Using Fuzzy Goal Programming for Long Range Production Planning in Agricultural Systems

Bijay Baran Pal and Bhola Nath Moitra

INTRODUCTION

The purpose of this paper is to present how the fuzzy goal programming (FGP) in the area of mathematical programming (MP) can be efficiently used for modeling and solving agricultural planning problems to achieve the target levels of production of various crops cultivated in different seasons in a year and make the profit by allocating the cultivable land properly and utilising the available productive resources carefully throughout the planning year.

Mathematical programming (MP) models to agricultural planning systems have been widely used since Heady (1954) demonstrated the use of linear programming (LP) for land allocation to crop planning problems. From mid-1960s to 1980s, LP approach to different agricultural planning problems have been deeply studied by Beneke and Winterboer (1973), Dhawan and Kahlon (1977), Nix (1979), Black and Hlubik (1980) and widely circulated in the literature (France and Thornley, 1984). The potential use of LP to farming has been surveyed by Glen (1987).

Although LP has been successfully implemented to farm problems, the limitation of LP is that of only optimizing (either maximizing or minimizing) a single objective problem subject to limited supply of productive resources. But the increasing demand of different types of agricultural products in society and involvement of a number of socio-economic goals to satisfy today’s most of the decision problems in agricultural systems are concerned with more than one objective, i.e., most of the farm problems are multiobjective in nature.

In the multiobjective decision making (MODM) area, goal programming (GP) (Charnes and Cooper, 1961) has appeared as a robust tool for multiobjective decision analysis and widely applied to different decision problems. The application of GP approach to several real-life problems has been surveyed by Romero (1986). The GP approach to farm planning was first introduced by Wheeler and Russell (1977). Thereafter, GP and its variants have been widely implemented to different planning problems in farm sector by the pioneer researchers in this field [Dobbins and Mapp (1982), Dryzan (1985), Rehman and Romero (1987), Rodriguez-Ocaña et al. (1996), Gómez-Limón et al. (1996)]. The use of pre-emptive priority based GP to land use planning problems has also been demonstrated by Pal and Basu (1996).

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Although GP has been widely accepted as a promising tool for multiobjective decision analysis, the main weakness of GP formulation is that the resource parameters of the problem need to be specified precisely in the decision-making environment. But, in most of the real-world problems, they are often ill defined (i.e., imprecisely defined). So, assigning of definite aspiration levels to the goals of a problem often creates decision trouble in most of the practical decision situations due to the expert’s ambiguous understanding of the nature of the parameters involved therein.

To overcome the above difficulty, the concept of fuzzy sets, initially proposed by Zadeh (1965), has been introduced to the field of MODM problems, where aspiration levels of the goals are imprecisely assigned, and the goals are then termed as fuzzy goals. Here, in the decision process, the goals are characterised by the membership functions by specifying the tolerance limits (upper and/or lower) for achievement of their respective aspired levels. A membership function actually represents the degree of achievement of a goal in the decision-making horizon. The membership value unity of a fuzzy goal means full achievement and zero means least achievement of it. All other membership values lie between 0 and 1.

The concept of fuzzy programming (FP) in general level was first introduced by Tanaka et al. (1974). Thereafter, FP approach to LP with several objectives was introduced by Zimmermann (1978). During 1980s, the use of fuzzy set theory in GP has been investigated by Narasimhan (1980), Hannan (1981a, b), Ignizio (1982) and Tiwari et al. (1987). In their approaches, MODM problems are actually converted into conventional LP problems by using max-min operator of Bellman and Zadeh (1970). The main drawback of such an approach is that the individual achievement of the goals on the basis of needs and desires of the decision maker (DM) cannot be obtained due to the inherent weakness of LP formulation.

In order to overcome such a problem, Hannan (1981a, b) has proposed a fuzzy goal programming (FGP) formulation where achievement of the elicited membership values (not unity) for each of the fuzzy goals has been taken into consideration. The computational difficulty with this approach is that the sensitivity analysis with the elicited membership values is required again and again to reach a satisfactory decision. The application of FGP approach to various real-life problems has been demonstrated by Chen and Tsai (2001).

Now, in the farm planning context, FP formulations of different types of problems have also been investigated [analysis of water use in agriculture – Owsinski et al. (1987), water supply planning – Slowinski (1986), farm structure optimization problem – Czyzak (1990), feed mix – Czyzak and Slowinski (1990)], and they are mainly concerned with single objective FP problems. But, in contrast to
the conventional LP and GP approaches to agricultural systems, FP approach has not been discussed that extensively. Again, FGP approach to farm planning problems is yet to appear in the literature.

