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QUEUE DISSIPATION SHOCKWAVE SPEED– A SIGNALIZED INTERSECTION CASE STUDY

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ABSTRACT

Queue formation and dissipation have been extensively studied in relation to traffic signalization, work zone operation, incident occurrence, and ramp metering. This study is an attempt to estimate the effect of vehicle mix, commute time, traffic direction, and road upgrade on queue dissipation speed (time). The data were collected at several intersections in Davis, California, U.S. and analyzed using regression models. The models were determined regressing several functional forms and considering the statistical significance and ease of interpretation of the included variables. The main findings are: 1) dissipation speed does not vary purely by location; 2) a heavy vehicle is faster to discharge than its passenger car size-equivalent is; 3) the queue in a left-turn lane discharges faster than that in a through lane; 4) an upgrade slope increases the queue dissipation time due to more rolling resistance to vehicle start-up and larger vehicle gaps for safety ; 5) morning queues generally discharge more slowly; 6) contrary to common delay estimation models, regression analysis shows that queue dissipation time is linearly related to the number of vehicles rather than quadratically or in other ways; and 7) the simple linear function performs well both in terms of explanatory power (R2) and consistency of signs.

INTRODUCTION

The dissipation speed of queues is an important aspect of traffic behavior, which has not been given sufficient attention in research studies. Precise estimation of traffic congestion and delay is extremely important in studying traffic flow characteristics (Rouhani and Niemeier, 2011) and developing effective traffic control strategies. Delays and other congestion-related measures have been widely used in devising and employing system-wide traffic signalization (Yin, 2008), environmental policies (Rouhani, 2013; Mirchi et al., 2012), fuel consumption optimization (Rouhani, 2010), network design (Poorzahedy and Rouhani, 2007), road privatization (Rouhani, 2009), work zones (Chitturi et al., 2008), incident conditions (Bertini and Leal, 2005), and ramp metering operations (Lu et al., 2011).

Analysis of various signalized intersections and various signal control strategies require more explanatory queue dissipation models. Including a more detailed queue discharge/delay model can improve the estimation power of the traffic assignment models, especially for the predictive dynamic assignment models (Chen and Hsueh, 1998; Huang and Lam, 2002). Optimizing traffic signals usually based on demand variations are another main body of research which simulates queue formation and discharge (Yin, 2008; Zhang et al., 2010).

Some studies have estimated delays of various signalized intersection types (Olszewski, 1993; Fambro and Rouphail, 1998). Some of these models consider the shockwave theory as it relates to queuing as their basic assumption (Michalopoulos, 1981; Wu and Liu, 2011). To model traffic flow dynamics in a signalized network for real-time applications, Wu and Liu (2011) developed a section-based approach following the shockwave theory and using finite road sections. The study simplified the queue build-up and dissipation to derive an analytical solution for queuing dynamics and verified the model using real-world traffic signal data (SMART-Signal on the Highway 55 in the Twin cities, Minnesota). For more information on models to estimate delays at signalized intersections, readers are referred to Dion et al. (2004).

Numerous studies have investigated the queue length estimation problem (Liu et al., 2009; Izadpanah, 2009). Cheng et al. (2011) proposed a queue length estimation model using sampled vehicle trajectories as the only input and the shockwave theory as its main concept. The main concept of the model is the critical point, the point representing the changing vehicle dynamics. Based on this concept, the model can provide both real–time and offline traffic conditions to travelers. Queue length estimation is also an important topic for ramp metering (Lu et al., 2011).

Also, queue formation and dissipation are a research topic for scheduling work zones' time of operation (Jiang, 1999; Chitturi et al., 2008; Ramezani and Benekohal, 2011). These studies typically make use of the shockwave theory to estimate queue length and corresponding delay in work zones. Also, bottlenecks resulting from lane drops, especially for freeways, are well studied topics in the queue formation and dissipation research area (Cassidy and Mauch, 2001; Bertini and Leal, 2005). Even though these studies are important for designing work zones and for easing the traffic operations, the studies consider only the basic parameters, and some of the variables that should be included are usually forgone. Incorporating information about queuing attributes in transportation models is another main research topic (Cetin and List, 2006).

