POLLUTION, MARKET FAILURE, AND OPTIMAL POLICY
IN AN ECONOMYWIDE FRAMEWORK

by

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Abstract

Pollution is an economic problem because it involves various kinds of market failure. First, much pollution generation is an unpriced externality from production and consumption activities. Second, some kinds of pollution represent a public good, with a "missing market" for pollution control activities. Starting from seminal work by Leontief, a number of economywide models have been developed which consider pollution within an input-output framework and capture the production externality. This paper provides a more general empirical modeling framework which can be used to analyze optimal policy choices when pollution is also treated as a public good. The framework involves three components. First, a multisector, economywide, computable general equilibrium (CGE) model is extended to include the externality characteristics of pollution generation. Second, pollution and cleaning activities are explicitly included in a social welfare function. Finally, the social welfare function and the CGE model are joined into a nonlinear programming model. The CGE model equations serve as the constraints in the program, and pollution taxes and a government cleaning activity are included as policy instruments. The programming model simultaneously determines the optimum levels of these instruments, as well as the resulting market equilibrium. The model is implemented with stylized data and is used to explore tradeoffs between different government policy instruments in controlling pollution in various first-best and second-best scenarios. The empirical results demonstrate the feasibility of including pollution within an optimizing framework in an economywide model and support the view that it is important to devise pollution control policies that work through market mechanisms.
Introduction

In a seminal article, Leontief (1970) incorporated pollution and pollution control activities in an economywide, input-output framework. Leontief treated pollution as an unpriced byproduct of sectoral production—a negative intermediate input—and also included a pollution removal or "cleaning" activity in the model. He showed that the standard open input-output model could be extended to show how different vectors of final demand, including demand for net pollutants, could be achieved. He also demonstrated that the input-output model could be used in dual form to solve for the implicit price of the unmarketed pollutant.

The input-output framework captures important features of pollution generation and control. First, much pollution is an unpriced byproduct of production activities and so generates externalities in the economy. Second, linkages among sectors through flows of intermediate inputs are important in sorting out the direct and indirect contributions of different sectors to overall pollution. The aggregate level of pollution is sensitive to changes in the structure of final demand, as well as to the demand for cleaning activities. Finally, cleaning or pollution removal activities use resources and are hence costly. Input-output models provide a consistent framework for evaluating the costs of alternative pollution control policies.

Since the Leontief article, there has been continuing work analyzing pollution with input-output models. The open Leontief model has some well-known drawbacks: final demand (including that for pollution control) must be fixed exogenously and the model is demand-driven, with no resource constraints. Some of the later work has sought to remedy these deficiencies while remaining within the linear framework. There has also been some work incorporating pollution into nonlinear computable general equilibrium (CGE) models. A CGE model simulates the workings of product and factor markets, solving for a set of prices that clear all markets, equating supplies and demands of goods and factors. Dufour, Harrington, and Rogers (1988) have built a CGE model which follows closely the specification of the Leontief model, using his stylized data.

1 Although Leontief did not do so, it would be very easy to extend his model to include pollution as a byproduct of consumption activities (e.g., air pollution generated by automobiles).

2 See, for example, Ahmed (1973) who introduces a symposium on the use of input-output models for analyzing pollution and energy. Miller and Blair (1985) review much of the literature.

3 See, for example, Flick (1974), Leontief (1974), Steenge (1978), and Rhee and Miranowski (1984).

4 Surveys of CGE models are provided by Dervis, de Melo, and Robinson (1982); Robinson (1989); and Shoven and Whalley (1984). Devarajan (1988) and Bergman (1988) survey CGE models focused on energy and resource issues. Jorgenson and Wilcoxen (1990) have built a dynamic CGE model of the United States to explore the impact on growth of environmental regulation policies. Bergman (1989) analyzes similar issues using a CGE model of Sweden which includes emissions and emission-
The model by Dufournand, Harrington, and Rogers is very close to "standard" CGE models in the literature, with the addition of sectoral pollution coefficients and a "cleaning" activity (following the treatment by Leontief). Their CGE model incorporates constant elasticity of substitution (CES) sectoral production functions and a single household with a Cobb-Douglas utility function, yielding an expenditure function with constant value shares. They assume that all pollution will be removed by the government, which demands the output of the cleaning sector. Thus, pollution is implicitly seen as a public good and there is no private demand for the cleaning activity. In their model, the cleaning activity is financed either by a lump-sum income tax or an indirect tax on polluting sectors with sectoral rates set so that each sector pays just enough tax to clean the amount of pollutants it generates. They make the extreme assumption that the correct level of pollution is zero, and the model is used to compare the effect of two different ways of financing the necessary cleaning activity.

