A NON-PARAMETRIC ANALYSIS OF AGRICULTURAL TECHNOLOGY

by

Jean-Paul Chavas

and

Thomas L. Cox*

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*Respectively Professor and Assistant Professor of Agricultural Economics, University of Wisconsin-Madison. The seniority of authorship is shared by the authors. The authors would like to thank two referees of the journal for helpful comments on an earlier draft of the paper. Department of Agricultural Economics, University of Wisconsin-Madison, Staff Paper No. 271, 1987.

A Non-Parametric Analysis of Agricultural Technology

Productivity measurement and technical change have received considerable attention in the literature (e.g. Jorgenson and Griliches; Sato, 1970; Stevenson; Binswanger; Peterson and Hayami; Antle; Lopez; Ball; Capalbo and Denny; Capalbo and Antle). Using a primal approach, technical progress is typically measured by changes in output that are not attributable to changes in inputs. Alternatively, the dual approach measures technical progress in terms of changes in cost (profit) not attributable to changes in input prices and output levels (output prices). This suggests a need to distinguish between the contributions of technical progress and those of returns to scale (e.g. Stigler) and input prices (e.g. Griliches).

However, this general distinction may not always be possible. For example, in the absence of a priori hypotheses on the structure of technical change, Diamond et al. have shown the non-identifiability of the elasticities of substitution and the bias in technical change.[1/] Similarly, Sato (1980) has shown that, under certain conditions, technical progress is undistinguishable from a scale effect. The suggestion that a set of production data can be generated by <u>more than one combination</u> of technology and technical change raises serious questions about the identifiability and measurement of technology and technical change.

Depending on the data as well as a priori hypotheses about the structure of technical change, the measurement of technical change may

be exactly identified, identified up to a range of indeterminacy, or not identified at all (Diamond et al.). Perhaps a particular parametric specification amounts to choosing a particular combination of technology and technical change, yielding model-dependent results.[2/] In this context, it is desirable to investigate technology and technical change in a way that does not depend on the parametric specification of the model.

This article presents a non-parametric analysis of agricultural technology and technical change under profit maximizing or cost minimizing behavior. This analysis extends the non-parametric approach of production decisions developed by Hanoch and Rothschild and Varian to include technical change (section II). The resulting non-parametric tests of production behavior, based on linear programming, are used to analyze U.S. agricultural technology from 1948 to 1983. This analysis provides some insights on the nature of U.S. agricultural technology and technical change and tests various separability hypotheses about the production function for U.S. agriculture. Although the non-parametric approach does not solve the identification problem raised by Diamond et al., the results have the advantage of not being dependent on a particular form of the production technology. The findings have implications for the more traditional parametric analysis of agricultural technology.

NON-PARAMETRIC TESTS OF PRODUCTION DECISIONS:

Consider a competitive firm that maximizes net return and faces the following decision problem

$$V(p,A,h) = \underset{x}{\text{Max}}\ p'x \quad \text{s.t. } g(x,A) \geq h \qquad . \tag{1}$$

where x is some netput decision vector (with positive elements corresponding to outputs while negative elements are inputs), p is the vector of corresponding prices, the function g(x,A) represents technology, A > 0 being a technology index, h is a scalar, and V(p,A,h) is the indirect objective function. The function g(x,A) is assumed to be strictly decreasing and concave in x. Likewise, the maximization hypothesis is maintained throughout the article.

The firm is observed choosing x T times, $x_1,\ldots,x_T$, and each observation is associated with a situation t characterized by market prices p_t and technology (h_t,A_t), $t = 1,\ldots,T$. The non-parametric approach to production analysis[3/] tests the consistency of the actual decisions $X = \{x_1,\ldots,x_T\}$ with the optimization problem (1) (i.e. whether x_t is equal to $x^*(p_t,A_t,h_t)$, the solution of max $\{p_t'x\colon g(x,A_t) \geq h_t\}$) without ad hoc specification of functional form for g(x,A), V(p,A,h) or $x^*(p,A,h)$.

The non-parametric testing of production decisions is based on the following proposition (see the proof in the Appendix).

