Scope and Scale Economies for Multi-Product Farms: 
Firm-Level Panel Data Analysis

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Abstract

This study used the flexible fixed cost quadratic function to analyze the cost structure of multi-product farms using farm-level panel data. The robustness of estimated parameters are examined using four panel data estimators. Results suggest that scale economies remain significant in Illinois farming. An increase in soybean acreage reduces the marginal cost of producing corn. Firm-specific effects, that indicate the levels of fixed costs, are found to be positive and significant.

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1. Introduction

The growth in farm size in the U.S. agriculture suggests that the average farm is exploiting its cost economies. That is, long-run average costs fall with increasing crop acreage due to the effects of declining per unit fixed cost. Without an understanding of size and scope economies, it is difficult to identify cost economies that may be achieved by multi-product farms. Increase in farm size may lead to cost economies, but the presence of scope economies in diversified versus specialized farms may tend to lower costs in terms of comparable level of output.

According to Baumol, Panzar and Willig (1982) there are two conditions that may lead to economies of scope in multi-product farms: the cost complementarity between two crops and/or the sharing or joint utilization of quasi-fixed inputs by crops. Cost complementarity may arise when adding a new crop into production reduces the marginal or average incremental cost of producing another crop. Shareable quasi-fixed inputs such as land, labor, management, buildings, tractors, tillage, planting, and harvesting machinery and equipment are also sources of economies of scope. They generate fixed costs such as depreciation, interest, taxes, and insurance that may help reduce costs per unit of output as the output level is increased. To ignore impacts of quasi-fixed input costs on farm cost structure analysis may result in the false estimation of scope and size economies.

Previous studies (Hallam, 1993) on cost structure of farms did not isolate the effects of quasi-fixed costs to scale and scope economies. They computed cost statistics that do not provide information on significant benefits from sharing fixed costs. These studies also aggregated diverse crop outputs into a single measure of outputs and determine whether there are scale economies. The presence of scale economies for an aggregate measure of crop outputs does not imply the presence of scale economies for any of components of the aggregate measure of crop output.

Beyond that, aggregation avoids some of the major questions that need to be answered: Are scale economies specific to corn or soybeans, or do they act jointly among a set of crops?
Does a set of specialized farms (corn or soybean farms) or a single diversified corn-soybean farm efficiently produce the set of crops more? Is the cost function subadditive for corn and soybeans production? How production costs of corn are affected by change in the acreage of soybeans? Despite the theoretical importance of these questions, the empirical evidence has been limited due primarily to the unavailability of sufficient data on the stand-alone cost of producing each crop in isolation. Farmers, policy makers and researchers are interested in knowing whether scale economies are specific to corn or soybeans. The following section presents the theoretical framework of the study. The next section discusses panel data estimators. The fourth section describes the construction of farm-level panel data. The fifth section presents the empirical results. The last section discusses policy implications and provides a conclusion to the research.

2. Conceptual Framework

Multi-product Cost Functions

Assuming that the production technology of multi-product farms is described by a product transformation function $T(Q_{fit}, X_{fit}, Z_{fit})$ that satisfies all conditions for the existence of a unique dual Cost function. For the $f$th farm in time period $t$, we define a vector of outputs $i (Q_{fit})$, vectors of variable input $i$ quantities and prices ($X_{fit}$ and $W_{fit}$), and vectors of quasi-fixed input $i$ quantities $Z_{fit}$ (and their respective prices, $R_{fit}$). If farms are assumed to be in short-run total cost (SRTC) function is defined as:

$$C_{short}^{fit}(Q_{fit}, W_{fit}, Z_{fit}) = R_{fit} Z_{fit} + \min_{X}
\{W'X : T(Q_{fit}, X_{fit}, Z_{fit})\} = R_{fit} Z_{fit} + C_{var}^{fit}(Q_{fit}, W_{fit}, Z_{fit})$$

In this case, multi-product farms are assumed to minimize the joint short-run total cost function with respect to the variable input $X_{fit}$ conditional on the levels of quasi-fixed inputs $Z_{fit}$ subject to
a product transformation function \( T(Q_{fit}, X_{fit}, Z_{fit}) \). The optimal solution yields the dual short-run variable cost (SRVC) function, \( C_{var}^{fit}(Q_{fit}, W_{fit}, Z_{fit}) \) that is consistent with the economic theory.

In the long run, when a farmer buys or rents additional land and machinery, he/she alters the size and scope of the farm enterprises. The long-run equilibrium values of fixed inputs are optimized through the minimization of the SRVC function with respect to fixed inputs, \( Z_{fit} \):

\[
(2) \quad \frac{\partial C_{short}^{fit}(.)}{\partial Z_{fit}} = R_{fit}' + \frac{\partial C_{var}^{fit}(.)}{\partial Z_{fit}} = 0 \iff R_{fit}' = -\frac{\partial C_{var}^{fit}(.)}{\partial Z_{fit}}
\]

provided that the variable cost function is decreasing and convex in \( Z_{fit} \). \( R_{fit} \) is the reduction in variable costs incurred by buying or renting additional unit of \( Z_{fit} \). The above partial derivative gives the optimal level of the fixed input, \( Z_{fit}^* = F(Q_{fit}, W_{fit}, R_{fit}) \). By substituting the value of \( Z_{fit}^* \) in the short-run cost function, the long-run multi-product long-run total cost function is derived as:

\[
(3) \quad C_{long}^{fit}(Q_{fit}, W_{fit}, R_{fit}) = -\frac{\partial C_{var}^{fit}(.)}{\partial Z_{fit}} * F(Q_{fit}, W_{fit}, R_{fit}) + C_{var}^{fit}(Q_{fit}, W_{fit}, F(Q_{fit}, W_{fit}, R_{fit})).
\]

The study of economies of size means an examination of the shape of the long-run average cost (LRAC) curve that reflects the presence of economies and diseconomies of scale. For the multi-product farm, the behavior of costs depends not only on the size of the output but also on crop mix.

