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MEASUREMENT OF CONSUMER GAINS FROM MARKET STABILIZATION

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Abstract

This paper demonstrates that there is little difference between exact measures of consumer gains from market stabilization and approximations like expected change in consumer surplus for most cases of practical significance, although if there is a difference its relative importance does not disappear as the degree of stabilization diminishes. Careful specification of the nature of stabilization is much more crucial to the accuracy of welfare measurements. It is important to determine whether the stabilization is in price or quantity, whether a partial or general equilibrium setting applies, whether supply responds to the stabilization or not, and whether the demand curve is linear or not. In any case, improved analytical approximations or easily implementable numerical methods make it unnecessary to rely on suspect measures of consumer gains.

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Unfortunately, there remains much confusion about the way to model the marketwide effects of stabilization policies and about the way to value monetarily their effects on consumers. In studies of stabilization of exchange rates in small open economies and of schemes for commodity buffer stocks, for example, the favored measure of welfare effects has been some version of expected social surplus. But Anderson (1979, p. 1165) has asserted that "No version of expected social surplus can be more than a poor approximation" to the correct measure, which he calls ex ante compensation. As apparent confirmation, Helms (1985) has presented examples where expected consumer surplus and expected equivalent variation grossly mismeasure the rigorous construct, and may even have the wrong sign. Nevertheless, the appropriate question is not whether expected surpluses can be inexact in special examples but whether there are conditions, as Willig (1976) has shown in the case of certainty, under which an approximation to the rigourous construct is acceptably close.

For typical applications, involving goods with low budget shares, there is little difference among the various measures. Using analytical approximations, we show that expected consumer surplus and expected equivalent variation converge on ex ante compensation as the budget share approaches zero. The convergence and accuracy of the expected surplus measures are confirmed with numerical examples.

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But when the budget share is sufficiently large, the sensitivity of the marginal utility of income to changes in price does become important in determining the relative error of the expected surplus measures, regardless of the change in the variance of price. In the case of a change from one certain price to another, the relative errors of different surplus measure disappear the price change approaches zero. But we show that no analogous convergence is seen as the change in variance approaches zero. Even so, when the budget share is large enough for consumers' risk aversion to matter, it may well be inappropriate to treat the stabilized market as isolated from the rest of the economy. General equilibrium effects are likely to dominate those attributable to risk aversion.

Not only can the mistake of ignoring general equilibrium effects swamp that of using the wrong measure of welfare, but so does inappropriate representation of uncertainty in the market. For example, if a mean-preserving change in the dispersion of consumption is represented as a mean-preserving change in price, the calculated effects on comsumers may easily have the wrong sign. Similarly, the error from overlooking possible changes in mean supply, as when supply is incorrectly assumed to be perfectly inelastic, can dominate that from using expected consumer surplus.

1. Measures of Consumer Compensation in Uncertain Markets

We assume that preferences of the identical consumers are nonstochastic and state independent and that markets for contingent claims do not exist. Although for simplicity the model has only one period, the effects of stabilization must be evaluated using an information set Ω that contains the distribution of states of the world $\tilde{w}$ but not the realization w. (Let a ~ denote a random variable.) That is, evaluation takes place "before" the realization is known. In contrast, the competitive representative consumer maximizes utility conditional on the realization w (that is, "after" w is observed), by choosing the consumption set q conditional on income y and the current price vector P. The corresponding utility function is

$$(1)\quad v(P,y) = \max_{q \geq 0} U(q) \quad \text{subject to } P'q \leq y$$

We confine our attention to cases in which the only price that is stochastic is the price of the commodity of interest (commodity j), so we can simplify notation by avoiding subscripts and denoting the price and quantity of good, j namely the jth elements of P and q, as P and q respectively. The price of good is in turn a function P(M,y) of marketed surplus M of good j, which is entirely consumed, and income y. The marketed surplus is provided by producers, a group distinct from consumers. We defer consideration of economic responses of producers till a later section. Marketed surplus is a measureable real-valued function $M(\Psi,\tilde{w})$ where Ψ is a binary variable indicating the presence (Ψ=1) or absence (Ψ=0) of uncertainty. The price of good j in the unstabilized case is denoted by $\tilde{r} = P(M(1,\tilde{w}),y)$. In the stabilized market, the price is $\tilde{s} = P(M(0,\tilde{w}),y)$.