This paper presents a FGP formulation for optimal production of crops by proper allocation of the cultivable land and utilizing the available productive resources in the decision environment. The potential use of the approach is demonstrated by a case study.

II

FORMULATION OF FP PROBLEM

In a fuzzy MODM situation, the objectives are always described fuzzily, whereas, the resource constraints may be crisp or fuzzy and that depend on the fuzziness of the decision-making environment.

Let \( b_k \) be the aspiration level assigned to the \( k \)-th objective \( f_k(X) \) (\( k = 1, 2, \ldots, K \)). Then, the fuzzy goal may appear as one of the following forms:

\[
\begin{align*}
& f_k(X) \geq b_k & \quad \ldots(1) \\
& f_k(X) \leq b_k & \quad \ldots(2) \\
& f_k(X) = b_k & \quad \ldots(3)
\end{align*}
\]

where \( X \) is the vector of decision variables, and where \( \geq, \leq \) and \( = \) indicate the fuzziness of the aspiration levels and these are to be understood as “essentially greater than or equal”, “essentially smaller than or equal” and “essentially equal”, respectively in the sense of Zimmermann (1978). The aspiration level \( b_k \) signifies that the DM will be satisfied even for a value larger than \( b_k \) upto a certain tolerance limit and/or down to the same or different tolerance limit, and that depends on the fuzzification of an objective.

Definition of Membership Function

Let \( t_{uk} \) and \( t_{lk} \) be the upper and lower tolerance ranges, respectively, for the achievement of the aspired level \( b_k \) of the \( k \)-th fuzzy goal. Then, the tolerance intervals for achievement of the aspired levels of the fuzzy goals with \( \geq, \leq \) and \( = \) types of given restrictions are determined as: \( (b_k - t_{lk}, b_k), (b_k, b_k + t_{uk}) \) and \( (b_k - t_{lk}, b_k + t_{uk}) \), respectively, where \( (b_k - t_{lk}) \) and \( (b_k + t_{uk}) \) are called the lower and upper tolerance limits, respectively.

Now, the membership function, say \( \mu_k(X) \), for the \( k \)-th fuzzy goal can be characterised as follows (Zimmermann, 1978):

For \( \geq \) type of restriction, \( \mu_k(X) \) takes the form:
\[
\mu_k(X) = \begin{cases} 
1 & \text{if } f_k(X) \geq b_k \\
\frac{f_k(X) - (b_k - t_{1k})}{t_{1k}} & \text{if } b_k - t_{1k} \leq f_k(X) < b_k \\
0 & \text{if } f_k(X) < b_k - t_{1k}
\end{cases} \quad \ldots (4)
\]

For \( \leq \) type of restriction, \( \mu_k(X) \) becomes

\[
\mu_k(X) = \begin{cases} 
1 & \text{if } f_k(X) \leq b_k \\
\frac{(b_k + t_{uk}) - f_k(X)}{t_{uk}} & \text{if } b_k < f_k(X) \leq b_k + t_{uk} \\
0 & \text{if } f_k(X) > b_k + t_{uk}
\end{cases} \quad \ldots (5)
\]

For \( \geq \) type of restriction, \( \mu_k(X) \) can be expressed as

\[
\mu_k(X) = \begin{cases} 
1 & \text{if } f_k(X) = b_k \\
\frac{(b_k + t_{uk}) - f_k(X)}{t_{uk}} & \text{if } b_k < f_k(X) \leq b_k + t_{uk} \\
\frac{f_k(X) - (b_k - t_{1k})}{t_{1k}} & \text{if } b_k - t_{1k} \leq f_k(X) < b_k \\
0 & \text{if } f_k(X) > b_k + t_{uk} \\
or & \text{if } f_k(X) < b_k - t_{1k}
\end{cases} \quad \ldots (6)
\]

Here, it may be noted that the membership functions defined for \( \geq \) and \( \leq \) type of fuzzy goals are actually the particular forms of the membership function defined for \( \approx \) type of a fuzzy goal.

The membership functions for the fuzzy resource constraints can also be defined in an analogous way.

III

FGP FORMULATION

In a fuzzy decision making environment, achievement of a fuzzy goal to its aspired level means achievement of the associated membership function to its highest degree (unity). So, the aspiration level of a membership goal in FGP can be considered as 1.

It may be mentioned here that in the case of a crisp goal either under- or over-deviational variables or both of them are minimized to achieving the aspired level of the goal and that depends on the decision-making situation. But in case of achieving
the aspired level of a membership goal, any over-deviation from it means full achievement of the associated fuzzy goal (see, Dyson, 1981). As such, only under-deviational variables are required to minimize for achieving the aspired levels of the membership goals to the extent possible in a decision-making context.