In one of the very few studies on queue dissipation models, Lin and Cooke (1986) developed a simulation model based on the car-following model that can estimate queue dissipation characteristics. In specific, the model was set up to analyze alternative signal control strategies. Although sophisticated in terms of driver behavior, the simulation model does not take into account various topographical and time period characteristics.

In another study of this infrequently examined topic, Vlahogianni et al. (2007) investigated the statistical characteristics of the short-term flow patterns of queues and the spillovers by implementing Bayesian augmented networks. They showed that the observed transitions in queuing conditions impose a set of prevailing traffic flow patterns with distinct statistical characteristics. However, the emphasis was mainly on traffic flow characteristics and other attributes were not considered. Based on data from 18 intersections in Sydney and Melbourne, Australia, Akcelic and Besley (2002) examined the queue discharge speed models but the emphasis was on headways, and some other main parameters were neglected.

The queue formation and dissipation problem can be simplified by making assumptions on road conditions, drivers' reaction times, vehicle dynamics characteristics and lengths, etc. However, studies based on such simplifications are usually unable to link the dissipation behavior with some difficult-to-model factors such as vehicle mix. The goal of this research is the development of a model, calibrated based on traffic survey data, to better understand the effects of vehicle mix, commute time, traffic direction, and road upgrade on queue dissipation speed.

This study tries to identify only the important parameters, which are usually ignored when modeling delay/queuing, and proper functional forms for estimating queue dissipation time (speed). The integration of this model or a resulting model into traffic assignment/simulation models requires further study. Although there are many types of traffic queues, the research scope is limited to signalized intersections. The data were collected for several intersections in Davis, California, U.S. and analyzed using regression models. The models are determined using few functional forms and considering the statistical significance and ease of interpretation of the included variables.

In the following sections, the methodology implemented in this study is explained. The data collection process is then described. The Discussion section represents and interprets the models in an attempt to produce some useful understandings of traffic behavior of queues in signalized intersections. Lastly, the "Conclusions" section summarizes the main findings.

METHODOLOGY

At an intersection, a red light usually causes the formation of vehicle queues, which will dissolve after the light turns green. In this research, one lane of one approach from the studied intersections was selected for survey. At a fixed location (survey point), multiple data cycles were surveyed. For each cycle, the data for the number and type of vehicles in the queue, the green light time, and the time when the first vehicle behind the survey point starts moving were collected. The distance between the survey point and the stop line, the traffic direction (through or left turn), the road slope (upgrade or horizontal), and the commute time (morning or afternoon) were also documented.

The target dependent variable is the queue dissipation speed. For convenience, the dissipation time, defined as the time duration between the green light start and the moving start of the first vehicle behind survey point (which is chosen far enough from the intersection line), is used as the dependent variable in place of the calculated queue dissipation speed.

All the other variables included in the models are treated as candidate explanatory variables and are initially all included in the linear regression model. The final models are determined based on the logical signs for and statistically significant coefficients.

DATA

A total of 65 cycles of data (at least 15 cycles from each intersection) were collected in 2007 from the 3 intersections (4 directions), which are listed in Table1 and shown in Figure 1. The binary values of 7 variables have been used in the models: the location (2 dummy variables), the traffic direction (through or left), the road slope (upgrade or horizontal), and the commute time (morning or afternoon). There are 3 location variables, one of which was not included when in model fitting, because of independency. Some of the data points were dropped to avoid including erroneous data. The errors are usually related to the queue formation; for some cycles, the queue did not form according to the study's requirements.

