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by

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Decomposing Input Adjustments under Price and Production Uncertainty

Robert G. Chambers[1] and John Quiggin[2]

March 10, 2000

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[1] Professor and Chair, Department of Agricultural and Resource Economics, University of Maryland, College Park, MD

[2] Australian Research Council Fellow, Faculty of Economics, Australian National University, Canberra

Abstract

A decomposition of input adjustments for stochastic technologies is developed and applied to the case of actuarially fair production insurance. The decomposition consists of a pure-risk effect and an expansion effect which are analogous to the Hicks-Allen decomposition familiar from consumer theory.

Keywords: uncertainty, duality, state-contingent technology, input use

Duality applies under uncertainty. In particular, Chambers and Quiggin (1998) have shown that dual cost structures exist for the continuous, stochastic technologies most familiar to agricultural economists. Beyond merely demonstrating existence, however, this finding has important implications for the analysis of stochastic decisionmaking. Agricultural economists long have intensively studied decisionmaking by producers facing stochastic technologies. And yet, no commonly accepted body of 'stylized facts' exists for most truly interesting formulations of this problem. Some have even questioned the relevance of the cost minimization hypothesis for risk-averse decisionmakers (Pope and Chavas). More generally, apart from a number of results that have been established for trivially stochastic situations, e.g., price but not production uncertainty, there is no common agreement as to what one can expect from a risk-averse producer facing a stochastic world.

For example, almost nothing of consequence is known about how the input utilization of risk-averse farmers differs from that of risk-neutral farmers or about the closely related question of how input utilization responds to the provision of insurance or income support. Heuristically, one expects risk-neutral farmers to undertake riskier production activities that bring with them the promise of higher return. Similarly, one also expects that insuring farmers or providing them government income support encourages them to undertake riskier production activities. If the worst happens, they always have the government or the insurer to fall back on. Reasoning thus, one expects that inputs that might be perceived as enhancing the riskiness of the production outcome would be more heavily utilized. Conversely, inputs which do little to enhance productivity, but which do act as damage-control agents, would be expected to be used less intensively. Stated in this manner, this would seem almost self evident. However, the existing literature suggests that this is not generally the case even if attention is restricted to single-output, single-input technologies (Quiggin 1992; Ramaswami; Horowitz and Lichtenberg; Hennessy). Because such technologies are highly restrictive, the natural implication seems to be that little, if anything, can be said for more realistic technologies.

Our contention is that much of this indeterminancy arises from the way in which agricultural economists have modelled production uncertainty in the past. Because of the confusion that has arisen over whether risk-averse producers minimize cost or whether duality applies under uncertainty, agricultural economists have overlooked a decomposition of input adjustments under uncertainty that sheds light on these issues. The goal of this paper is to demonstrate the importance of the duality between cost and stochastic technologies by suggesting such a decomposition of input adjustment under uncertainty.

The basic model is a state-contingent formulation, which encompasses both production and price uncertainty, that allows full exploitation of the duality between the technology and the cost structure in comparative-static analyses. We use this duality and a stochastic version of Shephard's lemma to suggest a method for examining input adjustments under uncertainty in a new and informative manner which closely parallels the familiar Hicksian and Slutsky decomposition from consumer theory, but which does not rely on the single-input, single-output stochastic production function model that has dominated many previous studies. Hence, it can intuitively illustrated with familiar graphical techniques. After formulating this decomposition, we illustrate its usefulness by applying it to study input utilization for the simplest possible crop insurance problem, actuarially fair crop insurance.

1 The State-Contingent Technology

Following Chambers and Quiggin (1996, 1997, 2000), the stochastic technology is represented by a multi-product, state-contingent input correspondence. To make this explicit, suppose that the states of nature are given by the set $\Omega = \{1, 2, ..., S\}$, let $\mathbf{x} \in \Re_{+}^{N}$ be a vector of inputs committed prior to the resolution of uncertainty, and let $\mathbf{z} \in \Re_{+}^{M \times S}$ be a vector of state-contingent outputs. So, if state $s \in \Omega$ is realized (picked by 'Nature'), and the producer has chosen the *ex ante* input-output combination $(\mathbf{x}, \mathbf{z})$, then the realized or *ex post* output vector is $\mathbf{z}^{s}$ corresponding to the sth column of $\mathbf{z}$. In other words, the observed output is an M-dimensional vector $\mathbf{z}^{s}$ where z_{m}^{s} corresponds to the $m-$th output that would be produced in state s.

More formally, the technology is represented by an input correspondence, $X : \Re_{+}^{MxS} \rightarrow \Re_{+}^{N}$, which maps matrices of state-contingent outputs into input sets that are capable of producing that state-contingent output matrix. It is defined

$$X(\mathbf{z}) = \{\mathbf{x} \in \Re_+^N : \mathbf{x} \text{ can produce } \mathbf{z}\}.$$

We impose the following axioms on $X(\mathbf{z})$:

X.1 $X(\mathbf{0}_{MxS}) = \Re_+^N$ (no fixed costs), and $\mathbf{0}_N \notin X(\mathbf{z})$ for $\mathbf{z} \geq \mathbf{0}_{MxS}$ and $\mathbf{z} \neq \mathbf{0}_{MxS}$ (no free lunch).

X.2 $\mathbf{z}' \leq \mathbf{z} \Rightarrow X(\mathbf{z}) \subset X(\mathbf{z}')$.

X.3 $\mathbf{x}' \geq \mathbf{x} \in X(\mathbf{z}) \Rightarrow \mathbf{x}' \in X(\mathbf{z})$.

X.4 $\lambda X(\mathbf{z}) + (1-\lambda)X(\mathbf{z}') \subset X(\lambda\mathbf{z} + (1-\lambda)\mathbf{z}') \qquad 0 \leq \lambda \leq 1$.

X.5 $X(\mathbf{z})$ is closed for all $\mathbf{z} \in \Re_+^{MxS}$.

The first part of X.1 says that doing nothing is always feasible, while the second part of X.1 says that realizing a positive output in any state of nature requires the commitment of some inputs. X.2 says that if an input combination can produce a particular matrix of state-contingent outputs then it can always be used to produce a smaller matrix of state-contingent outputs. X.3 implies that inputs have non-negative marginal productivity. X.4 tells us that the state-contingent technology is convex, and intuitively it leads to diminishing marginal productivity of inputs. X.5 is a technical assumption that ensures the existence of the revenue-cost function that we develop next.

2 The revenue-cost function

Denote by $\mathbf{p} \in \Re_{++}^{M\times S}$ the matrix of state-contingent output prices corresponding to the matrix of state-contingent outputs. The interpretation of $\mathbf{p}$ is basically the same as $\mathbf{z}$. If 'Nature' picks $s \in \Omega$, then the vector of realized spot prices is $\mathbf{p}^s \in \Re_{++}^M$. We assume that producers are competitive, they take these state-contingent output prices and the prices of all inputs, denoted by $\mathbf{w} \in \Re_{++}^N$, as given. The state-contingent revenue vector $\mathbf{r} = \mathbf{pz} \in \Re_+^S$ has typical elements of the form $r_s = \sum_{m=1}^{M} p_m^s z_m^s$.

Producers will be concerned with state-contingent revenue rather than output *per se*, and it is useful to consider the *revenue-cost function* defined as

$$C(\mathrm{w},\mathrm{r},\mathrm{p}) = \min\left\{ \mathrm{w}\cdot\mathrm{x} : \mathrm{x} \in X(\mathrm{z}), \sum_m p_{ms} z_{ms} \geq r_s, s \in \Omega \right\},$$

if there exists a feasible state-contingent output array capable of producing r and ∞ otherwise. The properties of $C(\mathrm{w},\mathrm{r},\mathrm{p})$ that follow from X.1-X.5 (Chambers and Quiggin, 2000) are:

Properties of the Revenue-Cost Function (CR):

CR.1$C(\mathrm{w},\mathrm{r},\mathrm{p})$ is positively linearly homogeneous, non-decreasing, concave, and continuous in $\mathrm{w} \in \Re^N_{++}$.

CR.2 Shephard's Lemma.

CR.3 $C(\mathrm{w},\mathrm{r},\mathrm{p}) \geq 0$ with equality if and only if $\mathrm{r} = 0$.

CR.4 $\mathrm{r}' \geq \mathrm{r} \Rightarrow C(\mathrm{w},\mathrm{r}',\mathrm{p}) \geq C(\mathrm{w},\mathrm{r},\mathrm{p})$.

CR.5 $\mathrm{p}' \geq \mathrm{p} \Rightarrow C(\mathrm{w},\mathrm{r},\mathrm{p}') \leq C(\mathrm{w},\mathrm{r},\mathrm{p})$.

CR.6 $C(\mathrm{w},\mathrm{r}_{-s},\theta r_{s,},\mathrm{p}_{-s},\theta \mathrm{p}_s) = C(\mathrm{w},\mathrm{r}_{-s},\theta r_{s,},\mathrm{p}_{-s},\theta \mathrm{p}_s), \quad \theta > 0.$

CR.7$C(\mathrm{w},\mathrm{r},\mathrm{p}) = C(\mathrm{w},\mathrm{r}/k,\mathrm{p}/k), \quad k > 0.$

CR.8 $C(\mathrm{w},\mathrm{r},\mathrm{p})$ is convex in r.

We shall typically assume that $C(\mathrm{w},\mathrm{r},\mathrm{p})$ is smoothly differentiable in all state-contingent revenues and input prices. By assuming a differentiable in revenues cost structure, we, therefore, rule out the stochastic-revenue function approach and the non-stochastic production approach of Sandmo.

