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# A DYNAMIC MODEL OF PERSONAL WEALTH AND INCOME DISTRIBUTION IN A GROWING CLOSED ECONOMY

by

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# A Dynamic Model of Personal Wealth and Income Distribution in a Growing Closed Economy*

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## Abstract

A two-sector, two-class growth model is developed incorporating the following features: (a) the capital good is more capital intensive than the wage good, (b) the wage good is different from the consumption good of the capitalists, and (c) the saving/income ratio is a function of wealth holdings. In this model, we derive conditions under which a more egalitarian wealth distribution is associated with a more egalitarian income distribution. We demonstrate the difficulty of achieving a constant long-run wealth and income distribution by market forces alone. The savings behavior may be incompatible with such an objective and there may not exist any technological progress that would sustain a desirable wealth and income distribution. As a result, policy intervention in the form of direct wealth and income transfers may be necessary to attain a desirable wealth and income distribution in the long run.

# A Dynamic Model of Personal Wealth and Income Distribution in a Growing Closed Economy

## INTRODUCTION

The issue of wealth distribution has been a very important and sensitive one in discussions of social and economic reforms. This is because wealth is both an important source of income and a source of status and power. Most of the differences in policy prescription arise from differences in social philosophy. Nevertheless, an explicitly specified model may help clarify some of the purely technical issues surrounding the debate. In particular, it may help shed light on the long-run effects of redistributive policies upon the welfare of various social groups. The purpose of this paper is to move in this direction by specifying a model appropriate for the analysis of the long-run interactions among wealth distribution, income distribution, economic growth, and accumulation.

## 1. A BRIEF SURVEY OF RELATED STUDIES

In our survey we will restrict our attention to studies that are closely related to our interests here. We shall, therefore, focus solely upon studies that provide a theoretical framework that can be used in the discussion of production, income distribution, savings, wealth distribution, and economic growth in a closed economic system.

Although there was a surge of interest in distribution theories in the 1970s, the focus of them was mostly on the distribution of income.[1] The

existing literature on wealth distribution is motivated by the desire to justify the empirical observation of log normal or Pareto distribution. It is based upon models of stochastic processes and probability distributions. [For recent work along this line, see Pestieau and Possen (1979).] There is no question that random elements (e.g., in the distribution of ability and inheritance) are, in fact, present in the determination of wealth and income distribution. On the other hand, one may also wonder if there exist structural forces that set the parameters of the stochastic processes.

In contrast to the stochastic models of wealth distribution, there has been relatively little work on studying wealth distribution in a closed deterministic economic system. An early contribution was made by Stiglitz (1967, 1969), who introduced models which integrated factor and personal income distribution, inheritance, and economic growth. More recent contributions along similar lines include those by Schlicht (1975) and Höllander (1978). Whereas Stiglitz (1969) showed that, if the balanced growth path of an economy is stable, then "all wealth and income is asymptotically evenly distributed,"[2] Schlicht demonstrated an asymptotic tendency toward a two-class equilibrium even if all individuals exhibit identical behavioral equations." Höllander also found the same tendency as Schlicht did for any initially unequal distribution of wealth. The reason for these very different findings lies in the different specifications of saving behavior in the different models.[3] Finally, Bourguignon (1981) has shown that, where one or more unegalitarian wealth distributors exist within the stability interval of a stable egalitarian distribution, they are superior to the egalitarian one.

A major weakness of the four models mentioned above is that they are all one-sector models. By their very nature they are, therefore, unable to deal with relative prices. More interesting is the two-sector, two-class price endogenous model developed by Stiglitz (1967). In the light of the recent results of Schlicht and of Hollander, this two-class framework can be viewed as the limiting pattern of a general n-class model.[4] However, in his model Stiglitz employed some restrictive assumptions which limit its usefulness.

First, Stiglitz assumed given constant saving rates on the part of both capitalists and workers. This postulate, along with the other assumptions in the model, is responsible for his finding that, if there exists a non-degenerate steady state (namely, a steady state in which the amount of wealth owned by each class is constant and strictly positive), then it is unique. Furthermore, under certain conditions, the unique steady state is globally stable in the absence of limit cycles. These results imply that, under the qualifications of the model, wealth redistribution schemes can be effective only in the short run because in the long run there is a natural tendency for the distribution of wealth to return to its unique long-run equilibrium. However, Stiglitz's uniqueness proof would not go through if savings rates were allowed to depend on other variables in the model. In principle, multiple equilibria could then arise, and some redistribution schemes could have permanent effects.

Second, in proving stability of the unique long-run equilibrium discussed above, Stiglitz assumed that the consumption good is more capital intensive than the investment good. This assumption runs counter to empirical findings and was made only for mathematical convenience. It is sufficient but not

necessary for the stability of the equilibrium. In fact, Stiglitz gave an example in which stability prevailed despite the violation of the capital intensity condition.[5] Nevertheless, it seems useful to build into the general model a more realistic assumption about relative capital intensity, namely, that the consumption good is less capital intensive than the investment good.

Third, it was assumed by Stiglitz that both capitalists and workers consume the same good or—what is the same as far as modeling is concerned—that the consumption goods for capitalists and workers are similar in production. This assumption rules out the existence of a "backwash effect" discussed in the literature of economic development.

The model developed in the next section is a modification of Stiglitz's (1967) model, which relaxes all the restrictive assumptions mentioned above.

## 2. THE MODEL

In the model economy there are two goods (good 1 and good 2) and two classes (capitalists and workers). Good 1 is a consumption good for the capitalists and is also an investment good for capital accumulation. Capital is the only form of wealth. Good 2 is a consumption good for workers only. This setup is reminiscent of the classical tradition of introducing a wage good in models of income distribution.

Following Stiglitz (1967), we assume that workers also own capital. Let the number of labor and capitalists be L and C, respectively, at the beginning of the process with L > C. Suppose each worker owns $k_W'$ of capital and each

capitalist owns $k_c'$ of capital. Define $k_c$ as the ratio of total capital owned by the capitalists to total labor, i.e., $k_c = k_c'C/L$. For notational convenience we also define $k_w = k_w'$. Let $y_w$ and $y_c$ be similarly defined as the income/labor ratios for workers and capitalists, respectively. Let $S_w$ and $S_c$ be the saving/income ratios of workers and capitalists, respectively. Assume further that the number of workers and capitalists grows exogenously at the same rate, $\eta$; that capital depreciates at rate u; that there is no interclass marriage; and, finally, that wealth is bequested equally among the decendants. Under these conditions and by adopting good as numeraire, the following differential equations describe the behavior of wealth ownership over time[6]:

$$\frac{\dot{k}_c(t)}{k_c(t)} = S_c(t)\, R(t) - \eta - u \tag{1a}$$

$$\frac{\dot{k}_w(t)}{k_w(t)} = S_w(t)\left[R(t) + \frac{W(t)}{k_w(t)}\right] - \eta - u \tag{1b}$$

$$k_c(0) = k_c^o \geq 0;\ k_w(0) = k_w^o \geq 0 \tag{1c}$$

where $k_c^o$ and $k_w^o$ are given initial wealth holdings and R(t) and W(t) are the rental and wage rates, respectively, both of which are determined endogenously in the economy at time, t.

