Is the Irrigation Water Demand Really Convex?

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Abstract

The seasonal irrigation water demand under uncertainty, which lies at the core of this paper, is still very roughly known. We know, however, that irrigated agriculture accounts for a large proportion of water use, especially in many water-scarce areas. In this paper, we estimate the irrigation water demand, for various climatic conditions characterizing the distribution of the necessarily stochastic, demand functions under uncertainty. We use a dynamic programming model to represent the farmer's decision program under uncertainty. A crop-growth simulation model (EPIC-PHASE), provides the response function to the decisions taken and climatic events and is linked to a CRRA utility function representing the farmer's objective function. This model is used to generate the data allowing the estimation of irrigation water demand by a nonparametric procedure. An application to irrigation water demand is proposed in the South-West of France. We show that the estimated demand functions present four main areas: For very small quantities, where the farmer considers water as an essential input to crop growth, the demand is inelastic. The second area corresponds to mean quantities where the plant has reached a satisfactory level of growth; water is no more an essential input and is not yet a risk reducing input. The farmer is more responsive to change in water price. But, we find a third, non-intuitive, area for larger quantities where the water is a risk reducing input and the demand becomes inelastic again. The last area is classic, the water demand is obviously elastic for important total water quantities. This result is of great importance to analyze a regulation policy.
Keywords: Seasonal irrigation water demand, uncertainty, regulation policy, decision

1 Introduction

In many countries of the world, agricultural activities are risky, and irrigation affects these risks. Different types of risk exist: price risk, production risk, climatic risk; this last one seems to be the most important in agriculture. Stochastic weather conditions affect considerably the production of farmers and therefore their revenue. Thus, risk considerations may have important effects on irrigation decisions of risk-averse farmers because irrigation is a risk reducing input and a certain source of water unlike rainfall.

In France irrigated agriculture accounts for a large proportion of water use, especially in many water-scarce areas. This position can induce imbalance between water needs and resources that are likely to cause conflicts between different categories of users (rural, urban, industrial and other users). Agriculture is presented as the main cause of this desequilibrium. In these situations, the regulator may force all water users to pay the water at its real value. The application of this policy is inherent to the knowledge of the intertemporal water demand functions for each user. In France, farmers are charged for water. The fees are fixed at low levels, compared to the ones paid by the others consumers and it is well known that low prices induce over-consumption. Moreover the knowledge of farmers water consumption remains imprecise. In this context, estimating farmer's water demand over a season is difficult and several questions are still unanswered.

The problem of evaluating irrigation water demand is not recent and has become a growing field of research in the last few years. There is an important literature assessing how farmers react to changes in the price of water. Two approaches on irrigation water demand estimation exist. If data relating to observed water consumption exist then the authors use econometric models (Ogg and Gollehon [23]; Moore and Negri [20]; Moore *et al.* [21] and [22]; Hassine and Thomas [12]). However, in France as in many countries of the world, they are imprecise data on these consumptions. This point has induced the use of programming models for the estimation of the water demand. Demand estimates are derived from simulations of profit maximizing behavior. These modeling procedures require the use of standard mathematical techniques such as linear programming (Shunway [26]; Montginoul and Rieu [19]), or quadratic programming (Howitt *et al.* [13]). These authors seems to conclude that irrigation water demand is completely inelastic below a threshold price, and elastic beyond (Montginoul and Rieu [19]; Garrido *et al.* [9]; Varela-Ortega *et al.* [27]; Iglesias *et al.* [14]).

The programming method studies are based on the mathematical formalization of the farmer's behavior. This latter is assumed to maximize his final profit. The water demand function is derived from the following scheme. For a given price, one estimates the quantity of water maximizing the farmer's profit. Variations in water prices induce different levels of optimal water quantities. The authors use this information directly to represent the derived demand for irrigation water.

The weaknesses of these models are due to the formalization of the farmer's program and to the necessarily simplifying assumptions. Therefore, the demand estimates obtained strongly depend on the specifications made and these results can be biased.

One aim of this paper is to estimate the seasonal irrigation water demand using programming methods and complementing the previous studies. The approach to derive irrigation water demand by programming methods needs to be precisely defined and the farmer's maximization problem has to be detailed.

Our paper makes four contributions to the literature on seasonal irrigation water demand under risk.

First, we use dynamic programming to describe the farmer's program. The advantage of our approach is that it allows to represent precisely the problem of allocation of a limited water supply on a given crop field under risk and to show the impact of multiple applications of the water during the irrigation season. Few papers in this literature have dealt with the subject of irrigation scheduling (Bontemps and Couture [1] or [2]).

Second, we use a crop growth simulation model, EPIC-Phase, for estimating crop yield response function to irrigation water. A major advantage of this model compared to the pre-specified functions performed in the literature is that it represents more precisely the biological and physical process of plant growth.

Third, we specify a Constant Relative Risk Aversion (CRRA) utility function as objective criterion which appears appropriate to describe the farmer's behavior(Chavas and Holt [7]; Pope and Just[24]). The previous studies are based on the strong assumption that the farmer is risk-neutral and he maximizes his profit while it is recognized in the literature that farmers are risk averse (Bouzit [5]). Neglecting the risk-averse behavior in agricultural models can lead to important overstatement of the output level and to biased estimation of the irrigation water value, as well as incorrect prediction of choices. Moreover, there have been few attempts in the literature to prove the important role of information in the decision making process of farmers under uncertainty (Bontems and Thomas [4]). To our

knowledge, taking into account risk aversion and information value for estimating irrigation water demand is rarely found in the literature.

Finally, the scheme used to derive irrigation water demand is based on the evaluation of the value of water for the farmer. We define this value as the maximum amount of money the farmer would be willing to pay for the use of one additional unit of the resource under water scarcity.

