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AN ANALYSIS OF ACREAGE DECISIONS UNDER RISK: THE CASE OF CORN AND SOYBEANS

By

Jean-Paul Chavas and Matthew T. Holt*/
University of Wisconsin-Madison

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*/ Respectively, professor and assistant professor in the Department of Agricultural Economics, University of Wisconsin-Madison.

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I - Introduction:

Much research has been done on the analysis of acreage response functions in agriculture. Following the relative success of the Nerlovian approach (e.g. Askari and Cummings), recent developments have attempted to strengthen the link between empirical supply response and economic theory either in a static framework (e.g. Weaver; Shumway; Antle) or in a dynamic framework (e.g. Vassavada and Chambers; Howard and Shumway). At the same time, there is increasing evidence that risk or risk behavior is important in aggregate agricultural production decisions (e.g. Behrman; Just; Lin et al.; Traill). However, the implications of decision theory under risk have typically played only a minor role in the empirical analysis of aggregate supply response. In other words, there appears to be a wide gap between the economic theory of risk behavior and the empirical specification and estimation of risk responsive aggregate supply functions. It is desirable to reduce this gap in order to refine the usefulness of our conceptual tools and to improve our understanding of economic behavior under uncertainty.

The objective of this paper is to develop an acreage supply response model under the expected utility maximization hypothesis and to investigate its empirical implications in the context of U.S. corn and soybean acreages. After the presentation of an expected utility model for acreage decisions (section II), testable hypotheses of economic behavior under risk are presented in section III. Importantly, multiple

sources of revenue uncertainty are incorporated into the analysis. Furthermore, linkages between government price support programs and the subjective probability distributions of uncertain output prices for decision makers are investigated (section IV). This is done by modifying the bounded price variation models considered by Maddala (1983a, 1983b), Shonkwiler and Maddala, and others to include multivariate price distributions. The implications of the theory are then incorporated into the specification and estimation of a system of risk responsive acreage decision functions for corn and soybeans in the U.S. (section V). In this context, a number of hypotheses suggested by the theory are tested and the results provide useful information on the nature of risk behavior in U.S. agriculture (section VI). Implications of the results are then discussed.

II - The Model:

Consider a farm household producing n crops, A_i being the number of acres devoted to the $i^{\underline{th}}$ crop and Y_i being the corresponding yield per acre, $i=1,\ldots,n$. Letting p_i be the market price of the $i^{\underline{th}}$ crop, then agricultural revenue is given by $R = \sum_{i=1}^{n} p_i Y_i A_i$. Denoting the cost of production per acre of the $i^{\underline{th}}$ crop by c_i, then the total cost of agricultural production is $C = \sum_{i=1}^{n} c_i A_i$. In the present case, revenue (R) is a risky variable since production lags dictate that both output prices $p = (p_1,\ldots,p_n)$ and crop yields $Y = (Y_1,\ldots,Y_n)$ are not observed by the household at the time production decisions are made.

Alternatively, input prices and per acre costs (c_i) are known at the time crop acreages are allocated.

The household then faces the budget constraint

$$I + R - C = q\,G$$

or

$$I + \sum_{i=1}^{n} p_i\,Y_i\,A_i - \sum_{i=1}^{n} c_i\,A_i = q\,G \qquad (1)$$

where I denotes exogenous income (or wealth) and G is an index of household consumption of goods purchased with corresponding price index q, q G denoting household consumption expenditures. Equation (1) simply states that exogenous income (I) plus farm profit (R-C) is equal to consumption expenditures (q G).

Let the constraints on acreage decisions be represented by

$$f(A) = 0 \qquad (2)$$

where $A = (A_1, \ldots, A_n)$. Assume that the household preferences are represented by a von-Neumann Morgenstern utility function U(G) satisfying $\partial U/\partial G > 0$. If the household maximizes its expected utility under competition, then the household's decision model is

$$\max_{A,G} E\,U(G) \text{ s.t. (1) and (2)}$$

where E is the expectation operator over the random variables reflecting uncertainty facing the decision-maker. After substituting the budget constraint into the utility function, the maximization problem can be expressed alternatively as:

$$\max_{A} \{EU[\frac{I}{q} + \sum_{i=1}^{n} (\frac{p_i}{q} Y_i - \frac{c_i}{q})A_i]\} \text{ s.t. } (2),$$

or

$$\max_{A} \{EU(w + \sum_{i=1}^{n} \pi_i A_i)\} \text{ s.t. } (2) \qquad (3)$$

where $w = (I/q)$ is normalized initial wealth and $\pi_i = (p_i/q) Y_i - (c_i/q)$ denotes normalized profit per acre of the $i^{\underline{th}}$ crop, $i=1,\ldots,n$, all prices being deflated by the consumer price q.

The above formulation illustrates that the acreage decision A is made under both price and production uncertainty. Thus, both yields Y and output prices p are random variables with given subjective probability distributions, yield uncertainty reflecting the influence of weather and other unknown factors on crop production.[1/] Consequently, the expectation E in (3) is over the uncertain variables p and Y and is based on the information available to the household at planting time.