3 Preferences

Following Yaari (1969) and Quiggin and Chambers, the producer's preferences are represented by a continuous and increasing function, $W : \Re^s \rightarrow \Re$, of his vector of state contingent net returns

$$\mathrm{y} = \mathrm{r} - (\mathrm{w}\cdot\mathrm{x})\,\mathbb{1}_s,$$

where $\mathbb{1}_S$ is the S-dimensional unit vector. The producer's preferences can thus be expressed in terms of the revenue-cost function as

$$\mathrm{y} = \mathrm{r} - \mathbb{C}(\mathrm{w},\mathrm{r},\mathrm{p})\mathbb{1}_s.$$

The producer is *risk-averse with respect to the probability vector* $\boldsymbol{\pi}$ if

$$W(\bar{y}\mathbf{1}^S) \geq W(\mathbf{y}), \forall \mathbf{y},$$

where $\bar{y}\mathbf{1}^S$ is the state-contingent outcome vector with $\bar{y} = \sum_{s\in\Omega} \pi_s y_s$ occurring in every state of nature. Both the usual decision-theoretic approach due to Savage and that employed here may be contrasted with the assumption, common in applied work, that there exist known objective probabilities. Here and in the Savage approach the probabilities, because they depend on preferences, are inherently subjective. In some cases (e.g., climatic uncertainty) where stable relative frequencies can be derived from long runs of historical data, the assumption of known objective probabilities may be appropriate. In such cases, we assume that all individuals would possess the same subjective probabilities.

If preferences are smoothly differentiable, the vector of subjective probabilities is unique and proportional to the marginal rate of substitution between state-contingent incomes along the equal-incomes vector. More concretely, without loss of generality, if preferences are smoothly differentiable (subscripts on functions denote partial derivatives),

$$\pi_s = \frac{W_s\left(c\mathbf{1}^S\right)}{\sum_{t\in\Omega} W_t\left(c\mathbf{1}^S\right)}, \quad s \in \Omega, \quad c \in \Re.$$

Pictorially, therefore, the *fair-odds line*, which gives the locus of points having the same expected value and whose slope is given by minus the relative probabilities is given by the slope of the tangent to the producer's indifference curve at the bisector. Figure 1 illustrates.

In order to impose some structure upon preferences other than simple aversion to risk, consider the partial ordering $\preceq_\pi$ of risky outcomes which possess a common mean for the probability vector $\boldsymbol{\pi}$. This partial ordering is defined by

$$\mathbf{y} \preceq_\pi \mathbf{y}'$$

if and only if $\mathbf{y}$ and $\mathbf{y}'$ have the same mean and $\mathbf{y}$ is less risky than $\mathbf{y}'$ in the sense of Rothschild and Stiglitz. Chambers and Quiggin (1997) define a function $W : \Re^S \rightarrow \Re$ to be *generalized Schur-concave* for π if $\mathbf{y} \preceq_\pi \mathbf{y}' \Rightarrow W(\mathbf{y}) \geq W(\mathbf{y}')$.

A comment about generalized Schur concavity is worthwhile. Unlike the assumption of expected-utility maximization, generalized Schur concavity doesn't impose additive separability across states of nature. Consequently, it does not rely upon the independence axiom which has proved vulnerable to a variety of criticisms. Even so, the expected-utility functional with concave u is generalized Schur-concave as can be recognized from the result due to Rothschild and Stiglitz that if $\mathrm{y} \preceq_{\pi} \mathrm{y}'$ then y would be preferred to y' by all individuals with risk-averse expected-utility preferences. More generally, generalized Schur concavity characterizes a number of preference classes, which are consistent with risk-aversion in our sense, but which are not consistent with expected utility. An example is given by individuals with maximin preferences

$$W(\mathrm{y}) = \min\{y_1, ..., y_S\}.$$

This class of preferences is risk-averse in our sense for all possible probability vectors (note it is not differentiable), and it is also generalized Schur concave. Another class of generalized Schur concave preferences is the mean-variance class. More generally, virtually all preference functions currently in use, including the rank-dependent models (Quiggin 1982, Yaari 1987) and weighted-utility models are consistent with generalized Schur concavity.

When W is smoothly differentiable, a basic result due to Chambers and Quiggin (1997), will prove useful:

Lemma 1 If $W : \Re^S \to \Re$ is generalized Schur-concave and once continuously differentiable everywhere on its domain, then

$$\left(\frac{W_s(\mathrm{y})}{\pi_s} - \frac{W_r(\mathrm{y})}{\pi_r}\right)(y_s - y_r) \leq 0,$$

for all s and r.

4 Risk-Neutral and Risk-Averse Production Equilibria

We first present some basic results on the production choices of risk-neutral and risk-averse producers.[1] Suppose the risk-neutral producer's subjective probabilities are given by the vector π. Then her first-order conditions on r may be written in the notation of complementary slackness as

$$\pi_s - C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \leq 0, \quad r_s \geq 0, \quad s \in \Omega.$$

That is, the marginal cost of increasing revenue in any state is at least equal to the subjective probability of that state. Pictorially, therefore, we represent the producer equilibrium by a hyperplane being tangent to her isocost curve. Figure 2 illustrates. The slope of the hyperplane is determined by the ratio of the producer's subjective probabilities, *the fair-odds line*, and the isocost curve is determined by the equilibrium level of revenue-cost. This is exactly analogous to the representation of production equilibrium in the non-stochastic, multi-product case. Instead of determining an optimal mix of outputs as in the non-stochastic multi-product case, however, the producer equilibrium now determines the optimal mix of state-contingent revenues.

Summing the first-order conditions on $\mathbf{r}$ yields an *arbitrage condition*

$$(1) \qquad \sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq \sum_{s\in\Omega} \pi_s = 1.$$

$\sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is the marginal cost of increasing all state-contingent revenues by the same small amount in each state of nature, i.e., it is the marginal cost of a sure increase in revenue of one unit. Hence, (1) requires this cost be at least as large as the associated sure increase in returns. If it were not, the decisionmaker could increase profit with probability 1 by either expanding or decreasing all revenues equally. For an interior solution, (1) must hold as an equality.

We refer to the set of revenue vectors $\mathbf{r}$ satisfying (1) as the *efficient set*, denoted $\Xi(\mathbf{w}, \mathbf{p})$,

$$\Xi(\mathbf{w}, \mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq 1 \right\}.$$

The boundary of $\Xi(\mathbf{w}, \mathbf{p})$ is the *efficient frontier*. Its elements are given by:

$$\Xi^o(\mathbf{w}, \mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) = 1 \right\}.$$

By the homogeneity properties of $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$, $\Xi(\theta\mathbf{w}, \theta\mathbf{p}) = \theta\Xi(\mathbf{w}, \mathbf{p})$ and $\Xi^o(\theta\mathbf{w}, \theta\mathbf{p}) = \theta\Xi^o(\mathbf{w}, \mathbf{p})$, $\theta > 0$ (Chambers and Quiggin, 2000). The efficient set and the efficient frontier are positively linearly homogeneous in input and output prices.

Different risk-neutral producers may hold different subjective probabilities. Regardless of the individual's subjective probabilities, a revenue vector $\mathbf{r}$ is potentially optimal for some risk-neutral decision-maker only if (1) holds. $\Xi^o(\mathbf{w}, \mathbf{p})$ is thus naturally interpreted as the collection of state-contingent revenues which are potentially expected-profit maximizing. To see why, suppose that (1) holds as an equality for an arbitrary revenue vector, call it $\hat{\mathbf{r}}$. Now construct a set of probabilities by setting $\hat{\pi}_s = C_s(\mathbf{w}, \hat{\mathbf{r}}, \mathbf{p})$ for all s. Because they belong to the efficient frontier and are derived from a non-decreasing revenue-cost function, these probabilities are positive and sum to one. Moreover, a risk-neutral individual having such probabilities would choose $\hat{\mathbf{r}}$ as the expected-profit maximizing vector of state-contingent revenues. The correspondence of the producer's subjective probabilities with these state-contingent marginal costs then determines the optimal point on the efficient set.

Turn now to the case of generalized Schur-concave preferences. The producer chooses state-contingent revenues to maximize:

$$W(\mathbf{y}) = W(\mathbf{r} - C(\mathbf{w}, \mathbf{r}, \mathbf{p})1_S).$$

So long as the preference function is smoothly differentiable in state-contingent revenues, then the first-order condition on r_s is:

$$W_s(\mathbf{y}) - C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \sum_{t \in \Omega} W_t(\mathbf{y}) \leq 0, \quad r_s \geq 0,$$

with complementary slackness.

The arbitrage condition (1) can be derived as a consequence of summing these first-order conditions

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq 1. \tag{2}$$

We conclude from (2) that a producer with generalized Schur-concave preferences chooses a revenue vector that is in the efficient set. Hence, as observed by Chambers and Quiggin (1997), there always exists a vector of probabilities that will induce a risk-neutral individual to choose the same production pattern as that chosen by one with generalized Schur-concave preferences. In general, however, these probabilities derived from the efficient frontier will not correspond to the producer's subjective probabilities unless she is herself risk-neutral.

Pictorially, this production equilibrium is illustrated by a tangency between the producer's indifference curve and one of her isocost curves as illustrated in Figure 3. This implies, for example, that when preferences are of the maximin form, producers completely stabilize revenues and produce where the efficient frontier intersects the equal-revenue curve (the bisector).

Several points should be made here to facilitate comparison of the risk-neutral production pattern with that associated with generalized Schur-concave preferences. For an interior solution, a risk-neutral producer chooses his state-contingent revenues so that for all $t, s \in \Omega$

$$\frac{C_s(\mathrm{w},\mathrm{r},\mathrm{p})}{\pi_s} = \frac{C_t(\mathrm{w},\mathrm{r},\mathrm{p})}{\pi_t}.$$

Moreover, summing the first-order conditions for a risk-neutral producer, it follows by complementary slackness that

$$\sum_{s\in\Omega}\pi_s r_s - \sum_{s\in\Omega} C_s(\mathrm{w},\mathrm{r},\mathrm{p}) r_s = 0. \tag{3}$$

Expression (3) requires that the marginal profitability of increasing the optimal state-contingent revenue vector radially is zero for a risk-neutral producer.