This model is used to characterize and to quantify the irrigation demand functions under uncertainty for various climatic conditions. We use this climatic variability to draw the distribution of this demand function for two different information sets. In the first case an *ex-ante* or *open-loop* decision rule[1] for input use (i.e. computing irrigation water applications before random shocks to weather conditions are observed) is performed while an *ex-post* or *feedback* strategy (i.e. when successive information components are processed by the farmer at different stages) characterizes the second case. The derivation of the demand functions for these two main strategies under various climatic conditions is of great interest and answers to the following questions: *What is the shape of the irrigation demand function under uncertainty? What is the distribution of this random function? How sensitive is it to the strategy used (or to the information set)?* and finally, *Is the irrigation water demand function convex?*

We have used this model to estimate the irrigation water demand for a season under stochastic weather conditions in the South-West of France[2]. We obtain this function for various climatic conditions characterizing the distribution of the necessary stochastic demand functions under uncertainty. We have used a nonparametric estimation procedure in order to have a precise information on the shape of these functions. We show that irrigation water demand depends obviously on two variables: climate and information set. All the estimated demand functions have the same shape, presenting four main areas. For very small quantities, where the farmer considers water as an essential input to crop growth, the demand is inelastic. The second area corresponds to mean quantities where the plant has reached a satisfactory level of growth; water is no more an essential input and is not yet a risk reducing input. The farmer is more responsive to change in water price. But we find a

[1]The farmer decides not to use all the information available at every stage in the decision process.

[2]The climate in our region is more humid than in the Mediterranean areas, but much more drier than in the north of France.

third, non-intuitive, area for larger quantities where the water is a risk reducing input and the demand becomes inelastic again. The last area is classic, the water demand is obviously elastic for important total water quantities. This result is of great importance in terms of regulation policy, since the real price for the region considered lies in the third inelastic area. If the regulator imposes a price (or a quota) regulation, the impact of the change in the price (or in the quota) will strongly depend on the location of the initial and final prices (quotas) within the four areas. Finally, we have provided a parametric estimation of the demand functions and compared their shapes to the nonparametric ones. The fit seems quite good but the parametric curves are convex and present only two well known areas. The use of these parametric representations of the demand functions for policy analysis may therefore be misleading.

The paper is structured in the following manner. Section 2 describes the procedure for evaluating the irrigation water demand function under uncertainty. We present the theoretical framework for calculating demand functions, and then describe the dynamic model of the farmer's decisions. Finally the numerical procedure of resolution and the nonparametric estimation are presented. In section 3 we present an application in the South-West of France. The main results and estimations are reported, as well as the graphical representation of the demand functions. The policy regulation implications of these results are developed and analyzed. Section 4 concludes the paper.

2 Evaluating irrigation water demand under uncertainty

2.1 Definitions

The methodology for evaluating irrigation water demand is based on the evaluation of the value of water for the farmer. The farmer uses water as long as the benefit from the use of an additional unit of the resource exceeds its cost. As water becomes scarce, the value of water for the farmer appears greater than the real water price. Therefore, the farmer would be ready to use more water. Under limited water supply, the farmer's water value is *"the maximum amount of money the farmer would be willing to pay for the use of an additional unit of the resource"*. For a given quantity of water allocated for the season, its value, noted $\lambda(Q)$ is the derivative of the maximized objective function evaluated for this given quantity. Under stochastic weather conditions, the objective criterion of the risk-averse farmer is the

expected utility of the profit, $U(\Pi(Q))$.

This opportunity cost $\lambda(Q)$ is defined as the derivative of the optimized utility function $U^*(\Pi(Q))$, these two functions depend on the total quantity of water Q:

$$\lambda(Q) = \frac{dU^*(\Pi(Q))}{dQ} \tag{1}$$

The knowledge of $\lambda(Q)$ for any total quantity of water, gives the *willingness to pay* function of the farmer. This function is just the inverse of the *irrigation water derived demand.* Therefore, the irrigation water demand function is completely derived once its inverse, the willingness to pay, is known.

Our goal is to characterize the distribution of this demand function over the climatic alea, meaning that we will have as many different functions as we have different climates. We will present in section 3 several demand function according to the stochastic variability of the climatic factors.

To characterize the stochastic demand and willingness to pay functions we use the *"mean demand function"* and *"mean willingness to pay"*. These functions are defined using the above procedure on the *"mean utility function"*, noted $E_\Omega\Big[U^*\Big(\Pi(Q)\Big)\Big]$ over the range of the climatic distribution, denoted Ω.

The *mean willingness to pay*, noted $\lambda_E(Q)$, is computed as :

$$\lambda_E(Q) = \frac{d\,E_\Omega\Big[U^*\Big(\Pi(Q)\Big)\Big]}{dQ} \tag{2}$$

2.2 Decision model

2.2.1 General framework

We distinguish Ω, I_t, and ω_t. Ω is the stochastic climate of the whole season. I_t is the farmer's characterization of this climate over the period t ; ω_t is the vector of real weather factors such as wind, rain, temperatures, and radiation, realized during the period t.

Consider a farmer facing a sequential decision problem of irrigation under uncertainty. At date $t = 1$, the farmer knows the total quantity of water available for the season, Q, the initial water stock in soil, $\bar{V}$, and the state of crop biomass, $\bar{M}$. The farmer has to take decisions on irrigation at each date $t = 1, ..., T - 1$, and must choose the quantity of irrigation water

denoted q_t. Therefore, we have a dynamic model of sequential choice under limited water supply with uncertainty, integrating three state variables $(M_t, V_t, Q_t) \quad for \quad t = 1, ..., T-1$.

$$M_{t+1} - M_t = f_t(M_t, V_t, \omega_t) \tag{3}$$

$$V_{t+1} - V_t = g_t(M_t, V_t, q_t, \omega_t) \tag{4}$$

$$Q_{t+1} - Q_t = -q_t \tag{5}$$

The change in the level of the biomass at any date (equation 3) is a function (f_t) of the current date state variable, water stock in soil, and climatic conditions during the period. The change in water stock in soil (equation 4) depends moreover on the decision taken at the current date. The total quantity of water has a simple decreasing dynamic (equation 5).

The irrigation water supply is constrained as follows:

$$\sum_{t=1}^{T-1} q_t \leq Q \tag{6}$$

The application level, q_t - if this quantity is selected positive - is subject to technological and institutional constrains:

$$\underline{q} \leq q_t \leq \bar{q} \qquad for \quad q_t > 0 \tag{7}$$

with $\underline{q}$ and $\bar{q}$ exogenous[3].

The final date ($t = T$) corresponds to harvesting when actual crop yield becomes known. Let Y denote the crop yield function ; that quantity depends only on the final biomass at date T and is denoted $Y(M_T)$.

The farmer's profit per hectare can be written as:

$$\Pi = r \cdot Y(M_T) - C_{FT} - \sum_{t=1}^{T-1} (c \cdot q_t + \delta_t \cdot C_F) \tag{8}$$

where r denotes the output price; C_{FT} denotes fixed production costs; c is the variable cost for each m^3 of water ; δ_t is a dummy variable taking the value 1 if the farmer irrigates and 0 if not. C_F represents the fixed costs for each irrigation done due to labor and energy costs. We assume in the following that there is no uncertainty on output price.