If there exist $\overline{k}_w$ and $\overline{k}_c$ such that both $\dot{k}_w(t)$ and $\dot{k}_c(t)$ are identically equal to zero for all t, we say that $\overline{k}_w$ and $\overline{k}_c$ constitute a long-run equilibrium (steady state). To study the behavior of wealth distribution, one should examine the equilibrium positions (if they exist) as well as the dynamic paths. In this model the effect of a redistributive policy can be viewed as the effect on the equilibrium positions and the dynamic paths of a change in initial positions.

At any moment in time, $k_c(t)$ and $k_w(t)$ are given; other variables, in particular R(t), W(t), and, possibly, $S_c(t)$ and $S_w(t)$, are determined endogenously by the equilibrium condition that supply equals respective demand. Let us refer to the equilibrium as momentary equilibrium. The equilibrium value of these variables is then used in (1) to generate new values of $k_c(t + \Delta)$, $k_w(t + \Delta)$, and $\Delta > 0$; and the process repeats itself *ad infinitum*. It is clear that differential equation (1) plus the conditions of momentary equilibrium fully describe the possible paths of the economy including the dynamics of wealth ownership.

Now let us specify the structure of the economy which determines its momentary equilibrium. Since our interest here is primarily in wealth

distribution, we assume that momentary equilibrium in unique. Conditions under which this is true will be discussed later. For the sake of notational simplicity, dependence on t will be suppressed.

## 2.1. Production and Supply

There are two factors of production—labor, L, and capital, K. Let $Y_i$ be the total output of good i, i = 1, 2. Assume that the production functions, $F_i(K_i, L_i)$, i = 1, 2, exhibit constant returns to scale so that they can be written in the conventional intensive form:

$$y_i = f_i(k_i) \qquad i = 1, 2 \tag{2}$$

where

$y_i = Y_i/L_i$, output/labor ratio in industry i

$k_i = K_i/L_i$, capital/labor ratio in industry i

and

$$f_i = F_i(k_i, 1).$$

Because our interests are mainly in long-run equilibria, we assume for simplicity that the Inada derivative conditions hold[7]:

$$f_i' > 0;\ f_i'' < 0;\ f_i'(0) = \infty;\ f_i'(\infty) = 0 \qquad i = 1, 2 \tag{3}$$

where $f_i'$ and $f_i''$ are the first- and second-order derivatives of $f_i$, respectively. We may also want to add that $f_i(0) = 0$, i = 1, 2.

Let us also assume that both the factor markets and the product markets are competitive, so that producers take output prices ($P_1$ and $P_2$) and

factor prices (W and R) as given and employ factors up to the point where the value of the marginal product is equal to factor price, i.e.,

$$P_i \frac{\partial F_i}{\partial K_i} = P_i f_i' = R$$

$$P_i \frac{\partial F_i}{\partial L_i} = P_i \left(f_i - k_i f'_i\right) = W \qquad i = 1, 2. \tag{4}$$

The assumption of constant returns to scale implies that profit in any momentary equilibrium is always zero. From (4), the following condition follows:

$$w \equiv \frac{W}{R} = \frac{f_i}{f_i'} - k_i \qquad i = 1, 2. \tag{5}$$

It is easy to show that $\partial w/\partial k_i > 0$ for all $k_i \in [0, \infty)$ so that we may invert (5) to obtain the optimal capital/labor ratio, $k_i$, as a function of the wage/rental ratio, w, i.e., $k_i(w)$, i = 1, 2. Let us assume that the first industry is more capital intensive than the second industry for all positive wage/rental ratios:

$$k_1(w) > k_2(w) \qquad \text{for all } w > 0. \tag{6}$$

The assumption of labor-intensive consumption goods for the workers is especially appropriate for less-developed economies.

Let us also assume full employment of both factors. Let $\lambda$ be the fraction of labor force in industry 1, i.e., $\lambda = L_1/L$. Then, full employment of labor implies that the fraction of labor in industry 2 is equal to $(1 - \lambda)$. Full employment of capital means that the weighted average of capital/labor intensity in the economy is equal to total capital outstanding per laborer, k.

Since good 2 is a consumption good for the workers only, the total demand/labor ratio for good 2 is given by (10):

$$D_2 = [1 - S_w(k_w)]\, y_w/p$$
$$= [1 - S_w(k_w)]\, (R\, k_w + W)/p \tag{10}$$

where p is given by $f_1'(k_1)/f_2'(k_2)$ in view of (4).

## 2.3. Momentary Equilibrium

As the supply of good 2 (per laborer) is given by $(1 - \lambda)y_2$, momentary euilibrium is determined by the following condition:

$$(1 - \gamma)y_2 = [1 - S_2(k_w)]\, (R\, k_w + W)/p. \tag{11}$$

Thanks to the Walras law, the market clearing condition for good 1 follows from that of good 2.

In general, $S_w$ depends on, *inter alia*, $k_w$, $y_w$, and p. If $S_w$ is a given constant fraction, (11) is a special case of Stiglitz's (1967) momentary equilibrium. It is well known that, in this case, the momentary equilibrium is unique if the sum of the Hicksian elasticities of substitution equals or exceeds 1, i.e.,

$$\sigma_1 + \sigma_2 \geq \quad \text{for all } w > 0 \tag{12}$$

where $\sigma_i = (\partial k_i/\partial_w) \cdot (w/k_i)$ and $k_i(w)$ is obtained by inverting (5).

If $S_w$ is a function of $k_w$ alone, then (12) is also sufficient for the uniqueness of momentary equilibrium because our model is functionally

In summary, the assumption of full employment of both factors amounts algebraically to (7):

$$\lambda k_1 + (1 - \lambda) k_2 = k \equiv k_w + k_c. \tag{7}$$

We have now completely specified the supply conditions in the economy. Once the relative price of the goods is given, the other variables in the system—namely, $y_1$, $y_2$, $k_1$, $k_2$, $\lambda$, W, and R—are determined. Alternatively, if the wage/rental ratio is known, all of the other variables follow. In fact, given our assumption about capital intensity in (6), the relative good price and the wage/rental ratio bear a one-to-one relationship. Let p be the relative price of good 2 in terms of good 1, i.e., $p = p_2/p_1$; it can be shown easily that p is a monotonically increasing function of w:

$$p = p(w), \; p' > 0 \tag{8}$$

where p' is the derivative of p with respect to w.[8]

## 2.2. Demand

To close the model of momentary equilibrium, let us bring in the demand conditions. Define y as the per-laborer national income in terms of good 1, i.e.,

$$\begin{aligned} y &= \lambda y_1 + (1 - \lambda) p \, y_2 \\ &= y_w + y_c \end{aligned} \tag{9}$$

where

$y_w = W + R k_w$, workers' average income

$y_c = R k_c$, capitalists' income per laborer.

identical to the Uzawa model as extended by Inada. However, if $S_w$ depends, as well, on the endogenous variables $y_w$ and p, stricter conditions are required, for example, gross substitutability. In this paper, we assume that $S_w$ and $S_c$ are increasing functions of $k_w$ and $k_c$, respectively. This is so both because the rates of return on investment in the real world are higher for higher absolute amounts of investment and because the marginal rate of substitution between future and present consumption is higher the larger the current level of income and wealth. Alternatively, one can point for justification to the desire for power [as in Höllander (1978)] or to the motives of bequest.