For an interior solution, it follows from the risk averter's first-order condition and Lemma 1 that:

$$\left(\frac{C_s(\mathrm{w},\mathrm{r},\mathrm{p})}{\pi_s} - \frac{C_t(\mathrm{w},\mathrm{r},\mathrm{p})}{\pi_t}\right)(r_s - r_t) \leq 0. \tag{4}$$

Expression (4) implies an inverse covariance between the elements of the state-contingent revenue vector r and the vector with typical element, $\frac{C_s(\mathrm{w},\mathrm{r},\mathrm{p})}{\pi_s}$. Hence, we conclude:

$$\sum_{s\in\Omega} C_s(\mathrm{w},\mathrm{r},\mathrm{p})\left(r_s - \sum_{t\in\Omega}\pi_t r_t\right) \leq 0. \tag{5}$$

Expression (5) and the arbitrage condition imply that a risk averter with generalized Schur-concave preferences will choose an optimal state-contingent revenue vector that is characterized by the fact that a small radial expansion of it will lead to an increase in expected profit.

Generally speaking, therefore, the risk-averter does not equate his marginal rate of transformation between state-contingent revenues to the ratio of probabilities as a risk-neutral individual would. Furthermore, the risk averter operates on a smaller scale than a risk-neutral producer in the sense that the former can radially expand his optimal state-contingent revenue vector and increase profit while the latter cannot. In a word, the risk averter trades off expected return in an effort to provide self insurance against the price and revenue risk that he faces. And because the preference function is generalized Schur concave, then, in the neighborhood of the equilibrium, the revenue-cost function must behave as though it, too, were generalized Schur concave. Accordingly, in that neighborhood, there must a negative correlation between marginal cost and the level of the state-contingent revenues.

5 Decomposing Input Adjustments

Our next task is to specify an algorithm for examining how input utilization responds to the provision of crop insurance. Our starting point is the recognition via CR. 2 (Shephard's Lemma) that optimal input demands can be recaptured directly from the revenue-cost function as

$$\mathbf{x}\left(\mathbf{w}, \mathbf{r}^*\left(\mathbf{w}, \mathbf{p}\right), \mathbf{p}\right) = \nabla_{\mathbf{w}} C(\mathbf{w}, \mathbf{r}^*\left(\mathbf{w}, \mathbf{p}\right), \mathbf{p}),$$

where $\mathbf{r}^*\left(\mathbf{w}, \mathbf{p}\right)$ denotes the producer's optimal state-contingent revenue vector and $\nabla_{\mathbf{w}}$ denotes the gradient of the function with respect to $\mathbf{w}$. So, for example, if input and output prices remain constant, comparing input demands for a risk-neutral individual with those of an individual with strictly generalized Schur-concave preferences, assuming both share the same technology, is simply a matter of comparing the same input demand function evaluated at two different optimal state-contingent revenue vectors. More generally, comparing different input demands arising from the same technology requires the ability to compare different state-contingent revenue vectors.

In an uncertain world, different state-contingent revenue vectors may usefully be compared in two dimensions. The first compares their relative expected returns, while the latter contains some measure of their riskiness. A risk-averse individual is, by definition, willing to trade off some increase in expected returns in return for an (appropriately defined) reduction in riskiness. Hence, it is imperative that any decomposition should recognize these two dimensions of the decisionmaker's problem. Therefore, we decompose all comparisons of different revenue vectors, and hence their associated input demands, into two effects. The first is a *pure risk effect* which keeps means constant but allows riskiness to vary, and the latter is an *expansion* effect which measures the difference in expected returns.[2]

Figure 4 illustrates our proposed decomposition for revenue changes. Let point A in that figure be the risk-neutral individual's optimum and point B be the risk-averter's optimal point. Now suppose that we want to compare these two optima and their associated input demands. For the purpose of discussion, we will make the comparison in terms of moving from B to A. However, it is also perfectly plausible to consider the move from A to B, but we defer that analysis to the reader's initiative.

The decomposition we employ breaks that move down into two effects. The first is the movement from B to the point C which is on the same fair-odds line as B. Point C has the same expected revenue as at B, but the same revenue mix as at A (is on the same ray as A). Because comparing points B and C involves comparing outcomes with the same mean, then in some sense (which we define precisely in a minute) the difference between B and C must be solely a difference in the riskiness of the two prospects. We shall call that comparison the *pure risk effect.*

The second effect, which measures the difference in the means of the two prospects, is associated with the movement (in this case) outward along the ray from C to point A. (A is arrived at by deflating point C by the ratio of C's mean to B's mean.) We shall refer to this movement in the revenue vector as the (radial) *expansion effect.* Combining these two effects allows us to arrive at a mean-compensated decomposition of revenue and input adjustments.

To make the mean-compensated decomposition meaningful, we need a clear definition of what it means for one state-contingent revenue vector to be riskier than another which possesses the same mean. Following Chambers and Quiggin (2000), we define a risk ordering, denoted $\preceq_W$, directly in terms of the preference function W. Hence, if $\mathbf{y}$ and $\mathbf{y}'$ share a common mean, then $\mathbf{y} \preceq_W \mathbf{y}'$ if $W(\mathbf{y}) \geq W(\mathbf{y}')$. (Strictly speaking, $\mathbf{y} \preceq_W \mathbf{y}'$ should be read as $\mathbf{y}$ is less risky than $\mathbf{y}'$ for preferences W. However, we shall simply say that $\mathbf{y}$ is less risky than y'.) So, in terms of Figure 4, C is riskier than B if it lies on a lower indifference curve than B.

Now that we have defined a risk-ordering, the final piece that we need is a way to relate that risk-ordering to input utilization. In the past, considerable attention has been devoted to the notions of *risk-reducing* and *risk-increasing inputs.* Intuitively, these notions seem clear: risk-reducing inputs reduce the riskiness of output, and risk-increasing inputs increase the riskiness of state-contingent outputs. Clear as this intuition seems, writers on production under uncertainty have struggled with formalizing a definition of risk-increasing and risk-reducing inputs that matches this simple intuition and which accords with general notions of increases and decreases in risk.[3]

The state-contingent approach adopted in this paper, however, leads to a rather different perspective. Rather than thinking of input choices, in combination with random variation, determining a stochastic output, we consider inputs and state-contingent outputs to be chosen jointly, in a preference maximizing fashion subject to a state-contingent input correspondence. Hence, it is natural to think in terms of complementarity between input choices and more or less risky state-contingent output patterns rather than in terms of simple causal relationships between input choices and risk.

Therefore, following Chambers and Quiggin (1996, 2000),[4] we define input n as a risk complement (risk substitute) at r if

$$\mathrm{r} \preccurlyeq_W \mathrm{r}' \Rightarrow x_n(\mathrm{w},\mathrm{r}',\mathrm{p}) \geq x_n(\mathrm{w},\mathrm{r},\mathrm{p}) \quad (x_n(\mathrm{w},\mathrm{r}',\mathrm{p}) \leq x_n(\mathrm{w},\mathrm{r},\mathrm{p})).$$

The intuition is clear. Something is a risk complement if more of it is used with more risky state-contingent revenue vectors than with less risky state-contingent revenue vectors. Just the reverse logic applies for a risk substitute. Because we have been able to make our notion of 'more risky' precise and to compensate for mean differences in state-contingent revenue vectors, this intuition accords closely with the more commonly popular notion of a risk increasing (risk reducing) input. However, we prefer our terminology because it emphasizes the simultaneity between the input choice and the state-contingent revenue choice, and it is a proper risk comparison as a mean-compensation is involved.[5]

Several comments are in order. First, it's not a purely technological definition. It depends upon both the technology and the producer's objective function W. (Also recall that probabilities, which are required for the mean compensation, in our framework are subjective.) Second, it's a local notion as it's expressly given at a point in state-contingent output space. And third,

$$\mathbf{r} \preccurlyeq_{\pi} \mathbf{r}' \Rightarrow \mathbf{r} \preccurlyeq_{W} \mathbf{r}'$$

if W is generalized Schur concave. Thus, via Lemma 1 this definition leads to a natural characterization of risk complementarity and risk substitutes in terms of partial derivatives of input-demand functions

Lemma 2 Suppose that the revenue-cost function is twice differentiable in all its arguments.

Input n is a risk complement for a generalized Schur-concave preference structure at $\mathbf{r}$ only if for all $r, s \in \Omega$:

$$\left(\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_r}\frac{1}{\pi_r} - \frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_s}\frac{1}{\pi_s}\right)(r_r - r_s) \geq 0.$$

Input n is a risk substitute for a generalized Schur-concave preference structure at $\mathbf{r}$ only if for all $r, s \in \Omega$:

$$\left(\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_r}\frac{1}{\pi_r} - \frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_s}\frac{1}{\pi_s}\right)(r_r - r_s) \leq 0.$$

From Lemma 2, $\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_r}\frac{1}{\pi_r}$ is inversely (positively) correlated with r_r if x_n is a risk substitute (complement), whence for risk substitutes

$$\sum_{s\in\Omega}\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_s}\left(r_s - \sum_{r\in\Omega}\pi_r r_r\right) \leq 0,$$

with the reversed inequality for risk complements. In words, an input is a risk substitute if its responsiveness to state-contingent revenue variation is large and positive for the lowest revenue states and small, and possibly negative, for the highest income states.

This result may be interpreted intuitively as follows. If an input is a risk substitute, it will tend to be used the most in producing the least risky revenue distributions. It is, therefore, natural to expect it to be most positively responsive to the lowest revenue states and the least responsive to the highest revenue states because this type of flexibility will be associated with smoother (less risky) revenue distributions.

6 Producer Equilibrium with Actuarially Fair Insurance

To apply our decomposition to our case study, we must first determine how producers behave in the presence of actuarially fair insurance. We assume that the insurer is risk-neutral and competitive. For simplicity, we assume that the insurer has the same information set as the producer, and the producer is risk-averse for the insurer's subjective probabilities, which we continue to denote as π. Because the insurer can observe 'Nature's' draw from Ω, he can write state-contingent insurance contracts. An actuary employed by the insurer would regard as fair any contract for which

$$\sum_s \pi_s I_s = 0.$$

where I_s denotes the net indemnity paid by the insurance company in state s. Any equilibrium insurance contract offered by a competitive, risk-neutral insurance company must be *actuarially fair* in this sense. To be actuarially fair, therefore, the net indemnity schedule must involve positive payouts in some states and negative payouts in other states of nature.