<u>Proposition</u>: Given a set of decisions $X = \{x_1,\ldots,x_T\}$, each x_t corresponding with a situation (p_t,A_t,h_t), $t=1,\ldots,T$, then

<u>a/</u> if x_t is the solution of max $\{p_t'x\colon g(x,A_t) \geq h_t\}$, g(x,A) being a strictly decreasing and concave function of x, then $\lambda_t \varepsilon R$ exist such that

$$h_t + \frac{p_t'}{\lambda_t}(x_t - x_s) - g(x_s,A_t) \geq 0 \tag{2a}$$

$$\lambda_t > 0 \qquad (2b)$$

$$s,t=1,\ldots,T;$$

b/ if (2a) and (2b) are satisfied, then a function $G(x,A)$ exists such that x_t is the solution of $\max \{p_t'x: G(x,A_t) \geq h_t,\ x \in X\}$.

Equations (2a) and (2b) present a set of necessary and sufficient conditions for the decisions $X = \{x_1,\ldots,x_T\}$ to be consistent with the optimization problem (1) for some production technology. Consistency testing then checks for a solution to the set of inequalities in (2).[4/] By allowing for technical change, the above results extend the non-parametric analysis of production decisions proposed by Hanoch and Rothschild or Varian. Some special cases that are useful in the analysis of production technology are presented next.

Profit Maximization

Profit maximization is obtained from (1) when $h = 0$ and $g(x,A)$ denotes the implicit production frontier. Then, in the absence of technical change, $A_t = 1$, $t = 1,\ldots,T$, $g(x_s,1) = 0$ and expressions (2) become

$$p_t'(x_t-x_s) \geq 0,\ s,t=1,\ldots,T, \qquad (3)$$

which is Varian's weak axiom of profit maximization (Varian, p. 584). Note the "revealed preference" interpretation of (3): if profits have been maximized given p_t, then $p_t'x_t$ should be greater than or equal to the profits $p_t'x_s$ generated by any other set of outputs and inputs

(netputs) evaluated at p_t. In the presence of technical change, given h $= 0$, $x = (\bar{x},\bar{\bar{x}})$, $p = (\bar{p},\bar{\bar{p}})$, $\bar{x}$ being a scalar with corresponding price $\bar{p}$, and $g(x_t, A_t) = A_t \cdot f(\bar{\bar{x}}_t) - \bar{x}_t$ is the implicit production function, then (2) becomes

$$\bar{x}_s(1 - \frac{A_t}{A_s}) + \frac{\bar{p}_t'}{\lambda_t}(x_t - x_s) \geq 0, \ \lambda_t > 0$$

or, using $\bar{p}_t = \lambda_t$ (the first-order condition with respect to $\bar{x}$),

$$-A_t \cdot \bar{p}_t \bar{x}_s + A_s \cdot [\bar{p}_t \bar{x}_t + \bar{\bar{p}}_t'(\bar{\bar{x}}_t - \bar{\bar{x}}_s)] \geq 0, \ s,t=1,\ldots,T. \qquad (4)$$

In this case, $A_t \cdot f(\bar{\bar{x}}_t)$ represents the production function for $\bar{x}_t$. This specification implies Hicks neutral technical change since the marginal rate of substitution between any element of $\bar{\bar{x}}$ is not affected by technical change. Thus, equation (4) provides a generalization of the weak axiom of profit maximization under technical change. In the single output case where $\bar{x}$ denotes output, then $\bar{x} = A \cdot f(\bar{\bar{x}})$ corresponds to an output-augmenting specification of technical change commonly found in the literature. Although this specification of Hicks neutral technical change is not the most general (see below), its empirical validity can be investigated non-parametrically using equation (4).