TABLE1 Studied intersections' information

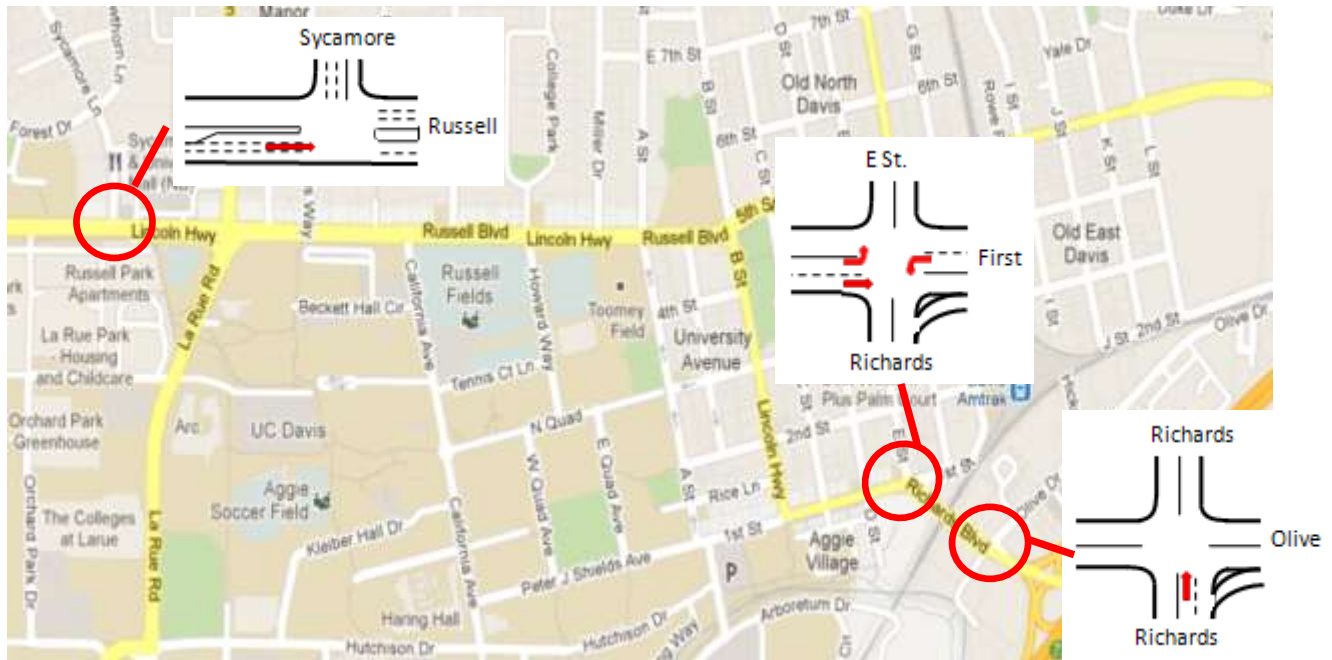| Intersection#	Location	Approach & Lane	Time	Road Slope:
1	Richards & Olive	From Richards to Cowell, through	17:00pm	Upgrade
2	First & E	From 1st to Richards, left turn	16:00pm	Horizontal
3	Russell & Sycamore	From Russell to Russell, through	8:00am	Horizontal
4	First & E	From Richards to 1st and E, *left turn and through*	9:00am	Horizontal