Let us summarize the model of momentary equilibrium. It may be represented by a system of six equations in six variables (R, W, $k_1$, $k_2$, p, and $\lambda$) and two parameters ($k_c$ and $k_w$):

$$R = f_1'(k_1) \tag{13a}$$

$$W = f_1(k_1) - k_1 f'(k_1) \tag{13b}$$

$$k_2 = k_2(w) = k_2 \left(\frac{W}{R}\right) \tag{13c}$$

$$\lambda = \frac{k_w + k_c - k_2}{k_1 - k_2} \tag{13d}$$

$$(1 - \lambda) f_2 (k_2) = [1 - S_w(k_w)] (k_w \cdot R + W)/p \tag{13e}$$

$$p = \frac{f_1'(k_1)}{f'_2(k_2)} \tag{13f}$$

where $k_2$ is obtained by inverting (5), and (13d) is obtained by solving (7).

## 2.4. Long-Run Equilibrium

From (13) we may solve the six variables in terms of the two parameters. In particular, we may express R and W as functions of $k_c$ and $k_w$. If the elasticity condition (12) holds, then the functional relationship is unique so that we can write $R(k_c, k_w)$ and $W(k_c, k_w)$. Suppose that both $k_c$ and $k_w$ are positive, we may substitute these expressions into the following two equations and solve for the long-run equilibrium:

$$S_c(k_c)\, R(k_c, k_w) = n + u \tag{14a}$$

$$S_w(k_w) \left[ R(k_c, k_w) + \frac{W(k_c, k_w)}{k_w} \right] = n + u. \tag{14b}$$

So, we have completely spelled out our basic model of wealth and income determination. In the next section we will present some comparative static results of the model just developed. In section 4 we will look into the question of multiple long-run equilibria.

## 3. COMPARATIVE STATICS

In this section we will derive some comparative static results of the momentary equilibrium with respect to its parameters, namely, $k_c$ and $k_w$. The endogenous variables we will look at include w, W, R, $y_w$, $y_c$, and y. Some of the derivations are quite laborious and are relegated to the appendices.

Thanks to the correspondence principle, we know that comparative static results are intimately related to the stability condition of the equilibrium. In Stiglitz (1967) and, more generally, in the growth literature, it was assumed that the economy was always in momentary equilibrium. Since, in most cases, interest was in the steady states, the question of stability of the momentary equilibrium was hardly raised. Moreover, stability of equilibrium often referred to that of the factor market (assuming that the good markets are always in equilibrium) while stability of the good markets (assuming that the factor markets are always in equilibrium) was seldom examined.[9]

### 3.1. Stability of the Good Markets

We will follow the literature in assuming that the economy is always in momentary equilibrium. However, since we want to derive comparative static results, we must know the stability conditions. In particular, we would like to obtain the stability conditions of the good markets because these markets are our focus of analysis.

The good markets are said to be stable in momentary equilibrium if, given any $k_c$ and $k_w$, raising the price of good 2 (or lowering the price of good 1) leads to an excess supply of good 2 (or an excess demand for good 1).

The left-hand side of (13e) represents the supply of, and the right-hand side represents the demand for, good 2, i.e.,

$$S_2 = (1 - \lambda)\, f_2(k_2)$$
$$D_2 = [1 - S_w(k_w)]\,(k_w \cdot R + W)/p. \tag{15}$$

Mathematically, the momentary equilibrium is stable if and only if $(\partial \log S_2)/(\partial \log p) > (\partial \log D_2)/(\partial \log p)$. But, we know from (8) that p and w are positively related. Thus, the good markets are stable if and only if $(\partial \log S_2)/(\partial \log w) > (\partial \log D_2)/(\partial \log w)$. Substituting the definition of $\lambda$ into (15), taking derivatives logarithmically, and using the relation in equation (16)[10] below, we obtain the stability condition: the good markets are stable if and only if (17) holds, i.e.,

$$\frac{\partial \log p}{\partial \log w} = \eta_1 - \eta_2, \tag{16}$$

where $\eta_i = (\partial \log y_i)/(\partial \log k_i) < 1$, $i = 1, 2$; and $\eta_1 > \eta_2$ because of (6),

$$\Delta \equiv \left(\eta_2 + \frac{k_2}{k_1 - k_2}\right)\sigma_2 + k_1\,\sigma_1\left(\frac{1}{k_1 - k} - \frac{1}{k_1 - k_2}\right)$$
$$+ (1 - \eta_2) - \frac{w}{w + k_w} > 0, \tag{17}$$

where $k \equiv k_w + k_c$.

In Appendix A, some sufficient conditions for stability in the good markets are derived. One such condition is $\sigma_2 > \eta_1 - \eta_2(1 - \eta_1)/(1 - \eta_2) \cdot \eta_2$,

which is somewhat weaker than what has been obtained in the literature—namely, $\sigma_2 > 1$—because $\eta_1$ is strictly less than 1.

### 3.2. The Basic Comparative Static Results

If we differentiate (13) logarithmically and totally with respect to w and make appropriate substitutions (see Appendix B), then we obtain the following equation:

$$\Delta \, d \log w = Z_1 \, d \log k_c + Z_2 \, d \log k_w \tag{18}$$

where

$\Delta$ is as defined in (17)

$$Z_1 = (k_c)/(k_1 - k) > 0$$

$$Z_2 = \left[ (k_w)/(k_1 - k) + (1 - \rho_w) - \epsilon_{S_w} (S_w)/(1 - S_w) \right].$$

In the expression for $Z_2$, $\rho_w$ is the share of wage income in workers' income, i.e.,

$$\rho_w = \frac{W}{W + R\,k_w} = \frac{w}{w + k_w},$$

and $\epsilon_{S_w}$ is the elasticity of the workers' saving ratio, i.e.,

$$\epsilon_{S_w} = \frac{\partial S_w}{\partial k_w} \frac{k_w}{S_w}.$$

Under certain conditions (see Appendix A) the good markets in momentary equilibrium are stable and, hence, $\Delta > 0$. From (18) the basic results follow:

$$\frac{\partial \log w}{\partial \log k_c} = \frac{Z_1}{\Delta} > 0 \tag{19a}$$

$$\frac{\partial \log w}{\partial \log k_w} = \frac{Z_2}{\Delta} \gtreqless 0. \tag{19b}$$

In (19) we see that an increase in $k_c$ unambiguously raises the wage/rental ratio. This can be seen more intuitively in the following way. Suppose that w remains unchanged when $k_c$ is increased, then p must also remain unchanged, because of (8). As a result, both $y_w$ and the demand for good 2 remain constant. However, with constant labor and constant relative price, the Rybczynski theorem dictates that, because of equation (6), the supply of good 2 must decrease when total capital increases. As demand exceeds supply, it is impossible for p to remain unchanged and, if the good markets are stable, p must rise to clear the markets. The movement of w follows immediately from (8).

