Optimal Dispatching Policy under Transportation Disruption

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Abstract

This paper proposes an optimal vehicle dispatching policy for transportation disruption. This policy determines the optimal vehicle capacity and dispatching time along a route. Transportation demand includes wiener demand and the unusual demand disruption, i.e. emergency evacuation. The cost function is consisted of waiting time cost and transporting cost. Sufficient conditions have been given with proof by quasivariational inequalities. Both finite planning horizon and an infinite horizon cases have been discussed. This proposed method can be applied into various scenarios, such as mass transit dispatching, freight transportation, aircraft shuttle scheduling and so on.

INTRODUCTION

Recently, transportation disruption topics have caused many researchers' interest due to the request of improved homeland security. This paper considers the transportation disruption problem in a continuous-time setting. A jump-diffusion model has been used to describe both dynamic transportation fluctuations and sudden transportation disruptions. The objective is to minimize the passengers waiting time cost and transport cost. Quasivariational inequalities approach has been used to derive sufficient conditions for this stochastic control problem. Optimal policies, i.e. optimal dispatching time and optimal dispatching capacity, have been found as well.

To authors' knowledge, no one has modeled the transportation disruption in a continuous time setting and found an optimal policy under transportation disruption explicitly. Most of research is to assess the effect by using simulation to compare different preselected policies. However, it is insufficient to show that the policy is optimal although it may perform well under certain conditions. Hence, to explicitly find an optimal policy is quite meaningful and significant.

In this paper we define the risk in two main parts: inherent risk and exogenous risk. The inherent risk is referred to the organizational intrinsic uncertainty, such as uncertain passenger arriving time, while the exogenous risk is referred as natural or man-made unusual disruption, such as earthquake, terrorism.

The idea of using jump-diffusion model into transportation disruption is that the number of passengers waiting along a route can be thought to be consisted of two parts: one is the small continuous changes due to the normal balance dynamics, such as uncertain people join the line or

leave the line arbitrarily, which can be seen as the counterpart of white noise in discrete time setting, and the other is the occasional jumps resulting from the release of new, significant information, such as emergency evacuation. The Wiener process can be used to represent the inherent transportation risk. On the other hand, the jump process can be used to denote the exogenous transportation disruption.

Literature Review

Transportation Disruption Literature Review – the transportation risk research usually is based in the context of the whole supply chain. Hence, the transportation risk is also part of supply chain risk. Tang (2006) has classified the supply chain risk into 4 main parts: supply, demand, product and information. In this paper, we mainly focus on demand disruption in transportation dispatching problems. Wilson (2007) investigates the effect of a transportation disruption on supply chain performance by using simulation. It has been found that the greatest impact occurs between tier 1 supplier and warehouse in a 5-echelon supply chain.

Newell (1971) studies a dispatching policy to minimize the total waiting time of all passengers. It has been found that for limited capacity vehicles, they should be dispatched as soon as they are full. Tapiero and Zuckerman (1979) consider a vehicle dispatching problem with competitions. Three dispatching policies have been compared for Poisson arrival processes. Karabuk (2007) describes a transportation problem involving scheduling of pickup and delivery of daily inventory movement between plants by integer programming.

Qi et al. (2004) analyze the supply chain coordination with demand disruption in a deterministic scenario and they show that changes to the original plan induced by a disruption will lead to significant cost across the system. Tomlin (2006) studies a single-product case in which a firm can source from two suppliers, one unreliable and one reliable but more expensive. He finds that a supplier's up time and the nature of the disruptions are key factors to determine the optimal strategy. He provides several contingency strategies and compares them in different settings.