Knowing that technological interactions that lead to scope economies are revealed in the cost side of profit function, the use of cost function seems to be a better descriptive way of estimating cost structure of farms. As discussed by Stefanou and Madden (1988) and applied by Weaver (1983), profit function may be useful in analyzing the structure of production but it does not provide new information about the size economies. The standard translog cost (TLC) function has been the most commonly applied functional form in multi-product farms cost structure analysis (Akridge and Hertel, 1986; Schroeder, 1992; Gallagher, Thraen, and Schnitkey, 1993). Its well-known disadvantage is its inability of modeling accurately the effects of specialization (Roller, 1990). Improvement of the TLC functional form has been made by substituting small positive values for zero outputs or by using Box-Cox transformation in computing costs measures.
(Moschini, 1988). These two modifications have yielded quite different scope economy results depending on how close the substituted positive values are to zero. Since the sample of this study contains farms that did not produce continuously soybeans from 1984 to 1994, the use of the TLC or its hybrid may provide biased estimates and lead to different policy conclusions if these farms are not accounted for in the empirical analysis.

**Generalized Quadratic Cost Model**

As a basis for developing short-run and long-run total cost functions of multi-product farms, the generalized quadratic cost (GQC) model is used (Featherstone and Moss, 1994):

\[
C_{ft} = \alpha_0 + \sum \alpha_i Q_{fit} + \sum \beta_i W_{fit} + .5(\sum \sum \alpha_{ij} Q_{fit} Q_{fjt} + \sum \sum \beta_{ij} W_{fit} W_{fjt}) + \sum \sum \gamma_{ij} W_{fit} Q_{fjt}
\]

Assuming cost minimization and using Shephard's lemma, a set of compensated input demand equations is derived:

\[
\frac{\partial C_{ft}}{\partial W_{fit}} = \beta_i + \sum \beta_{ij} W_{fjt} + \sum \gamma_{ij} Q_{fjt}
\]

Symmetry is imposed by restricting \( \alpha_{ij} = \alpha_{ji}, \beta_{ij} = \beta_{ji}, \) and \( \gamma_{ij} = \gamma_{ji} \) in the estimation procedure.

**Flexible Fixed Cost Quadratic Model**

Input prices are important in estimating cost functions. Due to the homogeneity of the farms location and little variation in input markets, input prices are not included in the model estimation (Hornbaker, Dixon and Sonka, 1989; Grosskopf, Hayes and Yaisawarng, 1992). Although the use of inputs differs across Illinois cash-grain farms, the sets of variable inputs (fertilizer, pesticides and seed) used by farmers are quite homogeneous. Therefore, the GQC sacrifices the linear homogeneity property of the cost function with respect to input prices. In addition, the use of a single intercept term (\( \alpha_0 \)) in quadratic cost specification is too restrictive. It
does not take into account product-specific stand-alone or incremental fixed costs and reduces the combined effects of the quasi-fixed costs (constant terms) with the quadratic interaction term of output on scope economies. The basic specification of the empirical model is a flexible fixed cost quadratic (hereafter FFCQ) function suggested by Lau (1974), embellished by Baumol, Panzar and Willig (1982), and applied by Mayo (1984). By assuming that farms are in the long-run equilibrium and denoting $D_i$ and $D_{ij}$ dummy variables that will be equal, respectively, to one for farms producing only one crop (corn or soybeans) and for farms producing two crops (corn and soybeans), and zero otherwise. The long-run FFCQ model may be written as follows:

$$C_{ft} = \alpha_0 + \sum \alpha_{0;i} D_i + \sum \alpha_{0;ij} D_{ij} + \sum \alpha_i Q_{fit} + .5 \sum \sum \alpha_{ij} Q_{fit} Q_{fjt} + e_{ft}$$

where

- $C_{ft}$ = Total cost of farm $f$ in year $t$;
- $Q_{fit}$ = Quantity of crop $i$ produced by farm $f$ in year $t$;
- $D_i$ = Dummy variable for farm $f$ that produces only crop $i$;
- $D_{ij}$ = Dummy variable for farm $f$ that produces crops $i$ and $j$;
- $e_{ft}$ = Residual error term for farm $f$ in year $t$.

Based on the estimated parameters of the FFCQ function, the cost structure measures such as scale, scope and product-specific scale, marginal and incremental costs are derived.

**Product-Specific Scale Economies**

Let assume that $S$ a set of two crops (corn and soybeans), $T$ a subset of one crop (e.g. corn) and $S-T$ another subset of one crop (e.g. soybeans). The product-specific scale economies of soybeans ($SCALES_{S-T}$) give information about changes in cost as the output of soybeans expands:

$$SCALES_{S-T} = \frac{[C(Q_S) - C(Q_{S-T})]}{[(Q_{S-T}) * \frac{\partial C(Q_S)}{\partial Q_{S-T}}]} = \frac{AIC_{S-T} * (MC_{S-T})^{-1}}{AIC_{S-T} * (MC_{S-T})^{-1}}$$


It is based on concepts of product-specific average increment cost (AICS-T) and marginal cost (MC_{S,T}), defined as:

\begin{equation}
AICS-T = \frac{C(Q_S) - C(Q_{S,T})}{(Q_{S,T})}
\end{equation}

\begin{equation}
MC_{S,T} = \frac{\partial C(Q_S)}{\partial Q_{S,T}}
\end{equation}

where AICS-T, C(Q_S) and C(Q_{S,T}) are, respectively, product-specific average incremental cost, joint cost of producing corn and soybeans and cost of producing soybeans. When \text{SCALE}_{S,T} is greater than 1 (AICS-T > MC_{S,T}), the average cost of producing soybeans falls as soybean output, Q_{S,T} increases reflecting economies of scale for soybeans. Notice that the average incremental cost of producing soybeans includes any product-specific fixed costs associated with the production of soybeans and depends on the assumed production of corn, Q_T. The AICS-T is decomposed into average incremental fixed cost (AIFC_{S,T}) and average incremental variable cost (AIVC_{S,T})

