COMMODITY BONDS WITH PUT OPTIONS FOR CONSUMPTION SMOOTHING BY COMMODITY-DEPENDENT EXPORTERS

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I. Introduction

Loans and other investment contracts are widely perceived as legally enforceable in lender countries but not in debtor countries. In that context, this paper shows how novel financing arrangements using commodity bonds with put options for the seller can be used for stabilization of risks associated with export prices.

Given the substantial instability in all primary commodity markets, one would expect countries that depend on a single primary export for most of their foreign earnings (for example, Mexico, Nigeria, Zambia, New Guinea and most OPEC countries) to experience especially sharp fluctuations in export earnings and their underlying wealth. To the extent that these fluctuations affect consumption, they are costly; and we would expect such countries to seek ways of managing these fluctuations and reducing their costs.

In many countries the nature of the resource endowment and its comparative advantage rule out production diversification as a significant near-term strategy, and we assume it away here. In addition, we rule out diversification via exchange of equity investments with foreigners. In this paper we consider the cost of export risk and show the potential contribution of commodity bonds in this context. We show that, in theory, appropriate commodity bonds can achieve optimal smoothing of i.i.d. export price disturbances—if that is what countries really want or need.

Commodity bonds ("c-bonds") are bonds whose principal repayment (and perhaps dividend payments) may be made in units of physical commodity (or the terminal value of some appropriate futures contract). Typically, the
bond buyer has the option to receive the nominal face value or the commodity bundle. In the finance literature, studies of the pricing of c-bonds (Schwartz; Carr; Priovolos) do not distinguish bonds issued by foreign governments from private corporate bond issues. However, the literature on foreign borrowing recognizes that the distinction is crucial.

II. Sovereign Borrowing and Default Prevention

The main distinction between corporate and sovereign borrowing, described in masterly fashion by Keynes and incorporated in the seminal work of Eaton and Gersovitz, is that collateral is generally unavailable to creditors of a sovereign borrower since the assets of the latter are located within its borders. Only in exceptional cases can they be attached by lenders in the event of default.

The absence of a final distribution of assets to creditors as seen in domestic bankruptcy also changes the nature of default. It arises in the context of a sequence of strategic moves by creditors and the sovereign debtor who retains (and, in fact, cannot credibly foreswear) the power to make subsequent decisions that affect the interests of creditors.

Here we focus on income-smoothing financial transactions between investors in developed countries (DCs) and a less-developed country (LDC) heavily dependent on a single commodity subject to substantial revenue fluctuations. The default penalty is enforcement of debt seniority clauses in the courts of all potential borrower-lender nations so that a defaulter’s foreign investments or servicing of new debt would be subject to seizure. Default means permanent elimination of foreign borrowing or lending opportunities.
III. The Costs of Income Variability

Consider a country that has economically unresponsive production ("zero supply elasticity") and seeks to maximize the expected utility of its representative consumer

\[ V_t = E \sum_{t=0}^{\infty} (1 + \delta)^{-t} u(c_t) \]

where \( E \) is the expectation operator, \( c_t \) is consumption in period \( t \), and \( u \) is felicity, \( u' > 0, u'' < 0 \). There is no storage. Output and price are each subject to one discrete i.i.d. random disturbance per period.

To dramatize the issues, assume that exports from a single commodity account for 33% of GNP on average, and suppose that the coefficient of variation (CV) of output and price of the commodity are both 30%, and that the correlation between output and price can be ignored. Suppose also that all other income is nonstochastic and that the country optimally shares risks internally. There is, however, no saving or borrowing or other intertemporal income smoothing. Using the standard formulas\(^1\) for the cost of risk, if the coefficient of relative risk aversion is \( R \) (defined for one-period variations in consumption), and if the CV of consumption is \( s \), then the annual cost of risk, \( \rho \), is defined implicitly by \( u(\bar{c} - \rho) = Eu(c_t) \), where a bar over a variable indicates its expected value, and the relative cost, \( \rho/\bar{s} \), is approximately (exactly if utility is quadratic in income per period) \( Rs^2/2 \). If consumption must be equal to income each year, then \( s = 0.33e \) where \( e \) is the CV of export revenue (and 0.33 is the average share of exports to GNP). If output and price are independently normally distributed, then \( e^2 = 0.19 \) (and this will hold approximately even if output and price are not normal). In this case, if \( R \) has
the not unreasonable value of 2, the cost of risk is approximately 2% of average income, the amount representative consumers would be willing to forego each year in return for a stabilized consumption stream of \( c \).