Based on literature, most of the research is conducted in deterministic scenario. Only a little research considers the disruption problem in a continuous setting. Yao (2008) compares inventory value among information sharing, continuous replenishment program (CRP), and vendor-managed inventory (VMI). Chen and Zhang (2008) discuss the effect of demand disruption on the performance of supply chain by using a jump-diffusion model in a continuous setting. A two-number inventory policy has been implemented in the supply chain and the optimal values have been searched by using Simulated Annealing (SA) method. They find that both jump size and direction will affect the supply chain performance and the total cost can be reduced by using the optimal values from SA method under jump disruptions.

Impulsive Control Literature Review – Impulsive control has been first studied by Bensoussan and Lions (1982) in stochastic control. Bensoussan and Tapiero (1982) discuss the applications of impulsive control in management. Impulsive control has been widely applied into many areas. Robin and Tapiero (1982) use impulsive control approach to the vehicle dispatching problem by incorporating Poisson noise. However, this model doesn't consider the transportation disruption scenarios and doesn't fit for large size arrival rate, i.e. mass transit dispatching. Brekke and

Oksendal (1994) applies impulse control for optimal switching in an economic problem. Bar-ilan and Sulem (1995) consider a continuous-time inventory problem which has been solved explicitly by using impulse control. Ohnishi and Tsujimura (2006) investigate a geometric Brownian motion with quadratic costs by using impulse control. Some mathematical finance problem could also be solved by impulse control, i.e. Korn (1999), Oksendal and Sulem (2002).

Paper Structure

The optimal dispatching problem has first been modeled, and then sufficient conditions have been given with proof by quasivariational inequalities. Both finite planning horizon and infinite horizon cases have been discussed. Optimal dispatching policy has been found next. Finally, conclusion has been given and future research has been pointed out as well.

PROBLEM MODELING

One reason to consider the model in continuous-time setting is that it could accurately capture the time when the disruptions happen and better describe the process evolution. At the mean time, it can also avoid some problems dealing with integration and differentiation in discrete time. Moreover, it can be thought as the generalization of Poisson noise in discrete- time, because as the arrival rate of Poisson noise increases, it can be approximated as normal noise (Casella and Berger 2001). Hence, this model fits for high-volume discrete system flow (Harrison 1985), in other words, it's appropriate for mass transit dispatching, freight dispatching aircraft shuttle scheduling and so on. Therefore, the model in this paper can be seen as a generalization of that in Robin and Tapiero (1982).

Wiener Process

In discrete-time time series model, the shocks are assumed to form a white noise process, which are not predictable. The counterpart of shocks in a continuous-time model is the increments of a Wiener process, which is also called a standard Brownian motion. If we focus on small change $\Delta w_t = w_{t+\Delta t} - w_t$ associated with a small increment Δt in time, then a continuous time stochastic process $\{w_t\}$ is a Wiener process if it satisfies:1) $\Delta w_t = \varepsilon\sqrt{\Delta t}$, where ε is a standard normal random variable, which indicates $\Delta w_t \sim N(0, \Delta t)$; 2) Δw_t is independent of w_j for all $j \leq t$.

The Wiener process is a special stochastic process with 0 drift and variance proportional to the length of time interval, which means that the rate of change in expectation is 0 and the rate of change in variance is 1. The generalized Wiener process is such that the expectation has a drift rate μ and the rate of variance change is σ^2. Denote the generalized Wiener process by x_t and use the notation dy for a small change in variable y. Then the generalized Wiener process for x_t is $dx_t = \mu dt + \sigma dw_t$, where w_t is a Wiener process.

Impulse Control

An impulse control is a double sequence (Bensoussan 1982)

$$v = (\tau_1, \tau_2, \cdots, \tau_j, \cdots; \zeta_1, \zeta_2, \cdots, \zeta_j, \cdots) \quad (1)$$

where τ_j is the stopping time and ζ_j is the corresponding impulses at the stopping time and it is assumed that ζ_j is $\mathcal{F}_{\tau_j}$-measurable for all j. In this paper, the stopping time can be seen as the dispatching time and the impulse represents the vehicle capacity. Suppose that when the state is x, the result of implementing the impulse ζ is that the state jumps immediately from $x = X(t^-)$ to $X(t) = \mathcal{T}(x, \zeta)$, where $\mathcal{T}$ is the operator such that $\mathcal{T}: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$.