FIGURE 1 Location of intersections

Figure 2 shows the data collection schematic procedure and the important variables for a left-turn lane on a hypothetical intersection. As can be seen in the figure, the length of the queue is fixed (Figure 2a), the start of green time (t_1) sets the time when the queue begins to dissipate (Figure 2b), the queue continues to dissipate (Figure 2c), and finally, the queue has completely dissipated (t_2 in Figure 2d).
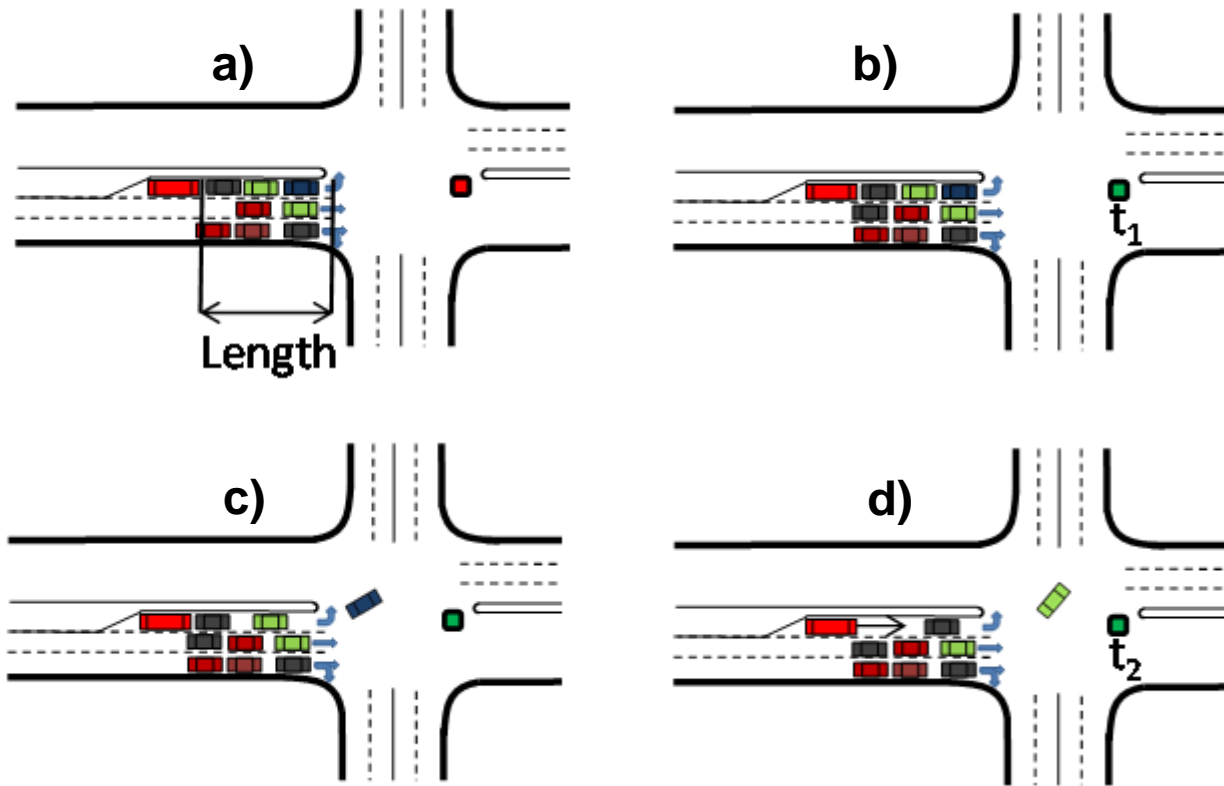

FIGURE 2

Schematic procedure of collecting data: a) when red; b) start of green-t_1; c) queue continues to dissipate; d) the complete dissipation of queue- t_2.

DISCUSSION

Linear Model

When all the candidate variables are included, the coefficients for location are all zeroes with very high t statistics, which implies that the dissipation speed does not vary purely by location. Therefore, the 3 binary location variables are dropped. The final linear model is as below:

dissipation time (in seconds)= 1.69 + 0.82 × (# of passenger cars)
+ 0.97 × (# of SUV or truck)
+ 1.20 × (# of heavy vehicles)
- 0.70 × (Is left turn?)
+ 1.74 × (Is upgrade?)
+ 0.45 × (Is morning commute?)

TABLE2 Linear model coefficients

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.6891	2.3007	0.7341	0.4658
No. of passenger cars	0.8174	0.341	2.3971	0.0198
No. of SUV, pickups	0.9684	0.3592	2.6957	0.0092
No of heavy vehicles	1.2034	0.7351	1.6371	0.107
Left Turn?	-0.6953	0.4412	-1.5757	0.1205
Upgrade?	1.7435	0.7265	2.3998	0.0196
Morning?	0.4527	0.4338	1.0434	0.3011

$N=65, R^2 = 0.65, \ adjusted \ R^2 = 0.62$

Table 2 reports the results of regressing a linear function on the data (adding that the length of the queues – Figure2a -- is around 50 meters for all the studied intersections). The adjusted R-squared is 0.62, meaning that a substantial share of variation in the data set is explained by the model. Nevertheless, the explanatory power of the model is defective because human (driver) factors are not included, which perceivably should have a significant impact on the dissipation time. The constant of the model is positive, which means that the average dissipation time due to the combined effect of all other factors not considered in this model is 1.69 seconds.

The coefficients of the 3 vehicle type variables are all positive, which is logical in that more vehicles in the queue would add to dissipation time. According to the observed patterns at the locations, the occurrence of heavy vehicles tends to reduce the dissipation time, which is counterintuitive, but may in fact be reasonable. Because of using a fixed survey point, the queue length is fixed. For constant queue lengths, a heavy vehicle usually takes the space of two passenger cars. Although heavy vehicles need more time to start up, the start-up time could be slower than the total time for two passenger cars, because a heavy vehicle could be approximated as two passenger cars attached together, meaning that the imaginary second car on the

back needs zero time to react and start up. In fact, the results show that a heavy vehicle is faster to discharge than its passenger car size-equivalent is.