[3]Farmers can face some limitations on the quantity q_t of water applied for each irrigation since the investments are fixed in the short term (see Bontems and Favard [3]).

The farmer is represented by a strictly monotonic, increasing and concave Von-Neumann-Morgenstern utility function, denoted U. We chose the most common CRRA utility function:

$$U(\Pi) = \left(\frac{1}{1-\beta}\right) \cdot \Pi^{(1-\beta)} \tag{9}$$

with β ($\beta \neq 1$), the relative risk aversion coefficient. We have assumed a risk aversion coefficient of 0.001, in accordance with the literature[4] (Jayet [16]).

The farmer's objective is to maximize the expected utility. We have to define now how the farmer does (or does not) incorporate the information he gets during the season. We focus here on two main procedures known as "*feedback*" and *"open-loop"* .

2.2.2 Information sets

• The feedback strategy

In this framework, the farmer incorporates all the information he gets during the decision process. At date 1, the farmer takes the decision q_1 according to his weather expectations. At date 2 he integrates the decision made at date 1 and the climate realized during period 1, he may revise his weather expectations using a bayesian rule:

Let $\Omega_c \subset \Omega$ for $c \in C$ denote a particular climate and let I_t be the subset of the climatic information on the period t for $t \in 1, \ldots, T$. Let's assume that the corresponding probabilities $P[\Omega_c]$ for $c \in C$ as well as the conditional probabilities $P[I_t|\Omega_c]$ are known[5]. Then from the Bayes's formula we find the *a posteriori* probability:

$$P[\Omega_c|I_t] = \frac{P[I_t|\Omega_c] \cdot P[\Omega_c]}{\sum_{c=1}^{C} P[I_t|\Omega_c] \cdot P[\Omega_c]} \tag{10}$$

This procedure can be repeated up to date $T-1$. The result of this classical process (see for example Sim [25]) is that the set of still possible states of the world is reduced via $I_t, I_{t+1}, \cdots, I_T$ until finally:

$$I_T \subseteq \Omega_c \;\; so\ that\ P[\Omega_c|I_T] = 1 \;\; for\ some \;\; c \in C \tag{11}$$

At this stage, the climate is finally known.

[4]The choice of this parameter is beyond the scope of this paper.

[5]In the procedure application (section3) we will use a 14 years database to compute these probabilities.

Through these computations the decision taken at date t clearly depends on the weather conditions observed during the period $[t-1,t]$ and on the past decisions $q_1,\cdots,q_{t-1}$. Formally, the farmer's sequential problem is:

$$Max_{q_1}E_{\Omega}Max_{q_2}E_{\Omega|I_1}...Max_{q_{T-1}}E_{\Omega|I_{T-2}}E_{\Omega|I_{T-1}} \quad \Big[U\Big(r\cdot Y(M_T)-C_{FT}-\sum_{t=1}^{T-1}(c\cdot q_t+\delta_t\cdot C_F)\Big)\Big] \tag{12}$$

$$s/c \qquad \begin{cases} M_{t+1}-M_t=f_t(M_t,V_t,\omega_t) \\ V_{t+1}-V_t=g_t(M_t,V_t,q_t,\omega_t) \\ Q_{t+1}-Q_t=-q_t \end{cases} \tag{13}$$

and subject to the technical constraint

$$and \quad s/c \qquad \begin{cases} \delta_t=\begin{cases} 0 \quad si \quad q_t=0 \\ 1 \quad si \quad q_t>0 \end{cases} \\ \underline{q}\leq q_t\leq \bar{q} \qquad iff \quad q_t>0 \\ M_t\geq 0, \quad V_t\geq 0, \quad Q_t\geq 0 \\ M_1=\bar{M}, \quad V_1=\bar{V}, \quad Q_1=Q \end{cases} \tag{14}$$

Where

E_{Ω} denotes the expectation over the climatic alea for the whole season or *a priori* distribution.

$E_{\Omega|I_{t-1}}$ represents the conditional expectation on Ω revised from the Bayes formula or *a posteriori* distribution.

- **The open-loop strategy**

On the contrary, the farmer's decision program is an *"open-loop"* one if he decides to choose all irrigations, $\{q_t\}_{t=1,...,T-1}$, before observing stochastic variables. In this case, all the decisions are made at date 1. At each period, the farmer does not revise his expectations. This procedure serves as benchmark since no information is incorporated during the season. The problem is the following:

$$Max_{\{\{q_t\}_{t=1,...,T-1}\}}E_{\Omega}\Big[U\Big(r\cdot Y(M_T)-C_{FT}-\sum_{t=1}^{T-1}(c\cdot q_t+\delta_t\cdot C_F)\Big)\Big] \tag{15}$$

subject to the above dynamics and technical constrains (13) and (14).

In this expression E_Ω represents, as in the previous section, the expectation on the whole climatic information set.

Under uncertainty, the two classes of strategies, *open-loop* and *feedback*, can be distinguished by the amount of information used and the anticipation of future knowledge. It is well-known that because information is never strictly useless, the farmer should prefer feedback to open-loop decisions. In real world situations, the farmer's strategy probably lies somewhere between these two extreme cases and some feedback must take place at some points in time.

2.3 Estimation procedure

2.3.1 Database

We need a database relating the total quantities of water to the maximized utilities in order to estimate the utility and the water demand functions. These data are obtained by solving the farmer's program described in the previous section (2.2) for different total quantities of water. Before solving the decision problem[6] for the two strategies, we need to characterize the production function $Y(M_T)$. That function is not pre-specified. We use an agronomic model, EPIC-PHASE (Cabelguenne and Debaeke [6]) to numerically represent it; this model also generates information relating to state variables previously represented by the functions $f_t(.)$ and $g_t(.)$. Using this crop growth simulation model it is possible to simulate yields for a large variety of soils and climatic conditions. The output from EPIC-PHASE is used as input in the economic model. Then the economic model evaluates utilities for various amounts of water available and for various climates. Finally the decision problem is solved using a global optimization framework and the optimized utilities are computed.