What are the economic implications of the optimization problem (3) for the acreage decision A? Letting A^* denote the optimal acreage choice in (3), such a choice in general depends on normalized initial income (or wealth) w, expected normalized profits per acre $\bar{\pi}_i =$

$E\{(p_i/q)\ Y_i - (c_i/q)\}$, as well as second and higher moments of the distributions of normalized profits per acre π_i, $i = 1,\ldots,n$, denoted here by σ. In other words, the optimal acreage decision can be written as $A^*(w;\ \bar{\pi};\ \sigma)$, where $\bar{\pi} = (\bar{\pi}_1,\ldots,\bar{\pi}_n)'$.

It can be noted from (3) that the acreage decision under risk $A^*(.)$ is homogenous of degree zero in (w, p, c, q), i.e. that a proportional change in all these variables would not affect economic decisions. While this result does not depend on risk preferences U(.), it should be emphasized that this well known homogeneity property involves output price p, input cost c <u>and</u> initial wealth w and consumer price q. It implies that the acreage decision can be expressed as a function of the relative prices w/q, p/q and c/q (or their probability distributions). However, unless additional restrictions are imposed on risk preferences (see Pope), it does <u>not</u> imply that the acreage function $A^*(.)$ is homogenous of degree zero in output and input prices (p,c). In other words, the classical result of riskless production theory stating that production decisions depend only on input-output price ratios does <u>not</u> hold in general under uncertainty.

III - <u>Properties of the Acreage Decision</u>:

The empirical implications of the expected utility maximization hypothesis have been investigated by Sandmo, Ishii, Chavas and Pope, Pope, and others. In this section we summarize some of these implications in the context of model (3). In particular, we focus on the theoretical restrictions implied by (3) which can be tested and/or

imposed in the empirical specification and estimation of the acreage decision $A^*(.)$.

First, Sandmo and others have examined the relationship between wealth effects, $\partial A^*/\partial w$, and the nature of risk preferences. In particular, a zero wealth effect, $\partial A^*/\partial w = 0$, corresponds to constant absolute risk aversion, i.e. to a utility function where the Arrow-Pratt risk aversion coefficient $-\frac{\partial^2 U}{\partial w^2}/\frac{\partial U}{\partial w}$ is a constant. Alternatively, $\partial A^*/\partial w \neq 0$ corresponds to non-constant absolute risk aversion. Non-zero wealth effects are of interest here to the extent that decreasing absolute risk aversion is a maintained hypothesis in much of the economic literature (e.g. Arrow).[2/]

Second, the optimization hypothesis (3) implies symmetry restrictions on the slopes $\partial A^*/\partial \bar{\pi}$. These symmetry restrictions take the form

$$\frac{\partial A^c}{\partial \bar{\pi}} = \frac{\partial A^*}{\partial \bar{\pi}} - \frac{\partial A^*}{\partial w} \cdot A^{*\prime} = \text{a (nxn) symmetric, positive semi-definite matrix} \qquad (4)$$

where A^c is the wealth compensated acreage decision, holding utility constant (e.g. see Chavas, 1987). Expression (4) states that the matrix of compensated effects $\partial A^c/\partial \bar{\pi}$ is symmetric, positive semi-definite. Also, it indicates that the slope of the uncompensated function $\partial A^*/\partial \bar{\pi}$ can be decomposed as the sum of two terms: the compensated slope (or substitution effect) $\partial A^c/\partial \bar{\pi}$ which maintains a given level of utility plus the wealth effect $(\frac{\partial A^*}{\partial w} \cdot A^{*\prime})$. These results are quite

general since equation (4) holds for any risk preferences.

As discussed above, the wealth effect vanishes under constant absolute risk aversion. Under such circumstances, $\partial A^c/\partial\bar{\pi} = \partial A^*/\partial\bar{\pi}$, i.e., compensated and uncompensated choice functions have the same slope with respect to $\bar{\pi}$ and $\partial A^*/\partial\bar{\pi}$ is a symmetric, positive semi-definite matrix from (4). This illustrates the influence of risk preferences on the properties of acreage choice functions since non-zero wealth effects are associated with a departure from constant absolute risk aversion. Also, note from (4) that non-negative wealth effects ($\partial A^*/\partial w \geq 0$) are sufficient conditions to guarantee that $\partial A_i^*/\partial\bar{\pi}_i \geq 0$, i.e. that an increase in expected returns per acre of the $i^{\underline{th}}$ crop will result in an increase in the optimal acreage of that crop.

Finally, Chavas and Pope (p. 229) and Pope have derived homogeneity restrictions in the context of the expected utility model (4). In particular, rewriting expression (2) as $f(A) = A_1 - g(A.) = 0$, where $A = (A_1, A.)$, Chavas and Pope have shown that the following restriction holds at the optimum under any risk preferences

$$\frac{\partial A^*}{\partial\bar{\pi}} \left(\frac{\partial f(A)}{\partial A}\right)' - \frac{\partial A^*}{\partial w} \cdot \frac{\partial f(A)}{\partial A} \cdot A = 0 \tag{5a}$$

