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IS THE IB PROPERTY RESTRICTIVE? EVIDENCE FROM TIME SERIES DATA USING DIFFERENT DEMOGRAPHIC SPECIFICATIONS

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Abstract
This study tests whether the IB property of equivalence scales is restrictive. We examine the statistical robustness of the test with respect to different demographic specifications. We also measure the economic costs of the IB restriction by examining its impact on the concavity of the cost function, on the elasticities, and on the differences between IB and non IB scales. Our results confirm the hypothesis that both the statistical test and the assessment of the economic costs of the IB property are sensitive to the choice of the demographic specification.

Key words: IB property, Equivalence Scales, Demographic Specifications.

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1. Introduction

The purpose of this paper is to test the IB property of equivalence scales as defined by Blackorby and Donaldson (1989) and Lewbel (1991). Is it restrictive and if so, to what degree? We hypothesize that the test results may be sensitive to the choice of the demographic specification.

The experiment consists of estimating several demand systems incorporating demographic information through different modifying techniques. We represent preferences according to the Gorman Polar specification of a cost function and modify them via Translating, Scaling, and Barten-Gorman as defined in Pollak and Wales (1981); Shifting as specified by Blundell and Lewbel (1991) and Blundell, Pashardes and Weber (1989); and Extended Barten-Gorman, nesting both Barten-Gorman and Shifting, as will be defined later in the paper.

The experiment uses time series data and estimates the modified models with and without the IB restriction. We examine the statistical robustness of the test with respect to different demographic specifications using the Likelihood Ratio test. We measure the economic cost implied by the IB restriction by examining its impact on the concavity of the cost function -- the minimal requirement for welfare comparisons -- on the derived compensated price and expenditure elasticities and by assessing the discrepancy between IB equivalence scales and the measurable component of non-IB scales (Blundell and Lewbel).

The next section sets the notation. In section 3, we present the Extended Barten-Gorman model and the related conditions necessary to impose the IB property. Section 4 discusses the issues related to estimation and the econometric identification of the demographic parameters. Section 5 comments on the results and the last section draws the conclusions.

2. Notation and Definitions

Set the basic notation according to the following definitions:

$h = (h_1,..,h_H) \epsilon \Re_+^H$ ($\Re$ the real numbers, $\Re_+$ the non negative reals) is the index vector for demographic profiles at year h=1,..,H: h=0 designates the reference profile, h=1 designates the profile chosen for comparison;

p_j = the price of commodity i=1,..,n assumed constant across profiles;

$p = (p_1,..,p_n) \epsilon \Re_+^n$;

q_i^h = the quantity of the ith commodity consumed by the hth demographic profile;

$q = (q_1,..,q_n) \epsilon \Re_+^n$;

w_i^h = the budget share of the ith commodity by the hth profile;

y^h = the total expenditures of the hth demographic profile (income in short);

d_r = the rth demographic characteristic;

$d = (d_1,..,d_R) \epsilon \Re_+^R$;

δ_{ir} = the scaling demographic parameter for the ith commodity and the rth characteristic;

$\delta_i = (\delta_1,..,\delta_R) \epsilon \Re^R$ the R vector of scaling parameters for the ith commodity;

$m_i(d;\delta_i):\Re_+^R \rightarrow \Re$ = the scaling function specific to the ith commodity;

τ_{ir} = the translating demographic parameter for the ith commodity and the rth characteristic;

$\tau_i = (\tau_1,..,\tau_R) \epsilon \Re^R$ the R vector of translating parameters for the ith commodity;

$t_i(d;\tau_i):\Re_+^R \rightarrow \Re$ = the translating function specific to the ith commodity;

C(u,p) = the minimum cost of attaining utility level u at prices p. By definition, y^h=C(u,p). This cost function is assumed to be twice continuously differentiable and theoretically plausible.

V(y,p) = the indirect utility function at income y and prices p.

$\Phi(u)$ = the level of utility of the reference demographic profile.

3. The Extended Barten-Gorman model

Following Lewbel (1985), consider the relation:

$$y = C(u,p,d) = f\{C^*[u,h(p,d)],z(p,d),d\} \tag{1}$$

where $C^*(u,p^*)$ is a well-behaved expenditure function, $y^*=C^*[u,h(p,d)] = C^*(u,p^*)$ is the minimum expenditure necessary to attain utility level u at some scaled prices $p_i^*=h_i(p,d)$ and translating prices $p_i^T=z_i(p,d)$ for some vector valued functions h and z.

Using the facts $y=C(.)=f(.)$ and $y^*=C^*(.)$, the Barten-Gorman specification[1] is obtained from equation (1) using the following f(y,p,d) modifying function:

$$C(u,p,d) = f(y^*,z(p,d),d) = y^* P^T \quad with \quad P^T = \prod_i \left(z_i(p_i,d)\right)^{t_i(d)} .$$

This expression corresponds to the Barten (1964) specification with the addition of fixed overheads P^T for "necessary" or "subsistence" quantities (Gorman, 1976). The different demographic specifications used in the experiment have been derived by making explicit assumptions about h(p,d) and z(p,d). The specifications are:

(a) $h(p,d)=z(p,d)=p$ ⇒ budget share Translating

(b) $h(p,d)=z(p,d)=pm=p^*$ ⇒ budget share Reverse Gorman

(c) $h(p,d)=p^*$ and $z(p,d)=1$ ⇒ budget share Scaling

(d) $h(p,d)=p^*$ and $z(p,d)=p$ ⇒ budget share Gorman.