We now consider how the farmer would optimally exploit the presence of a competitive crop insurance market. Given the freedom to choose any actuarially fair contract, the representative farmer's optimal production *cum* insurance scheme solves:

$$\max_{\mathbb{I},\mathbb{r}} \left\{ W\left(\mathbb{r} + \mathbb{I} - C\left(\mathbb{w},\mathbb{r},\mathbb{p}\right)\mathbb{1}_s\right) : \sum_s \pi_s I_s = 0 \right\}. \tag{6}$$

Recognizing that now

$$y_s = r_s + I_s - C\left(\mathbb{w},\mathbb{r},\mathbb{p}\right),$$

shows that (6) can be rewritten after a simple change of variables as

$$\max_{\mathbb{y},\mathbb{r}} \left\{ W\left(\mathbb{y}\right) : \sum_s \pi_s y_s = \sum_s \pi_s r_s - C\left(\mathbb{w},\mathbb{r},\mathbb{p}\right) \right\}.$$

Therefore, regardless of whether her preferences are smoothly differentiable or not, the farmer chooses her state-contingent revenue vector to

$$\max_{\mathbb{r}} \left\{ \sum_s \pi_s r_s - C\left(\mathbb{w},\mathbb{r},\mathbb{p}\right) \right\}.$$

Suppose that the farmer has chosen a particular state-contingent revenue vector, an indemnity vector, and thus a net-returns vector which is not consistent with this strategy. The farmer can then hold her net-return vector constant while rearranging her production choices to generate a larger amount of income than before. This extra income could then be used to raise all state-contingent net returns thus making the farmer better off with certainty. Because her objective function is non-decreasing in these state-contingent net incomes, she'll choose her production vector to maximize expected profit.

Presuming she chooses the expected profit maximizing state-contingent revenue vector, notice that by her risk aversion the indemnity schedule (evaluated at the expected profit maximizing state-contingent revenue)

$$I_s = \sum_t \pi_t r_t - r_s, \;\; s \in \Omega.$$

dominates all others because it guarantees her a certain income of

$$\max_{\mathbf{r}} \left\{ \sum_s \pi_s r_s - C(\mathbf{w}, \mathbf{r}, \mathbf{p}) \right\},$$

which is the best that she could possibly hope for. Even a risk-neutral individual would at least weakly prefer this contract to all others. Moreover, this indemnity schedule breaks even for the insurer.

So we've established that: *Risk-averse farmers who face an actuarially fair insurance contract will produce in the same fashion as a risk-neutral farmer.* In the presence of an actuarially fair insurance market, a risk-averse farmer's production pattern is independent of her risk preferences. An immediate implication is that a farmer's optimal revenue choice in the presence of actuarially fair crop insurance contract belongs to the efficient set. These results confirm, for our more general preference and production structure, the full-insurance result of Nelson and Loehman.

Figure 5 illustrates pictorially for the case of smooth preferences.[6] The isocost curve in that figure represents the level curve of $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ as evaluated at the optimal level of state-contingent production. It is drawn as tangent to the fair-odds line at the optimal state-contingent production point (r_1^*, r_2^*) reflecting the fact that the farmer picks her revenue vector to maximize expected net income. The farmer will now trade with the insurance company along the fair-odds line until her marginal rate of substitution between state-contingent incomes is the same as the insurer's. And since this equalization occurs at the bisector for smooth generalized Schur-concave preferences, the producer ultimately locates there.

7 The effect of insurance on input use

Assessing the impact that actuarially fair crop insurance has on input utilization, thus, reduces to comparing the input decisions made by a risk-neutral producer and a risk-averse producer. Generally speaking, there are four possible outcomes when expressed in terms of the expansion effect and the pure-risk effect. Both the expansion effect and the risk effect can be positive, in which case the overall effect on an input's use is positive. The expansion effect can be positive and the risk effect can be negative, in which case the overall effect is ambiguous. The expansion effect can be negative and the risk effect negative, in which case the overall effect is negative. And finally, the expansion effect can be negative and the risk effect positive, in which case the overall effect is ambiguous.

More finely, however, there exists an even larger number of possibilities. For example, the expansion effect on input utilization could be positive because the expansion effect on revenues is positive and the input is non-regressive to radial expansions in revenues. Alternatively, the expansion effect on input utilization could be positive because the expansion effect on revenues is negative, but the input is regressive in radial expansions of revenues. Similarly, a negative effect could emerge from a positive (negative) expansion effect on revenues and non-regressivity (regressivity) to radial expansions of revenues.

Similar ambiguities arise from the risk effect as well. For example, an input could be a risk substitute and the risk effect in terms of revenues could be associated with an increase in risk. The input risk effect would then be negative. The other obvious possibilities can be enumerated by the reader.

Our strategy for sorting through the possible results is somewhat different than the strategy typically pursued in previous studies. There the approach is to impose additional structure upon the producer's preferences, for example, constant absolute risk aversion or decreasing absolute risk aversion. The results, thus obtained, are limiting for at least two reasons. First, compared to the preference structure used here, the preference structure most typically used in other studies (expected utility) is quite restrictive because it imposes additive separability across states of nature. Moreover, it is widely recognized to rely on a weak conceptual basis because empirical evidence routinely refutes the crucial independence axiom underlying expected-utility theory. Thus, these studies are in the position of imposing additional structure on a model that has already been demonstrated to be empirically flawed.

Second, the production structure that underlies all these studies is even more restrictive than the preference structure. It imposes an extreme form of non-substitutability between state-contingent outputs (Chambers and Quiggin, 1998). And as Chambers and Quiggin (2000) demonstrate, the differences between a risk-neutral producer's production pattern and a risk-averter's production pattern in that framework ultimately reduce to determining whether the risk-averter produces more or less of a single reference state-contingent revenue which automatically determines all other revenue levels and the level of input utilization. In effect, the stochastic production function model can always be reduced to a trivial single-input problem. Given the extreme restrictiveness of the production model and the fact that input commitment really plays no role in determining the inherent riskiness of the state-contingent revenue vector, it's not surprising, therefore, that one is forced to place even more stringent restrictions on preferences to obtain results on input use.

We pursue an alternative strategy and place no restrictions on preferences other than that they be consistent with risk aversion and generalized Schur concavity. Instead, we examine restrictions on the shape of the isocost frontiers for the state-contingent technology.

The first restriction that we consider is what Chambers and Quiggin (2000) have referred to as *constant relative riskiness* of the revenue-cost function.[7] Constant relative riskiness, in the current context, is equivalent to requiring that the revenue-cost function be homothetic in state-contingent revenues. The main economic consequence of this fact is that the expansion path in state-contingent revenue space, defined by the locus of points which are expected-revenue maximizing for fixed levels of revenue cost, is linear. This happens because isocost frontiers for this technology are radial blow-ups of a reference isocost curve.[8]

Let the farmer's revenue vector in the absence of insurance be denoted by $\mathbf{r}^A$ and the farmer's revenue vector in the presence of actuarially fair insurance be denoted by r^F. Then, as discussed earlier, notice that the effect of providing insurance can be broken down into two parts, the mean-compensated move from r^A to $\frac{\sum_{s\in\Omega}\pi_s r_s^A}{\sum_{s\in\Omega}\pi_s r_s^F}\mathrm{r}^F$, which we refer to as the *mean-compensated revenue vector*, and the radial movement from this mean-compensated revenue vector to r^F. The mean-compensated revenue vector corresponds to point C in Figure 4.

Because the the mean-compensated revenue vector is either a radial expansion or a radial contraction of r^F, it lies on the producer's expansion path. Therefore, the mean-compensated revenue vector must be the most profitable state-contingent revenue combination for the revenue-cost level

$$C\left(\mathrm{w}, \frac{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{A}}}{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{F}}}\mathrm{r}^{\mathrm{F}}, \mathrm{p}\right).$$

The mean-compensated revenue vector, however, has the same expected revenue as r^A. Thus, the cost associated with r^A must be at least as large as that associated with the mean-compensated revenue vector. If it were not, the same expected revenue could be obtained from r^A at a cost level lower than $C\left(\mathrm{w}, \frac{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{A}}}{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{F}}}\mathrm{r}^{\mathrm{F}}, \mathrm{p}\right)$. But this contradicts the fact that the the mean-compensated revenue vector lies on the firm's expansion path. Figure 6 illustrates this fact by having the point of intersection between the fair-odds line through r^A and the risk-neutral expansion path lie below the firm's isocost curve for r^A.

Revealed-preference arguments, therefore, lead to the conclusion that

$$\mathrm{r}^{\mathrm{A}} - C\left(\mathrm{w}, \mathrm{r}^{\mathrm{A}}, \mathrm{p}\right) \preceq_W \frac{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{A}}}{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{F}}}\mathrm{r}^{\mathrm{F}} - C\left(\mathrm{w}, \mathrm{r}^{\mathrm{A}}, \mathrm{p}\right).$$

If this ordering of the outcomes did not hold, then r^A would not be the optimal choice for a risk averter. Notice that the preceding arguments have established that the mean-compensated revenue vector is less costly than r^{A}, thus it represents a feasible choice for this level of revenue cost. Hence, we conclude that

$$\mathrm{r}^{\mathrm{A}} \preceq_W \frac{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{A}}}{\sum_{s\in\Omega}\pi_s \mathrm{r}_s^{\mathrm{F}}}\mathrm{r}^{\mathrm{F}}.$$

From this observation we can state the following result.

Result 1 If the producer's revenue-cost function exhibits constant relative riskiness, the pure-risk effect of the provision of actuarially fair insurance on an input is positive if the input is a risk complement and negative if the input is a risk substitute.

