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A considerable amount of research has been devoted to studying the welfare consequences of stabilizing prices of basic internationally traded commodities. A widely held theoretical conclusion in the price stabilization literature is that complete price stabilization is preferred to no stabilization, e.g., Samuelson, Hueth and Schmitz, and Turnovsky. However, these results are usually derived using the assumption of zero storage costs. Also, partial stabilization is not generally considered as an alternative to complete price stabilization.

For these reasons, the theoretical stabilization literature has had a limited impact on the policy-making process. For example, if one considers complete price stabilization in reality, very high storage levels are required to enforce the policy; thus, storage costs usually become prohibitive. This is because artificially high price stability leads to high quantity variability of stock operations.

As an exception to the above, Massell (1969) showed that gains could be made from stabilization when storage costs are positive by using a "price band" type of stabilization policy. With the price band policy, the buffer stock authority sets upper and lower price limits; buffer stock transactions are then made such that price can vary freely (without intervention) between the two limits, but it cannot move outside of these limits.

Since Massell's work appeared, the price band concept has become very popular in the many proposals developed in response to the world food crisis of the 1970's. For example, at the International Wheat Council discussions in 1975 and 1976 (see Sarris for a discussion of these), many of the large wheat trading nations such as Canada, Australia, EEC, India, Egypt, and Japan proposed world wheat stabilization policies which operate according to a price band mechanism. Many economists have also advocated the price band proposed in world food price stabilization problems (e.g., Hillman, Johnson, and Gray). In addition, many of the recent simulation studies on the effect of price stabilization focus on price band policies (e.g., Sarris, Cochrane and Danin, and Sharples and Walker).

A major purpose of this paper is to show that price band policies are, in general, inferior mechanisms for achieving optimal social welfare through stabilization (using the same criterion of social welfare as has been used previously). This is done by developing an alternative to the price band policy which turns out to provide a global optimum with respect to the form of the buffer stock intervention policy. For this reason, it is found that the policy proposed herein is also preferred to buffer stock rules which operate with production triggers such as the one proposed by Tweeten et al. and the U. S. proposal presented at the International Wheat Council discussions as an alternative to the plans cited above.

The buffer stock policy suggested in this paper is one which modifies the demand curve by both rotation and stabilization and possibly altering the curvature such that buffer stock transactions make up the difference in modified and actual demand. It is found, however, that the optimum policy among this class of policies can be implemented by a simple rule such that buffer stock transactions are a constant multiple of the difference in actual price and "normal" price. Furthermore, a procedure for modifying the buffer authority's declared normal price is determined so that buffer stocks will not be excessively accumulated or depleted over sustained periods of time. It is interesting to note that a policy with these features was evaluated in a simulation study by Cochrane and Danin and was found to be preferable to no stabilization and also to the popular price band policy. The present paper, however, shows that their result is true analytically (simulation leaves room for doubt) and, furthermore, derives analytically the optimum policy among this general class of policies.

The Free-Market Model

Suppose, as did Massell, that industry demand and supply are linear and stochastic. Let demand be represented by

$$p = D(q) = a - bq + \delta \tag{1}$$

where p is price, q is quantity, both a and b are fixed, and δ is random at the time of decision making. Let supply (or short-run industry marginal cost) be represented by

$$C(q) = \phi + \beta q + \varepsilon \tag{2}$$

where β and ϕ are fixed and ε is random at the time of private decisions. Assuming ordinary supply-and-demand conditions, both b and β are positive.

Suppose, also, that production is random so that actual production q differs from planned production q_0 because of weather or some other stochastic influence. Where the difference in planned and actual production is represented by

$$\zeta = q - q_0, \tag{3}$$

the probability distribution of $e = (\delta\ \varepsilon\ \zeta)'$ will be characterized by[1]

$$E(e) = 0,\ E(ee') = \begin{bmatrix} \sigma_\delta & & 0 \\ & \sigma_\varepsilon & \\ 0 & & \sigma_\zeta \end{bmatrix} > 0.$$

Assuming competition, planned production in a free market is thus determined where expected price equals expected marginal cost,

(4) $$E(p) = E[C(q)].$$

Use of (4) implies from (1) and (2) that

(5) $$q_0 = \frac{a - \phi}{\eta}$$

where $\eta \equiv \beta + b$. Hence, free-market quantity and price are evident from (3) and (1):

(6) $$q = \frac{a - \phi + \eta\zeta}{\eta}$$

(7) $$p = \frac{\gamma + \eta(\delta - b\zeta)}{\eta}$$

where $\gamma = \beta a + b\phi$.