**Scope Economies**

The behavior of costs is also observed when the scope of farms changes. If scope economies exist, the cost of the joint production, C(Q_S) is less than the sum of costs of separately produced individual crops or subset of crops, C(Q_T) + C(Q_{S,T}):

\begin{equation}
C(Q_T) + C(Q_{S,T}) > C(Q_S)
\end{equation}

The degree of SCOPE, the percentage increase in costs from specialized production, is defined as:

\begin{equation}
SCOPE^O = \left[ \frac{C(Q_T) + C(Q_{S,T}) - C(Q_S)}{C(Q_S)} \right]^{-1}
\end{equation}

If SCOPE^O greater than 0, then scope economies exist and farms can be more cost efficient by diversifying cropping activities.
**Product-Specific Cost Complementarity and Quasi-Fixed Costs**

Dividing the total joint cost function, \( C(Q_S) = F_C + C_{var}(Q_S) \), into quasi-fixed input costs, \( F(S) \) and variable input costs, \( C_{var}(Q_S) \), two conditions leading to scope economies are identified. The first condition is the existence of cost complementarity (COMP) or jointness in variable input (e.g. fertilizer) between two crops \( Q_T \) (e.g., corn) and \( Q_{S-T} \) (e.g., soybeans). That is, the marginal costs of producing two crops are dependent. This condition may be defined as:

\[
\text{COMP} = \frac{\partial^2 C(Q_S)}{\partial Q_T \partial Q_{S-T}} = \frac{\partial^2 C_{var}(Q_S)}{\partial Q_T \partial Q_{S-T}} \iff \frac{\partial MCT}{\partial Q_{S-T}} = \frac{\partial AIC_T}{\partial Q_{S-T}}
\]

If COMP is less than zero, there are gains in diversification or economies of scope.

The second condition is the presence of product-specific fixed costs that can overcome the absence of cost complementarities. It is expressed as:

\[
\text{FC}_T + \text{FC}_{S-T} > \text{FC}_S
\]

As long as the fixed cost of producing all or a subset of products jointly (FC\(_S\)) is less than the sum of the fixed cost of producing two subsets of products (FC\(_T\) + FC\(_{S-T}\)) in different farms, two disjoint subsets of crops share quasi-fixed inputs cost function that is subadditive. According to Pulley and Humphrey (1993), the FFCQ function has the ability to provide information on the decomposition of scope economies into fixed-cost (SCOPE\(_{FC}\)) and variable-cost (SCOPE\(_{VC}\) or COMP) components. Extending the work of Baumol, Panzar, and Willig (1982), Gorman (1985) shows that, even when SCOPE\(_{VC}\) is equal to 0, the existence of subadditive product-specific fixed costs (SCOPE\(_{FC}\)) is a sufficient condition for presence of economies of scope. If corn and soybeans share variable inputs, the SCOPE\(_{VC}\) (\(\alpha_{ij}\)) coefficients would be expected to be negative. There is no reason to believe that farm's costs of producing corn are unaffected by the nature and scale of soybeans. Shumway, Pope and Nash (1984) stated that allocatable quasi-fixed inputs cause economies of scope when the marginal allocation of variable inputs depends upon the allocation of the fixed input and generate product-specific fixed costs.
Multi-product Scale Economies

Multi-product scale economies (SCALE) measure the cost implications of varying all crops (corn and soybeans) simultaneously while holding the mix of crops constant. It is defined as:

\[
\text{SCALE} = \frac{C(S)}{[MC_{S,T} \times Q_{S,T}} + (MC_{T} \times Q_{T})]
\]

Multi-product scale economies (diseconomies) exist if SCALE is greater (less) than unity. It is proved by Fernandez-Cornejo et al. (1992) that overall scale economies result from product-specific scale and/or scope economies. That is, strong scope economies may lead to overall scale economies that can be greater than one even if there are constant or decreasing product-specific scale economies. Declining average incremental or marginal costs \((\alpha_{ii} < 0)\) and cost complementarities \((\alpha_{ij} < 0)\) are conditions needed for overall multi-product scale economies.

3. Panel Data Model Estimators

Since panel data are used, unobserved heterogeneity among farms has to be accounted when using the OLS regression model. The robustness of estimated parameters are examined using four estimators: ordinary least squares (OLS), between-firm (BF), least-squares dummy variable (LSDV), and generalized least-squares (GLS) estimators.

Ordinary Least Squares Estimator

Consider the following linear regression model:

\[
TC_{ft} = G_{f}(Q_{fit}, \beta) + \epsilon_{ft}
\]

For the fth farm at year t, \(TC_{ft}\) is the total cost; \(G_{f}\) is the production technology; \(Q_{fit}\) is the vector of product outputs; \(\beta\) is a vector of \(k\) unknown production parameters; \(\epsilon_{ft}\) is the error term which
represents the effects of the omitted variables that are specific to n farm and T years. The ordinary least squares (OLS) estimator, $\beta$, is the value of $\beta$ that minimizes the sum of squared errors.