On the other hand, the effect of an increase in $k_w$ is ambiguous without further assumptions. This is because an increase in $k_w$ may raise the desire for accumulation so much that its negative effect on p (and, hence, on w) may more than offset the positive effect brought about by a higher aggregate capital/labor ratio, k.

### 3.3 Some Derivative Comparative Static Results

From (19) we may derive other interesting comparative static results. Knowing from (4) that W and R always move in opposite directions, it is not surprising to find that

$$\frac{\partial \log W}{\partial \log k_c} = \eta_1 \frac{Z}{\Delta} > 0 \tag{20a}$$

$$\frac{\partial \log R}{\partial \log k_c} = (\eta_1 - 1) \frac{Z_1}{\Delta} < 0 \ . \tag{20b}$$

Although the workers' rental income declines as R falls, it is more than offset by a rise in W. Thus, the workers' income also rises:

$$\frac{\partial \log y_w}{\partial \log k_c} = (1 - \eta_1)\ \eta_1 \frac{Z_1}{\Delta}\left(\frac{k_1 - k_w}{\eta_1 k_w + k_1(1 - \eta_1)}\right) > 0 \tag{21a}$$

because $k_1 > k > k_w$.

On the other hand, the effect of $k_c$ on capitalists' income is ambiguous as the rental rate, R, moves in opposite direction to $k_c$:

$$\frac{\partial \log y_c}{\partial \log k_c} = 1 - (1 - \eta_1) \frac{Z_1}{\Delta} \gtreqless 0. \tag{21b}$$

It may be rather unexpected that the effect of an increase in $k_c$ on national income is unambiguously positive in spite of (21b).

$$\frac{\partial \log y}{\partial \log k_c} = (1 - \eta_1)\ \eta_1 \frac{Z_1}{\Delta}\left[\frac{k_1 - k}{k_1(1 - \eta_1) + \eta_1 k}\right] + \frac{\eta_1 k_c}{\eta_1 k + (1 - \eta_1)k_1} > 0. \tag{21c}$$

This can be understood in the following way. First, the production frontier moves out when $k_c$ goes up with $k_w$ held constant. Second, we have shown that the equilibrium relative price, p, must go up. Thus, the national income must go up because both forces point in the same direction.

Since $(\partial \log w)/(\partial \log k_w)$ cannot be signed unambiguously, it is only natural that the effect of an increase in $k_w$ on R, W, $y_w$, $y_c$, and y is also ambiguous. If we want to obtain more definite results about $(\partial \log w)/(\partial \log k_w)$ and its derived entities, we must introduce more structure into our model—in particular, some characteristics of good 2. To this end, in the next subsection we will introduce the concept of wealth normality.

### 3.4. Wealth Normality

A good is said to be wealth normal (or wealth noninferior) if, at given good prices and factor prices, an increase in wealth does not lead to a decrease in the expenditure on that good. Let it be noted in passing that wealth normality is not equivalent to income normality in our model. Because $S_w$ is assumed to be a function of $k_w$ alone, a wealth-induced income change would differ from other forms of income changes in the effect on demand for goods.

According to our definition of wealth normality, it follows that good 2 is a wealth-normal consumption good for the workers if and only if (22) holds:

$$\frac{\partial}{\partial k_w}\left\{\left[(1 - S_w(k_w)\right]y_w\right\} \geq 0. \tag{22}$$

If good 2 is wealth normal, then it can be shown that $Z_2 > 0$ (see Appendix C for a proof); and the ambiguity of some of the comparative static results is eliminated:

$$\frac{\partial \log w}{\partial \log k_w} = \frac{Z_2}{\Delta} > 0 \tag{23a}$$

$$\frac{\partial \log W}{\partial \log k_w} = \eta_1 \frac{Z_2}{\Delta} > 0 \tag{23b}$$

$$\frac{\partial \log R}{\partial \log k_w} = (\eta_1 - 1) \frac{Z_2}{\Delta} < 0 \tag{23c}$$

$$\frac{\partial \log y_w}{\partial \log k_w} = (1 - \eta_1)\eta_1 \frac{Z_2}{\Delta} \left( \frac{k_1 - k_w}{\eta_1 k_w + k_1(1 - \eta_1)} \right) + \frac{k_w}{k_w + w} > 0 \tag{23d}$$

$$\frac{\partial \log y_c}{\partial \log k_w} = (\eta_1 - 1) \frac{Z_2}{\Delta} < 0 \tag{23e}$$

$$\frac{\partial \log y}{\partial \log k_w} = (1 - \eta_1)\, \eta_1 \frac{Z_2}{\Delta} \left( \frac{k_1 - k}{\eta_1 k + (1 - \eta_1)k_1} \right) + \frac{\eta_1 k_w}{\eta_1 k + (1 - \eta_1)k_1} > 0 \tag{23f}$$

Therefore, if good 2 is not wealth inferior, an increase in $k_w$ would raise w, W, and $y_w$; and it would depress R and $y_c$. The net effect on national income is positive implying that the workers gain more than what the capitalists lose. The impact of $k_w$ on w can also be explained more intuitively by applying the Rybczynski theorem as in 3.2 above.

## 4. MULTIPLE EQUILIBRIA

As we mentioned in section 1, Stiglitz (1967) showed that the long-run equilibrium (steady state) is unique if $S_w$ and $S_c$ are constants. However,

if they are functions of $k_w$ and $k_c$, respectively, the proof of uniqueness no longer goes through; and there may be multiple long-run equilibria although momentary equilibrium will remain unique as long as condition (12) holds. The questions of multiplicity and stability of long-run equilibria in our model have been addressed elsewhere (Adelman and Cheng). Here we will present only two simple results about the properties of these equilibria.

Proposition 1: In any long-run equilibrium, a capitalist must own more wealth than does a worker (i.e., $k_c' > k_w'$) if $S_w(k_w')$ and $S_c(k_c')$ are identically increasing functions, that is, if $S_w(k_w') = S_c(k_c')$, for all $k_w' = k_c'$, and $(\partial S_c/\partial k_c') > 0$, for all $k_c'$, where $k_w'$ and $k_c'$ are defined in Section 2.