Partial Stabilization by Buffer Stock Operations

Initially, the buffer policy is assumed to have the effect of modifying the demand curve both by reducing instability and by altering demand elasticity as in the earlier Massell (1970) paper. That is, the demand curve in (1) is assumed to be altered by buffer authority intervention obtaining the modified demand given by

(8) $$p = a_0 - b_0 q + \delta_0$$

where a_0 and b_0 are alternative parameters set by the authority. Also, suppose $\delta_0 \equiv k\delta$ for some fixed k which is also controlled by the buffer authority. Since the quantity demanded for consumption according to (1) is

(9) $$q^d(p) = \frac{a - p + \delta}{b}$$

and the modified quantity demanded for both consumption and storage from (8) is

(10) $$q^*(p) = \frac{a_0 - p + \delta_0}{b_0},$$

the net change in stocks required to enforce (8) is

(11) $$s = q^*(p) - q^d(p).$$

Since the demand now perceived by producers is (8) rather than (1), planned production with partial stabilization q_0^* can be determined by analogy with (5):

(12) $$q_0^* = \frac{a_0 - \phi}{\nu}$$

where $\nu = \beta + b_0$. The quantity traded by producers and price are thus evident from (3) and (8):

(13) $$q^* = \frac{a_0 - \phi + \nu\zeta}{\nu}$$

(14) $$p^* = \frac{\theta + \nu(\delta_0 - b_0\zeta)}{\nu}$$

where $\theta = \beta\, a_0 + b_0\, \phi$. The quantity demanded by consumers in (9) at p^*, however, is

(15) $$q^d(p^*) = \frac{-\theta + \nu[a + (1 - k)\delta + b_0\zeta]}{b\nu}.$$

Hence, from (11), net change in stocks is

(16) $$s = q^* - q^d(p^*) = \mu_s + \frac{(n - \nu)\zeta + (1 - k)\delta}{b}$$

where

(17) $$\mu_s = \frac{\theta - \gamma + a_0\, b - a\, b_0}{b\nu}.$$

The short-run variance of stock transactions is thus

(18) $$\sigma_s = \frac{(n - \nu)^2\, \sigma_\zeta + (1 - k)^2\, \sigma_\delta}{b^2}.$$

Buffer Stock Transactions Costs

The buffer authority in this model will generally incur transactions losses (or gains) by purchasing excess supplies at prices different from those at which excess demands are satisfied. Purchases of buffer stocks are given by $P \equiv p^*\, s$ (sales are represented by $P < 0$) and have expected value

(19) $$E(P) = \mu_t + \frac{k(k - 1)\sigma_\delta + b_0(b_0 - b)\sigma_\zeta}{b}$$

where

(20) $$\mu_t = \frac{\theta \mu_s}{\nu}.$$

Consumer Effects

To evaluate the individual sector benefits of partial price stabilization, the change in consumer and producer surplus must again be calculated. Following Massell (1969), the gain in consumer surplus is[2]

$$G_c = \frac{1}{2} (p - p^*) \; [q^d(p) + q^d(p^*)]$$

which from (6), (7), (14), and (15) has short-run expected value

(21) $$E(G_c) = \mu_c + \frac{(\nu^2 - \eta^2)\sigma_\zeta + (1 - k)^2\sigma_\delta}{2b}$$

where

(22) $$\mu_c \equiv \frac{1}{2} \left(\frac{\gamma}{\eta} - \frac{\theta}{\nu}\right) \left(\frac{a - b_0}{\eta} + \frac{\nu a - \theta}{b\nu}\right).$$

Producer Effects

Producer gains from partial stabilization are

$$G_p = p^* \, q^* - pq - \int_q^{q^*} (\phi + \beta x + \varepsilon)dx$$

and have expected value

(23) $$E(G_p) = \mu_g + (\eta - \nu)\sigma_\zeta$$

where

$$\mu_g \equiv \frac{\theta}{\nu}\left(\frac{a_0 - \phi}{\nu}\right) - \frac{\gamma}{\eta}\left(\frac{a - \phi}{\eta}\right) - \phi\left(\frac{a_0 - \phi}{\nu} - \frac{a - \phi}{\eta}\right)$$