Problem Formulation

Let $X(t)$ be the number of passengers or goods waiting on a line and they need to be dispatched along a route. Consider the state variable $X(t)$ following a jump-diffusion model

$$dX(t) = \mu(t)dt + \sigma(t)dw(t) + \int zN(dt, dz) \quad (2)$$

From (2), it shows that the Wiener process denotes the normal dynamics of $X(t)$ without exogenous disruption, while the integral part presents the exogenous disruption. Note that the integral is Lebesgue integral (Folland 1999), not the usual Riemann integral in the calculus. z denotes the jump size, $N(\cdot, \cdot)$ represents the Poisson random jump measure. In other words, the disruptions come from a non-memory and independent Poisson measure, which appropriately describe the property of disruptions in practice.

Considering the effect of impulse on the state variable, we have the following state equation

$$dX^{(v)}(t) = \mu(t)dt + \sigma(t)dw(t) + \int zN(dt, dz), \tau_j < t < \tau_{j+1} \quad (3)$$

where

$$X^{(v)}(\tau_j) = \mathcal{T}(X^{(v)}(\tau_j^-) + \Delta_N X(\tau_j), \zeta_j) = X^{(v)}(\tau_j^-) - \zeta_j \quad (4)$$

Note that $\Delta_N X^{(v)}(t)$ is the jump of $X^{(v)}$ which comes from the random jump measure $N(t, \cdot)$. Hence, $X^{(v)}(\tau_j^-)$ represents that the state variable contains the information flow from disruption and $X^{(v)}(\tau_j)$ includes the effects of impulse control at stopping time τ_j. The objective is to minimize the total cost which is consisted of waiting cost and transport cost. And we consider it in two cases: infinite planning horizon and finite planning horizon.

Infinite Planning Horizon – can be seen as the discount cost, and the objective is to minimize the expected total discount cost $J^{(v)}(x)$, which can be written as

$$J^{(v)}(s, x) = E^{s,x}\left[\int_0^\infty e^{-\rho(s+t)} bX^{(v)}(t)dt + \sum_{k=1}^{N} e^{-\rho(s+t)}(c + \lambda\zeta_k)\right] \quad (5)$$

where s and x are the initial values of time t and state variable X(t), respectively. And $bX^{(v)}(t)$ denotes the waiting time cost, where b is the unit cost coefficient per person per waiting time, $c + \lambda\zeta_k$ denotes the transport cost, where c is the fixed cost and λ is the proportional cost coefficient, ζ_k is the dispatching vehicle capacity.

Finite Planning Horizon – is to minimize the total cost $J^{(v)}(x)$in a certain time interval, which is given as follows

$$J^{(v)}(s, x) = E^{s,x}\left[\int_0^{T-s} bX^{(v)}(t)dt + \sum_{k=1}^{N}(c + \lambda\zeta_k)\right] \quad (6)$$

Then the next step is to solve this stochastic optimal control problem. We'll derive sufficient conditions, and then lead to a set of inequality conditions which are called quasivariational inequalities.

SUFFICIENT CONDITIONS DERIVATION

We will consider the finite planning horizon case, since the derivation is very similar for infinite planning horizon case. Let V denote the set of all impulse controls. We want to find value function $\Phi(s,x) = \inf\{J^{(v)}(s,x) \mid v \in V\} = J^{(v^*)}(s,x)$, $v^* \in V$ and v^* refers to optimal impulse control. Consider more general objective function $J^{(v)}(s,x)$ as follows

$$J^{(v)}(s,x) = E^{s,x}\left[\int_0^T f(s,X^{(v)}(t))dt + g(s,X^v(T)) + \sum_{\tau_j \le T_s} K\left(s,X^{(v)}(\tau_j^-),\zeta_j\right)\right] \tag{7}$$

where $X(0^-) = x$ and state equations refers equations (3) and (4).