These definitions comply with Pollak and Wales (1981) terminology. For empirical convenience[2] the translating demographic function $t_i(d)$ is specified as $t_i(d) = \Sigma_r \tau_{ir} \ln(d_r)$ and the scaling demographic function $m_i(d)$ as $m_i(d) = \Pi_r d_r^{\delta ir}$, for r=1,..,n.

Assume quasi-homothetic preferences as described by the demographically modified Gorman Polar cost function:

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1 The Barten-Gorman Demographic specification that we refer to in this paper is the Reverse Gorman modifying structure.

2 It should be emphasized that the choice of the functional form of the demographic functions is not restricted to any particular form partly because only the relative magnitudes of the estimated demographic functions have a meaningful interpretation. The researcher, however, can specify a more complex form such as a translog if interested in modelling economies of scale.

$$C(u,p,d) = \left(A(p,d)\,(\phi(u))^{B(p,d)}\right) P^T \tag{2}$$

and the linear in logarithm analog:

$$\ln C(u,p,d) = (\ln A(p,d) + B(p,d) \ln \phi(u)) + \ln P^T$$

where:

$$\ln A(p,d) = \alpha_0 + \sum_i \alpha_i(d) \ln p_i^* + .5 \sum_i \sum_j \gamma_{ij}^* \ln p_i^* \ln p_j^*$$

$$B(p,d) = \beta_0 \prod_{i=1}^{n} (p_i^*)^{\beta_i(d)} ,$$

$$\alpha_i(d) = \alpha_i + \sum_r \varsigma_{ir} \ln d_r , \quad \beta_i(d) = \beta_i + \sum_r \rho_{ir} \ln d_r,$$

and ς_{ir} and ρ_{ir} are the R demographic parameters of the shifting functions $\alpha_i(d)$ and $\beta_i(d)$.

The corresponding Extended Barten-Gorman Almost Ideal Demand System (AIDS) indirect utility function is given by:

$$\ln V = \frac{\ln y^* - (\alpha_0 + \sum_i \alpha_i(d) \ln p_i^* + .5 \sum_i \sum_j \gamma_{ij} \ln p_i^* \ln p_j^*)}{\beta_0 \prod_i (p_i^*)^{\beta_i(d)}} \tag{3}$$

where $\gamma_{ij}=\gamma_{ij}^*+\gamma_{ji}^*$, $V=\phi(u)$ and $\ln y^*=\ln y - \Sigma_i t_i(d) \ln p^*$ from equation (2). The term "Extended" refers to the fact that both the α_i and β_i coefficients are also functions of demographic variables.[3] This extension permits distinguishing between the intercept shifting function and the translating function. Moreover, it allows deriving profile specific income elasticities. Roy's identity yields the Extended Barten-Gorman AIDS budget shares:[4]

$$w_i = \alpha_i(d) + t_i(d) + \sum_j \gamma_{ij} \ln p_j^* + \beta_i(d) \ln \left[\frac{y^*}{A'(p,d)}\right]. \tag{4}$$

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3 The system that we term Extended Barten-Gorman differs substantially to the Extended Reverse Gorman as specified by Bollino and Rossi (1989).

4 The term ln A'(p,d) is as ln A(p,d) with γ_{ij} in place of γ_{ij}^*. Henceforth, to simplify notation ln A(p,d) will be used in lieu of ln A'(p,d).

It is important to note that the system represented in equation (4) is not a unique specification of the Extended Barten-Gorman. Lewbel shows that a theoretically plausible specification of a modified Marshallian share demand system can be obtained from equation (1) by applying the following transformation (1985, Theorem 4):

$$w_i = \frac{\partial f(y^*,p,d)}{\partial y^*}\,\frac{y^*}{y}\sum_j^n \frac{\partial h_j(p,d)}{\partial p_i}\,\frac{p_i}{p_j^*}\,w_i^*(y^*,p^*) + \frac{\partial f(y^*,p,d)}{\partial p_i}\,\frac{p_i}{y}$$

$$= (1 - \sum_i t_i(d))\; w_i^*(y^*,p^*) + t_i(d) = w_i^* + t_i(d)\ ,$$

where $\Sigma_i t_i(d)=0$ due to the homogeneity restrictions. Many other specifications can be obtained by applying Lewbel's technique.

Consider, for example, the following exponential specification of the h(p,d) function $h_i(p,d)=\exp(p_i m_i(d))=\exp(p_i^*)$. The derived Extended Barten-Gorman shares are:

$$w_i = \alpha_i(d) + \frac{p_i^*\,t_i(d)}{y^*} + \sum_j \gamma_{ij}\ln p_j^* + \beta_i(d)\ln\left(\frac{y^*}{A(p,d)}\right). \tag{5}$$

This specification is interesting because the translation term in (5) looks much like the committed quantity term of the linear expenditure system. However, the overhead is not fixed. The supernumerary quantities increase as the ratio p^*/y^* also increases. Hence the degree to which a good is perceived as a necessity is subjective and varies from individual to individual (Lewbel, 1985).