*Proof* : Proposition 1 follows from (14). Eliminating $(n + u)$ from (14), we obtain

$$\left[S_c(k_c') - S_w(k_w')\right] = \frac{S_w(k_w')\, W}{k_w'\, R} > 0.$$

Proposition 2: If $\varepsilon_{S_c} < \varepsilon_R$, where $\varepsilon_{S_c} = \{[(\partial S_c)/(\partial k_c)] \cdot (k_c/S_c)\}$ is the elasticity of the capitalists' saving ratio and $\varepsilon_R \equiv -(\partial R)/(\partial k_c) \cdot (k_c/R)]$ is the absolute value of the elasticity of the rental rate with respect to $k_c$, and if good 2 is wealth normal, then for the set of long-run equilibria satisfying (14), every equilibrium is Pareto efficient. That is to say, compared to any other equilibrium in the set, if workers own more capital in an equilibrium, then capitalists must own less, i.e., for any $(\bar{k}_{w1}, \bar{k}_{c1})$ and $(\bar{k}_{w2}, \bar{k}_{c2})$ satisfying (14), $\bar{k}_{w1} > \bar{k}_{w2}$ implies $\bar{k}_{c1} < \bar{k}_{c2}$ and vice versa.

*Proof* : Let $k_c = g(k_w)$ be the locus satisfying (14a). We can derive the slope of $g(k_w)$ by applying the implicit function rule,

$$\frac{d\,g}{d\,k_w} = -\frac{S_c \dfrac{\partial R}{\partial k_w}}{\left(\dfrac{\partial S_c}{\partial k_c} R + S_c \dfrac{\partial R}{\partial k_c}\right)}.$$

$$= -\frac{S_c \dfrac{\partial R}{\partial k_w}}{\dfrac{S_c R}{k_c}\left(\varepsilon_{S_c} - \varepsilon_R\right)}.$$

From (23c) we know that $\partial R/\partial k_w$ is negative if good 2 is wealth normal. It follows that $(d\,g/d\,k_w) < 0$ if $\varepsilon_{S_c} < \varepsilon_R$. Since any equilibrium must satisfy (14a), every equilibrium is Pareto efficient if $(d\,g/d\,k_w) < 0$.

## 5. WEALTH DISTRIBUTION AND INCOME DISTRIBUTION

In our model developed in section 2, the behavior of wealth distribution (including the dynamic paths and the equilibrium positions) is completely characterized once the initial wealth distribution is specified. An important question often raised is whether a given wealth distribution is in some sense more desirable than another. To answer this question, we need to have some criterion as a basis for comparison. In this paper, we take the view that every individual is equal and that an <u>egalitarian</u> wealth distribution is achieved when every individual owns the same amount of wealth. For our simple two-class model, such a distribution is achieved if the following condition is fulfilled:

$$k_c = \frac{C}{L} k_w. \tag{24}$$

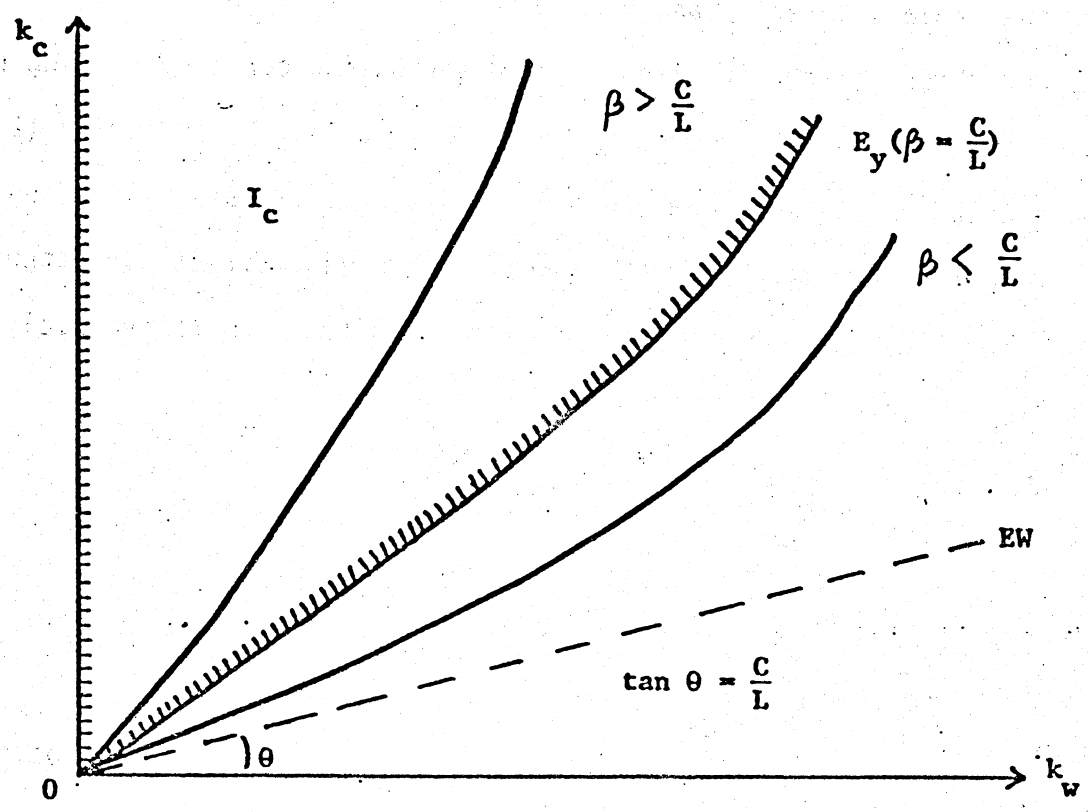
Let the locus of egalitarian wealth distribution in the $(k_c, k_w)$ space be denoted by EW (see Figure 1 below). From Proposition 1, we know that, under rather general conditions, the long-run equilibria will always lie to the left of EW. Furthermore, under the additional conditions in Proposition 2, the equilibrium that gives most wealth to the workers would be most egalitarian among the possible long-run equilibria. Therefore, under the principle of egalitarianism and restricting ourselves to the set of long-run equilibria, this particular equilibrium, which is most favorable to the workers, would be considered as the social optimum. In general, whether the optimum can be attained depends on the extent to which wealth redistribution can be effected. [For a more detailed analysis, see Adelman and Cheng (  )].

One may question why we put so much weight on relative wealth distribution. Would it be more meaningful to study the relative welfare positions of the two classes by focusing on their relative income distribution Is there a conflict between a favorable relative wealth distribution and a favorable relative income distribution?