Let the first-order conditions associated with (3) be $E(\frac{\partial U}{\partial w}\pi) + \lambda \frac{\partial f}{\partial A} = 0$, where λ is the Lagrange multiplier associated with the constraint (2) and $\frac{\partial f}{\partial A}$ is a (1 x n) of vector. Given $\lambda \neq 0$, substituting these conditions into (5a) yields

$$\frac{\partial A^*}{\partial \bar{\pi}} (\bar{\pi} + \delta) - \frac{\partial A^*}{\partial w} (\bar{\pi}' + \delta')A = 0 \tag{5b}$$

where $\delta = COV(\partial U/\partial w, \pi)/E(\partial U/\partial w)$ is a (nxl) vector. Under risk neutrality, $\partial A^*/\partial w = 0$ and $\delta = 0$, implying from (5b) that $\sum_{j=1}^{n} \frac{\partial A_i^*}{\partial \bar{\pi}_j} \bar{\pi}_j = 0$, i.e. that the acreage decision function A^* is homogeneous of degree zero in $\bar{\pi}_j$. This is the homogeneity restriction of classical production theory stating that production decisions are not affected by proportional changes in all input and output prices. However, under risk aversion, $\delta \neq 0$ and (5b) implies that this homogeneity-like restriction takes a different form.[3/]

Some empirical implications of specific forms of risk preferences have been presented by Pope. In particular, under constant relative risk aversion (i.e., where $(w + \sum_{i=1}^{n} \pi_i A_i).\frac{\partial^2 U}{\partial w^2}/\frac{\partial U}{\partial w}$ is a constant), a positive scaling of wealth does not alter optimal decisions (Sandmo). This implies that decisions functions are almost homogeneous of degree one in initial wealth, degree one in mean returns $\bar{\pi}$, degree two in moments of order two, and degree s in moments of order s of π. Similarly, under constant partial relative risk aversion (where $(\sum_{i=1}^{n} \pi_i A_i).\frac{\partial^2 U}{\partial w^2}/\frac{\partial U}{\partial w}$ is a constant), a positive scaling of profit $(\sum_{i=1}^{n} \pi_i A_i)$ does not alter optimal choices. This implies that decision

functions are almost homogeneous of degree one in mean returns $\bar{\pi}$, degree two in moments of order two of π, and degree s in moments of order s of π. (See Pope for details).

Finally, it is well known that $\partial A^*/\partial\sigma = 0$ and $\partial A^*/\partial w = 0$ under risk neutrality (i.e., the utility function U(w) is linear). Alternatively, $\partial A^*/\partial\sigma \neq 0$ and/or $\partial A^*/\partial w \neq 0$ implies a departure from risk neutrality. In particular, under risk aversion (where U(G) is concave), risk will influence production decisions and thus the allocation of resources in agriculture.

IV - An Application under Government Price Support Programs:

The acreage decision model (3) involves uncertainty about both prices p and yields Y. In this section we consider the influence of government programs on the subjective probability distribution of output prices p. In particular, we focus our attention on the case of a price support program which places a floor under the market price. Consequently, the price support truncates the subjective probability distribution of prices. This in turn will affect expected prices as well as second and higher moments of the price distribution. The result is that a price support program will influence both price expectations and the riskiness of revenue.

In order to investigate the truncation effects of a price support program, it is useful to consider particular specifications of the probability distributions. Also, since we are considering multiple activities, distributional assumptions help provide analytical results for a multivariate truncated distribution. Since the effects of

multivariate truncation are best understood in the context of a normal distribution (see Johnson and Kotz; Maddala, 1983a), we limit our discussion to the normal case.[4/]

Let $X = (X_1, X_2, \ldots)$ be a vector of normally distributed random variables with mean $\bar{X} = (\bar{X}_1, \bar{X}_2, \ldots) = E(X)$ and variance $V(X) = E(X-\bar{X})'(X-\bar{X}) = \{\sigma_{ij}\}$, where E is the expectation operator. Now, assume that each random variable X_i is truncated from below at a level H_i. Define the truncated random variables

$$x_i = \begin{cases} H_i \text{ if } X_i < H_i \\ X_i \text{ if } X_i \geq H_i \end{cases}, \quad i = 1, 2, \ldots$$

Consider the standardized random variable $e_i = (x_i-\bar{X}_i)/\sigma_{ii}^{1/2}$ and define $h_i = (H_i-\bar{X}_i)/\sigma_{ii}^{1/2}$. The mean and variance of e_i are derived in the Appendix. The expected value of e_i is

$$\bar{e}_i = E(e_i) = \phi(h_i) + h_i\,\Phi(h_i) \tag{6a}$$

where $\phi(.)$ and $\Phi(.)$ are the standard normal density function and distribution function, respectively. The second moments of e_i are given by (see Appendix)

$$M_{ii} = E(e_i^2) = 1-\Phi(h_i) + h_i\,\phi(h_i) + h_i^2\,\Phi(h_i) \tag{6b}$$

and

$$M_{ij} = E(e_i\ e_j) = F(h_i,h_j)\ \rho_{ij} + [(1-\rho_{ij}^2)/2\pi]^{1/2}\ \phi(Z_{ij})$$
$$+ h_i\ \phi(h_j)\ \Phi(k_{ij}) + h_j\ \phi(h_i)\ \Phi(k_{ji})$$
$$+ h_i\ h_j\ \Phi(h_i,h_j),\ i \neq j \tag{6c}$$