The optimization problem we are facing here is not a trivial one: for a given climate, and a given quantity of water Q, one has to find the irrigation schedule $(q_1, \cdots, q_{T-1})$ subject to (13) and (14) which maximizes the farmer's expected utility (equation (12) or (15)). Of course the farmer does not know the climate, and has to incorporate some information on it using an *open-loop* or a *feedback* strategy. We will need for that to incorporate the anticipated climate at beginning of the season or revised anticipations during the season.

[6]The numerical procedure of resolution integrating the agronomic model, an economic model, and an algorithm of search of the solution is detailed in Bontemps and Couture [1].

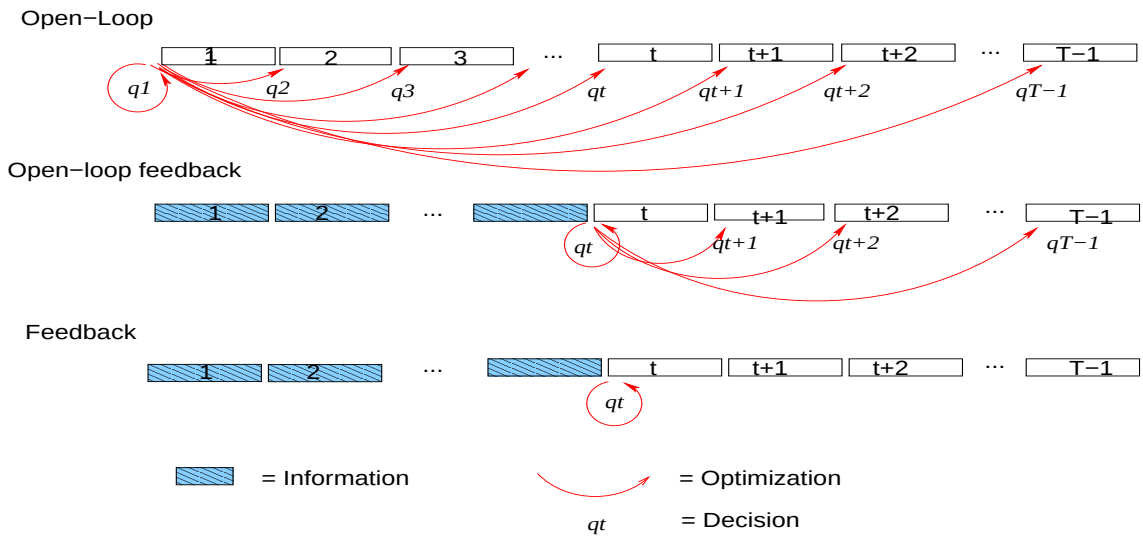


Figure 1: **Optimization process.**

The resolution of this problem is based on a method of global optimization over the set of possible irrigation schedules. In both cases, the set of constrains defined by (14) reduces the space of available irrigation schedules. Therefore the problem may be solved using an algorithm of search on all possible cases at each step. However, the objective functions being quite different, the procedure is different for the two cases.

In the *open-loop* case, the farmer has some anticipation on the climate and computes his optimal schedule at the beginning of the season. For any given value of the total quantity of water, and for any schedule, the model defined above may evaluate the utility function upon the anticipated climate. Because of the constrains inherent to the problem, the set of all possible irrigation schedules is not too large. One may then compute the expected utilities for each schedule. The optimal decision pattern is simply obtained by examining exhaustively the corresponding set of expected utilities. Finally, we repeat this procedure for different total quantities of water, and for various climates. Once the schedule maximizing the expected utility is found, we run the model on the real climate and find the real optimized utility, $U^*(Q)$.

In the *feedback* case, the problem is more tricky because at each decision step the farmer observes the climate and revises his anticipation before computing the schedule for the period. Because of the nature of our production function, which gives the output at the end of the

season, we use an approximation of the *feedback* program called *"open-loop feedback"* and assume that the farmer optimizes the schedule for the rest of the season. Since he does this operation at each decision period, it only retains the decision for the period considered, the approximation is therefore very close to the *"pure" feedback* strategy presented above (see Figure 1). The optimization program uses the same methodology than in the *open-loop* case. At each decision step we compute the set of the irrigation schedules still possible. We also compute the new anticipation using Bayes's formula and the observed climatic information. We run the crop simulation program over all these schedules taking into account the real observed climate at that period, and the expected ones. We find the optimal expected utility and schedule, take the corresponding decision and go to the next decision step[7] until we reach the final decision[8] and get the complete schedule. As in the latter case, we then run the real climate for that schedule and find the corresponding utility, $U^*(Q)$. We repeat this procedure for different quantities of water, and for various climatic years.

The database created in both situations consists in pairs (*quantity of water, optimized utility*) for various climates, and is well designed for estimating the utility and demand functions distributions over a climatic range. This database will be used through the non-parametric estimation procedure. An important feature of the demand functions we are estimating here is that they depend on stochastic climatic conditions, and therefore are stochastic. We will therefore estimate these functions for different climatic realizations and derive the main characteristics of the demand function distribution.

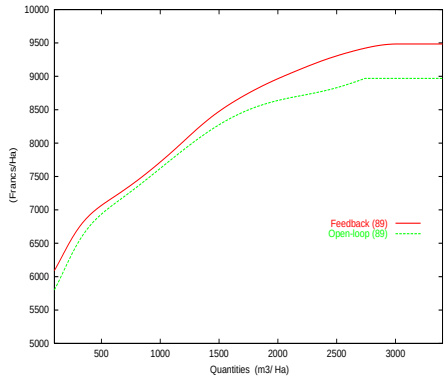
2.3.2 Estimation

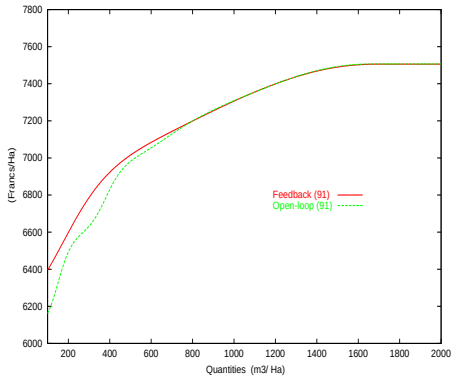
In order to estimate the water value, it is necessary to estimate the optimized utility function and its derivative. We use a nonparametric method to estimate these functions. A major advantage of nonparametric approach is that it allows to estimate an unknown function without assuming its form[9]. Another feature is that the estimation is only based on the data ; nonparametric estimators are all based on a weighted sum of functions of the data. The general procedure for estimating the utility function for a given climate is described in Appendix A; the procedure is the same for estimating demand functions.