On the other hand, if the survey point were not fixed but the total number of vehicles were, then the occurrence of heavy vehicles should have caused more dissipation time, which is consistent with the model result that the coefficient of "# of heavy vehicles" is greater than that of either "No. of passenger cars" or "No. of SUV, pickups" (although it is less significant). In fact, the three coefficient estimates rank logically according to the numerical magnitudes, meaning that bigger vehicles increase dissipation time.

The coefficient estimate for "Left Turn?" is negative, which seems counterintuitive, as left turn speeds are usually perceived as being lower than through speeds. With a relatively high t-statistic (-1.5757), one possible explanation is that drivers usually sense a shorter green light duration for a left turn than for a through lane. Vehicles in the left turn lane may try to closely follow the previous car in order to pass the stop line before the yellow light. The consequence is that, with everything else being equal, the queue in a left-turn lane discharges faster than that in a through lane. As shown in the next part, this result does not hold for all the models.

The positive coefficient for "Upgrade?" is intuitive, as upgrade creates more rolling resistance to vehicle start-up, and vehicle gaps tend to be larger for safety, both of which increase the queue dissipation time.

The coefficient estimate for "Morning?" is difficult to explain. This variable could be dropped because the t-statistic is not high. But the t statistic is not very low, either, which might imply the requirement for more or better data to further investigate the effect. Although it is a hypothesis, a tentative explanation is that morning commuters are generally busier in their vehicles, whether they are eating muffins, drinking coffee, still feeling sleepy, or thinking about the to-do-list for the day, which means they might need more reaction time to catch up with the previous vehicles. This could be a hypothesis for further study. However, the results are contrary to what known about the morning peak versus the PM peak, which is that evening peak volumes are more irregular and result in higher delays than do morning peak volumes.

Other Functional Forms

Table 3 reports the results of various functional form regressions (with the same dependent variable--t_2). The first observation is that nonlinear functional forms do not have a much higher R^2 or adjusted R^2 than linear forms. It should be noted that it is incorrect to compare R^2 across different functional forms because the total sum of squares of the transformed data (logged) is different than the total sum of squares for a linear function (Greene, 2003, page 344). However, even though most of the nonlinear functional forms have a higher number of variables than does the linear form, their explanatory power does not seem to be greater, which means a simple linear function can perform well.

Another important observation is that the signs of some variables are consistent across all functional forms. These variables include namely # of passenger cars (PC), upgrade binary variable (DUP), and the binary variable of "Morning?" (DMN). The signs of some other variables are not consistent e.g., the sign of the "Left Turn?" binary variable (DLT) is positive for the Trans-Log function and negative for all others. The coefficients for PC, for # of SUVs (SUV), and for # of heavy vehicles (HV) are usually significant and almost always consistent in terms of sign in all the regression models. The exceptions are models that have another term for the vehicle-mix variable (like HV^2 along with HV).

Contrary to common delay estimation models (Fambro and Rouphail, 1997; Heidemann, 1994), regression analysis shows that queue dissipation time is linearly related to the number of vehicles in the queue and that the quadratic and cubic coefficients for this variable are insignificant, which can be a result of the limited number of studied intersections and/or unsaturated conditions.

Some of the Trans-Log function's coefficients show the interaction effects between some of these variables and how important these interactions can be (terms like DMN. lnPC). For instance, the negative sign of DMN.lnPC shows that the sign and magnitude of the lnPC coefficient should be modified for the morning peak. Trans-Log functions can reveal some of these types of interactions between the variables and even though a linear function might be the final choice because of simplicity, Trans-Log functions can provide important insights of these types (Rouhani et al., 2013).