Define variational operator $\mathcal{L}\varphi(s,x)$ for function $\varphi(s,x)$ as

$$\mathcal{L}\varphi(s,x) = \frac{\partial\varphi}{\partial s} + \mu\frac{\partial\varphi}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2\varphi}{\partial x^2} + \int_{\mathbb{R}}\{\varphi(s,x+z) - \varphi(s,x) - \frac{\partial\varphi}{\partial x}z\}m(dz) \tag{8}$$

Define intervention operator $\mathcal{M}\varphi(s,x)$ for function $\varphi(s,x)$ as

$$\mathcal{M}\varphi(s,x) = \inf\{\varphi(\mathcal{T}(s,x,\zeta)) + K(s,x,\zeta)\} \tag{9}$$

Theorem 1 (Sufficient Conditions)

(I) Suppose we can find function φ such that

$$\text{(i)} \quad \mathcal{L}\varphi(s,x) + f(s,x) \ge 0 \tag{10}$$

$$\text{(ii)} \quad \varphi(s,x) \le \mathcal{M}\varphi(s,x) \tag{11}$$

$$\text{(iii)} \quad [\mathcal{L}\varphi(s,x) + f(s,x)][\varphi(s,x) - \mathcal{M}\varphi(s,x)] = 0 \tag{12}$$

Then

$$\varphi(s,x) \le \Phi(s,x)$$

(II) From (10) – (12), we have

$$\mathcal{L}\varphi(s,x) + f(s,x) = 0 \tag{13}$$

Suppose $\hat{v} \in V$, then

$$\varphi(s,x) = J^{(\hat{v})}(s,x) = \Phi(s,x)$$

and therefore $\hat{v} = v^*$ is optimal.

Proof: consider the initial time s is constant, in the following proof, we omit the argument s for convenience. By Dynkin formula (Dynkin 1965) to the stopping time τ_j, we obtain

$$\begin{aligned} E^x\left[\varphi(\hat{X}_{\tau_{j+1}^-})\right] &= E^x\left[\varphi(X_{\tau_j})\right] + E^x\left[\int_{\tau_j}^{\tau_{j+1}} \mathcal{L}\varphi(X(t))dt\right] \\ &\ge E^x\left[\varphi(X_{\tau_j})\right] - E^x\left[\int_{\tau_j}^{\tau_{j+1}} f(X(t))dt\right] \end{aligned}$$

Summing from j = 0 to j = m, we have

$$E^x\left[\int_0^{\tau_{m+1}} f(X(t))dt\right] \ge \varphi(x) + \sum_{j=1}^m E^x\left[\varphi\left(X_{\tau_j}\right) - \varphi(\hat{X}_{\tau_j^-})\right] - E^x\left[\varphi(\hat{X}_{\tau_{m+1}^-})\right] \tag{14}$$

By equation (9),

$$\varphi\left(X_{\tau_j}\right) = \varphi\left(\mathcal{T}\left(\hat{X}_{\tau_j^-},\zeta_j\right)\right) \ge \mathcal{M}\varphi\left(\hat{X}_{\tau_j^-}\right) - K(\hat{X}_{\tau_j^-},\zeta_j)$$

Subtracting $\varphi\left(\hat{X}_{\tau_j^-}\right)$ on both sides, we get

$$\varphi\left(X_{\tau_j}\right) - \varphi\left(\hat{X}_{\tau_j^-}\right) + K\left(\hat{X}_{\tau_j^-},\zeta_j\right) \ge \mathcal{M}\varphi\left(\hat{X}_{\tau_j^-}\right) - \varphi\left(\hat{X}_{\tau_j^-}\right)$$