[7] Note that for the first step, the procedure is exactly the one used in the *open-loop* case.

[8] The program may stop before the last period if there is no more choice, for example if the total quantity of water has been used totally in the first decision periods.

[9] The choice of the specifications of the considered functions, in particular the yield-water function, is always being debated at the present time.





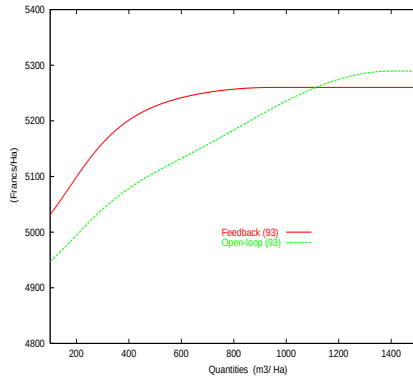


Table 1: Utility functions for *"Dry"*, *"Medium"* and *"Humid"* year.

3 An application in the South-West of France

Demand functions were estimated using the former procedure with data from the South-West of France (these data are described in Appendix B). In this area, agriculture is the largest water consumer with 2/3 of total water consumption. During low river flow periods there is a strong competition for water with urban and industrial uses. In this area, irrigated agriculture is quite recent and concerns most crops. Irrigation needs depend strongly on weather conditions. Irrigation water is generally drawn from rivers supplied by mountain reservoirs. The irrigation tools used in the South-West of France are generally sprinkler systems. The reference crop is corn because it remains the main irrigated crop in this area.

3.1 Results

The stochastic variability is presented in these results through three climates a "dry" one corresponding to real data of the year 1989, a "humid" year (1993) and a "normal" one (1991). We use these climatic years as high and low bounds of the distributions under study. We have run the simulation model for 9 quantities[10] of water for the irrigation season ($Q \in [0, 4000] m^3/ha$).

3.1.1 Utility

The first results we observe from the utilities estimations presented in Table 1 are in accordance with what could be expected: Within a climatic year the more information you have,

[10] This number is limited mainly because the computation time for the agronomic model EPIC is important and because the optimization procedure requires a great number of simulations.

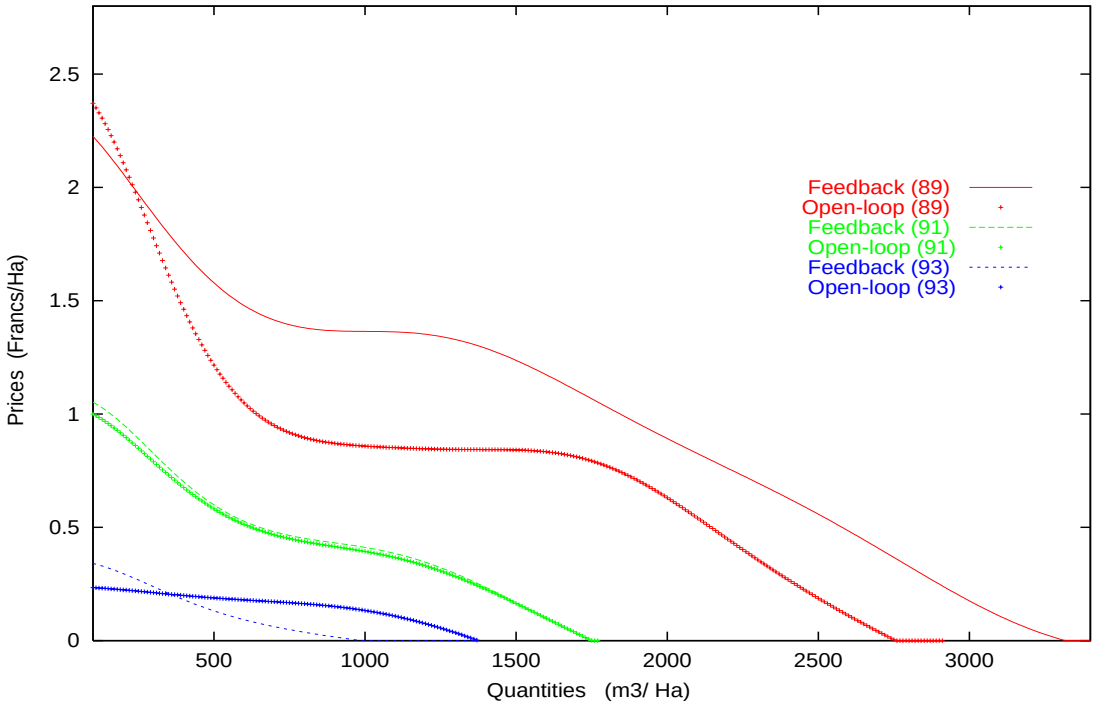


Figure 2: Demand functions for *"Dry"*, *"Medium"* and *"Humid"* year.

the higher the utility. Between the climatic years the drier the climate, the higher the utility. We may also notice that the shapes of the functions are quite identical.

3.1.2 Demand

Figure 2 reveals that the shapes of the demand functions are more or less the same. They present four areas: In the first one the curve is highly decreasing, becomes almost flat in the second, decreases greatly again in the third before changing its curvature once more at the end. Another common feature of the distribution presented, is that in each case we find a null price for some level of the total quantity of water and a maximum price for very small amounts of water. Note that, as for the utilities, the ordering of the functions is logical, between the three climates.

These curves give a good representation of what the distribution should be. We will not analyze in detail the general features of these curves, even if they represent the distribution bounds of the irrigation water demand under uncertainty, and focus in section (3.2) on the the shape of the mean willingness to pay functions (Figure 3). These curves are certainly the ones a regulator would closely look before setting either a price or a quota in situations where the water is scarce.