Now, it is to be followed that only two types of membership goals corresponding to the membership functions in equations (4) and (5) defined for ≥ and ≤ types of fuzzy goals are actually coming into consideration to formulate the FGP model of the problem.

The FGP model under a pre-emptive priority structure can be presented as:

Find X so as to

Minimize $Z = [P_1(d^-), P_2(d^-), ..., P_i(d^-), ..., P_f(d^-)]$

and satisfy

$$\frac{f_k(X) - (b_k - t_{1k})}{t_{1k}} + d_k^- - d_k^+ = 1$$

$$\frac{(b_k + t_{uk}) - f_k(X)}{t_{uk}} + d_k^- - d_k^+ = 1$$

$$d_k^-, d_k^+ \geq 0 \text{ with } d_k^- d_k^+ = 0, \ k = 1, 2, ..., K \quad \text{...(7)}$$

where $X$ is the vector of decision variables, $Z$ represents the vector of I priority achievement functions consisting of the under-deviational variables of the goals for minimizing them on the basis of the priorities of achieving the aspired levels of the associated goals, and $d_k^-, d_k^+$ are the under- and over-deviational variables, respectively of the $k$-th goal. $P_i(d^-)$ is a linear function of the weighted under-deviational variables at the $i$-th priority level, where $P_i(d^-)$ is of the form:

$$P_i(d^-) = \sum_{k=1}^{K} w_{ik} d_{ik}$$

$$w_{ik}, d_{ik} \geq 0, \ k = 1, 2, ..., K ; \ I \leq K,$$

where $d_{ik}$ is renamed for the actual deviational variable $d_k^-$ to represent it at the $i$-th priority level, $w_{ik}$ is the numerical weight associated with $d_{ik}^-$ and represents the weight of importance of achieving the aspired level of the $k$-th goal relative to others which are grouped together at the $i$-th priority level. The values of $w_{ik}$ ($k = 1, 2, \ldots, K$) are determined as:

$$w_{ik} = \begin{cases} 
\frac{1}{(t_{1k})_i} & \text{for the defined } \mu_k \text{ in equation (4)} \\
\frac{1}{(t_{uk})_i} & \text{for the defined } \mu_k \text{ in equation (5)}. 
\end{cases}$$
where \((t_{lk})_i\) and \((t_{uk})_i\) are used to represent \(t_{lk}, t_{uk}\), respectively, at the \(i\)-th priority level.

Again, the pre-emptive priority structure of the model (equation 7) signifies that the defined membership goals are in the order of pre-emptive priorities for achievement of their aspired levels. Actually, the notion of pre-emptive priorities holds that the \(i\)-th priority \(P_i\) is preferred to the next priority \(P_{i+1}\) regardless of any multiplier associated with \(P_{i+1}\).

The relationship of the priority factors can be explicitly presented as

\[ P_1 \gg\gg P_2 \gg\gg \ldots \gg\gg P_i \gg\gg \ldots \gg\gg P_I \]

which implies that the membership goals at the highest priority level (\(P_1\)) are achieved to the extent possible before the set of membership goals at the next priority level (\(P_2\)) is considered, and so forth.

Now, in the present FGP formulation of the problem, the objectives as well as the productive resources are considered as fuzzy, whereas the different coefficients involved with resource utilisation and production rate are crisply incorporated. The following is a list of different types of parameters and variables used in the formulated model:

**Fuzzy Productive Resources**

- \(L_s\) = Total area of land (in hectares (ha)) currently in use for cultivating the crop \(c\) in any season \(s\),
- \(M_h\) = Estimated total machine hours (in hours (hrs.)) required during the year,
- \(M_d\) = Estimated total man-days (in days) required during the year,
- \(W_s\) = Estimated total amount of water (in inch) required during the season \(s\),
- \(T_f\) = Estimated total amount of the fertiliser \(f\) (in quintals (qtls.)) required during the year,
- \(C_r\) = Estimated total amount of cash (in Rupees (Rs.)) required per annum for supply of the productive resources,
- \(P_c\) = Annual production level (in qtls.) of the crop \(c\),
- \(M_p\) = Estimated total market value (in Rs.) of all the yielding crops in different seasons in a year.