Models which follow Leontief and treat the demand for cleaning or pollution removal as exogenous cannot be used to determine the social demand for cleaning or optimal policy with regard to taxing polluters. This paper extends the economywide framework to include an explicit representation of pollution in a social welfare function. This social welfare function is then married to a CGE model that incorporates pollution as a production externality and also includes a cleaning activity. The CGE model differs from that of Dufournand, Harrington, and Rogers in its specification of sectoral production technology and in the assumed consumer behavior. The resulting model is then used to determine the optimal mix of taxes and cleaning, accounting for the production externality and public good properties of pollution and also for society's "taste" for pollution reduction.

Pollution, Welfare, and Demand

Pollution is a "bad" that detracts from utility. The simplest way to deal with such goods in demand analysis is to introduce a cleaning activity which has a positive effect on utility. Garbage is a bad; garbage collection is a good. Even abstracting from the public-good aspect of pollution, there are three problems with simply introducing the cleaning activity as a good in utility and demand analysis. First, there will no demand for the cleaning activity if there is no pollution. No garbage, no demand for garbage collection. Similarly, reductions in pollution should increase utility, even if they occur for reasons other than increased cleaning. Second, the consumer should be indifferent between purchasing the cleaning activity directly or having someone else provide it. A normal good generates utility by being directly consumed. Cleaning generates utility by removing dirt, and the consumer does not care who does the cleaning. Third, there is evidence that there are threshold effects in individual and social tolerance of pollution. Up to some critical level, either the pollution is

control activities. Devarajan (1990) discusses how CGE models of developing countries might be extended to include environmental issues.
not "damaging" or individuals tolerate it; and cleaning is not demanded until the flow of dirt reaches the critical value.\(^5\)

One approach is to write a utility function of the following form:

\[
U[C_1, \ldots, C_{n-1}, (C_n - D)]
\]

where \(C_k, k=1, \ldots, n-1\) are normal goods, \(C_n\) is the cleaning activity, and \(D\) is the flow of dirt.\(^6\) In this formulation, the consumer is indifferent between increasing \(C_n\) or decreasing \(D\). An obvious candidate for a specific functional form for the utility function is the Stone-Geary function:

\[
U = \prod_{i=1}^{n} (C_i - \gamma_i)^\gamma_i
\]

where the \(\gamma\) parameters are subsistence minima. In the normal Stone-Geary function, these minima parameters are constants. To incorporate pollution, the parameter for the cleaning good will be a function of the amount of dirt, \(D\). The \(\gamma\) parameters are defined as:

\[
\begin{align*}
\gamma_k &= \gamma_k \quad k = 1, \ldots, n-1 \\
\gamma_n &= D - D_0
\end{align*}
\]

where \(D_0\) is a threshold level of dirt.

Maximizing the Stone-Geary utility function subject to an income constraint generates a system of expenditure functions called the linear expenditure system (or LES).\(^7\) The system is given by:

\[
P_i \cdot C_i = P_i \cdot \gamma_i + \beta_i \cdot Y^S
\]

where \(P_i\) are prices, \(\beta_i\) are marginal expenditure shares, and \(Y^S\) is defined as "supernumerary" income, which is income not already committed to purchasing the minima. It is defined as:

\[
Y^S = Y - \sum_{i=1}^{n} P_i \cdot \gamma_i
\]

where \(Y\) is total income and the \(\gamma\) parameters are defined above.