Generally speaking, it seems that the expansion effect for a technology exhibiting constant relative riskiness can require either a shrinking or an expansion of the risk-neutral optimum depending upon the rate at which marginal costs of the state-contingent outputs rise. So, as a general matter, we cannot make a clear pronouncement as to what will be the effect of the provision of crop insurance for a producer facing such a technology without placing further structure upon the problem.

There does exist a class of technologies for which one can obtain clear results about both the expansion and the pure risk effects. That technology is the member of the class of translation-homothetic[9] technologies (Chambers and Färe), which Chambers and Quiggin (2000) refer to as exhibiting *constant absolute riskiness.* The technology exhibits constant absolute riskiness if

$$C(\mathrm{w},\mathrm{r},\mathrm{p}) = \hat{C}\left(\mathrm{w},\mathbb{T}\left(\mathrm{r},\mathrm{p},\mathrm{w}\right),\mathrm{p}\right)$$

where

$$T\left(\mathrm{r}+\delta 1^{S},\mathrm{p},\mathrm{w}\right) = T\left(\mathrm{r},\mathrm{p},\mathrm{w}\right)+\delta, \quad \delta \in \Re,$$

$$T\left(\lambda\mathrm{r},\lambda\mathrm{p},\mathrm{w}\right) = \lambda T\left(\mathrm{r},\mathrm{p},\mathrm{w}\right), \quad T\left(\mathrm{r},\mathrm{p},\lambda\mathrm{w}\right) = T\left(\mathrm{r},\mathrm{p},\mathrm{w}\right) \quad \lambda > 0,$$

and $\hat{C}\left(\mathrm{w},\mathbb{T}\left(\mathrm{r},\mathrm{p},\mathrm{w}\right),\mathrm{p}\right)$ is positively linearly homogeneous in input prices, homogeneous of degree zero in $T\left(\mathrm{r},\mathrm{p},\mathrm{w}\right)$ and p, non-decreasing and convex in $T\left(\mathrm{r},\mathrm{p},\mathrm{w}\right)$, non-increasing in p. $T\left(\mathrm{r},\mathrm{p},\mathrm{w}\right)$ is non-decreasing and convex in r.

Intuitively, technologies which exhibit constant absolute riskiness have isocost curves which are parallel to one another as one moves in a direction parallel to the bi-sector. Therefore, increasing revenue by the same amount in all states of nature has no effect on the rate at which state-contingent revenues substitute for one another in the technology. In that sense, constant absolute riskiness is the natural production analogue of general risk-averse preferences which exhibit constant absolute risk aversion (Quiggin and Chambers).

The most important property that technologies exhibiting constant absolute riskiness have for state-contingent technologies is that the cost level corresponding to the efficient set is unique for such technologies. Hence, in this special case, the efficient set corresponds exactly to a unique isocost contour. The easiest way to discern this property is to differentiate both sides of the expression

$$T\left(\mathrm{r}+\delta 1^{S}, \mathrm{p}, \mathrm{w}\right)=T(\mathrm{r}, \mathrm{p}, \mathrm{w})+\delta$$

with respect to δ and evaluate the resulting directional derivative at $\delta = 0$ to obtain

$$\sum_{s \in \Omega} T_{s}(\mathrm{r}, \mathrm{p}, \mathrm{w})=1.$$

Using this fact and our definition of constant absolute riskiness the arbitrage condition (1) can, in this case, be written as

$$\hat{C}_{T}(\mathrm{w}, \mathbb{T}(\mathrm{r}, \mathrm{p}, \mathrm{w}), \mathrm{p}) \geq 1.$$

Thus, assuming an interior solution the arbitrage condition determines a unique level of T and thus of revenue cost. In this context, notice that T may naturally be thought of as a revenue aggregate which has the property that increasing all state-contingent revenues by one unit increases it by one unit. For technologies exhibiting constant absolute riskiness, (1) simply reduces to equating the marginal cost of that revenue aggregate to one.

Because both risk averters and risk-neutral individuals produce in the efficient set, it follows that:

Result 2 If the technology exhibits constant absolute riskiness, the introduction of actuarially fair production insurance does not affect the level of revenue cost incurred by a risk-averse entrepreneur.

Accordingly, the only effect that production insurance has on the risk-averse entrepreneur is to change his optimal revenue mix to that associated with a risk-neutral individual. This brings with it an increase in expected revenue, at no additional cost, that can be used along with the production insurance to enhance the producer's overall welfare.

The production decisions for a risk-averse producer in the presence of insurance and in its absence can be illustrated graphically as in Figure 7 when the technology exhibits constant absolute riskiness. There the producer produces at r^F when insurance is provided and at r^A in its absence. It is pictorially obvious and generally true that

$$\frac{\sum_{s \in \Omega} \pi_{s} r_{s}^{A}}{\sum_{s \in \Omega} \pi_{s} r_{s}^{F}}<1. \tag{7}$$

The inequality follows from the fact that r^F must be associated with the highest expected revenue consistent with the constant level of cost.

Result 3 If the technology exhibits constant absolute riskiness, the introduction of actuarially fair production insurance increases the level of expected revenue produced by a risk-averse producer.

Moreover, a revealed preference argument exactly parallel to the one used in the discussion of constant relative riskiness reveals that

$$\mathbf{r}^{\mathbf{A}} \preceq_W \frac{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{A}}}{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{F}}} \mathbf{r}^{\mathbf{F}}. \tag{8}$$

Consequences of (8) and (7) are

Result 4 If the technology exhibits constant absolute riskiness, then the pure risk effect on an input is positive (negative) if the input is a risk complement (risk substitute).

Result 5 If the technology exhibits constant absolute riskiness, the expansion effect on an input is positive (negative) if the input is non-regressive (regressive) in radial expansions of revenue.

Because there exist unambiguous results for both the pure-risk and expansion effects on input utilization, we have:

Corollary 1 If the technology exhibits constant absolute riskiness, an input's utilization increases as a result of the introduction of actuarially fair insurance if the input is a risk complement and it is non-regressive in radial expansions of the state-contingent revenue vector.

Corollary 2 If the technology exhibits constant absolute riskiness, an input's utilization decreases as a result of the introduction of actuarially fair insurance if the input is a risk substitute and it is regressive in radial expansions of the state-contingent revenue vector.

8 Discussion of Results

The results that we have presented show that regardless of the preference structure, there are a number of things which can be said about the differences between input use for risk-neutral and risk-averse individuals or about input response to the provision of actuarially fair insurance. For example, consider the case of a technology that exhibits constant absolute riskiness. Then it follows from our discussion that any input which is both a risk complement and which is not radially regressive in revenues will be used more heavily in the presence of insurance than in its absence.

So, intuitively, one might think in terms of an input like chemical fertilizer which would seem to be a natural risk complement and which empirical evidence would also suggest is not regressive. Then, one could immediately conclude that an individual using a technology characterized by constant absolute riskiness would use more chemical fertilizer in the presence of insurance. This coincides nicely with popular wisdom on such inputs. Conversely, one sees that the pure-risk effect will lead the producer to utilize less risk-substitute inputs, such as pesticides. But more generally, the introduction of insurance might ultimately force even these inputs' utilization to rise as a result of the expansion effect if pesticides are not regressive to radial expansions of the state-contingent revenues. The most obvious criticism of traditional comparative-static analyses based on the notions of risk-increasing and risk-reducing inputs is that they confound risk effects with expansion effects.

For the class of technologies exhibiting constant relative riskiness, we see that the pure risk effect is always distinguishable and unambiguous. Thus the pure risk effect would push a farmer to use more risk-complementary inputs in the presence of insurance and fewer risk substitutes.

Perhaps the most important aspect of our results is that they establish that neither risk complementarity nor risk substitutability is sufficient to determine whether an input's utilization increases or decreases as a result of the provision of insurance. While this may seem counterintuitive, it is quite reasonable once one recognizes that provision of insurance evokes at least two responses on the part of producers. The first, which we have called the pure-risk effect, is the change in the mix of state-contingent revenues which changes the riskiness (from the producer's perspective) of the optimal state-contingent revenue bundle. Generally, we expect the producer to move to a more risky revenue bundle as market provision of insurance substitutes for the insurer's need to self insure. It is for this effect where the notions of risk complementarity and risk substitutability are most relevant. But providing insurance also influences the producer's scale of operation, and these scale adjustments can either reinforce or modulate the pure-risk adjustment depending upon the input's responsiveness to radial changes in the revenue vector.

Our results on input adjustment to insurance are most directly comparable with the results of Horowitz and Lichtenberg and Ramaswami who study input and supply responsiveness to the provision of insurance in the presence of moral hazard. Both of those papers report sufficient conditions for providing insurance to increase the use of a single scalar input. For example, Ramaswami shows that if that input is risk-reducing, in his sense, and preferences are expected-utility preferences exhibiting non-increasing absolute risk aversion, its use will fall as a result of an introduction of crop insurance. Notice, in particular, that this finding also implies under these circumstances that output or revenue will fall in every state of nature as a result of the introduction of insurance. This is a necessary consequence of the single-input assumption.

In our study, to concentrate our focus on the construction of an analytical framework we have abstracted from the moral-hazard problem by assuming that the insurer can write state-contingent contracts. However, it follows from results reported in Chambers and Quiggin (2000, Chapter 7) that provision of the production insurance of the type considered by Ramaswami moves the producer out of the efficient set. This is the natural extension of the Ramaswami result to the multiple-output, multiple-input technology that we consider. Moreover, one can show for technologies exhibiting constant absolute riskiness that revenue cost falls after such insurance is provided.[10] Given these results, one can then sort out the effects on individual inputs by using the methodology developed above. Of course, if one is willing to impose even more structure upon preferences (for example, constant absolute risk aversion) while still not requiring maximisation of expected utility, one can obtain even sharper results.

9 Concluding Remarks

This paper studies input adjustments by risk-averse decisionmakers using the state-contingent formulation of Chambers and Quiggin (1992, 1996, 1997, 2000). This framework allows us to rely on a version of Shephard's lemma for stochastic technologies that does not rely on the single-output stochastic production function model that has dominated most previous studies. The only restriction placed upon preferences is that they be consistent with a very mild form of risk aversion.