**Crisp Coefficients**

- \(M_{Hcs}\) = Average machine hours (in hrs.) required for tillage per ha of land for cultivating the crop \(c\) during the season \(s\),
- \(M_{Dcs}\) = Man-days (in days) required per ha of land for cultivating the crop \(c\) during the season \(s\),
- \(W_{cs}\) = Amount of water consumed (in inch) per ha of land for cultivating the crop \(c\) during the season \(s\),
- \(F_{fc}\) = Amount of the fertiliser \(f\) required per ha of land for cultivating the crop \(c\) during the season \(s\),
- \(P_{cs}\) = Estimated production of the crop \(c\) per ha of land cultivated during the season \(s\),
**Decision Variable**

\[ x_{cs} = \text{Allocation of the land for cultivating the crop } c \text{ during the season } s. \]

**Fuzzy Goal Description:**

The fuzzy goals that are taken into consideration in the model formulation are described as follows:

1. **Land Utilisation Goal**
   
   The fuzzy goals for utilisation of total land now under cultivation appear as:
   
   \[
   \sum_{c=1}^{C} x_{cs} \leq L_s, \ s = 1, 2, \ldots, S. 
   \]

2. **Productive Resource Goals**

   (a) **Machine-hour Goal**
   
   For tilling the land in proper time, it is highly desirable to provide estimated total machine-hours through the planning year.
   
   The machine-hour utilisation fuzzy goal takes the form:
   
   \[
   \sum_{s=1}^{S} \sum_{c=1}^{C} x_{cs} \cdot MH_{cs} \geq M_h. 
   \]

   (b) **Manpower Goal**
   
   To avoid the trouble with the hiring of more labourers and involvement of extra cost for it at different phases of crop yielding, a minimum number of labourers is to be employed for all the cropping seasons.
   
   The manpower fuzzy goal can be presented as:
   
   \[
   \sum_{s=1}^{S} \sum_{c=1}^{C} x_{cs} \cdot MD_{cs} \geq M_d. 
   \]

   (c) **Water Consumption Goal**
   
   A minimum level of supply of water is needed for smooth growing of the crops cultivated at different seasons.
   
   The fuzzy goals appear as:
   
   \[
   \sum_{c=1}^{C} x_{cs} \cdot W_{cs} \geq W_s, \ s = 1, 2, \ldots, S. 
   \]

   (d) **Fertiliser Requirement Goal**
   
   To achieve the aspired level of production of each of the crops, there is a need of using different fertilisers to maintain the productivity of soil.
The fuzzy goals appear as:
\[
\sum_{s=1}^{S} \sum_{c=1}^{C} x_{cs} \cdot F_{fc} \geq T_f, \quad f = 1, 2, \ldots, F_1.
\]

3. **Production Goal**

To meet the increasing demand of agricultural products in the society, the fuzzy production goals for yielding different types of crops in all the seasons can be presented as:
\[
\sum_{s=1}^{S} x_{cs} \cdot P_{cs} \geq P_c, \quad c = 1, 2, \ldots, C.
\]

4. **Cash Requirement Goal**

An estimated amount of cash is essentially involved for acquiring the productive resources. The fuzzy goal appears as:
\[
\sum_{s=1}^{S} x_{cs} \cdot A_{cs} \leq C_r.
\]

5. **Profit Goal**

Beyond the interest of achieving the target levels of the production of crops, making profit (in Rs.) to a certain level is highly desired in the agricultural planning horizon.

The fuzzy profit goal takes the form:
\[
\sum_{s=1}^{S} \sum_{c=1}^{C} (MP_{cs} \cdot P_{cs}) \cdot x_{cs} \geq M_p.
\]

Now, the construction of membership functions for the defined fuzzy goals and achievement of the associated membership goals when executed under the framework of pre-emptive priority based GP are demonstrated via the following case study.

**IV**

**A DEMONSTRATIVE CASE STUDY: CROPS PLANNING PROBLEM OF THE NADIA DISTRICT, WEST BENGAL, INDIA**

The land use planning problem for the production of principal crops of the Nadia district in West Bengal is considered to demonstrate the proposed FGP model. The data for the planning year 1999–2000 (publication year 2000–2001) for the model were collected from different agricultural planning units.

The data of the present total land area under cultivation and allocation of all other productive resources, except fertiliser, were collected from Bureau of Applied
Economics and Statistics (BAES), Government of West Bengal, *District Statistical Hand Book*, Nadia, 1999–2000. The data for requirements of fertilisers and estimated target levels of the production of crops were collected from the Office of the Principal Agricultural Officer, Nadia, West Bengal.

The expected total market value of all the yielding crops were collected from Office of the Superintendent of Agricultural Marketing Department, Nadia, Government of West Bengal.

The data of utilisation rates of different productive resources and production rate of each type of crops per hectare of land and the expected market value were obtained from different sources - water (G. O. No. 6533 / 2 (376) M. I.- I Branch, dated 1.5.83, Government of West Bengal), fertiliser recommendation (Basak, 2000), number of agricultural labourers (The Murshidabad Central Co-operative Bank Ltd., Berhampur, West Bengal), machine-hours (Office of the Executive Engineer, Agri Irrigation, Nadia, Government of West Bengal), production rate (BAES, Government of West Bengal).