Following the Leontief approach, dirt will be generated using fixed coefficients times sectoral output. Define \(D\) as the total amount of dirt net of any government cleaning. That is:

\[^5\]Note that, in this discussion, pollution is treated as a flow. Some models treat pollution as a stock which directly enters the utility function. Alternatively, one could view the stock of pollution as generating a flow of "pollution services" which detracts from utility. The models discussed below are all in flow terms. Devarajan (1990) discusses how pollution stocks might be incorporated into a dynamic CGE model.

\[^6\]There is an extensive literature on optimal growth models which include pollution in the social welfare function, usually as a stock variable. See Kamien and Schwartz (1982), who survey this literature and also briefly discuss the issue of incorporating critical values of pollution in the utility function. Theoretical models by Forster (1973) and Gruver (1976) treat pollution as a flow and appear close in spirit to the empirical model presented below.

\[^7\]See Lluch, Powell, and Williams (1977) for a detailed discussion of the linear expenditure system and the Stone-Geary utility function on which it is based.
\[ D = \sum_i d_i X_i - G_n \]

where \( d_i \) is the sectoral pollution coefficient, \( X_i \) is sectoral output, and \( G_n \) is the government purchase of the cleaning activity. Thus, \( D \) is the net dirt that has to be dealt with by private consumers. Given \( D \), private consumption of cleaning, \( C_n \), is determined from the LES expenditure function. For this expenditure function, one can prove the following proposition.

**Proposition:** The consumer is indifferent between consuming cleaning privately (\( C_n \)) or having the government do the cleaning (\( G_n \)). Proof: Assume initially that \( G_n = 0 \) and \( C_n \) is determined by the LES equation. Now, set \( G_n \) to some positive value and finance the expenditure by a lump-sum tax. Thus, income, \( Y \), falls by \( P_n \cdot G_n \), which should also decrease supernumerary income, \( Y^s \), by the same amount. However, \( D \) also falls by \( G_n \) and, hence, so does \( Y^s \), which will raise \( Y^s \) by \( P_n \cdot G_n \). The net result is that supernumerary income is unchanged. Hence, the demands for all goods other than cleaning are unaffected and the increase in \( G_n \) is simply matched by a decline in \( C_n \). Consumer utility is also unchanged.

Finally, note that assuming that the threshold parameter, \( D_0 \), is positive allows the possibility that \( C_n \) may be negative (if \( D < D_0 \)). From the expenditure function, it would then be possible for \( C_n = 0 \) for some \( D > 0 \). Indeed, it is possible to have \( C_n < 0 \), which is not so desirable. For actual applications, one generally expects to analyze situations in which \( D \) is much greater than zero and desired cleaning is well above zero, so the theoretical possibility that \( C_n \) might be negative should not cause any empirical problems.

The LES formulation with a \( y_n \) parameter depending on the level of pollution addresses the problems discussed earlier concerning the introduction of pollution and cleaning into the utility function. Utility depends on both the level of pollution and cleaning. The consumer is indifferent between consuming cleaning directly or having someone else provide the cleaning. And finally, a threshold is easily incorporated into the function. The only remaining issue is how to treat pollution as a public good. This issue is considered in the next section in the framework of a CGE model.

**A CGE Model with Pollution**

Table 1 presents the equations of a CGE model which incorporates pollution as a production externality, following Leontief, and also treats it as a public good. The model has one consumer, so utility is equivalent to social welfare. Alternatively, one could interpret the model as consisting of a set of consumers with identical utility functions, with good \( n \) being a public good whose aggregate value is an argument in each function. The result, which is reflected in the CGE model, is that there is no private demand for the cleaning activity because consumers cannot materially affect the level of pollution through private cleaning. Another alternative justification for the treatment in the CGE model is simply to assume that, for whatever reason, the private market for the cleaning activity is missing.8

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8For example, the existence of common property resources can lead to such missing markets. For a general discussion of pollution externalities, see Newbery (1980).
Table 1: Public Good Pollution Model With Production Externality