For a given state of the world w, the equivalent variation to make consumers as well off with price r as with price s is implicitly defined by

$$(2)\quad v(r,y+EV(r,s)) = v(s,y)$$

This compensation is state-dependent, that is, conditional on w. Expected contingent equivalent variation (often denoted <u>ex post</u> in the literature) for a change of the price regime from $\tilde{r}$ to $\tilde{s}$ is

$$(3)\quad E[EV(\tilde{r},\tilde{s})] = \int EV(\tilde{r},\tilde{s})f(\tilde{w})dw.$$

E[] is the expectation operator, conditional on the information set Ω. One can think of E[EV] as the expected cost of insurance payments that will render the welfare of the consumer facing price regime $\tilde{r}$ unaffected by any deviation from price regime $\tilde{s}$.

The correct measure for evaluating a policy of stabilization before the realizations of w, and by extension, M and P, are observed is the "noncontingent" equivalent variation, implicitly defined as EV*,

(4) $E[v(\tilde{r},y+EV^*)] - E[v(\tilde{s},y] = 0$

EV* is commonly denoted "_ex ante_" in the literature, a terminology that courts misinterpretation as a description of the timing of payment of compensation, which is actually irrelevant here. What is important is that EV* is conditional solely upon Ω, which does not contain the realization of $\tilde{w}$.

Noncontingent compensation holds the consumer's expected utility constant, over the comparison of price regimes, conditional on Ω.[1] But it does not prevent the change in regime from resulting in a welfare gain or loss upon the realization of $\tilde{w}$. Therefore, the consumer's expectation over the possible changes in his _ex post_ marginal utility contingent on w affect his calculation of EV*. Thus, EV*, as does the corresponding noncontingent compensating variation CV*, reflects the consumer's risk aversion.

In general the noncontingent (_ex ante_) measures EV* and CV* differ from the expected state-contingent (_ex post_) measures E[EV] and E[CV]. The differ-

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[1]Note that any effect of compensation EV* on v via P is ruled out, in accordance with normal practice. Obviously, if all consumers are to be compensated, they will rationally adjust EV* to account for the effect of this increase in income on P, even though they are competitive price takers. But if any costs of stabilization to be compared with EV* are of similar magnitude, the net change in income may be negligible. Consistency in the treatment of the feedback effects of benefits and costs on P is more important than whether or not the feedback is taken into account.

ence is the "option value" in the terminology of Weisbrod (1964) and others.[2] Where s is deterministic, the option value $EV^*(\tilde{r},s) - E[EV(\tilde{r},\tilde{s})]$ is the difference between the value of free insurance against price disturbances and the expected cost of the insurance payouts. In that setting the problem with expected state-contingent compensation as a welfare indicator is conspicuous. If an individual facing a fair bet is offered risk-free "insurance" with zero return, the contingent compensation values the insurance at its expected cost of zero, whereas the noncontingent correctly include any risk premium.[3]

In previous studies of stabilization, it has been common practice to use yet a third measure, expected Marshallian consumer surplus E[A], as an approximation to the effects on consumer welfare.

$$(5) \quad E[A] = \int [\int_{\tilde{r}}^{\tilde{s}} q(\tilde{P},\tilde{y})dP]f(\tilde{w})dw$$

where A is defined as having the same sign as EV. Rogerson (1980) has shown that the conditions for E[A] to represent consumer preferences (to have the same sign as EV^* or CV^*) are the same as under certainty, namely that the marginal utility of income be unaffected by the price changes. When this condition is violated, there is no reason to believe that E[EV] or E[CV] is in general a more accurate indicator than E[A], although usually E[A] is most maligned.

But the question of the accuracy of E[A] versus E[EV] raises a more important question. When, if ever, are E[A], E[EV], and E[CV], adequate approx-

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[2]One difficulty with the literature is that the term `option price', which is, of course, distinct from the usage in finance, is rarely precisely defined. Here we use the definition implicit in Schmalensee (1972).

[3]Helms (1984) provides another demonstration that contingent measures are not true indicators of welfare. He investigates a change from $\tilde{r}$ to $\tilde{s}$ where the two random variables have identical independent distributions. If the good in question is strictly normal, E[CV] will be positive and E[EV] will be negative, even though there is no substantive change.

imations for the correct measures EV^* and CV^*? To answer this question, we derive in the next section a general analytical approximation to EV^*, and show its specific form when applied to common representations of changes in market uncertainty.