IV. Consumption Smoothing by Borrowing and Lending

Can a country optimally smooth consumption by borrowing and lending from overseas sources? If the utility function is quadratic, then \( \delta \) can be interpreted as the rate at which future consumption is discounted; and if this is equal to the rate of interest abroad, \( r \), then the country would have no motive for saving or borrowing other than to smooth consumption. We make this assumption here to focus on the consumption smoothing aspect of international borrowing. We continue to assume that exports are subject to random i.i.d. price disturbances. Then the optimally "smoothed" consumption of a borrower committed to borrowing and lending only for smoothing and to meeting his interest payment obligations is (Newbery and Stiglitz, pp. 201 and 202) \( c_t = E_t(c_{t+1}) = \bar{y} - rL_t \). Under the scheme accumulated debt, \( L \), follows a discrete random walk with increment equal to the difference between income \( y_t \) and its mean, \( \bar{y} \). For permanent operation, there must be no limit on \( L \). But in finite time, \( L \) will pass the value at which repudiation becomes more attractive than continued interest payments, even if all borrowing and lending opportunities are then cut off.\(^2\) Thus, competitive lenders will not make unlimited loans. Any feasible loans would offer at best only suboptimal and/or impermanent smoothing.

The nature of the evolution of general obligation loan contracts for sovereign borrowers is a currently active research area.\(^3\) At this stage it seems clear that consumption smoothing by sovereign borrowers using conventional borrowing and lending is infeasible if the contract is not
renegotiated. If so the quest for a better instrument makes sense. Accordingly we now turn our attention to c-bonds.

V. Commodity Bonds Issued by Sovereign Lenders

To simplify the discussion, assume that the c-bond under discussion is a zero-coupon bond with payment upon maturity consisting only of a completely specified commodity bundle. We assume the issuer is competitive and market risk-neutral with respect to this bond (see O'Hara for analysis of the demand side of the market for c-bonds under other assumptions). As above, assume initially that all contracts are always honored.

Under these assumptions, if the country issues c-bonds (which in this model need only be one-period bonds) and if these can be issued (and indefinitely re-issued) at the present value of the expected price for next period, then their risk-reducing properties in the steady state are exactly the same as those of an optimal forward or futures hedge at the same price. Newbery and Stiglitz (p. 186) show that, in the case of stationary, uncorrelated output and price disturbances, the ratio of income variance with and without optimal forward hedging, is roughly $1/(1 + k^2)$, where $k$ is the ratio of the CVs of price and output. In our numerical example above, $k$ equals 1. If there is no other means of consumption smoothing by lending and borrowing, then c-bonds will halve the steady state costs of the risk—to 1% of GNP in our example. If the CV of income were the same, but only price were stochastic, then c-bonds eliminate risk, worth 2% of GNP.

Assume, henceforth, that no other borrowing is possible and that all income variation is due to price. Then with credible commitment, complete smoothing is achieved by selling c-bonds for the whole (deterministic) output.
The country then has constant income and consumption and delivers all output of random value to the lender.

In low-price states the smoothing raises income, so there is no incentive at all to default. But in high-price states, delivery to the lender reduces current income, \( y_t \), by \( (y_t - \bar{y}) \). This, plus the expected present value of autarkic future consumption, may in some high price states exceed the maximum expected present value of the consumption path given default does not occur now. Then in those states default will rationally occur; a no-default commitment is not credible.

The credibility of a no-default commitment by a c-bond issuer depends on the parameters of the model. Consider the simple case with a two-point probability density for the multiplicative income disturbance which is i.i.d., \( u = \pm v \), with probabilities of outcomes \( +v \) and \( -v \) equal to one-half. Assume mean income is unity and utility is quadratic over the consumption range, \( 1 - v \) to \( 1 + v \). Then the annual cost of risk in the stochastic steady state (and the value of access to c-bonds) is in this case with all uncertainty due to price: \( \rho^* = R v^2 / 2 \) and the present value is \( \rho^*/\delta = R v^2 / 2 \delta \). Now consider the stochastic steady state in which a fraction \( (1 - a) \) of output, \( 0 < a < 1 \), is delivered each period in payment for c-bonds issued one period earlier, and all consumption is financed from current sales of c-bonds and the uncovered fraction \( (a) \) of output. If the income draw is high at \( v \), then default is the expected-utility-maximizing decision if and only if the current period gain, \( v - av \), exceeds the present value of the risk cost incurred. If the c-bonds cover a fraction \( (1 - a) \) of output \( 0 < a < 1 \), the change in per period risk cost is \( R v^2 (1 - a^2) / 2 \). The one-shot gain from default is \( v - av \). Default occurs if \( \delta \geq R v (1 + a)/2 \), so full coverage is feasible if and only if \( \delta \leq R v/2 \); some fractional coverage is feasible.
if and only if $\delta < \frac{1}{2} Rv$; some fractional coverage is feasible if and only if $\delta < Rv$.