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( X_i = a_i \cdot L_i^\alpha \cdot K_i^{(1-\alpha)} )</td>
<td>Sectoral Cobb-Douglas production functions. ( X ) is output, ( L ) is labor, ( K ) is capital, and ( a ) and ( \alpha ) are parameters.</td>
</tr>
<tr>
<td>(2) ( D = \sum_i d_i \cdot X_i - G_n )</td>
<td>Net pollution after government cleaning. The ( d_i ) are fixed pollution coefficients.</td>
</tr>
<tr>
<td>(3) ( P_i^p = P_i - t_i - \sum_j A_{ji} \cdot P_j )</td>
<td>Value added or net price. ( P ) is the price, ( t_i ) is a specific tax, and ( A_{ji} ) are input-output coefficients.</td>
</tr>
<tr>
<td>(4) ( t_i = t_p \cdot d_i )</td>
<td>Sectoral pollution tax rate per unit output. The scalar ( t_p ) is the tax rate per unit of pollutant.</td>
</tr>
<tr>
<td>(5) ( W_L = P_i^p \cdot \alpha_L \cdot X_i ) ( L_i )</td>
<td>Sectoral labor demand equations. ( W_L ) is the wage of labor.</td>
</tr>
<tr>
<td>(6) ( W_K = P_i^p \cdot (1-\alpha_k) \cdot X_i ) ( K_i )</td>
<td>Sectoral capital demand equations. ( W_K ) is the capital rental rate.</td>
</tr>
<tr>
<td>(7) ( U = \prod_i (C_i - \gamma_i)^{\bar{h}_i} )</td>
<td>Social welfare function. Stone-Geary function.</td>
</tr>
<tr>
<td>(8) ( \gamma_k = \bar{\gamma}_k ) ( \gamma_n = D - D_0 )</td>
<td>Minima parameters in welfare function. ( D_0 ) is the pollution threshold parameter.</td>
</tr>
<tr>
<td>(9) ( \bar{P} = \prod \bar{p}_i^{\bar{h}_i} )</td>
<td>Numeraire price index. It is the cost of living index given the social welfare function. ( P ) is exogenous.</td>
</tr>
<tr>
<td>(10) ( Y^\alpha = \sum_i t_i \cdot X_i )</td>
<td>Pollution tax revenue.</td>
</tr>
<tr>
<td>(11) ( Y = \sum_i P_i^p \cdot X_i + Y^\alpha - \sum_i P_i \cdot G_i )</td>
<td>Total private income.</td>
</tr>
</tbody>
</table>
Equation Explanation

(12) \[ Y_s = Y - \sum_k P_k \gamma_k \]
Private supernumerary income.

(13) \[ P_k \cdot C_k = P_k \gamma_k + \mu_k Y^s \]
Private linear expenditure system.

(14) \[
\begin{align*}
G_k &= 0 \\
G_n &= \bar{G}_n
\end{align*}
\]
Government demand.

(15) \[ C_i = X_i - \sum_j A_{ij} \cdot X_j - G_i \]
Product market balance equation.

(16) \[ \sum_i L_i = \bar{L}^s \]
Labor market balance equation. \( \bar{L} \) is the exogenous supply of labor.

(17) \[ \sum_i K_i = \bar{K}^s \]
Capital market balance equation. \( \bar{K} \) is the exogenous supply of capital.

Notes:
- \( i = 1, \ldots, n \)
- \( k = 1, \ldots, n-1 \)
Good \( n \) is the "cleaning" activity.
- \[ \sum_i \beta_i = \sum_k \mu_k = 1 \]
- \[ \mu_k = \frac{\beta_k}{\sum_{j=1}^{n-1} \beta_j} \]

The social welfare function, equation 7, is the Stone-Geary function extended to include pollution, with all the desirable features discussed above. However, the expenditure functions, equation 13, are derived assuming that the consumer has no private demand for cleaning \( (C_n = 0) \) since the market for cleaning is assumed to be missing. These functions thus exclude \( C_n \) and are defined only over the normal goods \( (k = 1, \ldots, n-1) \). The marginal expenditure shares, \( \mu_k \), exclude the cleaning activity, even though pollution and cleaning affect private utility and social welfare.