Our principal contribution is to develop a framework for analyzing input adjustment under uncertainty that can be usefully illustrated with graphical techniques. Because the focus is on developing a method for decomposing and analyzing input adjustments, in our application of the method, we have only considered the simplest type of insurance market, one which is complete and actuarially fair. The analysis of incomplete and nonfair crop insurance has been addressed in Chambers and Quiggin (2000, Chapter 7). And while they have not directly analyzed the consequence of such insurance provision for input utilization, this paper's approach can be extended to their results. Moreover, using the cost-function framework pursued in this paper, Chambers and Quiggin (1996, 2000) have analyzed the design of insurance-like incentive schemes in the presence of moral hazard arising from both hidden action and hidden information. The decomposition procedure developed here can be directly applied to those results to determine the effect of insurance provision on input utilization.

Notes

[1] These results, not discussed in detail here, are elaborated in Chambers and Quiggin (2000, Chapter 5).

[2] There are an infinity of ways to compute the pure risk effect and expansion effect depending upon how one makes the adjustment in mean values. Here we always restrict attention to the case where the mean adjustment is consistent with a radial expansion because we think it would be most familiar to most economists. More generally, the expansion can be measured in any direction. Which direction is chosen will be typically determined by a number of factors. For example, if it is assumed that individuals have preferences exhibiting constant relative risk aversion then the radial decomposition we suggest is usually the most appropriate because the expansion effect involves no change in the riskiness of the revenue bundle. However, if preferences exhibit constant absolute risk aversion, then an expansion effect measured in the direction of the equal-incomes vector would be more appropriate as it would involve no change in the inherent riskiness of the revenue bundle.

[3] An input is typically called risk-increasing or risk-reducing depending upon the sign of a second partial derivative of stochastic production function (e.g., Quiggin 1992, Ramaswami).

[4] In Chambers and Quiggin (1996), the terms risk increasing and risk decreasing were used in place of risk complements and risk substitutes.

[5] The more usual notion of risk-increasing and risk-decreasing inputs familiar from the literature is not based on a mean-compensated risk comparison. Hence, it is not a proper risk comparison.

[6] More generally, since a dominance argument was used to establish these results, it follows that they apply regardless of whether preferences or the technology are smooth. Moreover, in the case of non-smooth preferences, Chambers and Quiggin (2000) following Segal and Spivak show that farmers can fully insure even in the absence of an actuarially fair insurance contract.

[7] Constant relative riskiness and constant absolute riskiness, as defined below, are defined analogously to constant absolute risk aversion and constant relative risk aversion for the general state-contingent preference function W as in Quiggin and Chambers (1998). A straightforward extension of those arguments leads to the maintained relationship between homotheticity and translation homotheticity.

[8] This result is completely analogous to the result that maximum revenue expansion paths for non-stochastic, multi-output technologies which exhibit output homotheticity are straight lines.

[9] Translation homothetic technologies are the class of quasi-homothetic technologies which can be represented by non-decreasing translations of a reference isoquant in a given direction (Chambers and Färe).

[10] As noted earlier, under constant absolute riskiness, the efficient set determines a unique level of cost. If the producer doesn't operate in the efficient set, it turns out that his cost must fall as a consequence of the properties of constant absolute riskiness.

References

Chambers, R. G. "Insurability and Moral Hazard in Agricultural Insurance Markets." *American Journal of Agricultural Economics* 71 (August1989): 614-26.

Chambers, R. G., and R. Färe. "Translation Homotheticity." *Economic Theory* 11 (1998): 629-41.

Chambers, R. G., and J. Quiggin. "Cost Functions and Duality for Stochastic Technologies." *American Journal of Agricultural Economics* 80 (May1998): 288-95.

———. "Nonpoint-Source Pollution Control As a Multi-Task Principal-Agent Problem." *Journal of Public Economics* 59 (1996): 95-116.

———. "Separation and Hedging Results With State-Contingent Production." *Economica* 64 (May1997): 187-210.

———. "A State-Contingent Approach to Production Under Uncertainty." University of Maryland, Working Paper 92-03, 1992.

———. *Uncertainty, Production, Choice, and Agency: The State-Contingent Approach.* New York: Cambridge University Press, 2000.

Hennessy, D. "The Production Effects of Income Support Policies Under Uncertainty." *American Journal of Agricultural Economics* 80 (February1998): 46-57.

Horowitz, J. K., and E. Lichtenberg. "Insurance, Moral Hazard, and Chemical Use in Agriculture." *American Journal of Agricultural Economics* (November1993): 926-35.

Nelson, C., and E. Loehman. "Further Toward a Theory of Agricultural Insurance." *American*

Journal of Agricultural Economics 69 (August1987): 523-31.

Pope, R. D., and J.-P. Chavas. "Cost Functions Under Production Uncertainty." *American Journal of Agricultural Economics* 76 (May1994): 196-204.

Quiggin, J. "Some Observations on Insurance, Bankruptcy and Input Demand." *Journal of Economic Behavior and Organization* 18 (1992): 101-10.

———. "A Theory of Anticipated Utility." *Journal of Economic Behavior and Organization* 3 (1982): 323-43.

Quiggin, J., and R. G. Chambers. "Risk Premiums and Benefit Measures for Generalized Expected Utility Theories." *Journal of Risk and Uncertainty* 17 (November1998): 121-38.

Ramaswami, B. "Supply Response to Agricultural Insurance: Risk Reduction and Moral Hazard." *American Journal of Agricultural Economics* 75 (November1993): 914-25.

Rothschild, M., and J. E. Stiglitz. "Increasing Risk I: A Definition." *Journal of Economic Theory* 2 (September1970): 225-43.

Sandmo, A. "On the Theory of the Competitive Firm Under Price Uncertainty." *American Economic Review* 61 (March1971): 65-73.

Savage, L. J. *The Foundations of Statistics*. New York: Dover, 1972.

Segal, U., and A. Spivak. "First-Order Versus Second-Order Risk Aversion." *Journal of Economic Theory* 51 (June1990): 111-25.

Yaari, M. "The Dual Theory of Choice Under Risk." *Econometrica* 55 (January1987): 95-115.

———. "Some Remarks on Measures of Risk Aversion and on Their Uses." *Journal of Economic Theory* 1 (October1969): 315-29.

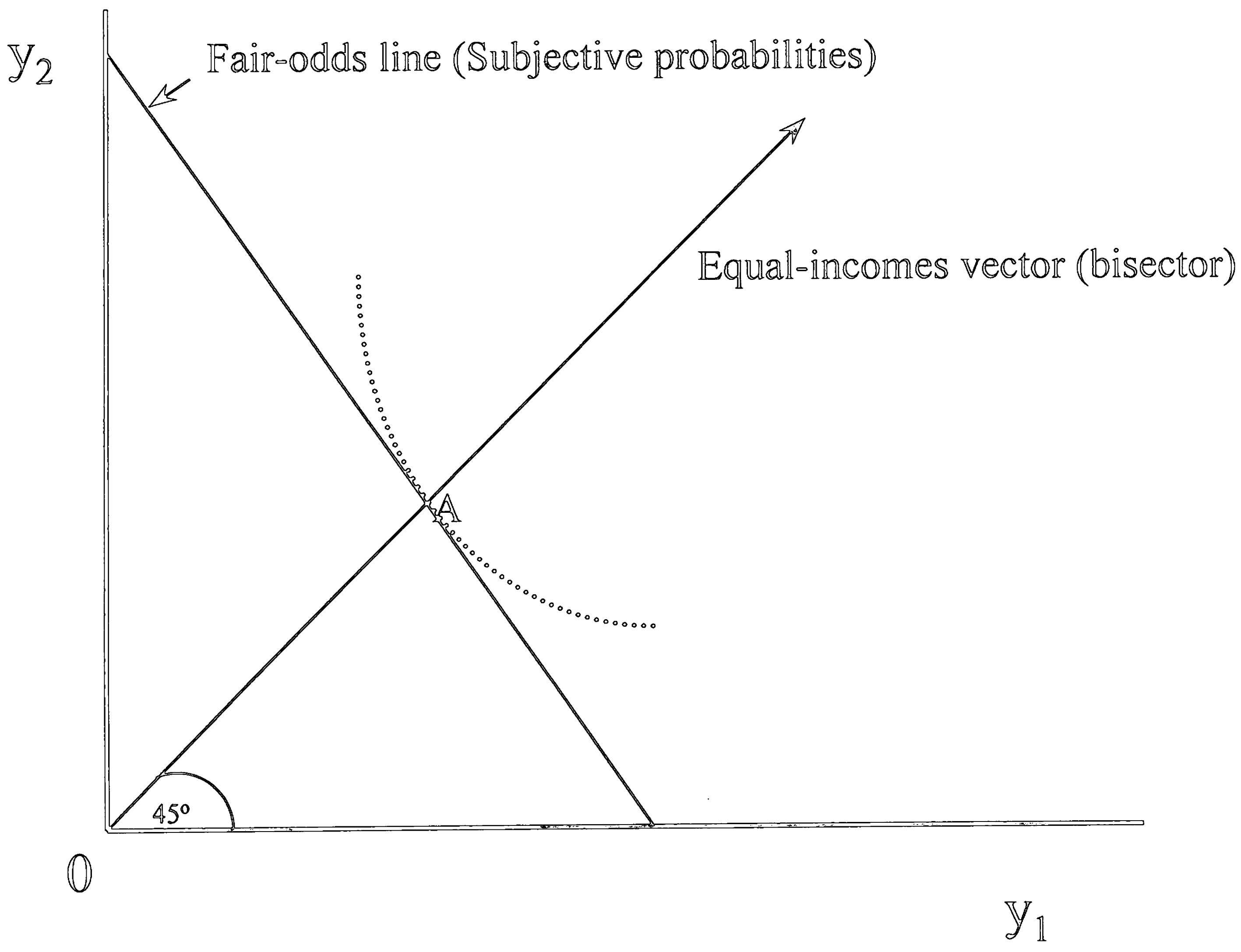
y_2
Fair-odds line (Subjective probabilities)
Equal-incomes vector (bisector)
A
45°
0
y_1