2. Analytical Approximation of Compensation Measures

Recent advances in numerical methods allow arbitrarily exact calculation of EV^* or CV^* given sufficient computational resources and knowledge of the ordinary demand curve and income risk aversion, as we demonstrate below. But first, to help understand the numerical results, we use analytical approximations to show the different welfare measures' sensitivity to various parameters under a given concept of stabilization, and to distinguish the effects of different concepts of stabilization under a given measure. It happens that these approximations are very accurate alternatives to the exact measures.

Consider a second-order Taylor series approximation to (4), which implicitly defines EV^*. We evaluate the approximation at the point $(\bar{r},y)$, where a $\bar{\ }$ denotes the arithmetic mean of $\tilde{r}$, $E[r(\tilde{w})]$.[4] Then the approximation for a change in the distribution of $\tilde{P}$ from $\tilde{s}$ to $\tilde{r}$, compensated by EV^* is:

$$(6)\quad 0 \approx E[v_pP_q(\tilde{s}-\bar{r}) - v_pP_q(\tilde{r}-\bar{r}) - v_yEV^* + 1/2((v_{pp}P_q^2 + v_pP_{qq})(\tilde{s}-\bar{r})^2 - (\tilde{r}-\bar{r})((v_{pp}P_q^2) + v_pP_{qq})(\tilde{r}-\bar{r}) + 2v_{py}P_qEV^*) - v_{yy}(EV^*)^2)]$$

This noncontingent equivalent variation EV^* can be thought of as comprising a component from pure stabilization, EV^*_S, and another component from a deterministic change in the mean of P between regimes, EV^*_D. The same

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[4]For valid application of the Taylor series approximation to power and logarithmic utility functions, it is necessary (Loistl (1976)) to assume that the distributions of y and x are such that $0<y<2\bar{y}$, $0<p(x)<2p(\bar{x})$ for all realizations of y and x. Our numerical examples satisfy these restrictions.

second-order approximation (6) implicitly defines EV_D^*, except that all the terms involving a deviation of a random variable from its mean drop out. Then the pure stabilization component, $EV_S^* \equiv EV^* - EV_D^*$, is

$$(7) \quad v_y EV_S^* \approx 1/2\ E[(v_{pp}P_q^2 + v_p P_{qq})((\tilde{s}-\bar{s})^2 - (\tilde{r}-\bar{r})^2) - 2v_{py}P_q EV_S^*(\tilde{r}-\bar{r}) - v_{yy}(EV_S^*)^2]$$

Since $E[\tilde{r}-\bar{r}] = 0$, and EV_S^* is state-independent, the first occurrence of EV_S^* on the right-hand side of (7) can be eliminated. Since $(EV_S^*)^2$ is of higher order than the variances of prices and incomes, it can be eliminated from the last term on the right-hand side. The analogous expansion for CV_S^* shows that, to this order of approximation, $-CV_S^* = EV_S^*$. Define the value of this approximation as V_S^*. Using Roy's identity, v_{py} and v_{pp} can be expressed as:

$$(8) \quad v_{py} = -v_{yy}q - v_y \frac{\partial q}{\partial y}$$

$$(9) \quad v_{pp} = -qv_{py} - v_y \frac{\partial q}{\partial p} = q^2 v_{yy} + qv_y \frac{\partial q}{\partial y} - v_y \frac{\partial q}{\partial p}$$

Substitute (8) and (9) into (7), eliminate the compensation terms of higher order, and divide through by v_y:

$$(10) \quad V_S^* = 1/2\ E[(((q(P(\bar{r}),y))^2 \frac{v_{yy}}{v_y} + q(P(\bar{r}),y) \frac{\partial q}{\partial y} - \frac{\partial q}{\partial P}))P_q^2 - qP_{qq})(\tilde{s}-\bar{s})^2$$

These approximations (7) and (10) force one to consider how marketwide stabilization feeds back on the responses of consumers and produces, whether the partial equilibrium response of that single market or the general equilibrium response of the entire economy. That is to say, the use of V_S^* to represent the effects of arithmetic-mean-preserving stabilization of M requires that EV_D^* equals zero. Whether or not EV_D^* is in fact zero depends on the nature of demand, the type of technology, and the form of market stabilization under consideration. The connection between a particular representation of stabilization and the fundamental structural change that stabilizes the market always exists, but is usually ignored. If it is recognized explicitly, then one can better judge whether the representation is appropriate to a particular problem. In general, to estimate the total effect of market stabilization on consumers one must consider both V_S^* and EV_D^*.