Cost Minimization:

Cost minimization is obtained from (1) when h_t denotes output (in the single output case), $g(x_t,A_t)$ is the explicit production function and x_t is the input vector (defined here to be negative). Again, if

technical change is output augmenting, i.e. $g(x_t,A_t) = A_t \cdot f(x_t)$, then we have the case of Hicks neutral technical change just discussed. In this context, equations (2) become

$$h_t + \frac{p_t'}{\lambda_t}(x_t - x_s) - \frac{A_t}{A_s} h_s \geq 0, \ \lambda_t > 0$$

or

$$\alpha_t h_t - \alpha_s h_s + \beta_t \cdot p_t'(x_t - x_s) \geq 0, \quad \beta_t > 0, \ \alpha_t > 0, \ s,t=1,\ldots,T, \quad (5)$$

where $\alpha_t = 1/A_t$ and $\beta_t = 1/\lambda_t A_t$. Equation (5) generalizes some non-parametric results found in the literature. In particular, in the absence of technical change (i.e., where $A_t = 1, t=1,\ldots,T$), then (5) reduces to

$$h_t - h_s + \beta_t \cdot p_t'(x_t - x_s) \geq 0, \ \beta_t > 0, \ s,t=1,\ldots,T$$

which is Varian's strong axiom of cost minimization (Varian, p. 583). This strong axiom in turn implies the weak axiom of cost minimization (see Hanoch and Rothschild; Varian)

$$p_t'(x_t - x_s) \geq 0 \text{ for } h_s \geq h_t, \ s,t=1,\ldots,T.$$

Again note the "revealed preference" interpretation of this non-parametric test: given prices p_t, if costs have been minimized by the netputs (negative inputs) vector x_t, then $-p_t'x_t$ should be less than or equal to $-p_t'x_s$, the cost generated by any other set of inputs producing

an output at least equal to h_t. Thus, equation (5) provides a basis for an empirical analysis of output augmenting (Hicks neutral) technical change under the cost minimization hypothesis.

Weak Separability:

A production function is weakly separable in x if it takes the form $f[g(x),A,z]$ where $g(x)$ is an aggregator function of x , f is strictly monotonic in g and "z" represents other arguments (besides x and A) of the production function. In this case, the choice of x for a competitive firm must satisfy $x_t = \text{argmax}\ \{p_t'x\colon g(x) \geq h_t\}$ for some h_t, which is a special case of (1). Then, equations (2a) and (2b) become

$$h_t - h_s + \gamma_t p_t'(x_t - x_s) \geq 0,\ \gamma_t > 0,\ s,t=1,\ldots,T, \tag{6}$$

where $\gamma_t = 1/\lambda_t$. This is the test for weak separability presented by Varian (p. 588). If the vector x is a subset of the production netputs, then (6) allows non parametric testing of the separability of some netputs from other netputs and technology in the production function. Note that this implies separability from technical change as well.

On the other hand, if the vector x includes all the production netputs, then (6) amounts to a test of the weak separability of all netputs from the technology index A. But this weak separability is equivalent to stating that the marginal rate of substitution between any two netputs is independent of technical change, which is the definition of Hicks neutral technical change. In fact, this is the characterization of Hicks neutral technical change in its most general

form (see Blackorby et al.). Thus, condition (6) allows the nonparametric testing of Hicks neutral technical change without an ad hoc specification of technology or changes in technology.

EMPIRICAL IMPLEMENTATION:

The tests of the consistency of production decisions with some underlying optimization model involves a solution to a set of inequalities (see (2) or its special cases (3), (4), (5) or (6)). Given a set of observations on prices and quantities, the tests are empirically implemented as follows.

The empirical implementation of (3) is straightforward since the inequalities in (3) involve only observable variables, p_t, x_t, $t=1,\ldots,T$. In this case, the procedure is to check whether the inequalities in (3) are satisfied for all observations, $t=1,\ldots,T$.

However, the inequalities (4), (5) or (6) involve variables that are not directly observable: the technology index A_t in (4), $\alpha_t = 1/A_t$ and β_t in (5), and h_t and γ_t in (6). In these cases, the nonparametric test consists in finding whether a set of values taken by the unobserved variables would satisfy the corresponding inequalities. Since the inequalities (4), (5), or (6) are linear in the unobserved variables, the test is formulated as a linear programming problem.

Let y be the vector of unobserved variables associated with T observations in equations (4), (5), or (6). Then, the inequalities (4), (5), or (6) can be written as $A'y \geq c$, given appropriate definitions of the matrix A and the vector c.