Specifications	Parameters	Demands Feedback	Open
(1) : $P = \alpha_1 + \beta_1 \cdot exp(-\gamma_1 \cdot Q)$	$\hat{\alpha}_1$	-0.043948	-0.058292
	$\hat{\beta}_1$	1.220422	1.216876
	$\hat{\gamma}_1$	0.000941	0.001006
	r^2	0.993415	**0.9951764**
(2) : $P = \alpha_3 + \beta_3/\sqrt{Q}$	$\hat{\alpha}_3$	-0.113706	-0.147926
	$\hat{\beta}_3$	13.498286	13.459313
	r^2	0.896510	0.898665
(3) : $P = \alpha_4 + \beta_4/\sqrt{Q} + \gamma_4/\sqrt[3]{Q}$	$\hat{\alpha}_4$	-1.388966	-1.4001683
	$\hat{\beta}_4$	-42.634237	-41.660033
	$\hat{\gamma}_4$	31.563734	30.994017
	r^2	**0.99467271**	0.994237

Table 2: Results of nonlinear regressions

3.1.3 Parametric versus nonparametric demand functions

The nonparametric estimation of irrigation water demand provides a precise figure of the demand function without assuming any parametric specification of this function. However it may be interesting to have a parametric, and more practical, form for this function. Moreover, almost all irrigation demand studies use some *ad-hoc* parametric specifications for the profit or production functions (Moore *et al.* [22] or Hassine and Thomas[12]) and therefore indirectly specify the demand function. We have estimated parametrically by nonlinear regression, the mean demand functions using the data generated by the nonparametric procedure. We have tested several specifications having the same shape than the nonparametric demand function. The table 2 gives some of the specifications we have tested and their associated R^2.

We have graphically represented the comparison of the best parametric specification and the nonparametric estimations of the mean demand functions in the table 4. We may notice two important points, first at this level[11], the parametric curves gives a good approximation of the nonparametric ones, but these parametric functions are, by construction, convex.

This means that the four areas we have discussed earlier and which are of great im-

[11] To have a better comparison and test between parametric and nonparametric curves one may use specific tests (see Härdle and Mammen [11]), we only provide here indications based on the R^2.

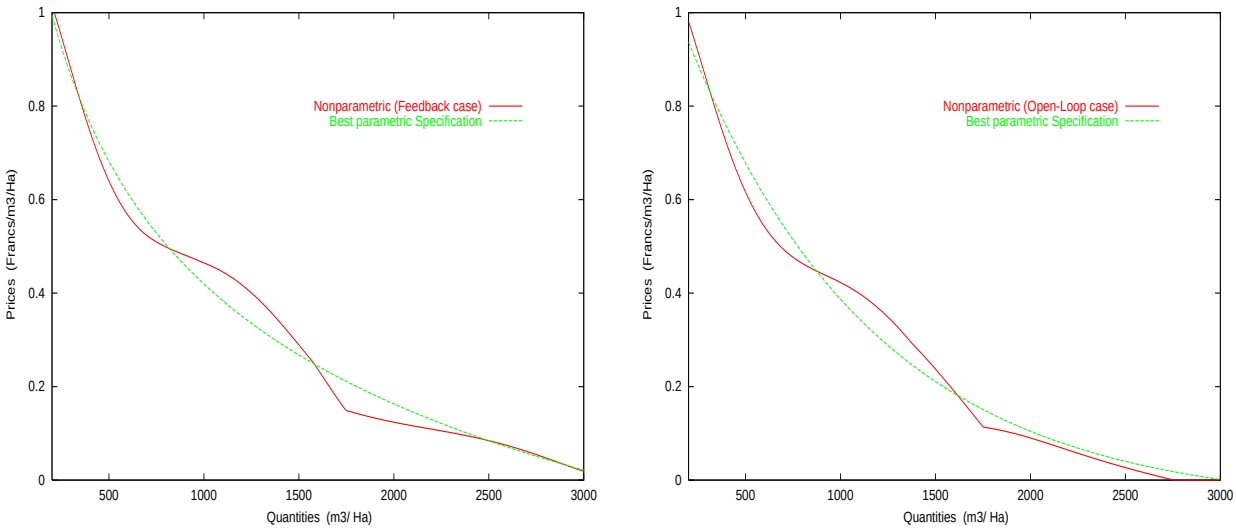


Table 3: Parametric versus Nonparametric comparison in the *feedback* and *open-loop* case

portance, are no longer present in the demand functions. The use of these parametric representations of the demand function for policy analysis may be misleading.

3.2 Economic analysis

3.2.1 Policy implication

The shape of the mean demand functions presented in figure 3 are similar whatever the information set. As previously mentioned, they present four main areas[12] schematically represented in figure 4.

For high prices (above 0.50 $francs/m^3$), the water irrigation demand is inelastic. This area corresponds to very small quantities where the farmer considers water as an essential input to crop growth; consequently, he will reduce his consumption for significant changes in water price. The second area (prices between 0.40 and 0.50 $francs/m^3$), corresponds to larger quantities where the farmer is more responsive to changes in water price. The plant has reached a satisfactory level of growth; water is no more an essential input and is not yet a risk reducing input. But, we find here a third, non-intuitive, area where the demand becomes inelastic again (prices beyond 0.40 $francs/m^3$). In this area the water is a risk reducing input, the farmer chooses large water quantities to insure a maximum and certain level of profit. Therefore to reduce the farmer's consumption the regulator has to increase

[12]In the figure 2, these areas appear even more clearly for some climates and cases.