The Extended Barten-Gorman model in equation (4) nests the following demographic specifications:

(a) Barten-Gorman

$$w_i = \alpha_i + t_i(d) + \sum_j \gamma_{ij}\ln p_j^* + \beta_i\ln\left(\frac{y^*}{A(p,d)}\right) \tag{6}$$

for $\alpha_i(d)=\alpha_i$, $\beta_i(d)=\beta_i$, and $\ln y^*=\ln y-\ln P^T$;

(b) Scaling

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j^* + \beta_i \ln \left(\frac{y^*}{A(p,d)}\right) \tag{7}$$

for $\alpha_i(d)=\alpha_i$, and $\beta_i(d)=\beta_i$,

(c) Translating

$$w_i = \alpha_i + t_i(d) + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln \left(\frac{y^*}{A(p)}\right) \tag{8}$$

for $\alpha_i(d)=\alpha_i$, $\beta_i(d)=\beta_i$, $h(p,d)=p^*=p$, and $\ln y^*=\ln y-\ln P^T$;

(d) Shifting

$$w_i = \alpha_i(d) + \sum_j \gamma_{ij} \ln p_j + \beta_i(d) \ln \left(\frac{y^*}{A(p,d)}\right) \tag{9}$$

for $P^T=1$, $h(p,d)=p$, and $\ln y^*=\ln y$.

Note that integrability requires that these specifications be estimated using y^* and without linearizing the deflating index A(.). This guarantees exact recovery of the modified cost and indirect utility functions which can then be used to derive equivalence scales and to make welfare comparisons that are fully cardinal. This is elaborated below.

4. The IB Property and Equivalence Scales

Let an Aggregate Equivalence Scale[5] (AES) be defined as the ratio of the cost function of family 1 with demographic profile described by the vector $\mathbf{d}^1$ to the cost function of the reference family 0 with demographic profile $\mathbf{d}^0$,

$$AES = \frac{C(u,p,d^1)}{C(u,p,d^0)} = \frac{C(V(y^0,p,d^0),p,d^1)}{C(V(y^0,p,d^0),p,d^0)} = \frac{C(V(y^0,p,d^0),p,d^1)}{y^0} \tag{10}$$

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5 The attribute "aggregate" stresses the distinction between general equivalence scales and group equivalence scales (Lewbel, 1989a).

where both families face the same prices p and share the same base utility level u or y.[6] The equivalence scale determines how much extra income is needed for a specific population profile at time h to reach the same level of utility as a reference profile r.

The definition of equivalence scales is similar to the definition of a Cost of Living Index (COLI) because the equivalence scales represent the ratio of cost of living indices for different demographic profiles. A True Cost of Living Index requires that the utility function is homothetic in order to ensure independence from the level of utility chosen for comparison. When this requirement is not satisfied the COLI is not uniquely determined and is described by a schedule.

The same analogy applies to the AES. However, in this case, a "True AES" in the sense that it is Independent of the Base utility level, or possesses the IB property (Blackorby and Donaldson 1988; Lewbel 1991), does not require the utility function to be homothetic. To illustrate, consider the schedule of Aggregate Equivalence Scales derived within the Extended Barten-Gorman scaled AIDS model:

$$AES(u,p,d) = \frac{C(u,p,d^1)}{C(u,p,d^0)} = \left[\frac{A(p,d^1)}{A(p,d^0)}\,\phi(u)^{[B(p,d^1)-B(p,d^0)]}\right]\left[\frac{p^T(p,d^1)}{p^T(p,d^0)}\right]. \tag{11}$$

This equivalence scale is not unique because it is not independent of the base level of utility u. This scale can be generated from the general class of cost functions $C(u,p,d)=m(p,d)G(u,p,d)$ for some functions $m(p,d)$ and $G(u,p,d)=\text{argmin}_q\{p'q \mid U(q,d)>u\}$.

Note that the same conditional demands $q((p,y)\mid d)$ also underlie the class of cost functions $C'(u,p,d) = m(p,d)G'(u,p,d)$ where $G'(u,p,d) = \text{argmin}_q\{p'q \mid F(U(q,d),d)>u\}$ for some monotonically increasing function $F(u,p,d)$. This is the cause of the identification problem. Blundell and Lewbel (1991) and Lewbel (1991, Lemma 1) claim that the class of cost functions with F independent from d such as $C(u,p,d)=m_0(p,d)G(u,p)$ generate

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6 This definition of Aggregate Equivalence Scale corresponds to Lewbel's (1991) cost of characteristics index.

unique conditional preferences q((y,p)|d), thus solving the identification problem, and producing equivalence scales that are independent of the base level of utility or income (IB):

$$AES^{IB}(u,p,d) = \frac{C(V(y^0,p,d^0),p,d^1)}{C(u,p,d^0)} = \frac{m_0(p,d^1)}{m_0(p,d^0)} . \tag{12}$$

In this strict sense, they are true and unique. Blackorby and Donaldson (1991) show that the Blundell and Lewbel claim is incomplete. In the case of Piglog preferences, such as the AIDS, scales are exact in the sense that they are independent of the base income or utility level but interpersonal comparisons cannot be determined uniquely. Thus, different scales are compatible with the same observed behavior. Both $C(u,p,d)=A(p,d)\phi(u)^{B(p,d)}$ and $C'(u,p,d)=A(p,d)[\phi(u)S(d)]^{B(p,d)}$ for some vector valued function S(d) generate different IB equivalence scales from the same observed demand behavior. We overcome this problem by assuming that S(d) is independent from d, as it is implicitly assumed in the study by Blundell and Lewbel.