The data of cash expenditure for utilisation of productive resources per ha of land were collected from different sources – irrigation expenditure (Rs. / acre) (G.O. No. 6533 / 2 (376), M. I. – I Branch, dated 1.5.83, Government of West Bengal), average annual daily wage rate for agricultural field labourers (Government of West Bengal, 2002), cost of seeds, fertilisers, insecticides (The Murshidabad Central Co-operative Bank Ltd., Berhampur, West Bengal). Here, the involvement of all types of costs per ha of land is taken together for model simplification.

Now, the range for the upper tolerance limit for land utilisation goal, i.e., the extra land that can also be brought under cultivation, if required, was obtained from BAES, Government of West Bengal.

Regarding the upper tolerance limit for total cash expenditure goal, the past record shows that approximately 10 per cent of the total cash allocated in the previous year can be increased for the next planning year. So, the amount of cash for an increase of 10 per cent of the present allocation is considered as the tolerance limit for the goal.

The lower tolerance limits for other fuzzy goals, the data of the previous planning year (1998–1999) are taken into consideration.

Now, the three successive crop seasons in West Bengal are pre-*kharif* (*s* = 1): jute, *aus* paddy and sugarcane; *Kharif* (*s* = 2): *aman* paddy and *Rabi* (*s* = 3): *boro* paddy, wheat, mustard and potato. The different types of crops can be numbered as *c* = 1 for jute, *c* = 2 for *aus* paddy, *c* = 3 for *aman* paddy, *c* = 4 for *boro* paddy, *c* = 5 for wheat, *c* = 6 for mustard, *c* = 7 for potato and *c* = 8 for sugarcane, respectively.

The data for the aspiration levels and tolerance limits of the fuzzy goals are presented in Table 1.

The data for productive resource utilisation, cash expenditure and market price are given in Table 2.

Now, for the given aspiration level data in Table 1 and the crisp coefficient data in Table 2, the fuzzy goals of the problem can easily be defined. Then, using the tolerance limit data presented in Table 1, the membership goals for the defined fuzzy goals can be constructed by following the goal expressions in equation (7).
### TABLE 1. THE DATA DESCRIPTION OF FUZZY GOALS AND THEIR TOLERANCE LIMITS

<table>
<thead>
<tr>
<th>Goal (1)</th>
<th>Aspiration level (2)</th>
<th>Tolerance Limit (3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Land utilisation (‘000 hectares)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Pre-kharif season</td>
<td>272.135</td>
<td>–</td>
<td>309.33</td>
</tr>
<tr>
<td>(ii) Kharif season</td>
<td>272.135</td>
<td>–</td>
<td>309.33</td>
</tr>
<tr>
<td>(iii) Rabi season</td>
<td>272.135</td>
<td>–</td>
<td>309.33</td>
</tr>
<tr>
<td>2. Productive resource</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Machine hours (hrs.)</td>
<td>37,843.75</td>
<td>29,912.80</td>
<td>–</td>
</tr>
<tr>
<td>b) Man-days (days)</td>
<td>46,510.66</td>
<td>43,596.18</td>
<td>–</td>
</tr>
<tr>
<td>c) Water requirement (inch)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Pre-kharif season</td>
<td>2727.84</td>
<td>2524.34</td>
<td>–</td>
</tr>
<tr>
<td>(ii) Kharif season</td>
<td>1490.40</td>
<td>1437.60</td>
<td>–</td>
</tr>
<tr>
<td>(iii) Rabi season</td>
<td>5675.00</td>
<td>5605.20</td>
<td>–</td>
</tr>
<tr>
<td>d) Fertiliser requirement (metric tonne)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Nitrogen</td>
<td>44.50</td>
<td>37.20</td>
<td>–</td>
</tr>
<tr>
<td>(ii) Phosphate</td>
<td>23.00</td>
<td>19.80</td>
<td>–</td>
</tr>
<tr>
<td>(iii) Potash</td>
<td>19.00</td>
<td>13.00</td>
<td>–</td>
</tr>
<tr>
<td>3. Production (‘000 metric tonne)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Jute</td>
<td>306.00</td>
<td>302.85</td>
<td>–</td>
</tr>
<tr>
<td>(b) Rice</td>
<td>870.00</td>
<td>843.70</td>
<td>–</td>
</tr>
<tr>
<td>(c) Wheat</td>
<td>136.26</td>
<td>112.50</td>
<td>–</td>
</tr>
<tr>
<td>(d) Mustard</td>
<td>60.54</td>
<td>54.40</td>
<td>–</td>
</tr>
<tr>
<td>(e) Potato</td>
<td>110.00</td>
<td>98.60</td>
<td>–</td>
</tr>
<tr>
<td>(f) Sugarcane</td>
<td>259.00</td>
<td>81.50</td>
<td>–</td>
</tr>
<tr>
<td>4. Cash expenditure (Rs.)</td>
<td>64,410,15.80</td>
<td>94,00,113.90</td>
<td></td>
</tr>
<tr>
<td>5. Profit (Rs.)</td>
<td>1,25,00,000.00</td>
<td>94,00,113.90</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 2. DATA DESCRIPTION OF PRODUCTIVE RESOURCE UTILISATION, CASH EXPENDITURE AND MARKET PRICE