As the CV, $v$, the relative risk aversion, $R$, or the uncovered portion $a$ decreases, the minimum $\delta$ consistent with default rises. Default on full coverage is not a problem in this case if income is risky enough and/or risk aversion is high enough.

VI. Optimal Dynamic Smoothing Strategies

Default Constraint Nonbinding

We have seen above that the c-bonds may be default-free in the stochastic steady state with an i.i.d. price disturbance in which consumption equals the mean value of output discounted one period. If so, one description of the optimal infinite horizon smoothing plan for implementation in period 0, given current income, $y_0$ (assumed for this exposition to be entirely from export of one commodity at price $p$), and the discount rate equal to the interest rate is as follows: Invest $\beta y_0$, where $\beta = 1/(1 + r)$, overseas for a certain periodic rate of return of $r$, issue a c-bond to cover all output, with current sale price $\beta \bar{y}$, and consume $r\beta y_0 + \beta \bar{y}$ in each period 0, 1, 2, ... Full consumption smoothing is immediately achieved forever. (A short forward contract plus a loan on the anticipated proceeds could replicate the c-bond contract. So could a short futures contract, with an additional line of credit to cover initial and variation margin.)

The opportunities for overseas investment at the (certain) market interest rate and for sale of c-bonds at unbiased prices are all the financial facilities needed for this plan. Furthermore, note that, if the initial income,
$y_0$, is invested where it can be collateralized for the c-bond loan (for example in the lending country), the default constraint is relaxed relative to the comparative static analysis above that assumed all income was from sales of c-bonds and none of the current income in the period in which c-bonds were introduced was saved. So, even if full c-bond coverage seemed infeasible in that analysis, the above strategy may work.

If one ignores transactions costs, as we do here, a number of different combinations of contracts could replicate the above arrangement, given the assumption of a nonbinding default constraint. One example is a short forward contract plus a loan on the anticipated proceeds of the contract. Several commentators have inferred that a combination of a futures contract and a loan would also be equivalent. If one takes seriously the assumption of one discrete decision instant before the later maturity date, then they are equivalent if the loan is adjusted to cover initial margins. But in a more general context the futures contract is marked to market as price varies over the time between commitment and maturity, and this leads to additional uncertain increases or decreases in credit requirements on the part of the hedger. In practice this can result in serious complications, especially if trading is obstructed by price move limits for significant periods, and/or interest rates move substantially and are not themselves hedged.

If the default constraint binds on hedging with commodity bonds or forward contract, the full smoothing described above is infeasible. The alternative of using futures markets is precluded because the variation margin requirements that make default unattractive cannot be met by a liquidity-starved borrower. Nor will they be satisfied by a third party lender because of the induced incentive of the borrower to default on the margin loans.
Default Constraint Binding

If the default constraint binds, the immediate transition to full consumption smoothing is precluded. We ask what the optimal consumption smoothing contract is in such cases, following the analysis of Worrall (and noting also Kletzer), and then see if it can be replicated by existing financial instruments.

Suppose the export price in any period \( t \) can take one of \( S \) values corresponding to \( S \) states of the world, \( p_t(s) = p(s) \ p(1) < p(2) < \ldots p(S) \), and associated with these values, the income of the country, valued at the spot price, is \( y(s) = p(s) \ q \ s = 1, 2, \ldots, S \). The optimal contingent borrowing contract is a level of borrowing, \( b \), and a schedule for repayment in the next period, \( M_{t,s} = M(y_t - m_t, p_{t+1}(s)) \), contingent on the price realization \( p_{t+1}(s) \) which maximizes the borrower's utility subject to his not wishing to default. If the optimal value function is \( V \), then \( V \) is the solution to the problem

\[
(2) \quad V(y_t - m_t) = \max_u (y_t - m_t + b_t) + E[V(y(s) - M_{t,s})](1 + r)
\]

where \( y_t \) and \( m_t \) are the levels of income at current price \( p_t \) and debt repayment in the current period \( t \), and consumption \( c_t = y_t + b_t - m_t \). This is to be maximized by choosing \([b_t, M_{t,s}]\) subject to the constraint that the borrower does not wish to default in any state \( s \), and consequently forego any future lending or borrowing opportunities:

\[
(3) \quad V(y_s - M_{t,s}) > u(y(s)) + E[U(y)]/r, \quad s = 1, 2, \ldots, S
\]

and subject to the zero profit constraint which, for risk-neutral lenders, is

\[
(4) \quad -b_t + \beta E[M_{t,s}] = 0.
\]
From the envelope condition, \( u'(y_t - m_t + b_t) = V'(y_t - m_t) \), \( V(o) \) is strictly concave, implying existence of a unique optimum. The first-order conditions from this constrained maximization problem are

\[
(5) \quad u'(c_s) = (1 + \mu_s) V'(y(s) - M_{t,s}), \quad s = 1, 2, \ldots, S
\]

where \( \mu_s \) is proportional to the multiplier on the default constraint in state \( s \), which will be zero if the constraint does not bind.

It is possible to show (Worrall, pp. 5-6, Results 1-3) that, if the default constraint binds when the scheme is implemented in period \( t \), with current repayment obligation \( m_t \), then the optimal loan has a contingent repayment schedule that sets a floor on net income in the next period, \( (y(s) - M_{t,s}) \), equal to current net income, \( (y_t - m_t) \), with repayment at higher income satisfying \( V[y(s) - M_{t,s}] = u(y(s)) + E[u(y)]/r \). Consumption \( (y(s) + b_{t+1} - M_{t,s}) \) is nondecreasing in net income, \( y(s) \).

Assuming the default constraint precludes complete smoothing, the optimal scheme could be operated using c-bonds as follows: In period \( t \) the lender issues to the borrower a loan \( b_t \) and a put option to cover fixed output with strike price \( P_t^* \) equal to

\[
(6) \quad P_t^* = (y_t - m_t + b_t / \beta + Z_t) / \bar{q}
\]

where the option premium, \( Z_t \), is determined by the zero-profit condition for the writer of the put

\[
(7) \quad (y_t - m_t + b_t / \beta) / \bar{q} = E\left\{ \min \left\{ P_{t+1}, (y_t - m_t + b_t / \beta + Z_t) / \bar{q} \right\} \right\},
\]

and \( b_t \) is the solution to the borrower’s optimization problem given the associated values of \( P_t^* \) and \( Z_t \) from (6) and (7).
In period $t+1$, the maximum repayment is $m_{t+1}^* = b_t / \beta + Z_t$. If the realized state $s$ in that period is such that $p_{t+1}^* \leq P_t^*$ and the option is exercised by delivery of $\bar{q}$ or equivalent trades, the borrower receives the option return less repayments, $P_t^* \bar{q} - b_t / \beta - Z_t$, and the lender is paid a net sum of $m_{t+1}^* = M_{t,s} = y(s) - y_t + m_t$, which may be negative. Income net of repayments is the same as last year, i.e., $y_{t+1} - m_{t+1} = y_t - m_t$. The smoothing arrangements of period $t$, $[b_t, P_t^* Z_t]$, are then replicated in period $t+1$.

If, however, $p_{t+1} > P_t^*$, the borrower repays the lender $m_{t+1}^*$, sells $\bar{q}$ on the market, and retains net income $\bar{q} p_{t+1} - m_{t+1}^* = y_{t+1} - m_{t+1}^* > y_t$ where

$$m_{t+1}^* = \max_s M_{t,s}.$$

Then the procedure is repeated for period $t+1$ and the new amount borrowed, $b_{t+1}$, is, in this case, less than $b_t$, but the strike price is higher to raise minimum net income to $(y_{t+1} - m_{t+1}^*)$.

If the default constraint is initially binding, the process thus evolves as follows. In the initial period (call it period 0), assuming no prior obligations, $m_0 = 0$, and $y_0 = p_0 \bar{q}$. Consumption is raised by c-bond sales to $y_0 + b_0$. In period 1, if the state is $j, 1 \leq j \leq S$, then $y_1 = y(j) = p(j) \bar{q}$, and $m_1 = M(y_0, p(j))$, so that consumption is $c_1 = y_1 + b_1 - m_1 \geq c_0$. Consumption never falls; assuming the maximum price $p(S)$ has positive probability, in finite time (period $w$), it occurs, and $c_{w+i} < p(S) \bar{q}$ is constant for $i = 0, 1, 2, 3, \ldots$ (A longer maturity offers no additional advantage in our model). In each period an instrument that can achieve this is a zero-coupon, one-period c-bond payable in dollars or
in a specified commodity bundle, at the seller's option. This instrument contrasts with the typical commodity convertible or commodity-linked bond which contains a call option for the purchaser rather than a put for the seller.