**Between-Firm Estimator**

The standard approach to obtaining the between-firms results is to regress the firm-specific means of the dependent variable on the firm-specific means of independent variables. The between-firm estimator is generally expressed as:

$$ (16) \quad \tilde{TC}_n = \{G_f(Q_{ft}, \beta)\} + \alpha + u_f + \varepsilon_f $$

**Least-Squares Dummy Variable Estimator**

The LSDV estimator is generally expressed as:

$$ (17) \quad TC_{ft} = G_f(Q_{ft}, \beta) + \alpha_f + e_{ft} $$

$\varepsilon_{ft}$ is decomposed into $\alpha_f$ and $e_{ft}$. $\alpha_f$ is farm-specific fixed-effect representing the cost of an unmeasured quasi-fixed input; $e_{ft}$, the stochastic costs of inputs that can not be controlled by any farm (e.g., weather, diseases). They are independently and identically distributed (i.i.d.) across farms and years and uncorrelated with the crop outputs. Product-specific quasi-fixed costs of machinery capture differences in technology between farm groups, which produce different crop mixes. They are assumed to be correlated with the crop outputs and their mixes.

**Generalized Least Squares Estimator**

In the case of the GLS or random effects (RE) model, the $\alpha_f$ is decomposed into $\alpha$ and $u_f$. $\alpha_f$ is assumed to be random draws from a distribution with mean $\alpha$. $u_f$ are i.i.d. $N(0, \sigma_u^2)$ and are assumed to be uncorrelated with the $e_{ft}$ and the outputs $Q_{ft}$. The RE model may be written as

$$ (18) \quad TC_{ft} = G_f(Q_{ft}, \beta) + \alpha_f + u_f + e_{ft} $$
The generalized least square estimator is obtained by applying OLS to the following equation:

\[(19) \quad TC_{fg} - \theta TC_f = (1 - \theta)\alpha + \{G_f (Q_{fit}, \beta) - \theta G_f (Q_{fit}, \beta)\} + e_f\]

where the quasi-deviation parameter is given by:

\[(20) \quad \theta = 1 - (\sigma^2_e/(T\sigma^2_u - \sigma^2_e)^{1/2})\]

The unknown variance components $\sigma^2_u$ (common noise) and $\sigma^2_e$ (farm-specific variance) have to be estimated first in order to compute $\theta$.

**4. Farm-Level Panel Data**

To estimate short-run variable and long-run total cost functions, corn farms and corn-soybean farms that participated in the Illinois Farm Business Farm Management (IFBFM) Association from 1984 to 1994 are used. The IBFM Association farms are highly representative of commercial agriculture. Their records are primarily year-end financial statements for individual farms. They are reliable and consistent across farms. They contain cross-sectional and time-series data on acreage, yields, prices, and on aggregate expenditures on variable and quasi-fixed inputs. In this study, variable inputs expenses include expenses on fertilizer, pesticide, seed, drying and storage and miscellaneous expenses. Quasi-fixed expenses involve machinery depreciation and repair expenses, and insurance expenses. Any econometric model, with total cost as dependent variable, that includes time series data, involves the problem of how to deal with the general level of cost. In this study, this difficulty is handled by deflating the total cost of producing crops, with an indicator of the price level such as the consumer price index.

The sampled farms have soil productivity rating and tillable acreage greater than or equal to 60 and 50, respectively. Table 1 presents summary statistics for econometric model variables. The sample contains 185 farms and has a mean tillable acreage of 671.37 acres. Farm size ranges from
102 to 2450 tillable acres. The average tillable acres of corn, soybean and set-aside for the observed period are, respectively, 326.1, 301.6 and 42.

In addition, the IFBFM data set reports the total variable costs according to input but does not allocate costs to the individual crops. This allocation is done by using the between-firm (BF) regression estimator with variable or total cost estimated as function of crop quantities. As opposed to considering all quasi-fixed expenses, the econometric model focuses on machinery fixed costs. Also set-aside and crop acreage, yields, and prices are included in the IFBFM data. The number of set-aside acres for any farmer is a function of the corn acreage. There is a strong linear dependence between corn acreage and set-aside acreage (Hornbaker, Dixon, and Sonka, 1989).

5. Empirical Results

Econometric Predictions

After the variables to be included into the cost functions were defined, a functional form of the cost function was selected that explicitly expresses the relationship between variable or total costs and the quantities of crops output that are produced in a farm. Then, different parameters of quadratic total cost function were estimated using four estimators: OLS, BF, LSDV, and GLS estimators. Corn and soybeans outputs, as explanatory variables, enter the quadratic cost specifications in three distinct ways: linearly, in quadratic form, and as cross products. The first-order coefficients ($\alpha_i$) for corn and soybeans are expected to be positive. The second-order coefficients ($\alpha_{ii}$) for corn and soybeans are also expected to be positive. But, the cross product coefficient ($\alpha_{ij}$) of corn and soybeans is expected to be negative showing the cost complementarity between both products. The FFCQ function involves quasi-fixed costs represented by different dummy variables associated with $\alpha_{oi}$ and $\alpha_{oij}$, and the variable cost which is a quadratic function of product output levels. These parameters are assumed positive since they represent product-specific quasi-fixed costs.
**Cost Model Estimates and Goodness of Fit Measures**

Table 2 presents parameter estimates for the GQC model and FFCQ model using farm-level panel data. The OLS estimator is used to derive parameter estimates of the GQC model. The BF, LSDV and GLS estimators are used to derive parameter estimates of FFCQ model for the multi-product farms. Total and variable costs are estimated as dependent variables. Overall, the results are consistent with our prediction. All specifications of the cost function using any estimator have explanatory power with adjusted R-square values over 0.73. This indicates that goodness of fit of the quadratic cost models is reasonably strong and that the independent variables have significant power in capturing variations in variable as well as total cost. The F-statistics for model regressions reject the hypothesis that all parameters are zero at 0.01 level for each of the estimator models. However, high R-squares provide no guarantee that the estimated models make microeconomic sense. Rather, one needs to examine the signs and statistical significance of the coefficients.