(24)
$$- \frac{\beta}{2}\left[\left(\frac{a_0 - \phi}{\nu}\right)^2 - \left(\frac{a - \phi}{\eta}\right)^2\right]$$

$$\equiv \frac{\beta\, \eta^2 (a_0 - \phi)^2 - \beta\, \nu^2 (a - \phi)^2}{2\nu^2\, \eta} .$$

Storage Cost

Although Massell (1969) had previously considered the effects of price stabilization with positive storage cost in the framework of this paper, his framework does not reflect the cost of carrying many successive years of high production into many successive years of low production versus the cost of carrying alternating high productions into immediately following years of low production. In a later work by Just, storage costs were assumed to depend at least indirectly on the length of time in storage. In this paper, however, this dependence is assumed explicitly so that storage costs during each time period are given by the storage cost per unit ρ times the quantity held in stocks during that time period,

(25)
$$C_s = \rho\, S.$$

Social Optimization

The approach in this section is to suppose a fixed stock goal $\bar{S}$ toward which the buffer authority continually attempts to move. The possibility of determining optimal buffer stock size $\bar{S}$ will then also be discussed. The fixed stock goal is met in expectations when

(26)
$$\mu_s = \bar{S} - S$$

where S is the current level of stocks. In other words the expected change in stocks should just bring buffer stocks back to their desired size. Hence, μ_s is fixed, and either a_0 or b_0 can be eliminated using (17). Solving for a_0 obtains

$$a_0 = \frac{b\,\nu\,\mu_s - \phi(b_0 - b) - a(\beta - b_0)}{\eta} \tag{27}$$

which implies that

$$\eta(a_0 - \phi) = \nu(a - \phi + b\,\mu_s) \tag{28}$$

$$\frac{\theta}{\nu} = \frac{\beta\,b\,\mu_s - b\,\phi + \beta\,a}{\eta}. \tag{29}$$

Hence, using (20), (22), (24), (28), and (29), it becomes apparent that

$$\mu_t = \frac{\beta\,b\,\mu_s^2 + (\beta\,a - b\,\phi)\mu_s}{\eta} \tag{30}$$

$$\mu_c = \frac{(2\phi - \beta\,\mu_s)\,(2ab - \beta\,b\,\mu_s)}{2\eta^2} \tag{31}$$

$$\mu_g = \frac{\beta\,b^2\,\mu_s^2 + 2\beta\,b(a - \phi)\mu_s}{2\eta^2}. \tag{32}$$

Before maximizing benefits with this policy, two additional considerations are needed. First, some constraint must be imposed to ensure that a buffer stock policy is not imposed which may (with nonnegligible probability) require more stocks than are currently available. Otherwise, the advertised policy may not actually be in effect anyway. Second, the repercussions of this period's buffer policy on future periods must also be considered. Suppose, for political reasons, that the buffer authority is constrained so that its attempted stock size must at least satisfy some constant Φ times the variance σ_s of buffer authority transactions with whatever policy is selected. If the distributions of ζ and δ are such that only finite ζ and δ are possible, then this constraint may simply require that stocks can never be depleted by chance and, thus, render the buffer stock policy inoperative. The maximization problem, ignoring repercussions in future periods, thus becomes

$$\max_{k, b_0 \geq 0} E(G) = E(G_c) + E(G_p) - E(P) - C_s$$

subject to the constraint imposed on the buffer authority $\bar{S} = \Phi\, \sigma_s$. The constraint can be imposed by substituting for C_s according to (25) and (26) which implies that

$$C_s = \rho(\bar{S} - \mu_s) = \rho\, \Phi\, \sigma_s - \rho\, \mu_s.$$

Finally, consider the extent to which future periods are affected by this period's policy. Future periods are affected by the amount of stocks carried over. From (16) and (26), it is apparent that expected stocks for the next period are $E(S + s) = S + \mu_s = \bar{S}$, and the variance is σ_s. Assuming the policy in (26) will also be followed next period, the desired stock transaction μ_s^* for the next period (corresponding to μ_s for this period) will be

$$(34) \qquad \mu_s^* = \bar{S} - (S + s) = \bar{S} - \bar{S} + (s - \mu_s) = \frac{(\eta - \nu)\zeta + (1 - k)\delta}{b}.$$