The model includes a pollution tax, \( t_p \), which is imposed per unit of pollution generated. This tax is translated to a specific output tax, \( t_o \), by multiplying by the pollution coefficient (equation 4). The inclusion of the pollution tax allows the government to force sectors to treat pollution as a priced cost of production. Through the net price equation, the pollution tax affects output supply and factor demand decisions by sectors. Setting \( t_p = 0 \) leads firms to ignore pollution costs, so it is then a pure production externality.
Private income is defined in equation 11. Note that any net balance between government expenditure and revenue from the pollution tax is financed by a lump-sum transfer to or from the single household. There is no requirement that government cleaning be financed by the pollution tax, either at the aggregate or sectoral levels. Given that there is no private demand for cleaning, the only cleaning is done by the government (G.). In terms of social welfare, of course, it does not matter whether cleaning is done privately or by the government.

Assuming that \( t_p \) and \( G_c \) are fixed parameters, the model is a well-behaved CGE model with as many independent equations as endogenous variables. The solution represents a market equilibrium, with prices and quantities reflecting supply-demand balance in all markets. However, the market equilibrium does not represent a welfare maximum. There are two problems: the existence of unpriced production externalities and the fact that cleaning is a public good with no private market demand. While linked in the general equilibrium system, these represent two distinct market failures which must both be addressed in order to achieve a welfare optimum.

In the model, the government has two instruments which it can use to affect the amount of pollution in the system: the pollution tax rate, \( t_p \), and the amount of government cleaning, \( G_c \). It can be proved that these two instruments can in fact be set so as to achieve a welfare optimum that is equivalent to a competitive solution in which the pollution externality is internalized and the demand for cleaning properly reflects its utility to the consumer. At this optimum, given that dirt and cleaning appear symmetrically in the utility function, the pollution tax will equal the price of cleaning. At the margin, society is indifferent between charging polluters for their pollution or paying to clean the dirt. In the next section, we explore the empirical properties of this model, starting from the stylized data presented by Leontief (1970).

A Stylized Empirical CGE Model

The model in Table 1 is implemented with the stylized data from the Leontief (1970) article. Table 2 presents the input output table including the cleaning activity and pollution generation. Table 3 presents a social accounting matrix (SAM) constructed from the Leontief data. The SAM shows expenditures from column accounts and receipts to row accounts. Corresponding columns and rows represent the receipt-expenditure accounts of the different actors, and must balance in sum, account by account. The units were chosen so that the wage and capital rental rates are one. Pollution does not appear in the SAM, since it is unpriced and hence generates no monetary transactions.
Table 2: Leontief Stylized Input-Output Table with Pollution

<table>
<thead>
<tr>
<th></th>
<th>Agric</th>
<th>Mfg</th>
<th>Clean</th>
<th>Total intermed</th>
<th>Consumption</th>
<th>Total Output</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>25</td>
<td>20</td>
<td>0</td>
<td>45</td>
<td>55</td>
<td>100</td>
<td>$2.00</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>14</td>
<td>6</td>
<td>6</td>
<td>26</td>
<td>30</td>
<td>56</td>
<td>$5.00</td>
</tr>
<tr>
<td>Cleaning</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>30</td>
<td>$3.00</td>
</tr>
<tr>
<td>Pollutant</td>
<td>50</td>
<td>10</td>
<td>0</td>
<td>60</td>
<td></td>
<td>60</td>
<td>$0.00</td>
</tr>
<tr>
<td>Labor</td>
<td>60</td>
<td>80</td>
<td>20</td>
<td>150</td>
<td></td>
<td>160</td>
<td>$1.00</td>
</tr>
<tr>
<td>Capital</td>
<td>20</td>
<td>130</td>
<td>40</td>
<td></td>
<td></td>
<td>190</td>
<td>$1.00</td>
</tr>
</tbody>
</table>

Notes:
All quantities are in physical units. Prices are given in the last column. The source is Leontief (1970). The gross production data have been adjusted to include consumption of cleaning in the balanced table. The input-output coefficients thus differ slightly from the Leontief data. Separate capital and labor inputs have also been added.