where $F(h_i,h_j) = \text{Prob}(X_i \geq H_i,\ X_j \geq H_j)$, $\rho_{ij} = \sigma_{ij}/(\sigma_{ii}\ \sigma_{jj})^{1/2}$, $Z_{ij} = \{(h_i^2 - 2\ \rho_{ij}\ h_i\ h_j + h_j^2)/(1-\rho_{ij}^2)\}^{1/2}$, $k_{ij} = (h_i - \rho_{ij}\ h_j)/(1-\rho_{ij}^2)^{1/2}$, and $\Phi(h_i,\ h_j) = \text{Prob}(X_i < H_i,\ X_j < H_j)$. It follows that the mean, variance, and covariance of $x = (x_1,\ x_2,\ \ldots)$ are

$$\bar{x}_i = E(x_i) = \bar{X}_i + \sigma_{ii}^{1/2}\ \bar{e}_i \tag{7a}$$

$$V(x_i) = E(x_i-\bar{x}_i)^2 = \sigma_{ii}\ (M_{ii} - \bar{x}_i^2) \tag{7b}$$

and

$$\text{COV}\ (x_i,\ x_j) = E(x_i-\bar{x}_i)\ (x_j-\bar{x}_j) = (\sigma_{ii}\ \sigma_{jj})^{1/2}\ (M_{ij} - \bar{x}_i\ \bar{x}_j) \tag{7c}$$

Expressions (7) provide an analytical evaluation of the truncation effect of a price support program on the mean, variance, and covariance of commodity prices. These results will be used next to investigate the influence of government programs on corn and soybean acreage decisions.

V - Data and Estimation:

In sections II and III, the acreage decision $A^*(w,\ \bar{\pi},\ \sigma)$ was derived and its properties discussed. Here, we propose to specify and

estimate the acreage function $A^*(.)$. This is done by analyzing annual time series data for U.S. corn (i=1) and soybean (i=2) acreage decisions from 1954-1977. The acreage variables A_1 and A_2 measure acreage planted to each crop (in thousands of acres) and were obtained from various USDA publications. The costs of production per acre (c_1 and c_2) and the market prices (p_1 and p_2) were obtained from Gallagher, as were the diversion payment and the support price (p_1^s and p_2^s) policy variables.[5/] The consumer price q was measured by the consumer price index as reported by the Bureau of Labor Statistics. Yields per acre were obtained from USDA publications for both corn and soybeans.

In order to investigate supply behavior under risk, it is necessary to make assumptions about the nature of expectations about prices and yields. As argued in section II, we use normalized prices, all prices being deflated by the consumer price index. We also adopt simple adaptive expectations for the untruncated normalized prices by assuming

$$E_{t-1}\left(\frac{p_{it}}{q_t}\right) = \alpha_i + \frac{p_{i,t-1}}{q_{t-1}} \tag{8a}$$

where $\alpha_i = E\left(\frac{p_{it}}{q_t} - \frac{p_{i,t-1}}{q_{t-1}}\right)$ as measured by the corresponding

sample mean, and

$$\mathrm{Var}\left(\frac{p_{it}}{q_t}\right) = \{3\left[\frac{p_{i,t-1}}{q_{t-1}} - E_{t-2}\left(\frac{p_{i,t-1}}{q_{t-1}}\right)\right]^2 + 2\left[\frac{p_{i,t-2}}{q_{t-2}} - E_{t-3}\left(\frac{p_{i,t-2}}{q_{t-2}}\right)\right]^2$$

$$+ \left[\frac{p_{i,t-3}}{q_{t-3}} - E_{t-4}\left(\frac{p_{i,t-3}}{q_{t-3}}\right)\right]^2 \}/6. \tag{8b}$$

The assumption stated in (8a) that expected prices are a function of the average price of the previous year has been successfully employed in previous research (e.g. Houck et al.; Chavas et al.). Expression (8b) states that the variance of price is a weighted sum of the squared deviations of past prices from their expected values, with declining weights. These measurements of price risk are also consistent with those used previously in the literature (e.g. Lin; Traill; Brorsen et al.). Expressions (8) give the untruncated mean and variance of the price distributions. These results, along with the expressions in (7), provide measures of the mean and variance of the truncated multivariate price distributions associated with price supports p_1^s and p_2^2.

To measure yield expectations, actual yields were regressed on a trend variable. The resulting predictions were taken as expected yields. Likewise, the estimated residuals were used to generate the variance of yield as well as the covariance between price and yield. For simplicity, both the variance of yield and the correlation between price and yield were assumed constant over time.[6/]

Finally, initial wealth w was measured as the farm value of proprietor equity. It was obtained by multiplying the U.S. farm value of proprietor equity by the percentage of U.S. farm acreage planted to corn and soybeans.