Consider the linear programming problem

$$\underset{y}{\text{Min}} \{b'y : A'y \geq c,\ y \in C\} \tag{7}$$

where the vector b is arbitrarily defined so that problem (7) is necessarily bounded and C is the cone domain of y. Either problem (7) has a solution, or it is infeasible. That is, if the inequalities (4), (5), or (6) have a solution, this solution is necessarily a feasible solution to problem (7). Alternatively, if problem (7) is infeasible, then the inequalities (4), (5), or (6) cannot be satisfied. Thus, verifying that equations (4), (5), or (6) have a solution is equivalent to checking for a feasible solution to the linear programming problem (7).

For $T > 3$, the number of constraints in the linear programming problem (7) will exceed the number of activities. In this case, it is computationally convenient to consider the linear programming problem dual to (7)

$$\underset{z}{\text{Max}} \{c'z : b-Az \in C^*,\ z \geq 0\} \tag{8}$$

where C^* is the polar cone of C. It is well known that (8) has an optimal solution if and only if (7) has an optimal solution (e.g. Luenberger; Sposito). Alternatively, if problem (7) is infeasible, then (8) is either unbounded or infeasible.

Here we solve the dual formulation (8) (using the simplex method) in the nonparametric testing of production decisions. If (8) has an

optimal solution

for some values of the unobserved variables ~~and the~~ THEN production data are consistent with the associated hypothesis. Alternatively, if (8) is either unbounded or infeasible, then the inequalities (4), (5) or (6) cannot be satisfied and the production decisions are not consistent with the corresponding hypothesis. Although these tests are not statistical tests (with associated probability statements), they can provide useful information in the analysis of technology, as illustrated next.

AN ANALYSIS OF AGRICULTURAL TECHNOLOGY:

The above methodology is applied to an aggregate time series analysis of U.S. agricultural technology. Implicit price and associated quantity indexes (1977=1.00) for the U.S. agricultural sector are taken from Capalbo and Vo. These annual data cover the years 1948 through 1983. The analysis includes 9 input measures (labor: (1) family and (2) hired; capital: (3) land, (4) structures, and (5) other capital; and materials: (6) energy, (7) fertilizers, (8) pesticides, and (9) miscellaneous), and six output measures ((1) small grains, (2) coarse grains, (3) field crops, (4) fruits, (5) vegetables, and (6) animal products).

The period 1948-83 is analyzed, as well as the sub-periods 1948-59, 1960-71, and 1972-83. The sub-periods isolate the economic context of each decade; in particular, the energy price shocks, high inflation, and surging export demand of the 1970's are isolated from the considerably more stable earlier time periods.

An empirically convenient dimension of the proposed non-parametric methodology is that groups of years can be explored for data consistency

with particular hypotheses. For example, if data consistency for 1948-83 is rejected, the 1948-59, 1960-71, and 1972-83 sub-periods are tested separately to help identify the years causing rejection. Hence, these non-parametric procedures can complement more traditional parametric analysis as heuristic tools for identifying time periods that are consistent with various maintained hypotheses.

The analysis begins with a non-parametric evaluation of various separability hypotheses via equation (6). This consists of investigating the existence of an aggregator function for particular groups of netputs in the characterization of the production function. In particular we test for the existence of output separability, the separability of capital inputs and labor inputs from other netputs and technology (as discussed by Capalbo and Denny), capital inputs separability (land, structures, and other capital), labor inputs separability (family and hired), and materials inputs separability (energy, fertilizer, pesticide, and miscellaneous inputs). These separability hypotheses are frequently used as maintained in empirical work. Furthermore, parametric tests of these hypotheses typically depend on the maintained functional form. In contrast, the non-parametric tests do not depend on the form of the production function. Table 1 summarizes the non-parametric results.

The rejection of the hypothesis of output aggregation for the years 1948-83 suggests that a single output characterization of U.S. agricultural technology is not consistent with the data and would involve aggregation bias. Rather, a multiproduct (versus single product) specification of U.S. agricultural technology is supported by

Table 1. Non-Parametric Separability Results for U.S. Agricultural Technology.*

Years	1948/83	1948/59	1960/71	1972/83
Output Separability:	R	C	C	C
Input Separability:	R	R	C	C
Capital and Labor Separability:	R	R	C	R
Capital Separability: K(land, structures, other capital)	C	-	-	-
Labor Separability: L(family, hired)	C	-	-	-
Material Separability: M(energy, fertilizer, pesticide, miscellaneous)	C	-	-	-

* R indicates rejection of data consistency with the appropriate weak separability test (6) (i.e. an unbounded or infeasible solution to the corresponding dual linear programming problem (8)), while C indicates data consistency with the non-parametric test.