Table 3: Social Accounting Matrix from Leontief Data

<table>
<thead>
<tr>
<th></th>
<th>Agric</th>
<th>Mfg</th>
<th>Clean</th>
<th>Labor</th>
<th>Capital</th>
<th>Household</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>50</td>
<td>40</td>
<td>0</td>
<td>110</td>
<td>150</td>
<td>160</td>
<td>200</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>70</td>
<td>30</td>
<td>30</td>
<td>150</td>
<td></td>
<td>190</td>
<td>280</td>
</tr>
<tr>
<td>Cleaning</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td></td>
<td></td>
<td>90</td>
</tr>
<tr>
<td>Labor</td>
<td>60</td>
<td>80</td>
<td>20</td>
<td></td>
<td></td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>Capital</td>
<td>20</td>
<td>130</td>
<td>40</td>
<td></td>
<td></td>
<td>190</td>
<td>190</td>
</tr>
<tr>
<td>Household</td>
<td></td>
<td></td>
<td></td>
<td>160</td>
<td>190</td>
<td></td>
<td>350</td>
</tr>
<tr>
<td>Total</td>
<td>200</td>
<td>280</td>
<td>90</td>
<td>160</td>
<td>190</td>
<td></td>
<td>350</td>
</tr>
</tbody>
</table>

Notes:
All entries are dollar flows.
Table 4 presents the input-output coefficients and three different sets of pollution coefficients. The first are the pollution/output coefficients ($d_i$). The second and third sets show the direct plus indirect effects on pollution of changes in final demand, either per unit of physical demand or per dollar spent. In the stylized Leontief data, agriculture is the main polluter. Indirect effects are important -- even final demand for cleaning generates some pollution. However, demand for agriculture, which represents 31 percent of aggregate final demand, generates 70 percent of total pollution. The CGE model will also reflect the same story, given the assumption of fixed input-output and pollution coefficients. The model makes the strong assumption that sectors cannot change their pollution coefficients.

**Table 4: Input-Output and Pollution Coefficients**

<table>
<thead>
<tr>
<th></th>
<th>Agriculture</th>
<th>Manufacturing</th>
<th>Cleaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>input-output coefficients</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.250</td>
<td>0.357</td>
<td>0.000</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.140</td>
<td>0.107</td>
<td>0.200</td>
</tr>
<tr>
<td>Cleaning</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>pollution coefficients</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Per unit production</td>
<td>0.500</td>
<td>0.179</td>
<td>0.000</td>
</tr>
<tr>
<td>Per unit final demand</td>
<td>0.761</td>
<td>0.504</td>
<td>0.101</td>
</tr>
<tr>
<td>Per dollar final demand</td>
<td>0.380</td>
<td>0.101</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Notes:
- Pollution coefficients per unit final demand and per dollar of final demand are based on the Leontief inverse, $d'(I-A)^{-1}$ where $d$ is the vector of pollution coefficients (per unit of production) and $A$ is the matrix of input-output coefficients.

The input-output table and SAM are used to calculate all the parameters in the CGE model, except for the subsistence minima and pollution threshold parameters in the utility function. The $\gamma_i$ parameters are set to zero for agriculture and manufacturing, and the threshold parameter, $D_0$, is set to 60, so that $\gamma_a$ equals zero in the base data set ($D = 60$). With these values for the minima parameters, the (social) marginal propensities to consume, $\beta_i$, are 0.31, 0.43, and 0.26 for agriculture, manufacturing, and cleaning, respectively. The initial value of the pollution tax, $t_\nu$, is set to zero. The CGE model assumes that cleaning is a public good, with no private demand. To generate the final demand for cleaning of 30, which appears in the data, government demand for cleaning, $G_n$, is set to 30. With these parameters, the solution of the CGE model yields a market equilibrium that exactly

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9 Obtained by multiplying the pollution coefficients times final demand (either in physical units or dollars).
replicates the real and nominal flows in Tables 2 and 3. The model is bench-marked to the Leontief data set.

**Optimal Policy Experiments**

The bench-mark solution of the CGE model represents a market equilibrium in which producers do not cost the pollution they generate --there is a production externality. Given the production externality, however, the bench-mark government demand for cleaning of 30 equals the demand that would be generated if cleaning were a private good, with the expenditure functions arising from the social welfare function. This solution, however, is in no sense optimal. It is possible to use the model to generate a first-best, welfare maximizing solution. Three different approaches will all yield the same answer.