Using these data, the acreage equations $A^*(w, \bar{\pi}, \sigma)$ were specified. Consider the first order Taylor series expansion

$$A_{it} = a_i + (\partial A_i/\partial w)\, w_{t-1} + \sum_{j=1}^{2} (\partial A_i/\partial \bar{\pi}_j)\, \bar{\pi}_{jt} + \sum_{k \geq j}^{2} \sum_{j=1}^{2} (\partial A_i/\partial \bar{\rho}_{jk})\, \bar{\sigma}_{jkt}$$

$$+ \theta_i A_{i,t-1} + u_{it},\ i = 1,2, \qquad (9)$$

where A_{it} is the number of acres planted to the $i^{\underline{th}}$ crop at time t, $\bar{\pi}_{jt} = E_{t-1}\{(p_{it}/q_t)\, Y_{it} - (c_{it}/q_t) \mid p_t \geq p_t^s\}$, is the truncated mean return per acre of the $j^{\underline{th}}$ crop, $\bar{\sigma}_{jjt} = \mathrm{Var}(p_{jt} \mid p_t \geq p_t^s)$ and $\bar{\sigma}_{jkt} = \mathrm{COV}(p_{jt}, p_{kt} \mid p_t \geq p_t^s)$ are the truncated variances and covariances of output prices, and u_{it} is an error term. Equation (9) is specified as a partial adjustment model where $(1-\theta_i)$ can be interpreted as the percentage of desired acreage adjustments that occurs from one year to the next. Letting $\beta_{ij} = \partial A_i^c/\partial \bar{\pi}_j$ be the compensated slopes with respect to $\bar{\pi}$ and using (4), it follows that equation (9) can be expressed alternatively as

$$A_{it} = a_i + \alpha_i\, (w_{t-1} + \sum_j A_j\, \bar{\pi}_{jt}) + \sum_j \beta_{ij}\, \bar{\pi}_{jt} + \sum_{k \geq j} \sum_j \gamma_{ijk}\, \bar{\sigma}_{jkt}$$

$$+ \theta_i\, A_{i,t-1} + u_{it},\ i=1,2, \qquad (10)$$

where $\alpha_i = \dfrac{\partial A_i}{\partial w}$ and $\gamma_{ijk} = \dfrac{\partial A_i}{\partial \sigma_{jk}}$. In the absence of <u>a priori</u> information about functional form, equation (10) provides a local approximation to the decision function $A^*(.)$. Also, note that the symmetry of (4) implies that $\beta_{ij} = \beta_{ji}$, $i \neq j$. Thus equation (10) is convenient for testing and/or imposing the symmetry restrictions (4).

Equation (10) can be used directly for an empirical analysis of acreage decisions for soybeans. The corn acreage equation (i=1) is

specified according to (10) except that corn diversion payments (DP) are also included as an intercept shifter, i.e., $a_1 = \bar{a}_1 + \delta$ DP. The model parameters are estimated by seemingly unrelated regression. This gives consistent and asymptotically efficient estimates and provides the basis for the empirical results presented next.

VI - Results and Implications:

The econometric model (10) is used to test various hypotheses about economic behavior under risk. The first hypothesis examined is the symmetry restriction (4) implied by expected utility maximization. Again, it should be emphasized that this test is general since the symmetry restriction holds for any risk preferences. The null hypothesis is associated with (4) is H_o: $\beta_{12} = \beta_{21}$ and the F-value for the test was found to be $F(1,25) = .433$. Thus, the symmetry restriction cannot be rejected at any usual levels of significance. The implication is that acreage decisions, as represented by equation (10), are consistent with the symmetry restriction implied by expected utility maximization. With the symmetry restriction imposed, the parameter estimates of equation (10) are presented in table 1. The estimated model explains historical variations in corn and soybean acreages well as indicated by the high R-squares. Also, many of the parameters are statistically significant and the corresponding revenue and risk elasticities appear in general to be of reasonable magnitude.

Having found evidence in favor of the expected utility model (3), we proceeded to test for the nature of risk preferences taking the symmetry restriction $\beta^*_{12} = \beta^*_{21}$ in (10) as maintained. As argued in

section III, the hypothesis of risk neutrality is tested as H_o: $\gamma_{ijk} = 0$ and $\alpha_i = 0$ for all i, j, k. The F-value for this test was found to be $F(8,26) = 5.193$, which implies that the null hypothesis can be rejected at the 5 percent level. In other words, the results indicate that agricultural risk preferences are not characterized by risk neutrality, a conclusion that is consistent with previous research (e.g. Just; Lin et al.).

If farmers are not risk neutral, what can be said about the nature of their risk preferences? As argued in section III, the hypothesis of constant absolute risk aversion can be tested as H_o: $\alpha_1 = \alpha_2 = 0$. The F-value for this test was $F(2,26) = 3.689$, indicating a rejection of the null hypothesis at the 5 percent level. This result provides evidence that the risk preferences of corn and soybean growers are not characterized by constant absolute risk aversion over the period of analysis.

The empirical results presented in table 1 also show positive wealth effects $\partial A^*/\partial w > 0$. In the single product case Sandmo has shown that a positive wealth effect in supply response can be interpreted as evidence of decreasing absolute risk aversion. To the extent that Samdmo's result holds in the multiproduct case, our analysis would therefore indicate that farmers are decreasingly absolute risk averse. In other words, higher initial wealth would tend to reduce the risk premium and stimulate production.[7]

The evidence of a positive wealth effect has several important implications. First, from equation (4), having wealth elasticities $\partial \ln A^*/\partial \ln w$ that are different from zero or one implies that the

uncompensated slope matrix $\partial A^*/\partial\bar{\pi}$ is <u>not</u> symmetric. Yet, the symmetry of uncompensated price slopes has been imposed as a maintained hypothesis in many previous studies of supply response (e.g. Shumway; Antle). The above result raises some important questions about the empirical findings reported in these studies: if the maintained hypothesis of symmetry of uncompensated price slopes is inappropriate, then the associated models are mispecified and the econometric results are suspect. By providing evidence that uncompensated price slopes are not symmetric, our analysis indicates that the implications of riskless production theory may not apply to supply response analysis under risk. It also suggests that more attention should be given to wealth effects in welfare and policy analysis (e.g., Pope and Chavas).