Then equation (14) can be represented as

$$\varphi(x) + \sum_{j=1}^{m} E^x\left[\mathcal{M}\varphi\left(\hat{X}_{\tau_j^-}\right) - \varphi\left(\hat{X}_{\tau_j^-}\right)\right] - \sum_{j=1}^{m} K\left(\hat{X}_{\tau_j^-}, \xi_j\right)$$

$$\leq \varphi(x) + \sum_{j=1}^{m} E^x\left[\varphi\left(X_{\tau_j}\right) - \varphi\left(\hat{X}_{\tau_j^-}\right)\right]$$

$$\leq E^x\left[\int_0^{\tau_{m+1}} f(X(t))dt\right] + E^x\left[\varphi(\hat{X}_{\tau_{m+1}^-})\right]$$

Then, we have

$$\varphi(x) \leq E^x\left[\int_0^{\tau_{m+1}} f(X(t))dt + \varphi(\hat{X}_{\tau_{m+1}^-})\right] + \sum_{j=1}^{m} K\left(\hat{X}_{\tau_j^-}, \xi_j\right)$$

$$- \sum_{j=1}^{m} E^x\left[\mathcal{M}\varphi\left(\hat{X}_{\tau_j}\right) - \varphi\left(\hat{X}_{\tau_j}\right)\right]$$

Since $\varphi\left(\hat{X}_{\tau_j^-}\right) \leq \mathcal{M}\varphi\left(\hat{X}_{\tau_j^-}\right)$,

$$\Rightarrow \quad \varphi(x) \leq E^x\left[\int_0^{\tau_{m+1}} f(X(t))dt + \varphi(\hat{X}_{\tau_{m+1}^-})\right] + \sum_{j=1}^{m} K\left(\hat{X}_{\tau_j^-}, \xi_j\right)$$

As $m \to M$ such that $\tau_m \longrightarrow T$ then we can obtain

$$\varphi(x) \leq E^x\left[\int_0^{T} f(X(t))dt + g(X(T))\right] + \sum_{j=1}^{m} K\left(\hat{X}_{\tau_j^-}, \xi_j\right) \leq J^{(v)}(x)$$

Hence,

$$\varphi(x) \leq \Phi(x)$$

which completes the proof of part I in Theorem 1.

Assume $\exists\, \hat{v} = (\hat{\tau}_1, \hat{\tau}_2, \cdots, \hat{\xi}_1, \hat{\xi}_2, \cdots)$, if $\mathcal{L}\varphi(x) + f(x) = 0$, by following the same procedure, all the inequalities will become equalities. Hence,

$$\varphi(x) = J^{(\hat{v})}(x)$$

$$\Rightarrow \quad \varphi(x) \leq \Phi(x) = \inf_{v \in V} J^{(v)}(x) \leq J^{(\hat{v})}(x) = \varphi(x)$$

Therefore,

$$\varphi(x) = \Phi(x) \text{ and } \hat{v} = v^* \text{ is optimal.}$$

And this completes the proof of part II in Theorem 1. □

Note that equations (10) – (12) are called quasivariational inequalities (QVI), and these conditions are the same for infinite planning horizon case by following similar proofing procedures.

OPTIMAL DISPATCHING POLICY

Now we want to solve these problems by using the theorem proved above. Considering the similar solving procedures for both finite and infinite horizon problem, we only solve infinite

planning horizon problem to demonstrate how to solve this class of stochastic optimal control problem.