One approach is to find the welfare maximizing solution by imposing the constraint that \( t_p = P_a \) and replacing the private expenditure functions by the social expenditure functions (using \( \beta_i \) instead of \( \mu_i \)). As noted above, at the welfare maximum, the cost to producers of generating dirt, \( t_p \), will equal the cost of cleaning, \( P_a \). The market equilibrium in this case is also the welfare optimum. A second approach is to embed the market CGE model into a nonlinear program with social welfare as the objective function. The policy instruments are \( t_p \) and \( G_a \). Solving the nonlinear program will yield values of \( t_p \) and \( G_a \) which generate a market equilibrium which maximizes social welfare. Finally, one can replace the expenditure equations with their social counterparts and maximize welfare with respect to only one instrument, \( t_p \).\(^{10}\) All three approaches are computationally feasible and yield the same solution.\(^{11}\)

Tables 5 and 6 present the results of a number of experiments designed to explore the sensitivity of the model economy and aggregate welfare to different policy scenarios. In the experiments reported in Table 5, the pollution tax is set to zero while in Table 6 the optimal pollution tax is determined in each experiment. In both sets of experiments, government demand for cleaning is set exogenously at values ranging from 15 to 45. In experiment 5 in both tables, the level of government demand for cleaning is set at the welfare-maximizing optimum, given also the optimum value of the pollution tax, \( t_p \). Thus, the solution for experiment 5 in Table 6 represents the first-best welfare maximum discussed above. The utility index for this experiment is set to 100, and all the other utility indices reported are

\(^{10}\)In this second approach, \( G_a \) is fixed exogenously to zero and \( C_a \) is solved endogenously. In fact, \( G_a \) could be set to any value less than the optimal value of \( C_a \), and the solution of the nonlinear program would be the same.

\(^{11}\)The CGE model and the various nonlinear programs are solved using software called GAMS (General Algebraic Modeling System). The program is described in Brooke, Kendrick, and Meeraus (1988). The GAMS program of the model is available on request.
<table>
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<th>Experiment</th>
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Notes:
Aggregate real GDP and consumption are defined using benchmark prices. Consumption is private consumption. Sectoral consumption and net dirt are in physical units.
<table>
<thead>
<tr>
<th>Experiment</th>
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**Notes:**
Aggregate real GDP and consumption are defined using bench-mark prices. Consumption is private consumption. Sectoral consumption and net dirt are in physical units. Sectoral pollution taxes are imposed as specific taxes in the model. They are presented here as ad valorem equivalents.
expressed as ratios to it. All the other solutions reported in Table 6 are second-best welfare maxima, with the optimal value of \( t_p \) given non-optimal values of \( G_n \). The solutions reported in Table 5 are simply market equilibria, with no optimization over any choice variable. Note that column 4 in Table 5, with \( G_n \) set to 30, is the bench-mark solution.

The experiments reported in Table 5 indicate how sensitive welfare is to different levels of cleaning, and how insensitive are prices. Low levels of cleaning have a dramatic impact on welfare, reducing aggregate utility to 74 percent of optimum, compared to 96 percent in the bench-mark solution. Once the level of cleaning reaches the bench-mark value of 30, more cleaning has little impact on welfare. A 50 percent increase in cleaning (from 30 to 45), leaves aggregate welfare virtually the same, even though it changes the structure of production and consumption. The structure of prices is very insensitive to changes in cleaning demand.

The experiments reported in Table 6 indicate the power of pollution taxes in affecting the amount of dirt generated in the economy and aggregate welfare. Experiment 4, which computes the optimal pollution tax given the bench-mark value of cleaning (\( G_n = 30 \)), yields about a third additional reduction in dirt (from 30.0 to 20.5). When government cleaning is low, the pollution tax rises and the net impact on dirt generation is relatively greater. For example, when \( G_n \) is 15, the net dirt in the system falls from 53.0 in the market equilibrium to 35.8 when an optimal pollution tax is imposed. In general, the use of the pollution tax yields much smaller variations in social welfare across the various second-best experiments. In the most extreme case, when \( G_n \) equals 15, social welfare is only 5.6 percent below its optimal value.