Source: Computations by the authors.

these results (e.g. Shumway; Antle). However, further analysis of data consistency indicates that the 1948-59, 1960-71, 1972-83 and 1960-83 sub-periods support the output separability hypothesis. These results indicate that interaction of data from the 1948-59 and 1960-83 periods causes the 1948-83 rejection since 1948-59 and 1960-83 are consistent with the output separability hypothesis. These non-parametric tests can easily accommodate adding earlier years to the 1960-83 data so as to identify the longest time span consistent with the hypothesis of interest.

Non-parametric tests of the input separability hypothesis yield similar results: the data do not support the existence of a single aggregator function for all inputs in the production function. This result is obtained for the whole sample period (1948-83) as well as the 1948-59 sub-period. However, the 1960-71 and 1972-83 periods are consistent with this hypothesis. Again, the 1948-59 period is the source of rejection of data consistency for the 1948-83 period.

Next the hypothesis of weak separability of capital and labor inputs from all other netputs and technology is evaluated. The data for 1948-83, 1948-59, and 1972-83 are inconsistent with this separability hypothesis. However, the 1960-71 (as well as 1960-78) period is consistent with the non-parametric test of this hypothesis.

Next we non-parametrically evaluate data consistency with hypotheses commonly used to justify input aggregations: capital separability, labor separability and materials separability. As indicated in Table 1, the 1948-83 U.S. data are consistent with each of these hypotheses. Hence, our analysis indicates the existence of single

measures of aggregate capital, aggregate labor and aggregate materials inputs. This provides support for analyzing U.S. agricultural technology using three aggregate inputs (labor, capital and material), a practice commonly found in the literature.

Table 2 summarizes the non-parametric tests of technical change in U.S. agriculture. Profit maximization without technical change is tested via equation (3) and Hicks neutral technical change (defined as netputs separability from the technology index) is tested using equation (6). Both hypotheses are tested using disaggregate, multiple outputs as well as a single, aggregate output specification. Aggregate output is measured using a Theil-Thornquist quantity index; the corresponding output price is an implicit price index obtained by dividing aggregate value by the quantity index. Given output aggregation, equations (4) and (5) yield non-parametric tests of output augmenting technical change under profit maximization and cost minimization, respectively. As noted previously, equation (4) corresponds to a more restrictive specification of Hicks neutrality than netput technical separability via equation (6). Similarly, equation (4) is more restrictive than equation (5) since the assumption of profit maximization is more restrictive than the assumption of cost minimization: the former implies the latter, but not vice versa.

The separability results reported above indicate a lack of data consistency with output separability over the 1948-83 period, but not for 1948-59, 1960-71, and 1972-83. Thus, the results of Table 2 for 1948-83 using aggregate output should be interpreted cautiously since the maintained hypothesis of output aggregation over this period is not

Table 2. Non-Parametric Tests of Technical Change for U.S. Agriculture.*

Years	1948/83	1948/59	1960/71	1972/83
		Multiple Outputs		
Profit Maximization (without Technical Change):	R	C	R	R
Hicks Neutral Technical Change:	C	-	-	-
		Aggregate Output		
Profit Maximization (without Technical Change:	R	R	R	R
Hicks Neutral Technical Change:	C	-	-	-
Output Augmenting Technical Change: Profit Max.	R	R	C	R
Output Augmenting Technical Change: Cost Min.	R	R	C	C

* R indicates rejection of the corresponding non-parametric test while C indicates date consistency with the tested hypothesis.

Source: Computations by the authors.

supported. However, the aggregate output results for the other time periods are supported by the previous output separability results.