One form of stabilization where EV_D^* equals zero, and hence where EV_S^* fully reflects the change in welfare is consumption stabilization when the supply elasticity of M is zero. Denote the consumer gain from such an arithmetic-mean-preserving (AMP) stabilization of quantity consumed as $V_S^*(AMPq)$.[5]

$$(11) \quad V_S^*(AMPq) = 1/2\ [(P_q)^2 \frac{\bar{r}}{P(\bar{r})} [\gamma(\eta^y - \rho_y) - \eta^D] - \bar{q}P_{qq}]\Delta\sigma_q^2$$

$$= \frac{P(\hat{q},y)}{2\hat{q}\eta^D} [\frac{\gamma}{\eta^D}(\eta^y - \rho_y) + C - 1]\Delta\sigma_q^2$$

where η^y and η^D are the income elasticity and uncompensated price elasticity of demand evaluated at $\hat{q} = \bar{r}$, ρ_y is the Pratt-Arrow coefficient of relative risk aversion with respect to income at $v(P(\hat{q},y),y)$, $\Delta\sigma_q$ is the change in the variance of quantity consumed, and γ is the budget share,

$$(12) \quad \gamma = \frac{P(\hat{q},y)\hat{q}}{y}$$

C is the coefficient of relative curvature of the demand curve (analogous to the Pratt-Arrow measure of relative risk aversion with respect to income)[6]

$$(13) \quad C \equiv -qP_{qq}/P_q$$

A related expression for expected consumer surplus is obtained by holding v_y constant in (11), so that $\eta^y = \rho_y$

$$(15) \quad E[A(AMPq)] \approx \frac{P(\hat{q},y)}{2\hat{q}\eta^D} (C - 1)\Delta\sigma_q^2$$

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[5]This formula was presented in Wright and Williams (1984) footnote 3, p. 171.

[6]Wright (1979) identified the importance of C in the welfare effects of consumption stabilization.

Few previous writers have used this concept of stabilization of quantity consumed. Instead they have viewed stabilization as an AMP reduction in price dispersion. The noncontingent (<u>ex ante</u>) compensating variation for this type of stabilization, $V_S^*(AMPP)$, for this concept of stabilization is obtained in an analogous fashion,

$$(15) \quad V_S^*(AMPP) = \frac{q(\hat{P},y)}{2\bar{P}} [\gamma(\eta^y - \rho_y) - \eta^D]\Delta\sigma_p^2$$

where $\Delta\sigma_p^2$ is the change in the variance of price. (See Anderson (1979) or Turnovsky, Shalit, and Schmitz (1980).)

In the case of linear consumption demand, $\Delta\sigma_p^2 = (\eta^D)^2(q/P)^2\ \Delta\sigma_q^2$, and $C = 0$, so (11) and (15) are equivalent. Otherwise, $V_S^*(AMPq)$ will almost always differ from $V_S^*(AMPP)$, and the concept of AMPP stabilization is, as discussed in section 4 below, inconsistent with the assumption of zero steady-state response of consumption (marketed surplus).

In the linear case, (11) shows that consumers can lose from consumption (or price) stabilization even if they are risk-averse ($\rho_y > 0$); judging from the expected surplus measure (14) one would conclude, as did Waugh (1944), that consumers definitely lose. If, on the other hand, $C > 1$, as in the case with demand of constant and strictly negative price elasticity, then (11) indicates that consumers can gain from consumption stabilization even if they are risk neutral with respect to income ($\rho_y = 0$), as indicated by the expected surplus calculations of Wright (1979) for the case of zero supply response. More important, comparison of (11) and (14) indicates that expected consumer surplus may be an accurate measure of the benefits of stabilization even if consumers are extremely risk averse, as long as the budget share γ is sufficiently small, a condition for accuracy of consumer surplus introduced by Hotelling (1938).