All the models have significant (at least at 10%) and positive parameters on the linear and quadratic output terms. Positive first-order crop output coefficients \( \alpha_i \) indicate that the cost surface appears to satisfy monotonicity in output quantities (Tables 2 and 3). The presence of positive estimates for the second-order output coefficients \( \alpha_{ii} \) of corn and soybeans indicates that, the quadratic cost function for the average farm is convex. That is, the positive parameter gives rise to U-shaped average costs for corn-soybean farms along each output axis, which is consistent with classical economic theory. The negative cross-product coefficient \( \alpha_{ij} \) of corn and soybeans indicate that the marginal cost of corn is an increasing function of the quantity of corn produced and decreasing function of the quantity of soybeans produced. Similarly, the marginal cost of soybeans is an increasing function of the quantity of soybeans produced and decreasing function of the quantity of corn produced. This result is robust across models and estimators, and confirms the advantages of joint production of corn and soybeans, a feature that characterizes Illinois farming.
Dummy variables have been included to allow the intercept to vary between different farm
groups observed in the sample. Using the LSDV estimator, farm-group specific dummy variables
are positive as expected and statistically significant at 1 percent. These coefficients illustrate how
quasi-fixed costs of farm machinery vary with crop output mix. It suggests that the underlying
technology facing each farm group (corn farms and corn-soybean farms) may be different.

Choice of the Appropriate Study Panel Data Estimator

There are several issues that need to be considered when making an appropriate choice
between the four estimators. This choice is made in order to compute cost economies measures.
When the number of farms n is large and the number of years T is small, as is the case in this
study, using the random effect model will result in more efficient estimates (Hsiao, 1986).
However, the GLS estimator is based on the assumption that the quasi-fixed cost ($\alpha_{oi}$ and $\alpha_{oij}$)
are assumed to be uncorrelated with crop output ($Q_{it}$). It may be incorrect to assume that farm
group or crop-specific machinery fixed costs are independent of the crop mixes since their
magnitude may vary depending upon which set of crops (only corn or corn and soybeans) is
being produced. Parameters’ signs of the four estimators are similar.

Table 1 shows the relative efficiency of estimators using degree of freedom, root-mean-
square-error (RMSE), adjusted $R^2$, and F-Value. The between-firm estimator is chosen to
compute measures of economies of scale and scope for the average farm for several reasons: (1)
The between-firm estimator results in the lowest RMSE and the highest adjusted $R^2$; (2) The
between-firm estimator uses cross-sectional data to estimate long-run cost statistics (Baltagi and
Raj, 1992); (3) The shape of the long-run average cost curve by size and scope of operation is
important in farming; (4) Farmers are also interested in how cost varies over alternative sizes and
scopes of operation, to indicate whether they will be a competitive advantage or disadvantage
relative to farmers with other sizes and scope of operation; and (6) With the between-firm
estimator, the problem of autocorrelation present in the time-series data is avoided.
Economies of Scale and Scope for the Average Multi-product Farm

Table 3 provides estimated derivative of marginal costs of corn and soybeans and coefficients of dummy variables for the average farm firm. Diagonal elements give information on the curvature of marginal cost curves for corn and soybeans, respectively. Positive values of diagonal elements suggest that marginal cost curves are increasing. Increasing marginal costs are consistent with decreasing returns to scale. Statistically significant negative off-diagonal elements provide evidence of a cost complementarity between any of two crops. This result suggests that corn or soybean farms (single-crop farm) would be at a cost disadvantage compared with diversified corn-soybean farms because of their inability to capture scope economies in variable input. This result reinforces the presence of jointness in the production of corn and soybeans (the marginal cost of one crop decreases as the output level of the other crop increases). Producing both corn and soybeans in the same farm results in total cost saving relative to producing the same quantities in two separate farm firms. Parameter estimates of dummy variables indicate that there may be significant differences in quasi-fixed costs structure of each farm group. In other words, the technology of a firm that produces corn or soybeans separately may be different than one that produces jointly corn and soybeans. If the corn farm suddenly decides to produce soybeans, it will have different product-specific fixed costs captured by a dummy variable. Results show that the product-specific quasi-fixed costs of producing corn and corn and soybeans jointly are $13,684 and $19,557, respectively. The derived incremental fixed cost of adding soybeans in a corn farm is $5,873.

Table 4 reports estimates of scale, scope and product-specific economies of scale at 1984-1994 production levels. Using total cost or variable cost as a dependent variable, the figures given in this Table suggest that there exist multi-product economies of scale in each year. The scale economies range from 1.002 to 1.030 with an average of 1.019 and 1.016 using total cost and variable cost, respectively. The existence of multi-product economies of scale in average corn-
soybean farms suggest a proportional increase in production of corn and soybeans simultaneously while holding the mix of corn and soybeans constant would entail a less than proportional increase in total and variable costs.

The existence of overall scale economies requires the presence of either scope economies or product specific scale economies. It is clear that scope economies serve to magnify the effects of product-specific economies of scale. All values of scope economies are positive, implying the presence of economies of scope. If the average farm combines the production of corn and soybeans, it can have a cost saving of 26.7 percent in total cost and 30.2 percent in variable cost, as given by degrees of 0.267 and 0.302 for overall scope economies.

Derivatives of marginal costs of corn and soybeans are positive (Tables 2 and 3) and indicative of product-specific diseconomies of scale for corn and soybeans. Table 4 also provides estimates of product-specific scale economies that measure the total or variable cost impact of increasing production of corn holding constant soybean output level, and vice-versa. Table 5 shows both crops, corn and soybeans, display product-specific diseconomies of scale. Marginal costs of corn and soybeans are greater than estimated average incremental costs throughout the observed period. This also implies diseconomies of scale associated with producing corn and soybeans separately. Soybeans has lower product-specific diseconomies of scale than corn. This is consistent with Ojemankinde, Lange and Zacharias (1989) who found that soybeans exhibited higher product-returns to scale than rice in southwest Louisiana.