Infinite Planning Horizon

Considering the general form in equation (7), for infinite planning problem, we have

$$f(s,x) = bxe^{-\rho s}, \quad \text{and} \quad K(x,\xi) = (c + \lambda\xi)e^{-\rho s}$$

Guess the continuation region $D = \{x : 0 < x < \bar{x}\}$ for some $\bar{x} > 0$ and the value function φ has the form $\varphi(s,x) = e^{-\rho s}\psi(x)$. Then by equation (13), we have

$$\mathcal{L}\varphi(x) + f(x) = \frac{\partial\varphi}{\partial s} + \mu\frac{\partial\varphi}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2\varphi}{\partial x^2} + \int_{\mathbb{R}}\{\varphi(s,x+z) - \varphi(s,x) - \frac{\partial\varphi}{\partial x}z\}m(dz) + bxe^{-\rho s}$$

$$= e^{-\rho s}\left[-\rho\psi(x) + \mu\psi'(x) + \frac{1}{2}\sigma^2\psi''(x) + \int(\psi(x+z) - \psi(x) - \psi'(x)z)m(dz) + bx\right] = 0$$

$$\Rightarrow -\rho\psi(x) + \mu\psi'(x) + \frac{1}{2}\sigma^2\psi''(x) + \int(\psi(x+z) - \psi(x) - \psi'(x)z)m(dz) + bx = 0$$

To solve this equation, try $\psi(x) = e^{rx} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}$

$$\Rightarrow -\rho e^{rx} - bx - \frac{b\mu}{\rho} + \mu r e^{rx} + \frac{b\mu}{\rho} + \frac{1}{2}\sigma^2 r^2 e^{rx}$$

$$+ \int\left(e^{rx+rz} + \frac{bx + bz}{\rho} + \frac{b\mu}{\rho^2} - e^{rx} - \frac{bx}{\rho} - \frac{b\mu}{\rho^2} - xre^{rx} - \frac{bz}{\rho}\right)m(dz) + bx = 0$$

$$\Rightarrow \quad e^{rx}\left[-\rho + \mu r + \frac{1}{2}\sigma^2 r^2 + \int(e^{rz} - 1 - rz)m(dz)\right] = 0$$

$$\Rightarrow \quad h(r) := -\rho + \mu r + \frac{1}{2}\sigma^2 r^2 + \int(e^{rz} - 1 - rz)m(dz) = 0$$

As $h(0) = -\rho < 0$ and $\lim_{r\to\infty} h(r) = \infty$, then there exists two solutions $r_2 < 0 < r_1$ with $|r_2| > r_1$, for such r_1 and r_2, we try

$$\psi(x) = A_1\left(e^{r_1 x} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}\right) + A_2\left(e^{r_2 x} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}\right)$$

Since $\psi(0) = 1 + \frac{b\mu}{\rho^2}$,

$$\Rightarrow (A_1 + A_2)\left(1 + \frac{b\mu}{\rho^2}\right) = 1 + \frac{b\mu}{\rho^2}$$

$$\Rightarrow A_1 + A_2 = 1$$

Assume $A_1 > 1, A_2 < 0$, let $A = A_1 > 0, A_2 = 1 - A < 0$, we get

$$\psi(x) = A\left(e^{r_1 x} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}\right) + (1 - A)\left(e^{r_2 x} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}\right)$$

Define $\psi_0(x) := A(e^{r_1 x} - e^{r_2 x}) + e^{r_2 x} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}$ for all $x \geq 0$.

To study $\mathcal{M}\psi$, we first consider

$$g(\xi) := \psi_0(x - \xi) + c + \lambda\xi, \qquad \xi > 0$$

The first order condition for a minimum can give

$$g'(\xi) = -\psi'_0(x - \xi) + \lambda = 0$$

$$\Longrightarrow \psi'_0(x-\zeta) = \lambda > 0$$

Now $\psi'_0(x) > 0\ \forall x$, then $\psi''(x) < 0$ iff

$$Ar_1^2 e^{r_1 x} + (1-A) r_2^2 e^{r_2 x} < 0$$

$$\Longleftrightarrow \quad x < \frac{1}{r_1 - r_2} \ln \frac{r_2^2 (A-1)}{r_1^2 A} := \hat{x}$$

