2,\rho)} - \frac{C}{\rho} = -l$$

$$\frac{1}{(\rho - \mu)} + \alpha(\mu,\sigma2,\rho) \cdot A \cdot H^{\alpha(\mu,\sigma2,\rho)-1} - \beta(\mu,\sigma2,\rho) \cdot B \cdot H^{\beta(\mu,\sigma2,\rho)-1} = 0$$

$$\frac{1}{(\rho - \mu)} + \alpha(\mu,\sigma2,\rho) \cdot A \cdot L^{\alpha(\mu,\sigma2,\rho)-1} - \beta(\mu,\sigma2,\rho) \cdot B \cdot L^{\beta(\mu,\sigma2,\rho)-1} = 0$$

$S := \mathrm{Find}(H, L, A, B)$

$$S = \begin{pmatrix} 23.165 \\ 11.597 \\ 2.522 \times 10^{4} \\ 0.07 \end{pmatrix}$$

Entry price $\quad$ Exit price

$S_1 = 11.597$ $\quad S_0 = 23.165$

$K = 37.59$ $\quad C = 14.93$

$l = -10$ $\quad FC = 3.007$

$TC = 17.937$

Now Graph the Solution, which entails finding where the option values are equal to the investment values. These are the value matching and smooth pasting requirements.

Price range to evaluate the two options $Pr := 5, 5.1 .. 40$

Estimate of the value of the option to invest evaluated at Pr

$$V0(Pr) := S_3 \cdot Pr^{\beta(\mu, \sigma 2, \rho)}$$

Estimate of the option value to exit evaluate at Pr

$$V1(Pr) := S_2 \cdot Pr^{\alpha(\mu, \sigma 2, \rho)}$$

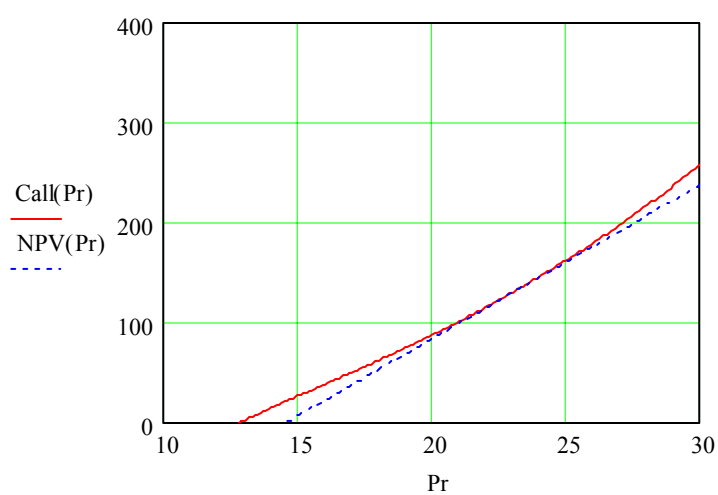
Compute the Net Present Value of the Investment

$$NPV(Pr) := \frac{Pr}{(\rho - \mu)} - \left(\frac{C}{\rho}\right) - K \qquad NPV(S_0) = 132.17$$

Compute the value of the call option, which is the value of the option to invest, minus the value of the option to exit

$$Call(Pr) := V0(Pr) - V1(Pr) \qquad Call(S_0) = 132.17$$

In the figure below the Call should be tangent to the NPV at the entry price of H computed above



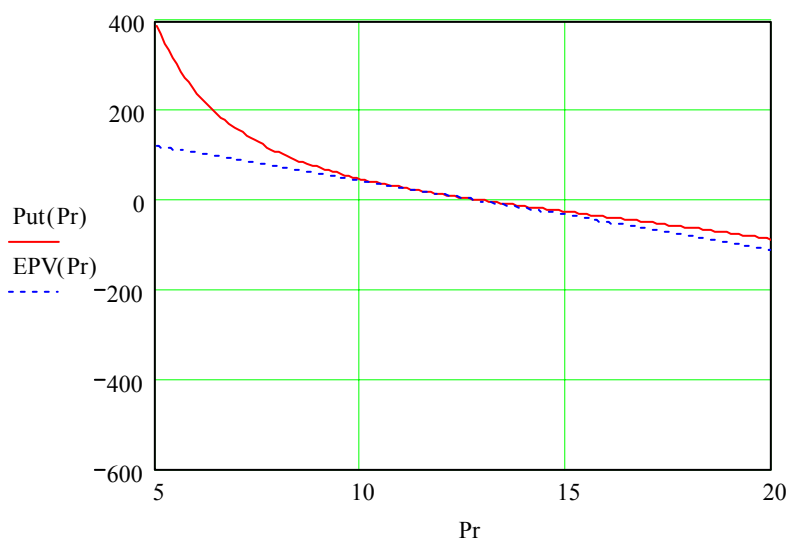
Exit Present Value

$$EPV(Pr) := -l - \frac{Pr}{(\rho - \mu)} + \frac{C}{\rho} \qquad EPV(S_1) = 18.213 \qquad S_1 = 11.597$$

Compute the value of the put option, which is the value of the option to exit, minus the value of the option to enter

$$Put(Pr) := V1(Pr) - V0(Pr) \qquad Put(S_1) = 18.213$$

In the figure below the Put should be tangent to the NPV at the entry price of L computed above



Computing the exit price without re-entry and the entry price without exit

Many times the option to re-enter after exit does not exist. Might be the case in farming if the family farm which has been in the family for generations is sold.

Below is the exit price if the farmer cannot re-enter the industry

$$l := -l \qquad Sa := C - \rho \cdot l \qquad Sa = 14.13$$

$$exitprice := Sa \cdot \left[\frac{(\rho - \mu)}{\rho}\right] \cdot \frac{-\alpha(\mu, \sigma 2, \rho)}{(-\alpha(\mu, \sigma 2, \rho) + 1)} \qquad exitprice = 8.28$$

For symmetry of results, below is the result if the option to exit after entry does not exit.
It is difficult to visualize that scenario occurring (since you can almost always walk away), although it is often computed.

Below is the entry price if the farmer cannot exit the industry

$$Ea := C + \rho \cdot K$$

$$entryprice := Ea \cdot \left[\frac{(\rho - \mu)}{\rho}\right] \cdot \left(\frac{\beta(\mu, \sigma 2, \rho)}{\beta(\mu, \sigma 2, \rho) - 1}\right) \qquad entryprice = 24.87$$

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