<table>
<thead>
<tr>
<th>(1)</th>
<th>Jute (2)</th>
<th>Aus (3)</th>
<th>Aman (4)</th>
<th>Boro (5)</th>
<th>Wheat (6)</th>
<th>Mustard (7)</th>
<th>Potato (8)</th>
<th>Sugarcane (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine hours (hrs. / ha)</td>
<td>61.02</td>
<td>61.02</td>
<td>40.52</td>
<td>38.51</td>
<td>36.36</td>
<td>36.36</td>
<td>36.36</td>
<td>61.02</td>
</tr>
<tr>
<td>Man-days / ha</td>
<td>124</td>
<td>84</td>
<td>89</td>
<td>111</td>
<td>74</td>
<td>47</td>
<td>119</td>
<td>247</td>
</tr>
<tr>
<td>Water requirement (inch / ha)</td>
<td>60</td>
<td>25</td>
<td>12</td>
<td>48</td>
<td>12</td>
<td>6</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Fertiliser (kg/ ha)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>20</td>
<td>40</td>
<td>20</td>
<td>100</td>
<td>100</td>
<td>80</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>Phosphate</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>50</td>
<td>50</td>
<td>40</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>Potash</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>50</td>
<td>50</td>
<td>40</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>Production (kg/ ha)</td>
<td>2538.00</td>
<td>2076.00</td>
<td>1885.00</td>
<td>3401.00</td>
<td>2301.00</td>
<td>795.00</td>
<td>17,779.00</td>
<td>59,283.00</td>
</tr>
<tr>
<td>Cash expenditure (Rs. / ha)</td>
<td>8,577.98</td>
<td>6,700.94</td>
<td>6,811.57</td>
<td>10,508.44</td>
<td>7,685.76</td>
<td>5,093.10</td>
<td>22,527.05</td>
<td>23,031.57</td>
</tr>
<tr>
<td>Market price (Rs. / qt.)</td>
<td>980.00</td>
<td>646.00</td>
<td>540.00</td>
<td>564.50</td>
<td>700.00</td>
<td>1,150.00</td>
<td>190.00</td>
<td>1500.00</td>
</tr>
</tbody>
</table>
**Construction of Membership Goals**

1. Land Utilisation Goals

Regarding utilisation of the total land it is worth mentioning here that all the crops are seasonal except *sugarcane*, which takes all the three seasons before harvesting.

So, the membership goals for utilisation of land in the three successive seasons appear as:

(i) \( \mu_1 : 8.3164403 - 0.0268853(x_{11} + x_{21} + x_{81}) + d_{1}^- - d_{1}^+ = 1 \)

(ii) \( \mu_2 : 8.3164403 - 0.0268853(x_{32} + x_{81}) + d_{2}^- - d_{2}^+ = 1 \)

\( \mu_3 : 8.3164403 - 0.0268853(x_{43} + x_{53} + x_{63} + x_{73} + x_{81}) \)

(iii) \( + d_{3}^- - d_{3}^+ = 1 \)

...(8)

2. Productive resource goals

(a) Machine hour goal

\( \mu_4 : 0.0076939(x_{11} + x_{21} + x_{81}) + 0.0051093x_{32} + 0.0048556x_{43} \)

\( + 0.0045845(x_{53} + x_{63} + x_{73}) - 3.771654 + d_{4}^- - d_{4}^+ = 1 \)

(b) Manpower goal

\( \mu_5 : 0.0425461x_{11} + 0.0288216x_{21} + 0.0305371x_{32} + 0.0380856x_{43} \)

\( + 0.0253904x_{53} + 0.0161263x_{63} + 0.0408306x_{73} + 0.0847492x_{81} \)

\( - 14.958476 + d_{5}^- - d_{5}^+ = 1 \)

(c) Water consumption goals

\( \mu_6 : 0.2948402x_{11} + 0.1228501x_{21} + 0.1474201x_{81} - 12.404619 + d_{6}^- - d_{6}^+ = 1 \)