In the case of the Extended Barten-Gorman model the schedule of AES(.) takes the following form:

$$\begin{aligned} AES(.) &= \frac{C(u,p,d^1)}{C(u,p,d^0)} = \frac{m(p,d^1)\ G(u,p,d^1)}{m(p,d^0)\ G(u,p,d^0)} = \\ &= \left(\frac{P^T(p,d^1)}{P^T(p,d^0)}\right)\left(\frac{A(p,d^1)\ \phi(u)^{[\beta_0 + \sum_i \beta_i(d^1)\ (\ln p_i + \ln m_i^1)]}}{A(p,d^0)\ \phi(u)^{[\beta_0 + \sum_i \beta_i(d^0)\ (\ln p_i + \ln m_i^0)]}}\right) \\ &= \frac{m(p,d^1)}{m(p,d^0)}\ \phi(u)^{[\sum_i \beta_i(d^1)\ \ln m_i^1 - \beta_i(d^0)\ \ln m_i^0 + \ln p_i(\beta_i(d^1) - \beta_i(d^0))]} . \end{aligned} \tag{13}$$

Proposition 1. In the Extended Barten Gorman model the function G(.) is independent of d and the scales are IB, if and only if either:

(1) $\beta_i(d)=0 \Rightarrow \beta_i=0\ \forall i$, which is necessary and sufficient for the AIDS cost function to be homothetic, or

(2) $m_i=m_j$ and $\beta_i(d)=\beta_i$ $\forall i,j$ and $\forall$ $d\epsilon R$, which is the case of Engel scales, or

(3) $\beta_i(d)=\beta_i$, $\Sigma_i\beta_i \ln m_i=0$ and (1) does not hold.

The last constraint is the least restrictive. The IB condition $B(p,d) \perp d$ is a necessary and sufficient condition for the equivalence scales $AES(p,d) \perp u$ and guarantees that the cost function is always separable in u and d as $C(u,p,d)=m_0(p,d)C(u,p)$.

Observe that if $\beta_i(d) = \beta_i$ and $\Sigma_i\beta_i \ln m_i = 0$, then $\Phi(u)$ in the expression for equivalence scale (13) is raised to zero. Consequently the AES do not depend on u. In the Engel case for which $m_i = m_j=m$, and $\beta_i(d) = \beta_i$, then $\Sigma_i\beta_i \ln m_i = \ln(m)\Sigma_i\beta_i = 0$, by the homogeneity of degree zero in p of $B(p,d)$. Observe that when the first and most restrictive constraint applies $\beta_i = 0$ $\forall i$, then β_0 is set equal to 1 (without loss of generality) implying that $B(p,d) = 1$ and $C(u,p,d)=A(p,d)[\Phi(u)]$. Given that the AES is a ratio of cost functions with different demographic profiles, $\Phi(u)$ cancels.

When the IB property is imposed either as $\Sigma_i\beta_i \ln m_i=0$ or $\beta_i=0$, for all i, the corresponding AES are algebraically the same since the term $A(p,d)$ does not depend on the β's. However, note that, implicitly, the constraints are substantially different. The constraint $\beta_i=0$ implies that $\Sigma_i\beta_i \ln m_i=0$, but the opposite is not true.

In the Extended Barten-Gorman and Barten-Gorman specification of preferences, the **IB Aggregate Equivalence Scale** takes the following form:

$$AES(.) = \frac{C(u,p,d^1)}{C(p,u,d^0)} = \frac{A(p,d^1)\ P^T(p,d^1)}{A(p,d^0)\ P^T(p,d^0)} \tag{14}$$

$$= \exp\left[\left(\ln A(p,d^1) + \ln P^T(p,d^1)\right) - \left(\ln A(p,d^0) + \ln P^T(p,d^0)\right)\right] =$$

$$= \frac{m_0(p,d^1)}{m_0(p,d^0)}.$$

In the scaling specification the m_0 function does not incorporate the overhead term P^T and $\alpha_i(d)=\alpha_i$. In the translating specification $p^*=p$ and $\alpha_i(d)=\alpha_i$.

Consistently with the IB hypothesis (which separates u from m), the AES for the translating case is given by the ratio of the overhead functions P^T as follows:

$$AES(.) = \frac{P^{T^1}}{P^{T^0}} = \frac{m_0(p,d^1)}{m_0(p,d^0)} = \prod_i p_i^{(t_i^1 - t_i^0)}. \tag{15}$$

Equation (15) is the ratio of the fixed costs of family 1 and 0.

Note that the AES in equation (15) is always IB. This needs clarification. Lewbel states: "Assume a cost function is demographically translated. Then the model does not possess an IB household scale." (1991, Lemma 5). In Lemma 5, Lewbel refers to a translated cost function modified through an additive linear demographic function as it should be specified in quantity space. In share space, as in the AIDS, translating is incorporated via P^T as a multiplicative term and IB scales are plausible (Ferreira and Perali 1992a).

To illustrate, consider the case of translating in quantity space. This case corresponds to a cost function modified via a linear additive function P^T. The AES is constructed as $AES=(C(u,p)+\Sigma_i p_i t_i^1)/(C(u,p)+\Sigma_i p_i t_i^0)$. By construction, it is not possible to make the scales independent of u unless $\Sigma_i p_i t_i=0$ which corresponds to the trivial case in which AES=1. These considerations show that the independence of B(p,d) from d is a necessary but not sufficient condition to guarantee IB equivalence scales. Proposition 2 follows naturally.

Proposition 2. A cost function of the form $C(u,p,d)=C((A(p,d)\Phi(u)^{B(p,d)}),P^T)$ generates IB equivalence scales if $B(p,d)\perp d \Rightarrow B(p)$ and overheads P^T are incorporated multiplicatively as $C(u,p,d)=(A(p,d)\Phi(u)^{B(p)})P^T$.

Observe that in the case of the shifting model, the problem is not present because the intercept term is a function of demographic variables as in equation (2).