Our response is that wealth distribution and income distribution are intimately related because wealth ownership gives rise to nonwage income. In our model, the distribution of income (functional and personal) indeed is uniquely determined for any given $k_c$ and $k_w$, provided that the elasticity condition in (12) is satisfied. By definition, capitalists' income, $y_c$, is equal to $Rk_c$; workers' income, $y_w$, is equal to $Rk_w + W$; and the relative income share can be expressed in the following manner:

$$\frac{y_w}{y_c} = \frac{k_w}{k_c} + \frac{W}{k_c}. \tag{25}$$

Figure 1


$k_c$
$\beta > \frac{C}{L}$
$E_y(\beta = \frac{C}{L})$
$I_c$
$\beta < \frac{C}{L}$
EW
$\tan\theta = \frac{C}{L}$
$\theta$
0
$k_w$

To relate relative income distribution to relative wealth distribution, let us introduce the concept of a constant relative income distribution curve. From (25), we can define a β-income-distribution curve as the locus of $k_c$ and $k_w$ which satisfies the relationship in (26):

$$k_c = \beta \left[ k_w + w\ (k_w, k_c) \right]. \tag{26}$$

where β is the ratio between $y_c$ and $y_w$.

We may now define an egalitarian income distribution curve by setting β in (26) equal to C/L and let it be denoted by Ey (Figure 1). By comparing (24) and (26), we know that Ey must lie above Ew. Since capitalists very often have a higher personal income than the workers, we will restrict our attention to the set of wealth distributions for which this is true. More precisely, we will focus on the following set:

$$I_c = \left\{ [k_w, k_c] \mid k_c = \beta\ [k_w + w\ (k_w, k_c)];\ \beta > \frac{C}{L} \right\}$$

which is the area to the left of Ey.

On the basis of the above difficulties, we will say that an income distribution is more egalitarian than another if the β coefficient associated with the former is closer to C/L than is that associated with the latter. Since $I_c$ lies to the left of Ey and, hence of Ew, it follows that in this set capitalists own more wealth per person than the workers do which is consistent with what we observe in the world.

To relate relative income distribution to relative wealth distribution, we also have to know the properties of the β-income-distribution curves. It is shown in Appendix D that, if good 2 is wealth normal and if the response of w to $k_c$ is not too strong, then an increase in $k_c$ would raise β and an increase in $k_w$ would lower it. Under these conditions, a typical family of β-income-distribution curves would have the property that the relative income distribution becomes more egalitarian as the economy moves in the south-east (including south and east) direction in $I_c$. At the same time, such a movement also brings the economy closer to the locus of egalitarian wealth distribution, Ew. Conversely, a movement in the opposite direction would lead to a less egalitarian income distribution and less egalitarian wealth distribution. These results are summarized more precisely below in Proposition 3.

Proposition 3. In the set of $I_c$, provided that good 2 is wealth normal and that $(\partial \log w/\partial \log k_c) < 1 + k_w/w$, a wealth distribution is more egalitarian if and only if the income distribution is also more egalitarian.

*Proof*: Proposition 3 follows from the result contained in Appendix D and the definition of egalitarian distributions.

Part of the condition in Proposition 3 is, in terms of w, an endogenously determined variable which, in principle, depends on all of the parameters relevant to (13). It would be useful if we could express the conditions explicitly in terms of the exogenous variables. This is done in Proposition 4.

Proposition 4. Consider the set of long-run equilibria in $I_c$. Suppose that both $S_c$ and $S_w$ are inelastic, i.e., $\epsilon_{S_c} < 1$ and $\epsilon_{S_w} < 1$. Then, a wealth

distribution is more egalitarian if and only if it generates a more egalitarian income distribution.

*Proof*: In any long-run equilibrium, $y_w/y_c = [S_c(k_c)k_w]/[S_w(k_w)k_c]$. It can be shown easily that $[\partial\ (y_w/y_c)/\partial\ k_w] > 0$ and that $[\partial(y_w/y_c)/\partial\ k_c] < 0$ under the hypotheses of Proposition 4. Clearly, these relationships are sufficient to identify more egalitarian wealth distributions with more egalitarian income distributions in $I_c$.

Since Proposition 4 only deals with long-run equilibrium in which both the relative wealth distribution and the relative income distribution are constant, the next question is under what conditions a constant relative wealth distribution is consistent with a relative income distribution outside of equilibrium. An answer is provided in Proposition 5.

Proposition 5. Aside from a long-run equilibrium, a constant relative wealth distribution is consistent with a constant relative income distribution only if there exists some rescaling of $k_c$ such that $S_c(k_c)$ is equal to $\gamma\ S_w(k_w)$ for some $\gamma > 0$ and over some interval in which $k_w = k_c$.

*Proof*: From (1) we have:

$$\frac{d\ k_w/dt}{d\ k_w} - \frac{d\ k_c/dt}{k_c} = S_w(k_w)\ \frac{y_w}{k_w} - S_c(k_c)\ \frac{y_c}{k_c}.$$

A constant relative wealth distribution implies that $(d\ k_w/dt)/(k_w) = (d\ k_c/d\ t)/(k_c)$, which in turn implies: $y_w/y_c = (k_w\ S_c)/(k_c\ S_c)$.

If $y_w/y_c$ is also constant, $S_c/S_w$ must be constant, i.e.,

$$\frac{\partial S_c/d\,t}{S_c} = \frac{\partial S_w/\partial t}{S_w}$$

or

$$\varepsilon_{S_c} \frac{\partial k_c/\partial t}{k_c} = \varepsilon_{S_w} \frac{\partial k_w/\partial t}{k_w}.$$

But, $(\partial k_c/\partial t)/(k_c) = (\partial k_w/\partial t)/(k_w)$ when $k_c/k_w$ is constant. Therefore, $\varepsilon_{S_c}$ must be equal to $\varepsilon_{S_w}$, which can hold outside a long-run equilibrium only if there exists some rescaling of $k_c$ such that $S_c(k_c)$ is equal to a positive multiple of $S_w(k_w)$ over some interval in which $k_w = k_c$.

## 6. TECHNOLOGICAL PROGRESS, WEALTH DISTRIBUTION, AND INCOME DISTRIBUTION

A question also arises as to what kind of technological progress would sustain a particular income and/or wealth distribution. Since the difficulties of analyzing technological progress are well known, we do not expect to be able to say much on this subject here. Our purpose is only to suggest conditions under which some of the well-known processes of technological change would sustain a given relative income distribution and a relative wealth distribution.

Consider a special case of Harrod neutral (i.e., purely labor-augmenting) technological change in which the time rate of change is constant so that effective labor per worker at t can be expressed in the following fashion:

$$z^e(t) = e^{\alpha t};\ a > 0. \tag{27}$$

Further, assume that the argument in the saving functions, $S_w$ and $S_c$ is given by physical wealth deflated by the index of technological progress, i.e.,

$$k_i^e(t) = \frac{k_i(t)}{e^{\alpha t}} \qquad i = w, c. \tag{28}$$

Under the above assumptions, a long-run equilibrium is achieved when $k_c^e(t) \equiv \overline{k}_c^e$ and $k_w^e \equiv \overline{k}_w^e$ for all $t \geq 0$. In the equilibrium, the relative wealth distribution and the capital/effective labor ratio are constant. So are $S_w$ and $S_c$. Let the wage rate per unit of effective labor be $W^e$. Since the relative demand and supply conditions are identical, the wage/rental ratio ($W^e/R$) remains unchanged. The wage rate per worker, W, and personal wealth, $k_c$ and $k_w$, all rise at the same rate, $\alpha$. Consequently, the relative income distribution remains constant.