Figure 1: Risk-averse preferences

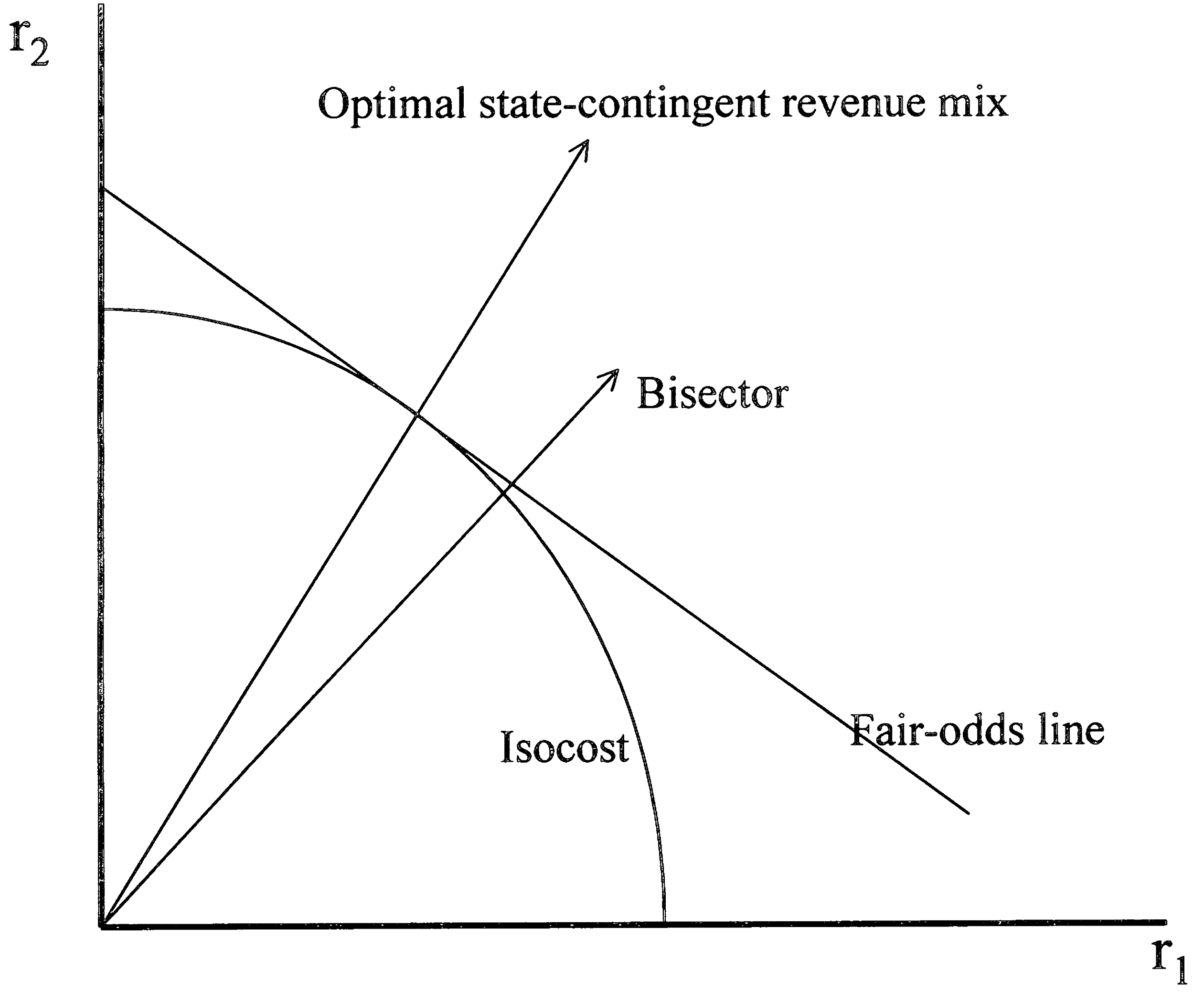
r$_2$
Optimal state-contingent revenue mix
Bisector
Isocost
Fair-odds line
r$_1$

Figure 2: Risk-neutral production equilibrium

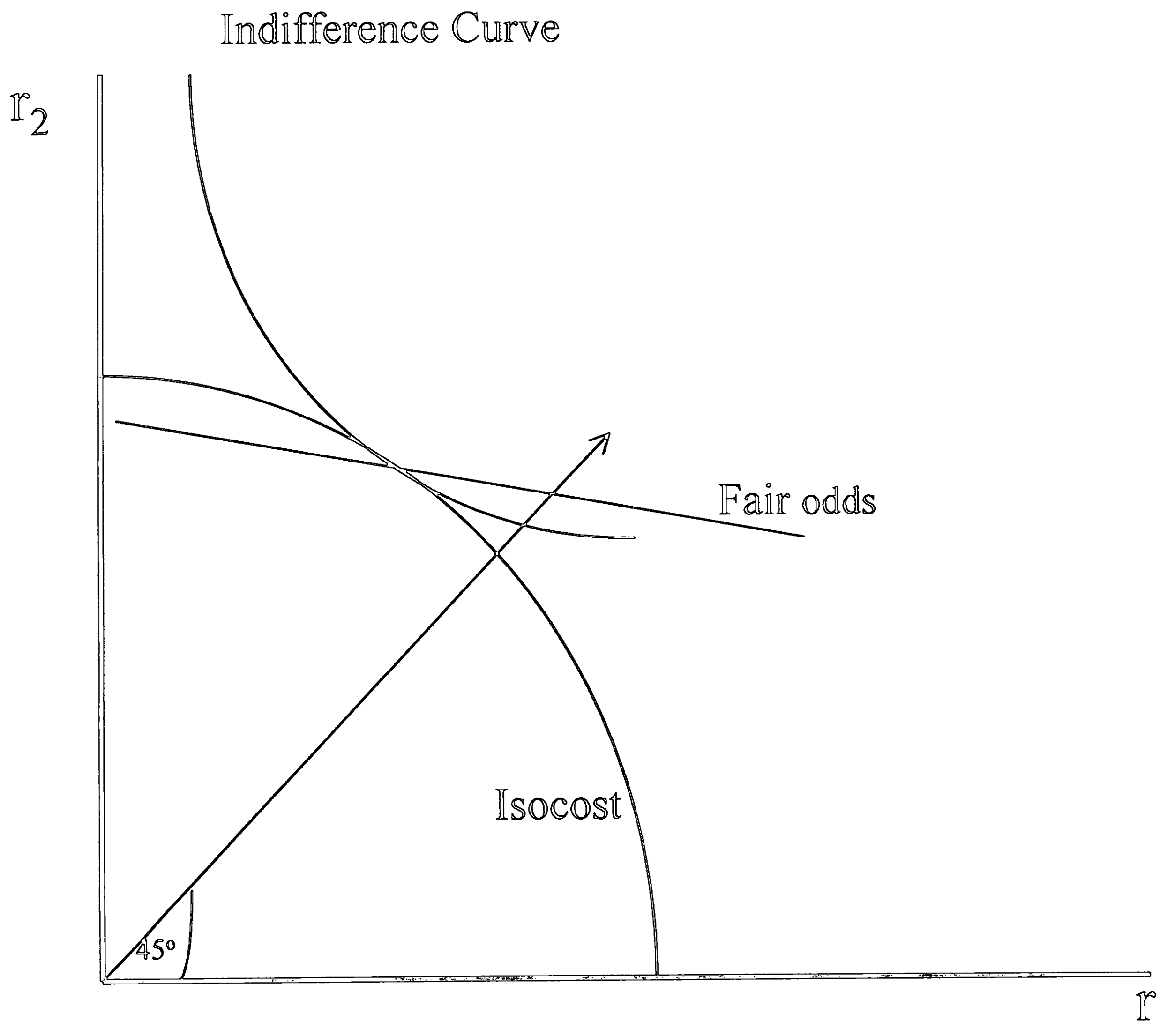
Indifference Curve
r_2
Fair odds
Isocost
45°
r_1