5. Estimation

The application is carried out by estimating a complete demand system over the period 1953-1988 whose separable components are food at home, food away from home and non food. In recent years the empirical examination of the food at home-food away from home issue has received increasing attention. For welfare measurement, the decomposition between food and non food is interesting because it is ethically in line with the Engel way of associating utilities with well-being.

In the data set, personal consumption expenditure represents income. Expenditure information was obtained from the National Income and Product Accounts of the United States as published by the United States Department of Commerce. Price indices were derived from the annual city averages of consumer price indices from the regular urban National Statistical Accounts with base years 1983-84. The demographic variables included in the model are the percentage of the U.S. population in the 0-15 age (D1) category and the percentage of U.S. population enrolled in schools in each year (D2). Demographic information was drawn from Current Population Reports of the U.S. Bureau of the Census. Descriptive statistics of the data used in the analysis are presented in Appendix 1.

The stochastic Extended Barten-Gorman model is given by:

$$E(w_i) = \alpha_i(d) + t_i(d) + \sum_j \gamma_{ij} \ln p_j^* + \beta_i(d) \ln \left(\frac{y^*}{A(p,d)}\right). \tag{16}$$

We assume that the errors across equations (ϵ_i) are normally distributed, with mean zero and a constant covariance matrix Ω. They are uncorrelated over time, but correlated in each period.

$$E(\epsilon_{ir}\epsilon_{js}) = \begin{cases} \sigma_{ij} & for \;\; r = s \\ 0 & for \;\; r \neq s \end{cases}$$

Moreover, all variables affecting demand are assumed to be exogenous.

The system of equations (16) formed by food at home (FH), food away from home (FAH) and non food (NF) was estimated jointly using maximum likelihood (ML) estimation. Because the adding up restrictions that were imposed to identify the parameters in the model, $\iota' w_i = 1$ and $\iota' \epsilon_i = 1$, the covariance matrix is singular and the system is estimated by invariantly dropping the non food equation.

Following Atkinson (1970), Davidson and MacKinnon (1981), and Pollak and Wales (1981) the Extended Barten Gorman model can be compounded in a parsimonious fashion that allows testing the specification of the nested and non-nested models. Define the demographic functions as follows:[7]

$$\alpha_i(d) = g_i \sum_r \delta_{ir} d_r = \sum_r \varsigma_{ir} d_r \quad \textit{for} \quad \varsigma_{ir} = g_i \delta_{ir} ,$$
$$\beta_i(d) = g_i \sum_r \rho_{ir} d_r ,$$
$$t_i(d) = (1 - v_i) \sum_r \delta_{ir} d_r = \sum_r \tau_{ir} d_r \quad \textit{for} \quad \tau_{ir} = (1 - v_i) \delta_{ir} ,$$
$$m_i(d) = v_i(1 - g_i) \sum_r \delta_{ir} d_r .$$

for some constant v_i and g_i. Observe that $g_i=1$ and $v_i=1$ implies the shifting model and $g_i=0$ implies the Barten-Gorman model that nests both scaling and translating. The estimation of the Extended Barten-Gorman model requires n+(nxr) extra parameters with respect to the Barten-Gorman model and only n extra parameters with respect to the shifting specification.

The models were estimated under the maintained hypothesis of homogeneity and symmetry. Adding up was explicitly imposed since the model is non linear. Homogeneity of degree 1 in p of the cost function implies the following restrictions which are used to ensure the identification of all demographic parameters:

$$\sum_j \delta_{jr} = 0 \quad \textit{and} \quad \sum_i v_i \delta_{ir} = \sum_i g_i \delta_{ir} = \sum_i g_i \rho_{ir} = 0, \quad \textit{for each } r.$$

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7 Note that it is also plausible to hypothesize that $\beta_i(d) = g_i \Sigma_i \delta_{ir} d_r$. We chose to allow for different parameters on $\alpha_i(d)$ and $\beta_i(d)$ to parallel the work by Blundell and Lewbel.

In matrix notation, the restrictions can be rewritten as $\Delta\iota=0$ and $\Delta T=0$, where Δ is an rxn matrix of demographic parameters for n being the number of equations, ι is a nx1 vector of ones, T is a n column vector of v_i or g_i parameters and 0 is a rx1 vector of zeroes. The theoretical justification for the restrictions is in Perali (1992).[8]

Given this set of restrictions, the artificial parameters, v_i, of the Barten-Gorman model are overidentified. To clarify, note that the condition $\Delta\iota=0$ is derived from the homogeneity condition of degree one in prices of the cost function. This implies that Δ is of rank n-1. Hence, the parameter v_n can take infinitely many solutions and there is not a unique way to reconcile the values of v_n. Remarkably, it is neither necessary nor interesting to recover the value of v_n uniquely from the product $v_n\delta_{nr}$. Due to the homogeneity of degree zero in prices of the demand system only n-1 of the artificial parameters, v_n , have to be uniquely identified to fully construct the translating and scaling effects in the Barten-Gorman framework.

The existence of at least one solution is guaranteed by the fact that the rank of Δ is $\leq$ r. Observe that the system would be otherwise consistent if all the v_i are equal. However, this option is not satisfactory since v would act as a normalization that could take any value.

The IB property is easily understood at the cost level. It is less clear, however, how to transfer such restrictions at the share level of the Extended Barten Gorman model.