As indicated in Table 2, these data are inconsistent with profit maximization without technical change for most time periods, using either a single or a multiple output specification. This is strong evidence of technical change in U.S. agriculture over the last few decades. In contrast, the general hypothesis of Hicks neutral technical change via netput technical separability using equation (6) is consistent with these data over the full 1948-83 period. This non-parametric result is obtained for both the single and the multiple output specification. This result implies a production function exhibiting netput technical separability from the technology index A. Thus, while the evidence of technical change in U.S. agriculture is strong, this change can be characterized by Hicks neutrality, where the marginal rate of substitution between any netput is independent of the technology index. This non-parametric result is obtained without making a priori assumptions about the form of the production function. It suggests that previous parametric evidence which rejects the Hicks-neutral hypothesis for U.S. agriculture (e.g. Binswanger; Antle) may be sensitive to functional form. This finding is consistent with Diamond et al.'s arguments concerning the non-identifiability of the bias in technical change without a priori knowledge of production structure. Thus, the parametric investigation of the bias in technical change and of Hicks' induced innovation hypothesis[5/] appears to be sensitive to the parametric specification of the model.

Finally, output-augmenting technical change is evaluated under profit maximization and cost minimization using the aggregate output specification. Neither of these hypotheses are consistent with the data over the 1948-83 and 1948-59 periods; thus, output augmentation is a fairly restrictive form of Hicks neutral technical change. In contrast, both hypotheses are consistent with the data over the 1960-71 period. While the cost minimization hypothesis of output augmenting technical change is also consistent with the data over the 1972-83 period, profit maximization is rejected for this time period. These results indicate that the non-parametric tests can distinguish maintained hypotheses with varying degrees of restrictiveness.

CONCLUDING COMMENTS

This article presents a non-parametric analysis of U.S. agricultural technology and technical change under profit maximizing or cost minimizing behavior. The non-parametric results of Hanoch and Rothschild, and Varian are extended to incorporate output augmenting (Hick's neutral) technical change. Given the identification and measurement difficulties confronting parametric approaches to technical change analysis (Diamond et al.; Sato), the proposed extensions of the non-parametric approach to include technical change hypotheses provide a powerful complement to the more traditional parametric approaches. The non-parametric approach presented here can be empirically implemented with a standard linear programming algorithm.

Application of this non-parametric approach to U.S. agricultural data from 1948-83 supports the (commonly used) specification of

agricultural technology as a function of three aggregate inputs: capital, labor, and materials. Some support for output aggregation is also found but only for the 1960-83 period. This indicates that agricultural technology is better represented by a multi-product specification (rather than a single product specification).

Profit maximization without technical change is rejected for most time periods and output specifications evaluated. This is interpreted as strong evidence of technical change in U.S. agriculture. In contrast, the non-parametric results support the hypothesis of Hicks neutral technical change, i.e. the existence of a production function where netputs are weakly separable from the technology index. These results suggest that previous parametric evidence concerning the bias in technical change is likely sensitive to the parametric specification used. This raises questions about the validity of some of the evidence presented in the literature in favor of the induced innovation hypothesis. However, a more restrictive formulation of Hicks neutrality, that is, output-augmenting technical change under profit maximization or cost minimization, is found to be inconsistent with the data for the 1948-83 period.

The non-parametric approach allows the testing of several maintained hypotheses which do not depend on a particular parametric specification of production relationships. The proposed non-parametric approach can provide a heuristic complement to traditional parametric specification/estimation methodology. In particular, the proposed methodology can be used to investigate particular hypotheses and identify time periods which support these hypotheses, prior to parametric analysis.

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Footnotes

1/ Technical change is Hicks neutral if the marginal rate of substitution between netputs is not affected by the change (see Blackorby et al.). Non-neutral technical change is said to be biased. For example, biased technical change is labor saving (capital using) if the marginal product of capital increases relative to the marginal product of labor, ceteris paribus. Diamond et al. show that in the absence of prior information, one may not be able to identify simultaneously the elasticities of substitution and bias in technical change. This means that there can exist more than one combination of elasticities of substitution and technical change that can equally well rationalize observed production data.

2/ The use of flexible functional forms (e.g. translog or generalized Leontief for a production, cost or profit function) does not help solve the identification problem for technical change. Rather a priori information on technology or on the nature of technical change is needed to solve the identification problem (e.g. see Diamond et al.). For example, assuming a specific technology would fix some parameters of the problem and can allow the identification of the bias in technical change. However, the technical change results would typically be dependent on the assumed nature of technology. To the extent that economists have weak a priori information about the nature of technology (e.g. the choice of functional forms), this can be quite undesirable.