Such an instrument does not satisfy the default constraint. The latter also precludes full coverage by forward contracts, which would achieve immediate full smoothing in the absence of the default constraint.

Before closing this section, we note that the theory used here assumes that sovereign defaults are penalized by withdrawal of all lending and borrowing opportunities. But the historical record (Lindert and Morton; Eichengreen) does not clearly show the expected differentiation, in availability of loans and their terms, between countries that have defaulted several times and those that have never done so. On the other hand, despite the apparently lenient treatment of sovereign defaulters, the overall ex post rate of return has substantially exceeded the return on lending within the creditor countries themselves (Lindert and Morton). Borrowers often appear to make net repayments in circumstances where it is difficult to demonstrate that their efforts are in their own self-interest, even where the latter is recognized as extending well beyond stabilization. Resolution of these puzzles is currently an active area of empirical investigation.

VII. Conclusions

Consumption smoothing could in principle be quite valuable to many countries in the absence of any other risk-reducing strategies. Commodity bonds (c-bonds) can achieve optimal consumption smoothing in the face of random export prices for commodity-dependent less-developed countries, dominating other international arrangements such as international buffer funds or attempts to create longer term futures markets. Depending on initial
conditions, the smoothing may be immediately complete, and use a straight c-
bond, or it might involve a nondecreasing consumption path, which becomes
constant if and when the highest income level is attained. In the latter case
the bond could be constructed as a conventional loan with attached put for the
seller; equivalently, a bond with a nominal face value at maturity and an
attached commodity value, delivery of either to be at the seller’s option. This
type of c-bond contrasts with the observed forms, which generally offer the
buyer a similar choice. The consumption-smoothing achieved reduces
downside exposure of the seller, while leaving him a sufficiently large share
of high realizations that he is not tempted to default.

Though we have shown this only in the case of pure price uncertainty
with i.i.d. disturbances (and, hence, no interperiod storage), availability of a
constant risk-free rate of return and market risk neutrality of lenders, our
results suggest further investigation of the smoothing possibilities of these
instruments in more general circumstances. If prices follow a random walk, it
is easy to show that bond-option packages like those discussed here can smooth
producers over a one-period production commitment under a default
constraint. (Of course, eventual perfect smoothing is not feasible in such
circumstances.) In a model with storage, prices tend frequently to be highly
correlated over short intervals. Nevertheless, the price process is stationary,
though complicated.6 The optimal smoothing contract in such a model is an
interesting topic for further investigation.

Whether the type of smoothing discussed here is what commodity
exporters want or need is another question. But continued access to the
benefit of income-smoothing is often identified as a major inducement for
honoring loan contracts originally motivated by other objectives such as
economic development (Eaton, Gersovitz, and Stiglitz), though the observed
procyclical nature of much borrowing raises questions about the smoothing objective (Gersovitz; see also Fishlow). Integration of this analysis with the extensive literature on swaps, renegotiation, and related matters is an obvious extension of this approach.
FOOTNOTES

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1 If consumption c is a random variable with coefficient of variation s,

\[ u(E(c) - \rho) = Eu(c). \]

Expand both sides in a Taylor series:

\[ u(E(c)) - \rho u'(E(c)) = u(E(c)) + 0.5s^2 E(c) u''(E(c)) \]

or \[ \rho / E(c) = 0.5s^2 R. \]

2 If only borrowing opportunities are lost, but the country may invest the payments it saves overseas at the same interest rate, it can actually achieve exactly the same consumption stream for periods beyond \( t + k \) as if it did not default (or never borrowed at all); see Bulow and Rogoff (1988). The partial smoothing is like that achieved by commodity storage (Wright and Williams).

3 See Eaton, Gersovitz, and Stiglitz for a recent survey. See also Kletzer and Bulow and Rogoff (1987). Alternative instruments are reviewed in Lessard.

4 There is a large literature following the pioneering work of Feder and Just on estimation (as distinct from explanation) of debt-service behavior.

5 See Finger and de Rosa for a cautionary analysis of the Compensatory Finance Facility of the International Monetary Fund. They find that, on average, it did not even stabilize the annual export incomes of participants.
See Wright and Williams 1982 for steady-state price distributions in a model of storage with rational expectations, and Williams and Wright (forthcoming) for details of price behavior with storage.
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-17-