When the IB restriction is imposed as $\beta_i=0$ at the share level, the ith share is no longer a function of income. One method to test such restriction is to estimate the short model and to perform a likelihood ratio test. A more efficient method, that does not require reestimating the model, is to use a t-test on the significance of each β_i parameter. It is sufficient to have one β_i significantly different from zero to reject the implied assumption of

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8 The term α_0 is normalized to 0. For further discussion on the identifiability of the intercept term refer to Blundell and Lewbel (1991).

homotheticity of the demand system. A similar approach can be used on the restrictions $\Sigma_i\beta_i \ln m_i = 0$ and $m_i = m_j \ \forall \ i,j$. This procedure is limited by the fact that it permits rejecting the IB property but not accepting it. Thus, such strategy would be interesting only as a pre-test for the IB property.

The most general approach is to impose the IB property by requiring that $\beta_i(d)=\beta_i$ and $\Sigma_i\beta_i \ln m_i = 0$. This corresponds to case 3 of Proposition 1. It is also the least restrictive way to impose the IB property in the AIDS framework modified with an Extended Barten-Gorman technique. In the Shifting model this reduces to $\beta_i(d)=\beta_i$. For the Barten-Gorman model only the component $\Sigma_i\beta_i \ln m_i = 0$ of case 3 applies. This implies that, for each r, we must have $\Sigma_i\beta_i\delta_{ir} = 0$. Hence, only (rxn)-2r demographic parameters have to be estimated.

6. Results

Four demographic specifications were estimated. Given the small data set, the estimation of the Extended Barten-Gorman model was difficult. Nevertheless, it is possible to compare the Shifting (S) and Barten-Gorman (BG) model using Pollak and Wales' (1991) dominance criterion. The shifting model dominates the Barten-Gorman since $V_S=367.14 > V_{BG}=364.37$, where V_S and V_{BG} are adjusted likelihood values.[9] However, the likelihood values are very close suggesting that both can be either accepted or rejected with some positive probability. Both are rejected if the likelihood value of the Extended Barten is greater than 367.14 at the 1% level of significance. Both models are accepted if the likelihood value is smaller than 364.37 at the same level of significance. If the likelihood value of the Extended Barten-Gorman lies in the closed interval delimited by the likelihood values of the nested

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9 V_i is equal to the likelihood value of the model (L) plus half of the value of the chi-square distribution at α% of significance with d degrees of freedom. The number of degrees of freedom is the additional number of parameters in the model that nests the models being compared.

specifications $V_{EBG} \in [364.4;367.1]$, then we reject the Barten-Gorman specification and accept the Shifting model at the 1% significance level.[10]

The null hypothesis that scaling and translating are the same cannot be rejected because the likelihood ratio test of translating against Barten-Gorman, and Barten-Scaling against Barten Gorman (Table 2) are too small. Thus, we will only present results for the Barten-Gorman and the Shifting model.

When equivalence scales are used to derive money metrics of utility for welfare comparisons, Blackorby and Donaldson (1988) and Lewbel (1989b) point out that the money metrics should be concave at all price levels. This means that the Slutsky matrix must be negative definite at all prices. Concavity ensures that social judgements do not contradict distributional judgments derived from a social welfare function which is quasiconcave when each of its arguments is concave.

We test for "single-peaked" preferences, by computing the eigenvalues of the Slutsky matrix incorporating demographic factors[11]. In all the four demographic specification, with and without IB, the test for the violation of the second order conditions, was performed at all data points and at the data means. The methodology used to test for concavity when demographic factors are incorporated is explained in detail in Ferreira and Perali (1992b).

Except for the BG model with the IB property imposed no violations were encountered. For this case, the test results violated the negativity condition for 75% of the cases. The highest eigenvalue takes the estimate of 0.0043. We did not compute confidence intervals. Therefore we cannot infer if the computed eigenvalues are statistically different from zero. Mathematically, global concavity cannot be claimed. However, a conclusive

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10 Note that the Extended Barten-Gorman model would include 3 extra parameters with respect to the Shifting model and 5 extra parameters with respect to the Barten-Gorman model.

11 Note that non-positive compensated elasticities is a necessary (minimal) but not sufficient condition for negative semi-definiteness of the Slutsky matrix.

statement does require precise knowledge of the behavior of the third term of the Taylor series approximation from which the A(p) term of the Gorman polar form is derived.[12]

The elasticities for the Shifting and Barten-Gorman models without and with the IB property are presented in Table 4 and Table 5. The comparison of the results allows us to determine whether the estimated elasticities are sensitive to the demographic specification chosen. The estimates also allow us to assess the statistical and economic costs of the imposition of the IB property in both models.

The results indicate that the statistical and economic differences between the estimated elasticities across demographic specifications and between the restricted and unrestricted model are significant. The most appreciable differences between BG and S, regardless of the imposition of the IB property, occur when comparing the demographic elasticities. It is remarkable that the differences are bigger when the IB property is imposed.

The S model is significantly less stable than the BG model when the IB property is imposed. As an example, consider Food Away from Home (fah) with respect to Enrollment (D2). The estimates differ significantly both statistically and economically when comparing S with BG. The elasticities do not differ appreciably when the IB property is imposed. In general, the signs of the elasticities from non-IB to IB change only for those elasticities that do not differ significantly from 0 at the 1% level.

Our experiment shows that the imposition of the IB property, though statistically rejected, imposes negligible losses of economic significance in terms of estimated elasticities. On the other hand, the variability of the estimated price, income and demographic elasticities across demographic specifications indicates that the choice of the modifying technique can crucially affect both the statistical and economic significance of the

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12 This issue could have been explored by "reinterpreting" the AIDS model as proposed by Nicol (1989).

results. These indications prove that the evidence constructed by Blundell and Lewbel using the S specification alone is not definitive.