3/ For a related non-parametric analysis of production efficiency, see Diewert and Parkan.

4/ The proposed non-parametric test is not a statistical test. Rather, it checks a set of inequalities which guarantees the existence of a production function that can rationalize a set of data in the context of the maximization hypothesis (1).

5/ Hicks' induced innovation hypothesis states that relative prices influence the direction of innovative activities and hence the direction of technical progress. For example, a rise in the price of labor with respect to the price of capital would induce a labor-saving (capital-using) technical change.

APPENDIX

Proof of the Proposition:

Consider the saddle point problem x^*, $\lambda^* \geq 0$ such that

$$L(x,\lambda^*,p,A,h) \leq L(x^*,\lambda^*,p,A,h) \leq L(x^*,\lambda,p,A,h), \ \forall x, \ \lambda \varepsilon R^+ \qquad (A1)$$

where $L(x,\lambda,p,A,h) = p'x + \lambda\ [g(x,A)-h]$.

If $g(x,A)$ is strictly decreasing and concave in x, then the saddle-point criterion (A1) is a necessary and sufficient condition for x^* to be a solution to the optimization problem Max $\{p'x\colon g(x,A) - h \geq 0\}$ (see Sposito; Karlin).

Denote the saddle-point of $L(x,\lambda,p_t,A_t,h_t)$ in (A1) by $\lambda_t = \lambda^*(p_t,A_t,h_t)$ and $x_t = x^*(p_t,A_t,h_t)$. The saddle point characterization (A1) implies that $\lambda_t\ [g(x_t,A_t)-h_t] = 0$, and

$$L(x_s,\lambda_t,p_t,A_t,h_t) \leq L(x_t,\lambda_t,p_t,h_t) = p_t'x_t.$$

This yields

$$p_t'\ x_s + \lambda_t[g(x_s,A_t)-h_t] \leq p_t'\ x_t, \ \lambda_t \geq 0.$$

Given that $g(x,A)$ is strictly decreasing in x, it follows that $\lambda_t > 0$, $h_t = g(x_t,A_t)$ and

$$h_t + \frac{p_t'}{\lambda_t}(x_t-x_s) - g(x_s,A_t) \geq 0, \ \lambda_t > 0 \qquad (A2)$$

This proves (a).

To prove (b), define the function

$$G(x_s,A_s) = g(x_s,A_s) + \underset{t}{\mathrm{Min}}\ \{h_t + \frac{p_t'}{\lambda_t}(x_t-x_s) - g(x_s,A_t)\} \qquad \text{(A3)}$$

Note that (A2) implies that $\lambda_s[g(x_s,A_s)-h_s] \le 0$. If x_s is feasible, this yields $\lambda_s[g(x_s,A_s)-h_s] = 0$, or $h_s = g(x_s,A_s)$ given $\lambda_s > 0$. Also, we have $G(x_s,A_s) \le h_s$ from (A3) while $G(x_s,A_s) \ge g(x_s,A_s)$ from (A2). Given $h_s = g(x_s,A_s)$, it follows that $G(x_s,A_s) = g(x_s,A_s)$, i.e. $G(x_s,A_s)$ gives a representation of $g(x_s,A_s)$ for all data points, $s=1,\ldots,T$.

From (A2) and (A3) and given $X = \{x_1, x_2,\ldots,x_T\}$, we have

$$\underset{x}{\mathrm{Max}}\ \{p_s'x\colon h_s \le G(x,A_s),\ x \in X\} \le \underset{x}{\mathrm{Max}}\ \{p_s'x\colon h_s \le g(x,A_s) + h_s$$

$$+ \frac{p_s'}{\lambda_s}(x_s-x) - g(x,A_s),\ x \in X\} = \mathrm{Max}\ \{p_s'x\colon p_s'x_s \ge p_s x,\ x \in X\} = p_s'x_s.$$

It follows that $x_s = \mathrm{argmax}\ \{p_s'x\colon G(x,A_s) \ge h_t,\ x \in X\}$.