Second, finding evidence against the hypothesis of constant absolute risk aversion raises questions about the appropriateness of a mean-variance quadratic programming approach to risk analysis. Indeed, the quadratic programming approach is typically motivated under constant absolute risk aversion and normality which imply zero wealth effects. Our results suggest a need to incorporate explicitly a wealth variable in programming models of risk issues. Moreover, nonlinear programming methods may be needed to investigate more completely the effects of wealth and risk in a programming context (e.g., Kaylen et al.).

Third, the existence of wealth effects has a number of important policy implications. If corn-soybean farmers exhibit decreasing absolute risk aversion, then higher private wealth tends to offset their need for income and price protection (by reducing the Arrow-Pratt risk premium). To the extent that tax payers are not risk averse, this

provides a justification for income transfers to corn-soybean farmers in situations where the farmers' initial wealth is low. In other words, the efficiency of risk allocation could be seen as a possible motivation for government farm programs.

In order to obtain additional insights into the nature of risk preferences, the tests proposed by Pope were performed at the mean values of the sample data. Testing the hypothesis of constant relative risk aversion (CRRA) consists of testing whether a rescaling of terminal wealth has a zero effect on acreage decisions. The F-value for the CRRA hypothesis was found to be $F(2,26) = 5.147$. Also, testing the hypothesis of constant partial relative risk aversion (CPRRA) consists of testing whether a rescaling of profit has a zero effect on acreage decisions. The F-value for the CPRRA hypothesis was found to be $F(2,26) = 12.561$. Using normal significance levels, these results indicate that neither CRRA nor CPRRA appear to characterize the risk preferences of corn-soybean producers. In short, many of the simple utility function representations are not supported by our data.

The elasticities of acreage with respect to a proportional increase (rescaling) of terminal wealth were found to be .013 for corn and .537 for soybeans when evaluated at the sample means. In the single product context, a positive effect of rescaling terminal wealth on supply decision can be interpreted as evidence of decreasing relative risk aversion (Sandmo). To the extent that this result holds in the multiproduct case, our analysis would therefore indicate that farmers exhibit decreasing relative risk aversion.[8/] An immediate implication

of this result is that a tax on terminal wealth would tend to reduce agricultural supply.[9/]

Finally, the supply models were simulated at alternative support price levels. Because of the truncation effects, changing the support price levels will influence the means, variances and covariances of producer prices (see (7)). In other words, the effects of support prices on crop acreage can be complex. Selected static simulation results for the effects of support prices on expected prices, price risk, including both the variance and covariance of prices, and acres planted are reported in table 2. As expected, increasing the support price of a crop tends to stimulate acreage for that crop, although the relationship is clearly non-linear. For example, when the support price is much below the expected market price, the truncation effect is negligible and the price support program has only a limited impact on acreage decisions. Alternatively, as support price levels are increased, the truncation effects become larger, and the resulting impact on acreage decisions is more pronounced.

The cross-commodity price effects reported in table 2 are of interest since increasing the support price for a commodity tends to increase its expected price which in turn tends to decrease the acreage of the substitute commodity. However, the risk reducing effect of a price support program also influences acreage substitution. The net effect of the soybean support price on corn acreage is found to be negative (table 2). However, the net effect of the support price for corn on soybean acreage is found to be positive for low price support levels (e.g., effective support prices less than $1/bu), and negative

otherwise (see table 2). Thus, within some price range, the risk reducing effect of corn support prices on soybean acreage is positive and dominates the mean price effect. This indicates that government programs and risk response can affect both quantitatively and qualitatively the nature of economic adjustments. In particular, it emphasizes the importance of variance and covariance effects across commodities and the role of risk in substitution relationships.

VII - Conclusions:

This study has presented a framework for analyzing multiple acreage decisions under uncertainty. A household decision model that includes both output price and yield uncertainty was developed and a number of testable hypotheses derived. The resulting behavioral relationships were tested with a system of corn-soybean acreage equations. In contrast to previous studies, careful attention was given to the truncation effects of government price supports on the bivariate distribution of corn and soybean prices. Expressions relating the truncated means, variances, and covariances for joint normally distributed random variables were developed. Moreover, the present study differs from previous work in that wealth variables were included in the estimated acreage equations. Among other things, this faciliates tests of specific utility functions commonly used in empirical work (e.g., CARA, CRRA, and CPRRA).