**Economies of Scale and Scope at Large versus Small Multi-product Farms**

Four farm size categories are considered here: very small farms with no more than 300 tillable acres, small farms with between 300 and 600 tillable acres, medium farms with between 600 and 900 tillable acres, and large firms with more than 900 tillable acres. The cost economies statistics reported in Tables 6 and 7 are evaluated at the mean values of the exogenous variables within each size range. These mean values are substituted into equations to compute estimates of different cost statistics. These statistics correspond to the average farm of each size class. Since
output mix varies among the farm size classes, a comparison across size classes is a comparison of changes in scale and scope. Corn-soybean farms are also characterized by increasing returns to scale in all four-size classes when total cost or variable cost is the dependent variable. Scale economies increase as the average size of farm class increases. Large corn-soybean farms show a total and variable cost advantage compared to very small farms. There is also a decrease in corn- and soybeans-specific scale economies from the very small farms to the large farms. Scope economies are smallest at the two smallest size classes. Increasing scale and scope economies mean that large, diversified corn-soybean farms are more cost efficient than small, specialized corn or soybean farms.

Tables 7 and 8 present estimated per acre marginal costs and average incremental costs of corn and soybeans by farm size. As the size of the average farm grows, the marginal costs of corn and soybeans decrease and increase, respectively. The average total and variable incremental costs of both crops decrease as the size of the average farm increases. This suggests that as farm size increase there is a decrease in product-specific economies of scale of both corn and soybeans. Therefore, it is less expensive to produce both crops in the same farm than in separate farms. Increasing economies of scope along with decreasing average incremental costs for corn and soybeans is a necessary and sufficient condition for sub-additivity in multi-product corn-soybean farms. There is no evidence that large farms in the sample exhaust the potential cost advantage of multi-product farms due to scale and scope economies.

Economies of Scale and Scope in terms of Total and Variable Costs

Economies of scale exist for total costs as well as for variable costs. Based on estimates of the period 1984-1994, the effect of scale is higher when considering total costs than when variable costs are considered for the average farm. Using farm size class results, the total cost function results also outperform the variable cost results. Otherwise, there are diseconomies of scale associated with corn and soybeans. During the study period 1984-1994, the effect of scope is lower when considering total costs than when variable costs are considered. These results
confirm the findings of Leathers (1992) that technological causes of jointness (cost complementarity between products) are important in agriculture. Finally, the existence of multi-product economies of scale suggests a proportional increase in corn and soybean production would entail a less than proportional increase in total and variable costs. In fact, it is precisely economies of scope that leads to the economic justification of predominance of corn-soybean farms in Illinois. This also justifies significant economies from rotating corn and soybeans. Fertilizer and pesticide costs rise for corn grown continuously on the same land. Yield decline for both continuous corn and soybean crops.

6. Policy Implications and Conclusions

In this study, the evidence about the existence of scope and scale economies at farming is derived using a multi-product function framework that explicitly disaggregates the crop output vector to take the heterogeneity of output. Results provide information that cannot be gained from an aggregate single-product analysis. Single-crop studies did not investigate the impacts of output mix and output level on variable or total cost of producing crops.

The existence of significant economies of scale and scope over a broad output range make it impossible for a number of small competitive farms to operate efficiently in Illinois agricultural market. Returns to scale remain significant for many of the largest farms in Illinois, making it impossible to identify a minimum efficient farm size. But these returns do not arise from the production of a particular crop (corn or soybeans). Corn-soybean farms are able to spread the quasi-fixed costs of machinery and equipment over corn and soybeans. The analysis also supports the notion that the cost functions of the largest multi-product farm firms examined are output-specific subadditive. In addition, they confirm the presence of multi-product economies of scale due to the existence of global economies of scope and cost complementarity between corn and soybeans. These production economies can be exploited by farms specialized only in single-crop production (corn or soybeans). These economies will arise from the joint usage of variable inputs
or quasi-fixed inputs. They are more fully employed throughout the growing season when corn and soybeans are grown in the same farm. This also suggests that the long-run configuration of Illinois agricultural industry is characterized by a sharp reduction in the number of farms.