TABLE3 All models' specifications

Type of function	R^2 (adj. R^2)	Coefficients of variables (t statistics)												
Linear	0.65 (0.62)	Intercept	PC	SUV	HV	DLT	DUP	DMN						
		1.69 (0.73)	0.82 (2.40)	0.97 (2.70)	1.20 (1.64)	-0.70 (-1.58)	1.74 (2.40)	0.45 (1.04)						
Quadratic	0.66 (0.60)	Intercept	PC	SUV	HV	DLT	DUP	DMN	PC^2	SUV^2	HV^2			
		0.10 (0.04)	1.38 (1.83)	1.26 (2.14)	0.50 (0.40)	-0.72 (-1.54)	1.63 (2.19)	0.42 (0.96)	-0.06 (-0.76)	-0.03 (-0.51)	0.62 (0.84)			
Cubic	0.68 (0.62)	Intercept	PC	SUV	HV	DLT	DUP	DMN	PC^2	SUV^2	HV^2	PC^4	SUV^4	HV^4
		0.33 (0.11)	1.58 (1.11)	-0.03 (-0.04)	1.15 (1.36)	-0.64 (-1.38)	1.60 (2.17)	0.35 (0.81)	-0.08 (-0.27)	0.35 (1.75)	drop	0.0001 (0.04)	-0.005 (-2.04)	0.11 (1.00)
Cobb-Douglas*	0.64 (0.60)	Intercept	lnPC	lnSUV	lnHV	DLT	DUP	DMN						
		1.33 (4.02)	0.24 (1.71)	0.22 (2.05)	0.11 (0.93)	-0.15 (-3.20)	0.30 (3.96)	0.06 (1.20)						
Trans-Log*,**	0.73 (0.66)	Intercept	lnPC	lnSUV	lnHV	DLT	DUP	DMN	$lnPC^2$	$lnSUV^2$	$lnHV^2$	lnPC.lnHV	DMN.lnPC	DMN.lnSUV
		-0.22 (-0.43)	1.00 (4.16)	drop	drop	0.43 (1.72)	0.18 (2.23)	2.26 (3.18)	drop	0.31 (4.41)	0.32 (1.23)	0.11 (0.95)	-0.86 (-2.95)	-0.62 (-2.85)

dependent variable is in Logs (Ln(t2)).

** Not all the coefficient and variables of the Trans-Log model are reported because of brevity.

*** To solve the problem of zero values for the log functions, one is added to all the logged variables.

CONCLUSIONS

This study was an attempt to identify the important but usually ignored parameters for estimating queue dissipation time (speed) at signalized intersections. The queue formation/dissipation data for several intersections in Davis, California, U.S. were analyzed using regression models. The modeling framework has some limitations. First, in its current format it cannot predict the dynamics of queue formation and dissipation, which is required for some traffic simulation and/or dynamic assignment models. However, by collecting additional data, it is possible to modify the modeling for this use. Second, some of the important variables are still missing due to the lack of data (e.g., width of the lanes and signal control strategies are important variables missing). Also, some of the variables can be used in a more precise format: instead of a binary variable for upgrade, slope of the direction could be measured and used. Lastly, the stopping space between vehicles can have a significant impact. Basically, drivers are different in terms of their driving behavior and safety measures, and incorporating these factors could improve the model although the data collection and its application can be hard to accomplish. Nevertheless, the models provide interesting results.

First, dissipation speed does not vary purely by location, meaning that even though intersections are different in design, control strategies, and etc., queue dissipation time is not significantly affected by these locational factors. Second, a heavy vehicle is faster to discharge than its passenger car size-equivalent is because a heavy vehicle could be approximated as two passenger cars attached together. This result is due to using a fixed queue length. Third, a left-turn queue discharges faster than a through queue (although the sign of DLT is not consistent, e.g. in the Trans-Log function, left-turn queue discharges slower) perhaps because drivers usually sense a shorter green light duration for a left turn than for a through lane and try to closely follow the car ahead in order to pass the stop line before the yellow light. Fourth, morning queues generally discharge more slowly (the binary variable DMN is consistent and significant across all functional forms). A tentative explanation is that morning commuters are generally busier in their vehicles, still feeling sleepy, or thinking about their to-do-list for the day, so they might need more reaction time to catch up with the queue. Fifth, contrary to common delay estimation models, regression analysis shows that queue dissipation time is linearly related to the number of vehicles rather than having quadratic or other functional forms, and the linear function performs well both in terms of explanatory power (R^2) and consistency of signs of the included variables.

The future studies can add some of the suggested missing variables, especially behavior-related variables, to improve the explanatory power of the models. Also, a multi criteria analysis (Madani et al., 2011) along with heuristic optimization techniques (Rouhani et al., 2010) can be used to improve the modeling. The integration of this modeling or more advanced versions of it into traffic assignment/simulation models requires further study involving the dynamics of queue dissipation and formation.

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