(pre-\( \text{kharif} \))

(ii) \( \mu_7 : 0.2272727x_{32} - 27.227272 + d_{7}^- - d_{7}^+ = 1 \) (\( \text{Kharif} \))

(iii) \( \mu_8 : 0.687679x_{43} + 0.1719197x_{53} + 0.0859598x_{63} + 0.2865329x_{73} \)

\( - 79.303724 + d_{8}^- - d_{8}^+ = 1 \) (\( \text{Rabi} \))

(d) Fertiliser requirement goals

(i) \( \mu_9 : 0.0027397x_{11} + 0.0054794x_{21} + 0.0027397x_{32} + 0.0136985(x_{43} + x_{53}) \)

\( + 0.109589x_{63} + 0.0205479(x_{73} + x_{81}) - 5.0958904 + d_{9}^- - d_{9}^+ = 1 \) (Nitrogen)

(ii) \( \mu_{10} : 0.00625(x_{11} + x_{21} + x_{32}) + 0.015625(x_{43} + x_{53}) + 0.0125x_{63} \)

\( + 0.0234375x_{73} + 0.03125x_{81} - 6.1875 + d_{10}^- - d_{10}^+ = 1 \) (Phosphate)

(iii) \( \mu_{11} : 0.0033333(x_{11} + x_{21} + x_{32}) + 0.0083333x_{43} + 0.0066666x_{53} \)

\( + 0.0125x_{63} + 0.0166666(x_{73} + x_{83}) - 1.66666 + d_{11}^- - d_{11}^+ = 1 \) (Potash)  

...(9)
3. Production goals
(a) \( \mu_{12} : 0.8057142x_{11} - 96.142857 + d_{12}^- - d_{12}^+ = 1 \) (Jute)
(b) \( \mu_{13} : 0.0789353x_{21} + 0.071673x_{32} + 0.1293155x_{43} - 32.079847 + d_{13}^- - d_{13}^+ = 1 \) (rice)
(c) \( \mu_{14} : 0.0968434x_{53} - 4.7348484 + d_{14}^- - d_{14}^+ = 1 \) (wheat)
(d) \( \mu_{15} : 0.1419642x_{63} - 9.7142857 + d_{15}^- - d_{15}^+ = 1 \) (mustard)
(e) \( \mu_{16} : 1.5595614x_{73} - 8.6491228 + d_{16}^- - d_{16}^+ = 1 \) (potato)
(f) \( \mu_{17} : 0.3339887x_{81} - 0.4591549 + d_{17}^- - d_{17}^+ = 1 \) (sugarcane) ....(10)

4. Cash expenditure goal
\( \mu_{18} : 3.1766814 - (0.2898886x_{11} + 0.0022642x_{21} + 0.0023019x_{32} + 0.0035512x_{43} + 0.0025973x_{53} + 0.0017211x_{63} + 0.0076128x_{73} + 0.0077833x_{81}) + d_{18}^- - d_{18}^+ = 1 \) ....(11)

5. Profit goal
\( \mu_{19} : 0.0175978x_{11} + 0.0108028x_{21} + 0.0142328x_{32} + 0.0201767x_{43} + 0.011396x_{53} + 0.0066595x_{63} + 0.0239002x_{73} + 0.0629162x_{81} - 7.8440538 + d_{19}^- - d_{19}^+ = 1 \) ....(12)

Now, the assignment of priorities to the membership goals for achievement of their aspired levels to the extent possible depends on the order of importance of achieving the aspired levels of the associated fuzzy goals defined in the decision-making environment.

In this planning context, the following priority structure is introduced to the membership goals for minimizing their under-deviational variables from the aspired levels : 
(i) The first priority is assigned to the goals for production of crops.
(ii) The second priority is assigned to the goals for land utilisation and profit.
(iii) The third priority is assigned to the goals for seasonal water consumption and fertilizer requirement.
(iv) The fourth priority is assigned to the goals for machine hour utilisation, manpower and cash expenditure.

The assignment of above priorities to the goals can be discussed as follows :

The primary aim of the DM is to meet the demand levels of the production of crops first and then profit. In this context, it may be noted that the utilisation of cultivable land as well as the other productive resources are fuzzily described and the corresponding membership goals are considered as flexible goals (not crisp / rigid constraints), so relaxation of their aspiration levels within the specified ranges can easily be accommodated, if needed.

Considering the above view points, minimization of under-deviations (indicated by \( d^- \)) of the membership values of the production goals is considered first, and the highest priority \( P_1 \) is given to them.
Again, it is to be kept in mind that the resources are very scarce. So, over-utilisation of them, specially land, is always undesirable. But, to reach a certain profit level by meeting the production levels of the crops with the utilisation of land, minimization of the under-deviational variables associated with the land utilisation goals as well as the profit goal should have to be considered next by the DM. Regarding this aspect in the model formulation of the problem, the second priority \( P_2 \) is given to the land utilisation goals and the profit goal.