Table 6 and Table 7 present the estimates of the Equivalence Scales for the Shifting and the Barten-Gorman model respectively for the first year after the base year 1953, the median year 1972 and for 1988, the last period considered. The first column reports the Aggregate Equivalence Scale (AES), as defined in equation (13), derived from a model without the IB property imposed *ex ante*.

In order to assess the 'economic cost' of imposing the IB property using the AES as a metric, we assume that this property holds and impose it *ex post*. By making $G(u,p,d) \perp d$, one can express the AES as the ratio of m functions, as in 13. This ratio is reported in the third column. Note that the definition as the ratio of m functions in equation (13) is the same as the ratio of m_0's as specified for AES^{IB} in equation (12). Nevertheless, because the parameters were estimated without imposing the IB property, there is no guarantee that G(u,p,d) will be independent of d. In equation (12) the IB property was imposed *ex ante*, which guarantees independence of G(u,p,d) from d, and the derived AES^{IB} are shown in the fourth column.

Blundell and Lewbel (1991) showed that the AES can be written as "the product of a ratio of household specific cost of living indices[13] ... times the corresponding equivalence scale in the base price regime". Therefore, define the Relative Aggregate Equivalence Scale (RAES) as:

$$RAES = \frac{C(u,p_0,d_0)}{C(u,p_1,d_0)} \frac{C(u,p_1,d_1)}{C(u,p_0,d_1)} = AES_{p_0} \frac{C(u,p_1,d_1)}{C(u,p_1,d_0)} = AES_{p_0} AES_{p_1}. \tag{17}$$

This shows that the RAES is also the product of the conditional AES at two different price regimes. Blundell and Lewbel suggest that when the IB property is statistically rejected only the RAES should be used. Table 6 and 7 show the RAES in the second column.

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13 Blundell and Lewbel term this ratio "relative equivalence scale."

Note that the differences between AES and AES^{IB} for the Shifting model are neither economically nor statistically significant. This is not the case for the Barten-Gorman model. This pattern is also present when comparing AES with RAES. In the last column we show the $RAES^{IB}$ that was computed as the product of $AES_{p0}{}^{IB}$ $AES_{p1}{}^{IB}$. The computation provides an interesting comparison with the RAES. Note that the values are neither statistically nor economically significantly different for both demographic specifications. This result is independent of the statistical acceptance of the IB property and of the demographic specification chosen since the relative measures are the ratios of cost of living indices.

It is interesting to measure how close the AES and the ratio of the m_0 functions are. If the difference between the two measures is statistically and economically significant, so will be the difference between the AES^{IB} and AES. The knowledge of this fact is important when the estimation of the IB restricted model is cumbersome or as a pre-test when the researcher is interested in assessing the statistical and economic cost of the IB property. Note further that the difference between the AES and the ratio of the m_0 functions for the Shifting model are not significant, while they generally are for the Barten-Gorman model. On the basis of this evidence, we would assert that the economic cost of imposing the IB property in terms of differences in scales is negligible for the Shifting model and substantial for the Barten-Gorman model.

These results should come as no surprise. Recall the different structure of the B(p,d) terms of the Shifting and Barten-Gorman model respectively:

$$\ln B(p,d)_S = \sum_i \beta_i(d) \ln p_i$$
$$\ln B(p,d)_{BG} = \sum_i \beta_i (\ln p_i + \ln m_i)$$

Assume the extreme case that the price indexes do not vary across goods so that $p_i=p_j=p$.[14] Then,

$$\ln B(p,d)_S = \ln p \sum_i \beta_i(d)$$
$$\ln B(p,d)_{BG} = \ln p \sum_i \beta_i + \sum_i \beta_i \ln m_i$$

By adding up, $\Sigma_i\beta_i(d)=0$ and $\Sigma_i\beta_i=0$. As a consequence, the term $\ln B(p,d)_S$ behaves as if it were independent of d, regardless of whether the IB restriction were imposed. This is not true for the term $\ln B(p,d)_{BG}$ since, by construction, the values of the demographic functions m_i do vary across goods.

Because of this construction, it is likely that the economic costs of the imposition of the IB property in non-Barten type models is underestimated independent of the statistical results of the IB test. Blundell and Lewbel statistically reject the IB hypothesis in a non-Barten environment, but they can say very little about the implicit economic costs.

7. Conclusions

The main purpose of this paper was to test whether the IB property of equivalence scales is restrictive and to what degree. As hypothesized the test results are sensitive to the subjective choice of the demographic specification. The IB property is clearly rejected only in the Barten-Gorman demographic specification. It should be stressed that the estimation performance for this demographic specification is expected to be relatively lower in small samples.

The results suggest that the statistical and economic differences between the estimated elasticities across demographic specifications and between the restricted and unrestricted model are significant. The empirical

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14 Notice that in empirical demand applications with time series data or with normalized prices in general the situation in which price indexes do not vary substantially across goods is quite frequent. As an example, consider the levels of the price indexes used in the present study shown in Appendix 1.

evidence of this study shows that the economic cost of the imposition of the IB property in terms of the estimated scales is negligible for the Shifting model and substantial for the Barten-Gorman model. These results are not surprising given the different structure of the Shifting and Barten-Gorman model. Therefore, it is not correct to infer the economic costs of the IB property using non-Barten like models. This indicates that the evidence constructed by Blundell and Lewbel using only the Shifting specification is not conclusive.