There are two main explanations of a preference for crop diversification among farmers. First, a preference for some degree of crop diversification among risk-neutral farmers can be explained by the existence of complementarity between crops. Second, uncertainty of net returns explains a preference among risk-averse farmers for crop diversification (Mafoua, Hornbaker and Sherrick, 1996). This is consistent with Pope and Prescott (1980) who found that there is a relationship between farm size and diversification that indicates the trade-offs between risk reduction and possible economies of scale. This crop diversification represents a potential means of overcoming some of the negative side-effects of monoculture of corn such as pest problems and soil erosion. Through legume-based crop rotations, soil fertility can be enhanced as nitrogen is collected from the air and recycled through nitrogen-fixing soybeans. Thus, the needs for purchasing fertilizer are reduced. Further, crop rotations enables farmers to make fuller use of field space and growing season time. Even though this study has examined the cost structure of a sample of farms in Illinois, the quadratic cost models have general applicability. For further research, there are several issues that can be analyzed: First, the effects of livestock production on scale and scope measures in cash-grain farms need to be addressed. Second, insight can be obtained from examining the cost structure in a dynamic framework and compare static and dynamic measures of production economies (Fernandez-Cornejo et al., 1992). Third, the ability of different functional forms (generalized translog or Leontief functions) to reveal the cost structure of the average farm may be analyzed using the same body of panel data. (Zhu, Ellinger and Shumway, 1995).
<table>
<thead>
<tr>
<th>Year</th>
<th>Variable Cost</th>
<th>Fixed Cost</th>
<th>Total Cost</th>
<th>CPI</th>
<th>Corn</th>
<th>Set-Aside</th>
<th>Soybeans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>$</td>
<td>$</td>
<td></td>
<td>(acre)</td>
<td>(acre)</td>
<td>(acre)</td>
</tr>
<tr>
<td>1984</td>
<td>47,235</td>
<td>6,735</td>
<td>53,970</td>
<td>94</td>
<td>314.88</td>
<td>21.71</td>
<td>268.75</td>
</tr>
<tr>
<td>1985</td>
<td>46,449</td>
<td>7,004</td>
<td>53,453</td>
<td>91</td>
<td>317.20</td>
<td>28.19</td>
<td>276.54</td>
</tr>
<tr>
<td>1986</td>
<td>45,982</td>
<td>7,400</td>
<td>53,382</td>
<td>86</td>
<td>287.45</td>
<td>67.91</td>
<td>275.46</td>
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<tr>
<td>1987</td>
<td>41,120</td>
<td>7,407</td>
<td>48,527</td>
<td>87</td>
<td>255.28</td>
<td>110.75</td>
<td>282.86</td>
</tr>
<tr>
<td>1988</td>
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<td>7,377</td>
<td>55,571</td>
<td>90</td>
<td>291.48</td>
<td>81.55</td>
<td>286.15</td>
</tr>
<tr>
<td>1989</td>
<td>50,711</td>
<td>8,194</td>
<td>58,906</td>
<td>95</td>
<td>334.79</td>
<td>35.45</td>
<td>305.75</td>
</tr>
<tr>
<td>1990</td>
<td>50,856</td>
<td>7,953</td>
<td>58,809</td>
<td>99</td>
<td>344.97</td>
<td>35.03</td>
<td>307.97</td>
</tr>
<tr>
<td>1991</td>
<td>53,386</td>
<td>7,740</td>
<td>61,126</td>
<td>100</td>
<td>350.44</td>
<td>25.48</td>
<td>317.63</td>
</tr>
<tr>
<td>1992</td>
<td>51,307</td>
<td>8,230</td>
<td>59,537</td>
<td>101</td>
<td>360.93</td>
<td>18.22</td>
<td>324.96</td>
</tr>
<tr>
<td>1993</td>
<td>53,005</td>
<td>8,739</td>
<td>61,744</td>
<td>103</td>
<td>348.07</td>
<td>37.51</td>
<td>331.31</td>
</tr>
<tr>
<td>1994</td>
<td>56,196</td>
<td>9,089</td>
<td>65,285</td>
<td>106</td>
<td>381.60</td>
<td>0.75</td>
<td>340.55</td>
</tr>
<tr>
<td>Mean</td>
<td>49,469.73</td>
<td>7,806.18</td>
<td>53,300.91</td>
<td>95.6</td>
<td>326.10</td>
<td>42.05</td>
<td>301.63</td>
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</table>
### Table 2: Parameter Estimates of the OLS, Between-Firm, LSDV and GLS Estimators

<table>
<thead>
<tr>
<th>Variable</th>
<th>Total Cost Function</th>
<th>Variable Cost Function</th>
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<tr>
<td></td>
<td>OLS</td>
<td>Between-Farm</td>
</tr>
<tr>
<td>Intercept</td>
<td>19012 (13.40)</td>
<td>-</td>
</tr>
<tr>
<td>Co_dum</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CB_dum</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Corn</td>
<td>0.70527 (10.78)</td>
<td>1.14591 (6.25)</td>
</tr>
<tr>
<td>Beans</td>
<td>1.51040 (6.31)</td>
<td>1.64585 (2.68)</td>
</tr>
<tr>
<td>Corn*Corn</td>
<td>4.7E-06 (4.14)</td>
<td>7.2E-06 (2.02)</td>
</tr>
<tr>
<td>Beans*Beans</td>
<td>7.9E-05 (6.11)</td>
<td>0.00011 (2.76)</td>
</tr>
<tr>
<td>Corn*Beans</td>
<td>-3.8E-05 (-5.26)</td>
<td>-6.2E-05 (-2.74)</td>
</tr>
</tbody>
</table>

---

Observations: 2035 185 2035 2035 185
RMSE: 20709.34 10635.14 20705 7678.81 9761.98
Adj. R-Square: 0.74 0.98 0.94 0.88 0.98
F-Value: 1168.95 2314.78 4743.54 2602.76 2234.31

T statistics in parentheses.
### Table 3: Product-Specific Quasi-Fixed Costs and Derivative of Marginal Costs

<table>
<thead>
<tr>
<th></th>
<th>Quasi-Fixed Costs</th>
<th>Derivative of Marginal Costs</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Corn</td>
</tr>
<tr>
<td>Corn</td>
<td>$ 13,684</td>
<td>8.1E-06</td>
</tr>
<tr>
<td>Soybeans</td>
<td>-</td>
<td>-6.2E-05</td>
</tr>
<tr>
<td>Corn-Soybeans</td>
<td>$ 19,557</td>
<td>-</td>
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</table>