In the similar way, the assignment of the next two priorities \( P_3 \) and \( P_4 \) to the remaining goals can be analysed.

Then, the resultant FGP formulation becomes

\[
\text{Find} \{x_{11}, x_{21}, x_{32}, x_{43}, x_{53}, x_{73}, x_{81}\} \text{ so as to }
\]

\[
\text{Minimize } Z = [P_1(0.3174603d_{12} + 0.0380228d_{13} + 0.0420875d_{14} + 0.1785714d_{15}) + 0.0877192d_{16} + 0.0056338d_{17})],
\]

\[
P_2(0.0268853d_{17} + d_{19} + d_{17}) + 0.000007d_{19} + P_3(0.004914d_{6}) + 0.01893d_{7} + 0.0143266d_{8} + 0.0001369d_{9} + 0.0003125d_{10}
\]

\[+ 0.0001666d_{17}, P_4(0.000126d_{4} + 0.0003431d_{2} + 0.0000003d_{18})] \]

and satisfy the given membership goal expressions in equations 8 - 12 where \( d_{i}, d_{i}^+ \geq 0 \) with \( d_{i}^{-} d_{i}^+ = 0, i = 1, 2, ..., 19 \).

The software LINGO (ver 6.0) is used to solve the problem.

The obtained solution is

\[
x_{11} = 120.5674, \quad x_{21} = 98.42553, \quad x_{32} = 124.20, \quad x_{43} = 126.8896, \quad x_{53} = 59.21773, \quad x_{63} = 75.47170, \quad x_{73} = 6.187075, \quad x_{81} = 4.368875.
\]

It is found that the membership values \( \mu_k \) for the fuzzy goals, except \( \mu_4 = 0 \) for machine hour, \( \mu_9 = 0 \) and \( \mu_{10} = 0.093 \) for fertilizers, nitrogen and phosphate respectively, are achieved to their respective aspired levels (unity). The results reflect that a satisfactory decision is achieved in the decision context.

The resulting model solution and the performance figure of the existing cropping plan recorded during the planning year 1999 – 2000 are presented in Table 3.

<table>
<thead>
<tr>
<th>Crop</th>
<th>Land allocation ('000 hectares)</th>
<th>Crop production ('000 metric tonnes)</th>
<th>Production achievement recorded in the year 1999 – 2000</th>
<th>Land allocation ('000 hectares)</th>
<th>Crop production ('000 metric tonnes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jute</td>
<td>120.5674</td>
<td>306.00</td>
<td>128.8</td>
<td>325.926</td>
<td></td>
</tr>
<tr>
<td>Rice</td>
<td>349.51513</td>
<td>870.00</td>
<td>314.9</td>
<td>816.80</td>
<td></td>
</tr>
<tr>
<td>Wheat</td>
<td>59.21773</td>
<td>136.26</td>
<td>52.20</td>
<td>120.10</td>
<td></td>
</tr>
<tr>
<td>Mustard</td>
<td>75.47170</td>
<td>60.00</td>
<td>66.50</td>
<td>52.80</td>
<td></td>
</tr>
<tr>
<td>Potato</td>
<td>6.187075</td>
<td>110.00</td>
<td>3.70</td>
<td>65.90</td>
<td></td>
</tr>
<tr>
<td>Sugarcane</td>
<td>4.368875</td>
<td>259.00</td>
<td>1.20</td>
<td>68.90</td>
<td></td>
</tr>
</tbody>
</table>

A comparison of the results presented in Table 3 shows that a better solution is obtained by using the proposed FGP model in terms of achieving the aspired levels of the production goals in the decision making environment.
V

CONCLUSION

The FGP approach to crop planning systems demonstrated in the paper provides a new look into the way of analysing the different agricultural activities in an imprecise decision-making environment. The main advantage of the approach is that the decision for achievement of the fuzzy goals is made on the basis of their tolerance limits in the planning environment as well the priorities assigned to them in the decision-making context. Under the framework of the proposed model, different environmental constraints (crisp as well fuzzy) can easily be incorporated without involving any computational difficulty. Again, under the flexible nature of the priority based FGP model, the priority structure can easily be changed to make the proper crop production decision on the basis of the needs and desires of the DM in the planning situation. In future studies, the proposed method can be extended to different farm related planning problems involving inexact data. However, it is hoped that the concept of solving production planning problems presented here can contribute to future research in the area of farm planning problems.

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