Hence, $\psi'_0(x) = \lambda$ has exactly two solutions provided $\psi'_0(\hat{x}) < \lambda < \psi'_0(0)$, and the solutions have the format

$$0 < \underline{x} < \hat{x} < \hat{x}$$

where $x^* = \hat{x}$ and $\hat{x} = \underline{x}$. We require $\psi(x) - \mathcal{M}\psi_0(x)$ for $x \geq x^*$, thus we get

$$\psi(x) = \psi_0(\hat{x}) + c + \lambda\zeta \text{ for } x \geq x^*$$

where $\hat{x} = x - \zeta$, which means the impulse size $\zeta = x - \hat{x}$ for $x \geq x^*$. Hence, we propose that ψ(x) has the form

$$\psi(x) = \begin{cases} A\left(e^{r_1 x} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}\right) + (1-A)\left(e^{r_2 x} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}\right) & 0 < x < x^* \\ \psi_0(\hat{x}) + c + \lambda(x - \hat{x}) & x \geq x^* \end{cases}$$

We need to find out $\hat{x}, x^*$ and A. By continuity at $x = x^*$, we have

$$A\left(e^{r_1 x^*} + \frac{bx^*}{\rho} + \frac{b\mu}{\rho^2}\right) + (1-A)\left(e^{r_2 x^*} + \frac{bx^*}{\rho} + \frac{b\mu}{\rho^2}\right)$$

$$= A\left(e^{r_1 \hat{x}} + \frac{b\hat{x}}{\rho} + \frac{b\mu}{\rho^2}\right) + (1-A)\left(e^{r_2 \hat{x}} + \frac{bx}{\rho} + \frac{b\mu}{\rho^2}\right) + c + \lambda(x^* - \hat{x}) \tag{15}$$

By differentiability at $x = x^*$, we have

$$A\left(r_1 e^{r_1 x^*} + \frac{b}{\rho}\right) + (1-A)\left(r_2 e^{r_2 x^*} + \frac{b}{\rho}\right) = \lambda \tag{16}$$

And $\hat{x}$ can be found by $\psi'_0(\hat{x}) = \lambda$, that is,

$$A\left(r_1 e^{r_1 \hat{x}} + \frac{b}{\rho}\right) + (1-A)\left(r_2 e^{r_2 \hat{x}} + \frac{b}{\rho}\right) = \lambda \tag{17}$$

Hence $\hat{x}, x^*$ and A can be solved from equations (15), (16) and (17).

Theorem 2 (Optimal Dispatching Policy)

If $\hat{x}, x^*$ and A are the solutions of equations (15) – (17) existing, then $\varphi^*(x) = \varphi(x; \hat{x}, x^*, A)$ satisfies QVI (10) – (12) and the optimal impulse control is $\zeta^* = x^* - \hat{x}$ and $\tau^* = \inf\{t > 0; X_t^{u^*} \notin (0, x^*)\}$.

Note that the mechanism of the optimal dispatching policy is that when the number of passengers or goods reaches certain upper limit x^*, at this particular stopping time τ^*, we dispatch the vehicle of capacity ζ^*. This policy can achieve the minimum total cost consisted of waiting cost and transport cost. And this policy works both for diffusion case (without disruption) and jump-diffusion case (with disruption).

CONCLUSIONS AND FUTURE RESEARCH

This paper models the transportation disruption scenarios as a stochastic optimal control problem. We discuss the problem in finite planning horizon and infinite planning horizon. Note that the

theory derived is fit for both cases and the solving procedures are very similar. To be concise, we prove the existence theorem for finite horizon by using quasivariational inequalities. And the optimal impulse control has been solved for infinite planning horizon case. This optimal policy could determine the optimal dispatching time and optimal dispatching vehicle capacity. Furthermore, it can also apply for different disruption sizes and jump directions.

One potential future research could be considering different cost structure, i.e. quadratic cost. If for some reason, the investigator is trying to keep the system at a stable level, then the cost structure in objective function can be modeled as quadratic form. And another direction is that if the value function is not in C^1, then we need to consider viscosity solution techniques.

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