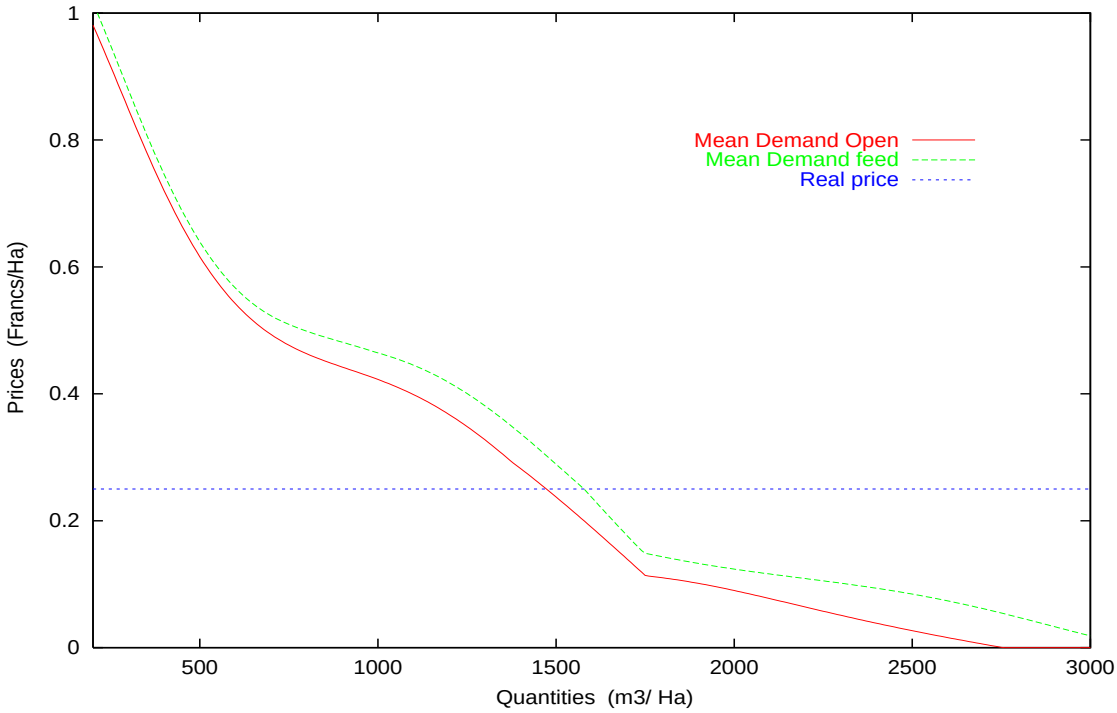


Figure 3: Mean demand functions in the *'open-loop"* and *"feedback"* case.

strongly the price. Finally, for very large quantities, the demand is elastic in the fourth area.

	Demands	
	Feedback	**Open-loop**
Price elasticity	-0.31	-0.34

Table 4: Price elasticities for a consumption of $1500 m^3/Ha$.

The real price of the water is in the third area ($0.25\ francs/m^3$), where the demand is quite inelastic. This *visual* analysis is confirmed by the computation of the price elasticities given in table 4. We find that the ratio of change in consumption is less than the ratio of change in price for the real water price.

All previous results are crucial information for the regulator in order to analyze the effects of a water regulation policy. There are two main ways of regulating, using quotas or prices.

Let's consider that the regulator imposes a quota, fixed[13] to 1500 m^3/ha. We can analyze

[13]This quantity corresponds to an average of the farmer's consumption in this area.

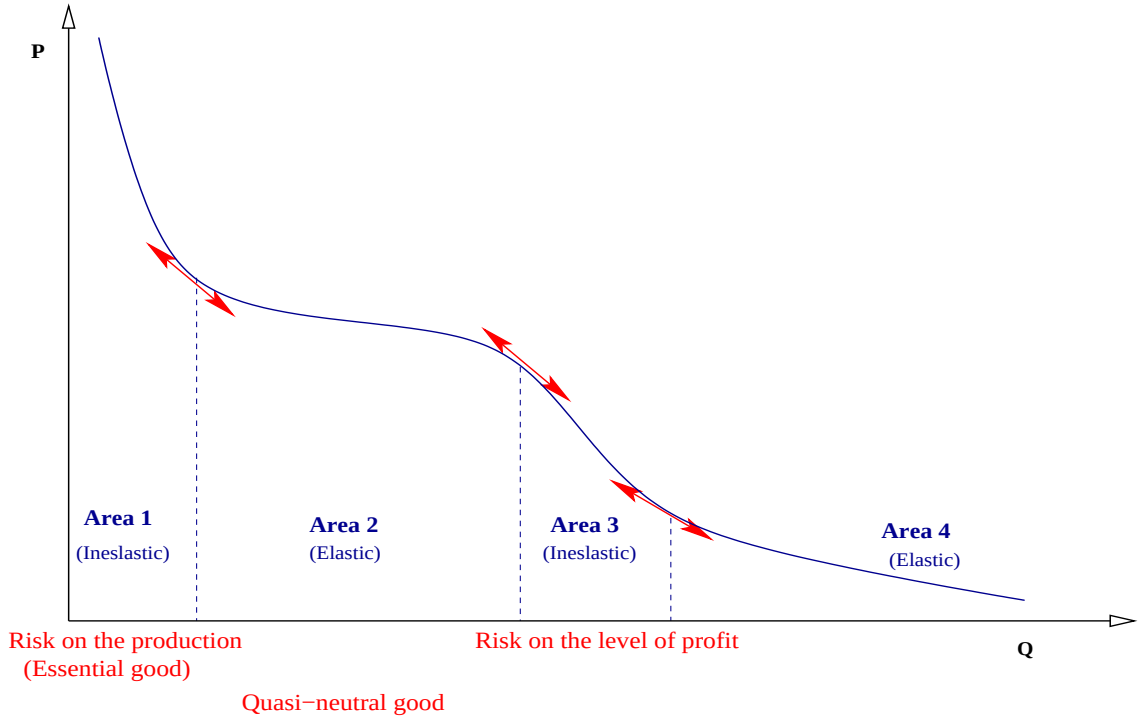


Figure 4: Schematic representation of the seasonal irrigation water demand curves.

the impact of the reduction of this quota down to 1350 m^3/ha (10% reduction) for the farmer. The loss in the farmer's surplus is around 50 $francs/ha$ (0.75%) whatever the case. If the regulator wants to maintain the farmer's revenue at the the initial level, he will have to subsidize the loss up to this sum.

If the regulator imposes a price regulation, the effects of the increase in price will depend on the area of the initial and final prices. For example, if we analyze the increase of 0.10 $francs/m^3$ starting with an initial price of 0.25 $francs/m^3$, in the third or *risk reduction* area, the total quantity of water is reduced by 230 m^3/ha (15.65% of the initial consumption) and the surplus is reduced by 94 $francs/ha$ (1.29% of the initial surplus). The same increase from 0.40 $francs/m^3$ up to 0.50 $francs/m^3$, in the *intermediate* area, leads to a much greater reduction of water by 420 m^3/ha (38.5%). The loss in terms of revenue is then 272$francs/ha$ (3.83% of the initial surplus). As the real price lies in the *risk reducing* area, the water pricing policy will be efficient if the increased price reaches the *intermediate area*, even more if it reaches the left border of this area. An increase of the real price leading to a new price within the same area will have few impacts on water consumption.

4 Conclusion

Our paper presents estimations of the seasonal irrigation water demand under uncertainty. We based our approach on the evaluation of the farmer's willigness to pay for an additionnal unit of the ressource under water scarcity and stochastic weather conditions. Utility functions are obtained through a sequential decision program for two main strategies, open-loop and feedback.

The farmer's dynamic program of decision is solved by a numerical procedure integrating a crop-growth model, and an optimization process linked to the economic model. A non-parametric estimation process is finally performed to derive the demand fonctions from the utilities. This procedure is applied to estimate demand functions for data from South-west of France. We obtain these functions for various climatic conditions, and thus characterizing the distribution of the demand functions under uncertainty.