For the more general case of technological progress and saving behavior, we cannot say anything in the affirmative; but we can obtain a nonexistence result as a corollary of Proposition 5.

*Corollary 1*: Unless there exists some rescaling of $k_c$ such that $S_c(k_c)$ is equal to $\gamma\, S_w(k_w)$ for some $\gamma > 0$ and over some interval in which $k_w = k_c$, there does <u>not</u> exist any technological change that would sustain a constant income and wealth distribution outside the long-run equilibria.

## 7. EXTENSIONS

Our model in section III can be easily extended to capture the phenomenon of "backwash effect" of economic growth. The assumption of class-specific

consumption goods is an artificial one. We may relax this assumption and allow both classes to consume both goods. In this case, if we assume further that the labor-intensive consumption good is inferior--a plausible assumption in a closed economy--then the phenomenon of "backwash effect" stands out clearly.

There is an asymmetry in the effect of income growth on capitalists and workers. Suppose that, at a certain momentary equilibrium, both classes consume both goods. Consider an exogenous increase in income on the part of the capitalists. If the labor-intensive consumption good (good 2 in our model) is assumed to be inferior, demand for it would decline while demand for the capital-intensive consumption good (good 1) would rise. Because of our capital-intensity assumption in (6), the capital rental rate would rise thus depressing worker's income and raising capitalists' income even further.

Now consider an exogenous increase in income on the part of the workers. Because good 2 is inferior, demand for it would fall while the demand for good 1 would rise. For the same reason, the rental rate would rise and the wage rate would fall. The effect on workers' income is ambiguous, but it would be negative if the workers' capital holding is small. On the other hand, the effect on capitalists' income is unambiguously positive.

Thus, there is a built-in asymmetry in the effect of income growth on distribution. When capitalists' income rises, there are forces to raise it even further. On the other hand, when workers' income rises, the economic forces may work in the opposite direction.

Another direction of possible extension is to relax the assumption of perfectly mobile labor. Once this is done, some workers may benefit from economic growth while others may be hurt by it.

Still one more direction of extension is the introduction of international trade. If we make the small-country assumption, the relative price, p, is given from outside. It follows that R and W are also given, and the dependence of R and W on wealth distribution disappears. Given an initial distribution of wealth--$k_c$ and $k_w$--the impact effect on personal income of opening up trade depends upon which good is exported. If good 2 is exported, the wage/rental ratio would rise. The rate of wealth accumulation of the workers would rise while that of the capitalists would fall. The opposite would happen if good 1, instead, were exported.

A final direction of extension would be to consider human capital in addition to physical capital. The two kinds of capital would not be perfect substitutes as inputs and might involve different production processes as outputs. For example, human capital may be more labor-intensive in production than is physical capital. If, in addition, human capital is complementary to physical capital, then a redistributive policy in favor of the workers would stand a better chance of benefiting both classes in the long run.

## 8. CONCLUDING REMARKS

In section 2 we developed a simple model which we believe offers a better framework for discussing issues related to wealth distribution and redistribution than what is currently available in the theoretical literature. We have shown that a redistribution toward the poor may hurt the rich even in the long run (Proposition 2). This is less rosy than what some reformists would suggest, namely, that both the poor and rich would gain from redistribution.

We have given conditions under which a more egalitarian wealth distribution is associated with a more egalitarian income distribution

(Propositions 3 and 4). The implication is straightforward: wealth redistribution is not just carried out for its own sake--it also brings about a more desirable income distribution.

It is difficult to achieve constant wealth and income distribution by market forces alone, particularly when the economy is out of long-run equilibrium. The saving behavior may be incompatible with such an objective (Proposition 5), and there may not exist any technological progress that would sustain a desirable wealth and income distribution (Corollary 1). As a result, policy intervention in the form of direct wealth and income transfer may be necessary before some desirable equilibrium is reached.

# APPENDIX A

For given $k_c$ and $k_w$, our model is equivalent to Uzawa's (1962) model in which the saving ratio out of wages is $S_w$ and the saving ratio out of profit is $\bar{S}_c$ [see Inada (1963)] if we define $\bar{S}_c$ as follows:

$$\bar{S}_c = S_w + (1 - S_w)\frac{k_c}{k} > S_w. \tag{A.1}$$

The total demand for investment good is then given by:

$$\bar{S}_c k + S_w w. \tag{A.2}$$

By hypothesis, the factor markets are in equilibrium; hence, k can be expressed as the following [see Drandakis (1963), p. 220]:

$$\begin{aligned} k &= \frac{k_1 k_2 [S_w k_1 + (1 - S_w)k_2 w]}{(1 - \bar{S}_c)k_1 + \bar{S}_c k_2 + w} \\ &= \frac{k_1 k_2 + [S_w k_1 + (1 - S_w)k_2] w}{\delta} \end{aligned} \tag{A.3}$$

where $\delta = (1 - S_w) k_w k_1/k + [S_w + (1 - S_w) k_c/k] k_2 + w$.

Substitute (A.3) into (17), and the stability condition becomes:

$$\begin{aligned} \Delta \equiv \left(\eta_2 + \frac{k_2}{k_1 - k_2}\right)\sigma_2 + \frac{k_1}{k_1 - k_2}\left\{\frac{\left[S_w + (1 - S_w)\frac{k_c}{k}\right]k_2 + S_w w}{(1 - S_w)\left[k_1\frac{k_w}{k} + w\right]}\right\}\sigma_1 \\ + (1 - \eta_2) - \frac{w}{w - k_w} > 0. \end{aligned} \tag{A.4}$$

Making use of the relationship, $k_i (1 - \eta_i)/\eta_i = w$, $i = 1, 2$, and after some tedious substitution and cancellation, we may rewrite (A.4) as

$$\frac{\Delta}{\eta_2} = \frac{(1-\eta_2)}{m}\sigma_2 + \frac{(1-\eta_1)}{m}\frac{\left[\frac{S_w}{\eta_2} + (1-S_w)\frac{k_c}{k}\right]}{(1-S_w)\left[\frac{1}{\eta_1} - \frac{k_c}{k}\right]}\sigma_1$$

$$- (1-\eta_2)\left[\frac{(1-\frac{k_w}{k_2})}{(1-\eta_2) + \frac{k_w}{k_2}\eta_2}\right] > 0 \qquad \text{(A.5)}$$

where $m = (1-\eta_2)\eta_1 - (1-\eta_1)\eta_2 > 0$ because of (7).