Table 1. Values of the Likelihood Functions

Demographic Specification	Non-IB	IB
Barten-Gorman (BG)	356.83	351.56
Barten Scaling (BS)	356.27	351.36
Translating (T)	356.21	356.21
Shifting (S)	361.47	356.10

Table 2. Likelihood Ratio Test for demographic specification

Demographic Specification	$LR=2(L^*-L)$	$X^2(.01;d.f.)$	$X^2(.05;d.f)$
BG vs BS d.f. = 2	1.12	9.21	5.99
BG vs T d.f. = 2	1.23	9.21	5.99

Note: L^* is the unrestricted log-likelihood value. $X^2(s;d.f.)$ where s=significance level and d.f.=number of restrictions.

Table 3. Likelihood Ratio Test for the IB property

Demographic Specification	$LR=2(L^*-L)$	$X^2(.01;d.f.)$	$X^2(.05;d.f.)$
Barten-Gorman d.f. = 2	10.54	9.21	5.99
Barten Scaling d.f. = 2	9.82	9.21	5.99
Shifting d.f. = 4	10.74	13.28	9.49

Note: L^* is the unrestricted log-likelihood value. $X^2(s;d.f.)$ where s=significance level and d.f.=number of restrictions.

Table 4. Elasticity Estimates for the Non-IB Shifting (S) and Non-IB Barten-Gorman (BG) Almost Ideal Model

Variables	Food at Home (fh)		Food \ Home (fah)		Non-Food (oth)	
	S	BG	S	BG	S	BG
p(fh)	-.629 (.119)	-.569 (.106)	.00936 (.213)	-.019 (.196)	.163 (.127)	.118 (.022)
p(fah)	.0029 (.066)	-.007 (.067)	-.135 (.481)	-.113 (.009)	.0101 (.026)	.009 (.018)
p(oth)	.626 (.099)	.575 (.108)	.125 (.302)	.132 (.258)	-.173 (.138)	-.128 (.015)
x	.519 (.234)	.396 (.083)	.840 (.097)	.879 (.101)	1.14 (.036)	1.133 (.013)
D1	-.021 (.07)	-.378 (.226)	-.014 (.046)	.105 (.041)	.158 (.072)	.07 (.218)
D2	.069 (.048)	.333 (.227)	-.019 (.034)	-.464 (.308)	-.094 (.147)	-.036 (.059)

Note: Asymptotic standard errors are in parentheses. The price elasticities are compensated.

Table 5. Elasticity estimates for the IB Shifting (S) and IB Barten-Gorman (BG) Almost Ideal Models

Variables	Food at Home (fh)		Food \ Home (fah)		Non-Food (oth)	
	S	BG	S	BG	S	BG
p(fh)	-.543 (.971)	-.484 (.079)	-.039 (.199)	-.078 (.206)	.112 (.021)	.104 (.021)
p(fah)	-.014 (0.699)	-.027 (.071)	-.078 (.460)	.012 (.473)	.008 (.019)	.005 (.020)
p(oth)	.557 (.106)	.511 (.102)	.118 (.280)	.067 (.281)	-.120 (.014)	-.109 (.015)
x	0.368 (0.08)	.529 (.046)	0.860 (0.09)	.842 (.098)	1.140 (0.01)	1.107 (.008)
D1	-.054 (.0237)	-.033 (.289)	.0069 (.0124)	.099 (.067)	.019 (.005)	-.0002 (.56)
D2	.0705 (.0248)	.153 (.275)	-.026 (.0128)	-.458 (1.34)	-.0148 (.0057)	.001 (.31)

Note: Asymptotic standard errors are in parentheses. The price elasticities are compensated.

Table 6. Equivalence Scales for the Shifting model

Year	AES	Relative AES	Ratio of mo's	AES^{IB}	Relative AES^{IB}
1954	1.00104 (.0009)	.999915 (.6E-5)	.998801 (.0034)	1.00102 (.0003)	.99992 (2E-5)
1972 Median	1.00505 (.0047)	.995499 (.0031)	.990745 (.0164)	1.00524 (.0014)	.99532 (9E-3)
1988	1.00405 (.0058)	.9946 (.0064)	.981654 (.0193)	1.00523 (.0021)	.99371 (.002)

Note: Asymptotic Standard Errors are in parentheses.

Table 7. Equivalence Scales for the Barten-Gorman model

Year	AES	Relative AES	Ratio of mo's	AES^{IB}	Relative AES^{IB}
1954	1.06476 (.0500)	1.00007 (.00011)	1.02958 (.04399)	1.0092 (.0028)	1.0000 (2E-5)
1972 Median	1.41371 (.8263)	1.00289 (.01354)	1.18536 (.22607)	1.0421 (.0163)	.9987 (.0006)
1988	1.46118 (2.4527)	1.00512 (.05418)	1.23825 (.30801)	1.0223 (.0249)	.9992 (.0010)

Note: Asymptotic Standard Errors are in parentheses.

Appendix 1. Summary Statistics - Years 1953 to 1988

Variable	Unit	Minimum	Maximum	Mean	Std Dev
total PC Expenditure	billn$	232.6	3235.1	1070.61	904.2203
share(food at home)	%	0.1151	0.2056	0.1604	0.0264
share(food away from home)	%	0.0529	0.0606	0.0554	0.0019
share(non food)	%	0.7347	0.8302	0.7843	0.0274
p(food at home)	$	29.5	116.6	50.2916	1.6450
p(food away from home)	$	21.5	121.8	44.6150	1.8094
p(non food)	$	23.3238	124.688	45.1786	1.7558
population 0-15 of age	%	0.2285	0.3306	0.2827	1.1421
pop enrolled in school	%	0.2047	0.2943	0.2584	1.0905

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