The empirical results indicate that risk is important in corn-soybean acreage allocation decisions and that the estimated wealth effects are also important empirically. Interestingly, the symmetry

restriction implied by expected utility maximization could not be rejected; however, the results also suggest that many commonly used utility functions, including CARA, CRRA, and CPRRA specifications, are not supported by the data. Not only do these results cast doubt on the use of CARA utility functions, but they also suggest that different policy approaches, such as targeting benefits directly towards low-income producers, may be warranted.

The importance of considering risk in a multicrop framework was illustrated by simulating the acreage models at various corn and soybean support price levels. The model simulations illustrate that cross-commodity risk reduction is potentially important since there is some range over which increasing the support price for corn will actually result in more acres planted to soybeans. It should be emphasized that such results could not be obtained by viewing risk in a single commodity context.

More work is required to determine whether these results are specific to the corn and soybean markets. For instance, different outcomes might be obtained for wheat where cropping alternatives are limited (Burt and Worthington). The present model could also be couched in a rational expectations framework. Various price support policies could then be evaluated in a market equilibrium context. Finally, the truncation results should be explored for different parametric forms of the joint price distribution. Such work would illustrate the extent to which the normality assumption influences the empirical results.

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TABLE 1 - Parameter Estimates[a]/

	Corn Equation	Soybean Equation
intercept	70140* (8556)	-17270* (4449)
corn diversion payments ($/acre)	-98355* (13989) [-.122]	--
$\bar{\pi}_1$ ($/acre)	2978 (8179) [.015]	-24309* (5245) [-.239]
$\bar{\pi}_2$ ($/acre)	-24309* (5245) [-.158]	47644* (7761) [.582]
σ_1	165578600 (436216500) [.010]	-1669877000* (420712100) [-.189]
$\bar{\sigma}_{22}$	63188210 (72515550) [.017]	-256865000* (70055070) [-.116]
$\bar{\sigma}_{12}$	-564663000 (608183100) [-.036]	2196321000* (618585600) [.266]
$w + \sum_j A_j \bar{\pi}_j$	.026275* (.011473) [.296]	.013037 (.010331) [.268]
A_{ijt-d}	-.012791 (.116184)	.916321* (.125198)
R^2	.9976	.9933

a/ Standard errors are in parentheses below the parameter estimates. Elasticities evaluated at mean value are presented in brackets. "*" indicates a coefficient significantly different from zero at the 5 percent significance level.

TABLE 2 - Simulation of the Effects of Support Prices for Corn and Soybean*

Support Price of Corn ($/bu)	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8
Expected corn price	1.24	1.24	1.24	1.24	1.25	1.31	1.43	1.61	1.80
Variance of corn price	.054	.054	.054	.051	.041	.022	.007	.001	.0001
Covariance corn/soybean prices	.047	.047	.047	.046	.042	.030	.014	.003	.0005
Corn acres	71980	71980	71980	71992	72130	72702	73777	74868	75716
Soybean acres	41607	41608	41637	41834	42289	41826	38849	34617	30773

Support Price of Soybeans ($/bu)	.4	.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
Expected corn price	2.75	2.75	2.75	2.75	2.76	2.82	2.98	3.25	3.61
Variance of corn price	.267	.267	.267	.261	.235	.168	.081	.024	.004
Covariance corn/soybean prices	.059	.059	.059	.058	.055	.049	.029	.012	.003
Corn acres	72600	72600	72597	72576	72483	72229	71699	70652	68931
Soybean acres	41496	41497	41506	41568	41794	42187	42645	43832	46716

* Untruncated expected prices are $1.24 for corn and $2.75 for soybeans while support prices are $1.0 for corn and $2.50 for soybeans. All other variables are set equal to their sample means.

Appendix

The mean of e_i:

Let $\phi(.)$ be the standard normal density function. We have

$$E(e_i) = h_i \int_{-\infty}^{h_i} \phi(y)\, dy + \int_{h_i}^{\infty} y\, \phi(y)\, dy$$

$$= h_i\, \Phi(h_i) + \phi(h_i)$$

where $\Phi(.)$ is the standard normal distribution function.

The second moments of e_i:

a/ M_{ii}:

$$E(e_i^2) = h_i^2 \int_{-\infty}^{h_i} \phi(y)\, dy + \int_{h_i}^{\infty} y^2\, \phi(y)\, dy \qquad \text{(A1)}$$

But the second term in (A1) can be shown to be equal to: $1-\Phi(h_i) + h_i\, \phi(h_i)$ (e.g., see Maddala, 1983a, p. 365). It follows that $E(e_i^2) = h_i^2\, \Phi(h_i) + 1-\Phi(h_i) + h_i\, \phi(h_i)$.

b/ M_{ij}, $i \neq j$:

Let $\phi(.,.)$ be the bivariate standard normal density function. Then,

$$E(e_i e_j) = h_i h_j \int_{-\infty}^{h_j} \int_{-\infty}^{h_i} \phi(y,z)\, dy\, dz + h_i \int_{h_j}^{\infty} \int_{-\infty}^{h_i} y\, \phi(y,z)\, dy\, dz$$

$$+ h_j \int_{-\infty}^{h_j} \int_{h_i}^{\infty} z\, \phi(y,z)\, dy\, dz + \int_{h_j}^{\infty} \int_{h_i}^{\infty} z\, y\, \phi(y,z)\, dy\, dz, \qquad \text{(A2)}$$