We show that irrigation demand functions depend on climate and information sets, but have the same shapes. They can be decomposed into four main areas. For small water quantities, the demand is inelastic and then becomes elastic while increasing water quantities. The demand becomes inelastic again and finally appears elastic for larger water quantities. This result (four-area decomposition of demand function) is a crucial information for the regulator to define a water regulation policy.

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A The nonparametric procedure of estimation

We will detail the general procedure for a given climate, the procedure is the same for estimating the *mean utility* and *mean demand* function.

A.1 Utility function estimation

For this almost final step, we use the data resulting from the optimization procedure described above, and simply represented for a given climate as a sequence of couples $(Q_i, U_i^*)_{i=1,\dots,n}$.

For a given climate, the unknown function, $U^*(\cdot)$, is estimated from n couples (Q_i, U_i^*). The kernel estimator of utility function evaluated for any value of Q, is a weighted sum of the observed responses Q_i, the weight being a continuous function of observed quantities, Q_i, and current evaluation point Q (see Härdle [10] for details). It is defined as:

$$\widehat{U^*}(Q) = \frac{\sum_{i=1}^{n} U_i^* \cdot K\left(\frac{Q_i - Q}{h}\right)}{\sum_{i=1}^{n} K\left(\frac{Q_i - Q}{h}\right)} \qquad \forall\, Q \in R \tag{16}$$

where $K(\cdot)$ is a kernel function, continuously differentiable. We use a Gaussian kernel function among existing kernel functions[14]. Note that $\widehat{U^*}(Q)$ will inherit all the continuity and differentiability properties of K. Therefore $\widehat{U^*(Q)}$ is continuous and differentiable. The bandwidth, noted h, determines the degree of smoothness of $\widehat{U^*(Q)}$; its choice will be discussed latter in this section.

A.2 Demand function estimation

In order to estimate the demand function, we will use the property that the utility function estimator is differentiable. If the estimate $\widehat{U^*}(Q)$ properly reflects the utility function, $U^*(Q)$, then the estimate of the utility function derivative is equal to the derivative of the estimate of the utility function (Härdle [10]). Therefore a derivation of (16) with respect to Q will give an estimator of the demand function[15].

In other words, the estimator $\widehat{\frac{\partial U^*}{\partial Q}}(\cdot)$ of the unknown function $\frac{\partial U^*}{\partial Q}(\cdot)$ is just the derivative

[14] Estimations based on Epanechnikov kernel slightly differ from the Gaussian kernel estimator.

[15] Note that the Mack and Müller's estimator [17] is easier to use for derivation since it has a denominator which does not depend on the derivative variable (Q here). Since the derivation calculus are quite obvious in our case, we have used the 'classic' kernel estimator, but we suggest to use this estimator for advanced derivation estimation.

of the estimator $\widehat{U^*(\cdot)}$. More precisely:

$$\frac{\widehat{\partial U^*}}{\partial Q}(Q) = \frac{\partial}{\partial Q}\left(\frac{\sum_{i=1}^n U_i^* \cdot K\left(\frac{Q_i - Q}{h}\right)}{\sum_{i=1}^n K\left(\frac{Q_i - Q}{h}\right)}\right) \qquad \forall\ Q \in R \tag{17}$$

This can be rewritten as:

$$\begin{aligned}\frac{\widehat{\partial U^*}}{\partial Q}(Q) &= \frac{1}{\left(\sum_{i=1}^n K\left(\frac{Q_i - Q}{h}\right)\right)^2} \cdot \Big(- \Big(\sum_{i=1}^n U_i^* \cdot \frac{1}{h} \cdot K'\big(\frac{Q_i - Q}{h}\big)\Big) \cdot \Big(\sum_{i=1}^n K\big(\frac{Q_i - Q}{h}\big)\Big) \\ &+ \Big(\sum_{i=1}^n U_i^* K\big(\frac{Q_i - Q}{h}\big)\Big) \cdot \Big(\sum_{i=1}^n \frac{1}{h} \cdot K'\big(\frac{Q_i - Q}{h}\big)\Big)\Big)\end{aligned}$$

We will not present the details of this calculus here, note however that since the kernel function $K(\cdot)$ is continuously differentiable, the estimator $\frac{\widehat{\partial U^*}}{\partial Q}(Q)$ is also continuously differentiable.

A.3 Smoothing parameter selection

Choosing the bandwidth, h, is always a crucial problem. If h is small, then we get an interpolation of the data. On the other hand, if h is high, then the estimator is a constant function that assigns the sample mean to each point. There exist several approaches to bandwidth selection (Vieu [28]) using theoretical considerations (*plug-in method*) or data-based method (*cross-validation method*).

A feature of these approaches is that the selected bandwidth is not fully adapted, particularly if observation data are small. We use as a benchmark the value obtained by cross-validation. The aim of this method is to choose a value for h minimizing the cross-validation criterion, defined as a sum of distances between the estimator $\widehat{U}^*(\cdot)$ evaluated at Q_i and the real data observed U_i^*. We denote the bandwidth selected by the cross-validation criterion by h^*. In practice, a refinement consists in using a slightly smaller bandwidth than h^* in order to limit oversmoothing. Following Härdle ([10] p. 160), the smoothing parameter selected for demand function estimator is the same that the one chosen for utility function estimator, even if this argument may be discussed.

B Data

A first set of data is required by the crop growth simulator model. This data set includes weather, soil, technical and irrigation practices, and crop data. The daily weather input file

Year	**Output price**	**Water price**	**Fixed Cost per irrigation**	**Fixed cost**
	r (Francs/Tonne)	c (Francs/m^3)	C_F (Francs)	C_{FT} (Francs)
1989	1049	0.25	150	2150
1991	1038	0.25	150	2150
1993	778	0.25	150	2150

Table 5: Source: ITCF [15]; Michalland [18] and Couture [8].

was developed from data collected at the INRA station in Toulouse, for a 14-years series (1983-1996). The soil characteristic data were included in the crop growth model. The soil is clayey and chalky.

Economic output and input price data are included as a secondary data set, see table 5. Output prices are farm-level producer prices. Input prices include irrigation variable costs and fixed costs by watering, and other fixed production costs. The fixed cost, (C_F), per irrigation includes energy and labor costs. The fixed production costs, (C_{FT}), are composed of fertilizer, nitrate, seed, and hail insurance costs.