The optimal pollution tax is generally high. For agriculture, the ad valorem equivalent rate varies from 55.8 to 26.1 percent, given the different values of \( G_n \). The total revenue from the pollution tax is also high. At the optimum (experiment 5), the gross tax revenue equals 41 percent of private income. The net tax, after paying for government cleaning, amounts to 15 percent of private income.\(^{12}\) The impact of the tax on relative prices is also dramatic. The price of agriculture at the optimum is 50 percent above its bench-mark solution value. When government cleaning is less than optimal, the differences in prices are greater. The model undoubtedly overstates this effect, since the pollution coefficients at the sectoral level are fixed, so producers cannot adjust their pollution coefficients in response to the tax. The adjustment comes solely through changes in final demand in response to price changes.

The experiments demonstrate the limitations in using standard macroeconomic aggregates to measure the impact of changes in pollution generation and cleaning. Real GDP, which includes value added in the cleaning sector, is virtually unchanged across all the experiments, even though there are large changes in social welfare. On the other hand, real private consumption is inversely related to the level of government cleaning, and is also hardly affected by changes in the pollution tax rate. Looking at these macro aggregates, one would conclude that cleaning detracts from welfare because it lowers real consumption and

\(^{12}\)Note that requiring cleaning costs to equal pollution tax receipts, as is done by Dufournand, Harrington, and Rogers (1988), would lead to a solution with lower welfare.
that the level of the pollution tax has no effect on welfare. In fact, aggregate welfare is very sensitive to the pollution tax, especially at low levels of cleaning. Welfare rises rapidly with increased cleaning starting from low levels and, at high levels of cleaning, welfare changes little with changes in cleaning levels. It would appear that it is safer to err on the side of demanding too much cleaning rather than too little.

Of course, all these results are sensitive to model specification. If price elasticities of demand were less elastic, the optimal pollution tax would be even higher. If producers could change technology in response to the pollution tax, the optimal rate would be lower. The qualitative results are probably pretty robust. Production and demand linkages certainly matter and the economist's traditional focus on incentive instruments such as pollution taxes is clearly justified. It is as important to change relative prices to account for the production externality as it is to engage in cleaning activities, given that pollution is a public good. Both kinds of market failure are important.

Conclusion

Pollution is a problem because it involves various kinds of externalities. Economists generally favor policies which work through changing prices and market incentives. There is an extensive theoretical and empirical literature on how to internalize the production, consumption, and public-good externalities that characterize most pollution generation, and so bring pollution control within the framework of markets. The model presented in this paper provides an empirical optimizing framework for analyzing pollution that captures these externalities. The framework involves three components. First, a multisector, economywide, computable general equilibrium (CGE) model is extended to include the externality characteristics of pollution. These include production externalities and pollution (and pollution control) as a public good. Second, pollution and cleaning activities are explicitly included in a social welfare function. Finally, the social welfare function and the CGE model are joined into a nonlinear programming model, with the CGE model equations serving as the constraints in the program. The resulting model is implemented with stylized data from Leontief (1970).

The results demonstrate the feasibility of including pollution within an optimizing framework in an economywide model and support the view that it is important to devise pollution control policies that work through market mechanisms. The explicit inclusion of cleaning and pollution in a social welfare function is important. There has been a great deal of work in a partial equilibrium context determine the social cost of pollution and the societal demand for pollution abatement and cleaning. From this work, it is potentially feasible to determine explicit social welfare functions which incorporate cleaning and pollution. Linking this work to the economywide CGE framework yields a model which simultaneously determines the optimum levels of cleaning and pollution taxes.

The CGE model used here is very simple and can easily be extended to include different functional forms and additional substitution possibilities in production and demand. An obvious extension would be to include other kinds of pollution externalities, such as pollution arising from consumption activities (e.g., cars and air pollution) or the negative
effects of pollution on production. Another important extension would be to include substitution possibilities in production which change the pollution coefficients at the sectoral level. One might also differentiate the cleaning sector to permit different costs of cleaning, depending on the demander. Firms thus can have the choice of changing their production technology or directly purchasing cleaning services in response to a pollution tax. While the basic mechanisms would be similar to those incorporated in the simple stylized model, the resulting models would allow a richer array of policy instruments and sectoral responses.
References