The effects of changes in the distribution of s on next period's expected gains can be examined by an analogy with this period's gains E(G) if μ_s is replaced by μ_s^* and ζ and δ are serially uncorrelated. The only changes in the next period's expected gains which depend on this period's policy (i.e., on the distribution of μ_s^*) relate to the terms μ_t, μ_c, and μ_g. Using asterisks to denote the next period's values in (30), (31), and (32), using (34) and taking expectations implies

$$E(\mu_t^*) = \frac{\beta\, b\, \sigma_s}{\eta}$$

$$E(\mu_c^*) = \frac{4ab\,\phi - b\,\beta^2\,\sigma_s}{2\eta^2}$$

$$E(\mu_g^*) = \frac{\beta\, b^2\, \sigma_s}{2\eta^2}$$

since $E(\mu_s^*) = 0$ in (34). Hence, the next period's expected gains are

$$E(G^*) = \kappa - \frac{\beta\, b}{\eta}\sigma_s - \rho\,\Phi\,\sigma_s$$

where κ is a constant with respect to this period's policy controls. Note that expected storage costs for the succeeding period are

$$E(C_s^*) = E[\rho(S + s)] = \rho\,\bar{S} = \rho\,\Phi\,\sigma_s$$

since $E(\mu_s^*) = 0$; thus, only $\rho\,\Phi\,\sigma_s$ enters $E(G^*)$.

Since the distributions of stocks beyond the next period are not affected by this period's policy when (26) is imposed, an appropriate objective is thus to maximize E(G) + E(G*)/(1 + r) subject to (33) where r is the social discount rate. Kuhn–Tucker conditions indicate optimum gains where

$$\frac{\partial E(G)}{\partial k} + \frac{1}{1+r}\frac{\partial E(G^*)}{\partial k} \geq 0,\quad k\left(\frac{\partial E(G)}{\partial k} + \frac{1}{1+r}\frac{\partial E(G^*)}{\partial k}\right) = 0$$

(35)

$$\frac{\partial E(G)}{\partial b_0} + \frac{1}{1+r}\frac{\partial E(G^*)}{\partial b_0} \geq 0,\quad b_0\left(\frac{\partial E(G)}{\partial b_0} + \frac{1}{1+r}\frac{\partial E(G^*)}{\partial b_0}\right) = 0.$$

Using (19), (21), (23), (30)–(32), and (18) obtains

$$\frac{\partial E(G)}{\partial k} = \frac{k\sigma_\gamma}{b} - \rho\,\Phi\,\frac{\partial\sigma_s}{\partial k},\quad \frac{\partial E(G^*)}{\partial k} = -\left(\frac{\beta\,b}{2\eta} + \rho\,\Phi\right)\frac{\partial\sigma_s}{\partial k},\quad \frac{\partial\sigma_s}{\partial k} = -\,\frac{2(1-k)\sigma_\delta}{b^2}$$

(36)

$$\frac{\partial E(G)}{\partial b_0} = -\,\frac{b_0\,\sigma_\zeta}{b} - \rho\,\Phi\,\frac{\partial\sigma_s}{\partial b_0},\quad \frac{\partial E(G^*)}{\partial b_0} = -\left(\frac{\beta\,b}{2\eta} + \rho\,\Phi\right)\frac{\partial\sigma_s}{\partial b_0},\quad \frac{\partial\sigma_s}{\partial b_0} = -\,\frac{2(b-b_0)\sigma_\zeta}{b^2}.$$

The implicit simple calculus conditions in (35) can be imposed for maximization so long as the indicated optimum controls are positive. One finds from

$$\frac{\partial E(G)}{\partial k} + \frac{1}{1+r}\frac{\partial E(G^*)}{\partial k} = 0,\quad \frac{\partial E(G)}{\partial b_0} + \frac{1}{1+r}\frac{\partial E(G^*)}{\partial b_0} = 0$$

using (36) that

(37) $$k = \bar{k} \equiv \frac{\beta b + 2\rho\eta\Phi(2+r)}{\beta b + (1+r)\eta b + 2\rho\eta\Phi(2+r)},\quad b_0 = \bar{b}_0\,\bar{k}\,b$$

where obviously $\bar{k}$, $\bar{b}_0 > 0$. It can also be verified that the appropriate second-order conditions hold throughout the necessary range (k, $b_0 \geq 0$).