Note that the second term in (A2) can be written as

$$h_i \int_{h_j}^{\infty} \int_{-\infty}^{h_i} y\, \phi(y,z)\, dy\, dz = h_i \int_{h_j}^{\infty} \int_{-\infty}^{\infty} y\, \phi(y,z)\, dy\, dz$$

$$- h_i \int_{h_j}^{\infty} \int_{h_i}^{\infty} y\, \phi(y,z)\, dy\, dz. \qquad \text{(A3)}$$

The first term on the right hand side of (A3) is equal to $h_i\, \phi(h_j)$. From Rosenbaum, and using the notation defined in the text, the second term on the right hand side of (A3) can be written as

$$-h_i\{\phi(h_j)[1-\Phi(k_{ij})] + \rho_{ij}\phi(h_i)[1-\Phi(k_{ji})]\}$$

which implies that

$$h_i \int_{h_j}^{\infty} \int_{-\infty}^{h_i} y\, \Phi(y,z)\, dy\, dz = h_i\phi(h_j) - h_i\{\phi(h_j)[1-\Phi(k_{ij})] + \rho_{ij}\phi(h_i)[1-\Phi(k_{ji})]\}.$$

By symmetry, the third term on the right hand side of (A2) is given by

$$h_j \int_{\infty}^{h_j} \int_{h_i}^{\infty} z\, \phi(y,z)\, dy\, dz = h_j\phi(h_i) - h_j\{\phi(h_i)[1-\Phi(k_{ji})] + \rho_{ij}\, \phi(h_j)[1-\Phi(k_{ij})]\}.$$

Likewise, following Rosenbaum the fourth term on the right hand side of (A2) can be shown to be

$$h_j \int_{h_j}^{\infty} \int_{h_i}^{\infty} zy\phi(z,y)dzdy = F(h_i,h_j)\rho_{ij} + \rho_{ij}h_i\phi(h_i)[1-\Phi(k_{ji})] + \rho_{ij}h_j\phi(h_j)[1-\Phi(k_{ij})] + [(1-\rho_{ij}^2)/2\pi]^{1/2}\ \phi(Z_{ij}).$$

After making the appropriate substitutions and collecting terms, (A2) can be shown to be

$$E(e_ie_j) = F(h_i,h_j)\ \rho_{ij} + [(1-\rho_{ij}^2)/2\pi]^{1/2}\ \phi(Z_{ij}) + h_i\ \phi(h_j)\ \Phi(k_{ij}) + h_j\ \phi(h_i)\ \Phi(k_{ji}) + h_ih_j\ \Phi(h_i,h_j)$$

where $F(h_i,h_j) = \Phi(h_i,h_j) + 1 - \Phi(h_i) - \Phi(h_j)$.

ENDNOTES

1/ The formulation in (3) is consistent with a yield function of the form

$$Y_i = \mathrm{Min}\{a_i(x_i),\ b_i(x_i)\},\ i=1,\ldots,n,$$

where x_i is a variable input (e.g. fertilizer), and $\mathrm{Min}\{a_i(x_i),\ b_i(x_i)\}$ is assumed to be a concave function of x_i. This is a kinked yield function if $\frac{\partial a_i}{\partial x_i} > \frac{\partial b_i}{\partial x_i} \geq 0$ at the point where $a_i(x_i) = b_i(x_i)$. Moreover, optimum input use x_i is not responsive to changing relative prices at the kink (at least within some range of prices). This formulation has been found to provide a reasonable representation of yield functions (e.g. Anderson and Nelson; Ackello-Ogutu et al.) In this context, letting r be the price of the input x_i, the variable cost of production per acre is $c_i = r\ x_i$ in equation (3) (within some range of prices).

2/ This is motivated by the intuition that higher initial wealth may tend to decrease the risk premium measuring the private cost of risk bearing. In other words, private wealth accumulation tends to reduce the need for insurance.

3/ Note that under constant absolute risk aversion, $\partial A^*/\partial w = 0$ and (5b) takes the form $\sum_{j=1}^{n} \frac{\partial A_i^*}{\partial \pi_j} (\bar{\pi}_j+\delta_j) = 0$. This illustrates the influence of risk preferences on the restrictions discussed by Chavas and Pope.

4/ The normal distribution has also been used widely for modeling truncation effects in a single commodity context. See, e.g., Shonkwiler and Maddala or Holt and Johnson.

5/ The support prices are "effective support prices" as constructed by Houck et al. and Gallagher. These are weighted averages of the target price and the loan rate, the weights being the percentage of planted acreage eligible for target price protection.

6/ The (untruncated) correlation between price and yield was estimated to be .2234 for corn and -.2788 for soybeans. The (untruncated) correlation ρ between p_1 and p_2 was also assumed to be constant for all years. The estimated value was $\rho = .576$.

7/ This is also consistent with much of the economic literature (e.g. Arrow; Binswanger).

8/ This result is also consistent with some of the empirical evidence obtained by Binswanger in rural India.

9/ The acreage elasticities with respect to a proportional increase (rescaling) of profit were found to be -.283 for corn and .269 for soybeans. Although our test indicates that farmers do not exhibit CPRRA, it is not clear from our results whether partial relative risk aversion is increasing or decreasing.