Equation (A.5) is the necessary and sufficient condition for the good markets to be stable. From the definition of m and the restrictions imposed on $\eta_1$ and $\eta_2$, it is clear that the coefficient of $\sigma_2$ exceeds 1. A sufficient condition for stability is $\sigma_2 > m/(1-\eta_2) = \eta_1\ \eta_2 - (1-\eta_1)/(1-\eta_2)$ because

$$(1-\eta)_2\left[\frac{1-\frac{k_w}{k_2}}{(1-\eta_2) + \frac{k_2}{k_2}\eta_2}\right]$$

is always smaller than 1 algebraically and equal to 1 only when $k_w = 0$. Should $k_w$ equal 0, it is easy to show that the coefficient of $\sigma_1$ also exceeds 1. Under this condition, $\sigma_1 + \sigma_2 \geq 1$ is sufficient for stability. Furthermore, because the coefficients of $\sigma_1$ and $\sigma_2$ are always positive, (A.5) is satisfied automatically if $k_w > k_2(k_w, k_c)$ where $k_2(k_w, k_c)$ is determined in (13).

# APPENDIX B

In this appendix we will derive some comparative static results of the momentary equilibrium described by (13). Before we proceed, it is useful to note some implications of profit maximization on the part of the producers—more specifically, some implications of (5).

*Fact 1.*

$$\epsilon_i \sigma_i + 1 - \eta_i = 0; \qquad i = 1, 2$$

where $\sigma_i$ and $\eta_i$ are as defined in the main text of this paper and $\epsilon_i \equiv (f_i'')/(f_i') k_i$ is negative in view of (3).

Fact 1 is obtained by taking the total derivative of (5) with respect to w and rearranging terms.

*Fact 2.*

$$k_i (1 - \eta_i)/(\eta_i) = w; \qquad i = 1, 2.$$

This is just a different way of writing (5).

Now, let us indicate how (18) is derived from (13). Using Fact 1, we may derive (B.1) from (13a):

$$\begin{aligned} d \log R &= \epsilon_i \sigma_1 \, d \log w \\ &= (\eta_1 - 1) \, d \log w \end{aligned} \tag{B.1}$$

By definition, W = R w. We may derive (B.2) by making use of (B.1):

$$\begin{aligned} d \log W &= d \log R + d \log w \\ &= \eta_1 \, d \log w. \end{aligned} \tag{B.2}$$

From (13c), the following is straightforward:

$$d \log k_2 = \sigma_2 \, d \log w. \tag{B.3}$$

Using (13f), we may derive (B.4):

$$\begin{aligned} \partial \log p &= (\varepsilon_1 \sigma_1 - \varepsilon_2 \sigma_2) \, d \log w \\ &= [(\eta_1 - 1) - (\eta_2 - 1)] \, d \log w \\ &= (\eta_1 - \eta_2) \, d \log w. \end{aligned} \tag{B.4}$$

Finally, substituting (13d) into (13e), taking total derivatives logarithmically of (13e) with respect to w, and making use of (B.1) through (B.4), we obtain:

$$\begin{aligned} &\left[ \frac{k_1}{k_1 - k} \sigma_1 + \eta_2 \sigma_2 - \frac{k_1 \sigma_1}{k_1 - k_2} + \frac{k_2 \sigma_2}{k_1 - k_2} - (1 \rho_w) \varepsilon_1 \sigma_1 \right. \\ &\qquad \left. - \rho_w \eta_1 + \eta_1 - \eta_2 \right] d \log w \\ &= Z_1 \, d \log k_c + Z_2 \, d \log k_w. \end{aligned} \tag{B.5}$$

Equation (B.5) may be further simplified to obtain (18) by using Fact 1 and the definition of $\rho_w$.

APPENDIX C

$$\frac{\partial}{\partial k_W}\left\{[1 - S_W(k_W)]y_W\right\} = (1 - S_W)\ R - \frac{\partial S_W}{\partial k_W}(W + R\ k_W).$$

The hypothesis of wealth normality implies that:

$$\frac{1}{W + k_W} \geq \frac{\partial S_W}{\partial k_W} / (1 - S_W).$$

But

$$Z_2 = \left(\frac{k_W}{k_1 - k} + \frac{k_W}{W + k_W} - \frac{S_W}{1 - S_W}\,\varepsilon_{S_W}\right)$$

$$= k_W\left(\frac{1}{k_1 - k} + \frac{1}{W + k_W} - \frac{\partial S_W/\partial k_W}{1 - S_W}\right) > 0$$

under the hypothesis for any $k_W > 0$.

# APPENDIX D

Let $k_c(k_w; \beta)$ be a solution to (26). Applying the implicit function rule, we obtain:

$$\frac{\partial k_c}{\partial \beta} = \frac{k_w + w}{1 - \beta(\partial w/\partial k_c)} = \frac{k_w + w}{1 - (\partial \log w/\partial \log k_c)\,[w/(k_w + w)]} \tag{D.1}$$

and

$$\frac{\partial k_c}{\partial k_w} = \frac{\beta[1 + (\partial w/\partial k_w)]}{[1 - \beta(\partial w/\partial k_c)]} = \frac{\beta[1 + (\partial w/\partial k_w)}{1 - (\partial \log w/\partial \log k_c)\,[w/(w + k_w)]}. \tag{D.2}$$

If $(\partial \log w/\partial \log k_c) < 1 + k_w/w$, the common denominator in (D.1) and (D.2) is positive; $\partial w/\partial k_w$ is positive if good 2 is wealth normal. Under these conditions, both $\partial k_c/\partial \beta$ and $\partial k_c/\partial k_w$ are positive which implies that an increase in $k_c$ raises $\beta$ and that an increase in $k_w$ depresses $\beta$.

Footnotes

[1]For a survey, see Sahota (1978).

[2]This is with the possible exception of a group with zero wealth.

[3]Stiglitz (1969) assumed that saving was a linear function of income; Schlicht assumed that the saving/income ratio was increasing in relative personal income; Höllander assumed that saving was increasing in the profit rate and the amount of personal wealth and decreasing in the wage rate.

[4]We were told by Stiglitz in a recent conversation that he had also studied the limiting two-class equilibrium in an unpublished paper.

[5]Production functions were assumed to be Cobb-Douglas thus satisfying another sufficient condition of stability: the elasticity of substitution in the second industry exceeds or is equal to unity.

[6]To write equation (1) it is not necessary to assume anything about the growth rate of capitalists. However, if the growth rate of capitalists differs from that of workers, the relationship between $k_c$ and $k_c'$ will change over time; and our analysis of $k_c'$ based on $k_c$ would be more complicated.

[7]For a generalization of such conditions and the necessary modification of our subsequent analysis, see Drandakis (1963).

[8]See equation (16) below.

[9]Drandakis has demonstrated to us that the two different stability conditions are equivalent for the Uzawa model and, hence, for our model (see Appendix A).

[10]See equation (B.4) in Appendix B.

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