### Table 4: Scale, Scope and Product-Specific Scale Economies by Time Period

<table>
<thead>
<tr>
<th>Year</th>
<th>Scale</th>
<th>Scope</th>
<th>Product-Specific Economies Corn</th>
<th>Scope</th>
<th>Product-Specific Economies Soybeans</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>1.017</td>
<td>0.211</td>
<td>0.866</td>
<td>0.612</td>
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<tr>
<td>1985</td>
<td>1.022</td>
<td>0.285</td>
<td>0.825</td>
<td>0.509</td>
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</tr>
<tr>
<td>1986</td>
<td>1.018</td>
<td>0.254</td>
<td>0.848</td>
<td>0.574</td>
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</tr>
<tr>
<td>1987</td>
<td>1.009</td>
<td>0.236</td>
<td>0.866</td>
<td>0.622</td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>1.002</td>
<td>0.149</td>
<td>0.922</td>
<td>0.749</td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>1.019</td>
<td>0.284</td>
<td>0.831</td>
<td>0.536</td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>1.020</td>
<td>0.282</td>
<td>0.831</td>
<td>0.531</td>
<td></td>
</tr>
<tr>
<td>1991</td>
<td>1.013</td>
<td>0.257</td>
<td>0.852</td>
<td>0.589</td>
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<tr>
<td>1992</td>
<td>1.027</td>
<td>0.321</td>
<td>0.798</td>
<td>0.412</td>
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<tr>
<td>1993</td>
<td>1.017</td>
<td>0.302</td>
<td>0.821</td>
<td>0.522</td>
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</tr>
<tr>
<td>1994</td>
<td>1.030</td>
<td>0.362</td>
<td>0.773</td>
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<tr>
<td>Mean</td>
<td>1.019</td>
<td>0.267</td>
<td>0.840</td>
<td>0.554</td>
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Table 5: Marginal, Average Incremental Variable and Total Costs by Time Period

<table>
<thead>
<tr>
<th>Year</th>
<th>Marginal Cost Corn ($/acre)</th>
<th>Marginal Cost Soybeans ($/acre)</th>
<th>Average Incremental Cost Corn ($/acre)</th>
<th>Average Incremental Cost Soybeans ($/acre)</th>
<th>Total Cost Corn ($/acre)</th>
<th>Total Cost Soybeans ($/acre)</th>
<th>Average Incremental Variable Cost Corn ($/acre)</th>
<th>Average Incremental Variable Cost Soybeans ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>131.09</td>
<td>55.90</td>
<td>130.19</td>
<td>36.04</td>
<td>108.29</td>
<td>34.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>154.03</td>
<td>81.53</td>
<td>145.54</td>
<td>44.20</td>
<td>119.28</td>
<td>41.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>143.23</td>
<td>78.27</td>
<td>140.33</td>
<td>47.47</td>
<td>114.93</td>
<td>44.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>129.46</td>
<td>78.27</td>
<td>131.28</td>
<td>44.10</td>
<td>106.57</td>
<td>52.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>69.62</td>
<td>53.43</td>
<td>75.89</td>
<td>41.34</td>
<td>62.36</td>
<td>40.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>131.66</td>
<td>82.10</td>
<td>126.23</td>
<td>46.66</td>
<td>102.64</td>
<td>43.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>131.18</td>
<td>77.92</td>
<td>125.45</td>
<td>43.96</td>
<td>102.32</td>
<td>41.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1991</td>
<td>105.23</td>
<td>78.39</td>
<td>104.47</td>
<td>48.69</td>
<td>84.72</td>
<td>46.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>166.11</td>
<td>68.86</td>
<td>149.89</td>
<td>30.63</td>
<td>123.29</td>
<td>27.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1993</td>
<td>126.87</td>
<td>85.59</td>
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<td>47.46</td>
<td>97.26</td>
<td>44.45</td>
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<tr>
<td>1994</td>
<td>173.25</td>
<td>74.25</td>
<td>150.68</td>
<td>27.80</td>
<td>123.25</td>
<td>24.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>132.22</td>
<td>75.20</td>
<td>128.09</td>
<td>44.10</td>
<td>104.71</td>
<td>41.57</td>
<td></td>
<td></td>
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</table>
Table 6. Scale, Scope and Product-Specific Scale Economies by Farm Size

<table>
<thead>
<tr>
<th>Farm Size Class (acres)</th>
<th>Farm Number</th>
<th>Total Cost Function</th>
<th>Variable Cost Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Corn</td>
<td>Soybeans</td>
</tr>
<tr>
<td>&lt; 300</td>
<td>14</td>
<td>1.006</td>
<td>0.079</td>
</tr>
<tr>
<td>300-600</td>
<td>76</td>
<td>1.012</td>
<td>0.177</td>
</tr>
<tr>
<td>600-900</td>
<td>59</td>
<td>1.020</td>
<td>0.291</td>
</tr>
<tr>
<td>&gt; 900</td>
<td>36</td>
<td>1.042</td>
<td>0.567</td>
</tr>
</tbody>
</table>

Table 7. Marginal and Average Incremental Total Costs by Farm Size

<table>
<thead>
<tr>
<th>Farm Size Class (acres)</th>
<th>Farm Number</th>
<th>Marginal Costs</th>
<th>Average Incremental Total Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Corn ($/acre)</td>
<td>Soybeans ($/acre)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Corn ($/acre)</td>
<td>Soybeans ($/acre)</td>
</tr>
<tr>
<td>&lt; 300</td>
<td>14</td>
<td>135.25</td>
<td>70.13</td>
</tr>
<tr>
<td>300-600</td>
<td>76</td>
<td>134.57</td>
<td>76.59</td>
</tr>
<tr>
<td>600-900</td>
<td>59</td>
<td>131.40</td>
<td>75.66</td>
</tr>
<tr>
<td>&gt; 900</td>
<td>36</td>
<td>130.49</td>
<td>76.99</td>
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</table>
### Table 8: Average Incremental Variable and Fixed Costs by Farm Size

<table>
<thead>
<tr>
<th>Farm Size Class (acres)</th>
<th>Farm Number</th>
<th>Average Incremental Variable Costs</th>
<th>Average Incremental Fixed Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Corn ($/acre)</td>
<td>Soybeans ($/acre)</td>
</tr>
<tr>
<td>&lt; 300</td>
<td>14</td>
<td>126.02</td>
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<tr>
<td>300-600</td>
<td>76</td>
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<td>53.89</td>
</tr>
<tr>
<td>600-900</td>
<td>59</td>
<td>101.65</td>
<td>39.30</td>
</tr>
<tr>
<td>&gt; 900</td>
<td>36</td>
<td>70.73</td>
<td>5.79</td>
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</table>

**References**