Figure 3: Risk-averse production equilibrium

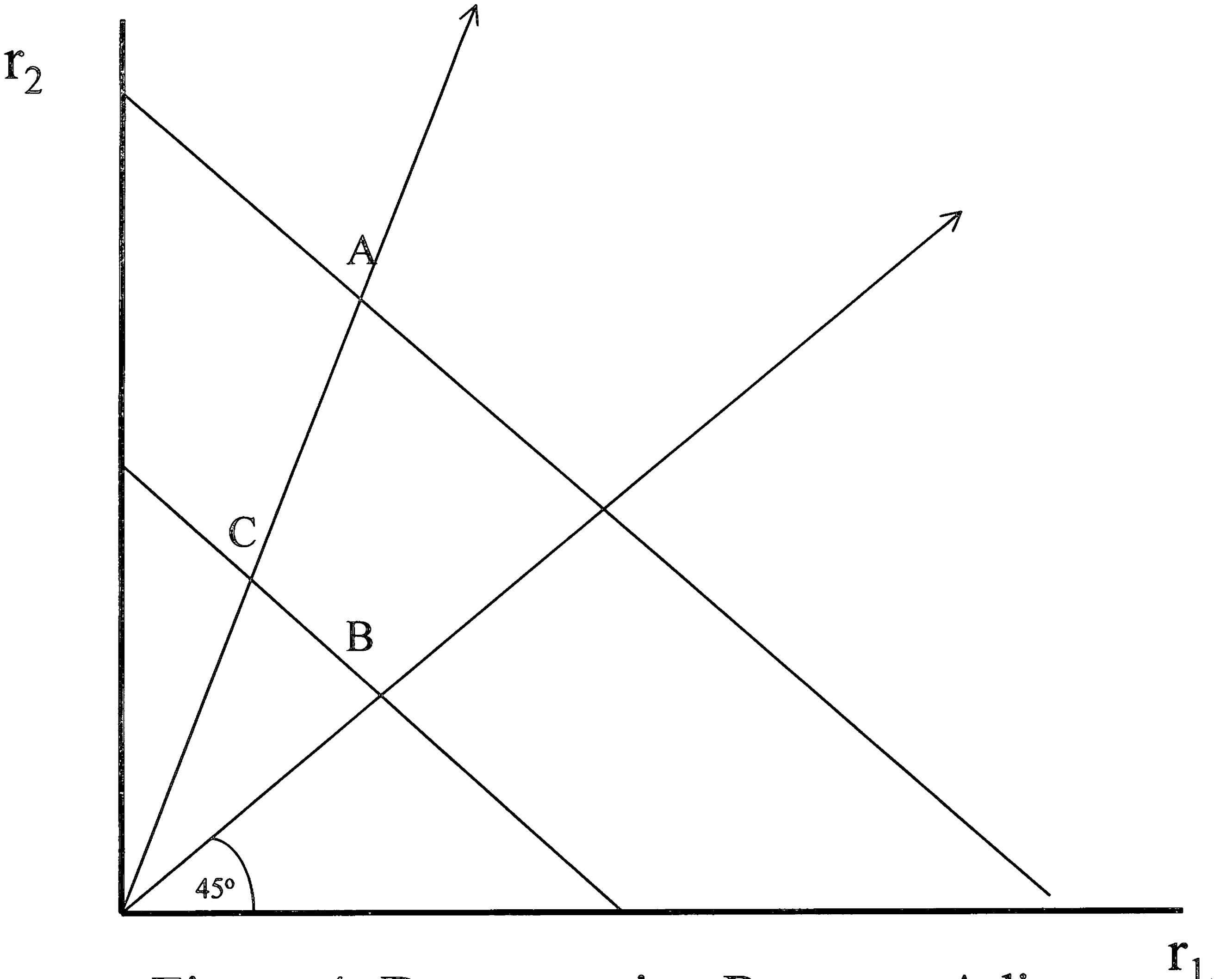

Figure 4: Decomposing Revenue Adjustments

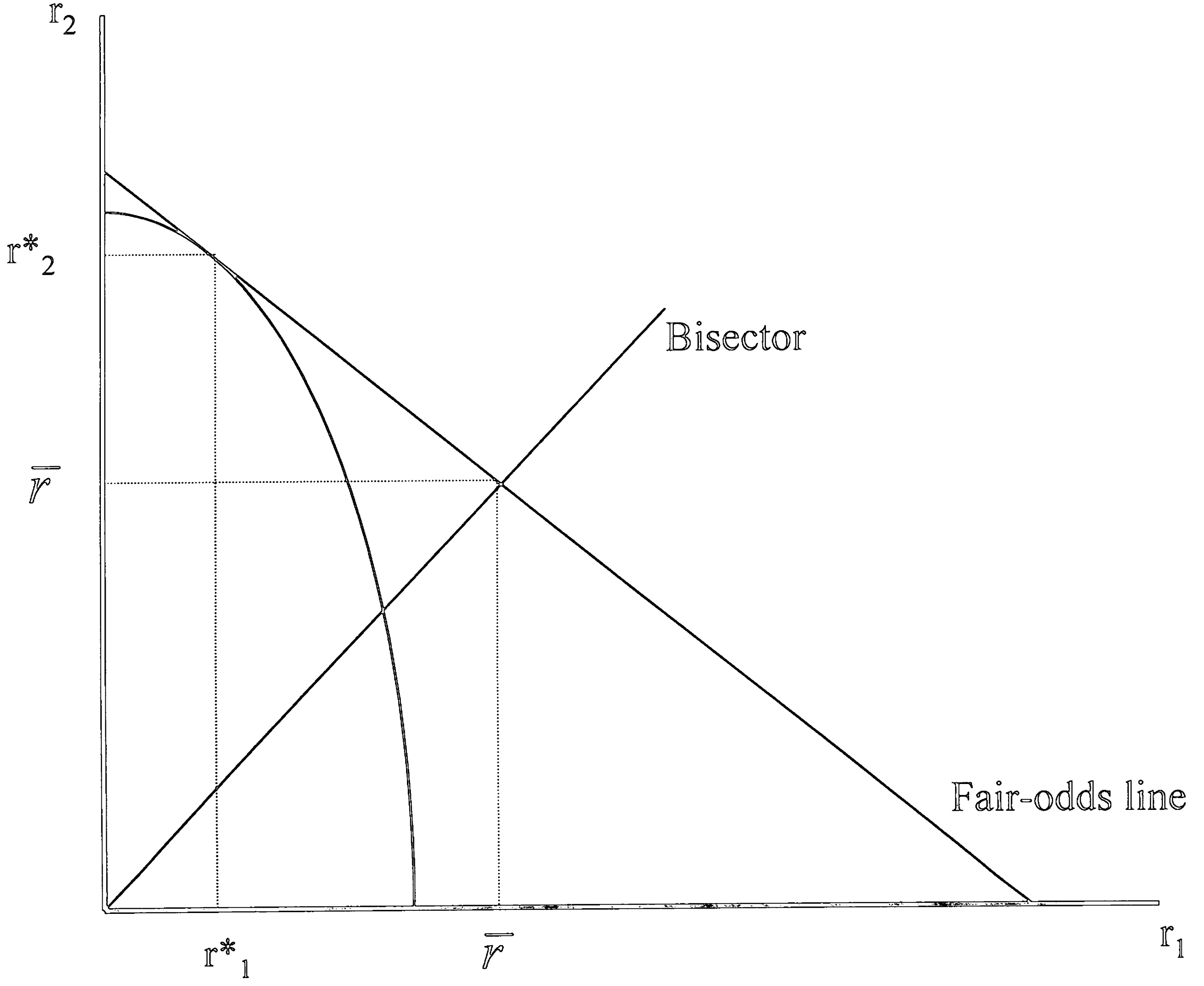
r_2
r^*_2
$\bar{r}$
Bisector
Fair-odds line
r^*_1
$\bar{r}$
r_1

Figure 5: Equilibrium with Fair Insurance

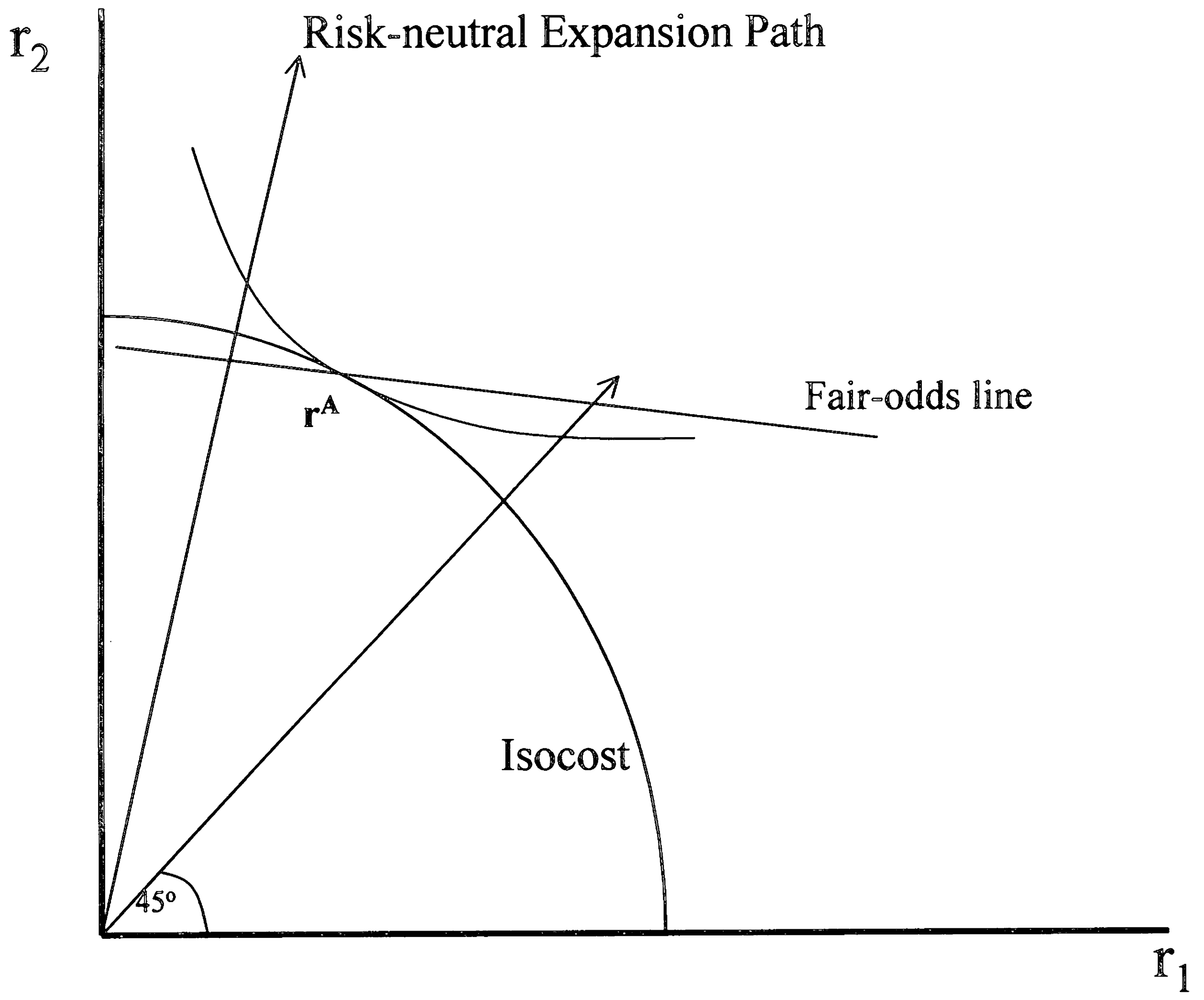
r_2
Risk-neutral Expansion Path
Fair-odds line
r^A
Isocost
45°
r_1

Figure 6: Constant relative riskiness and insurance

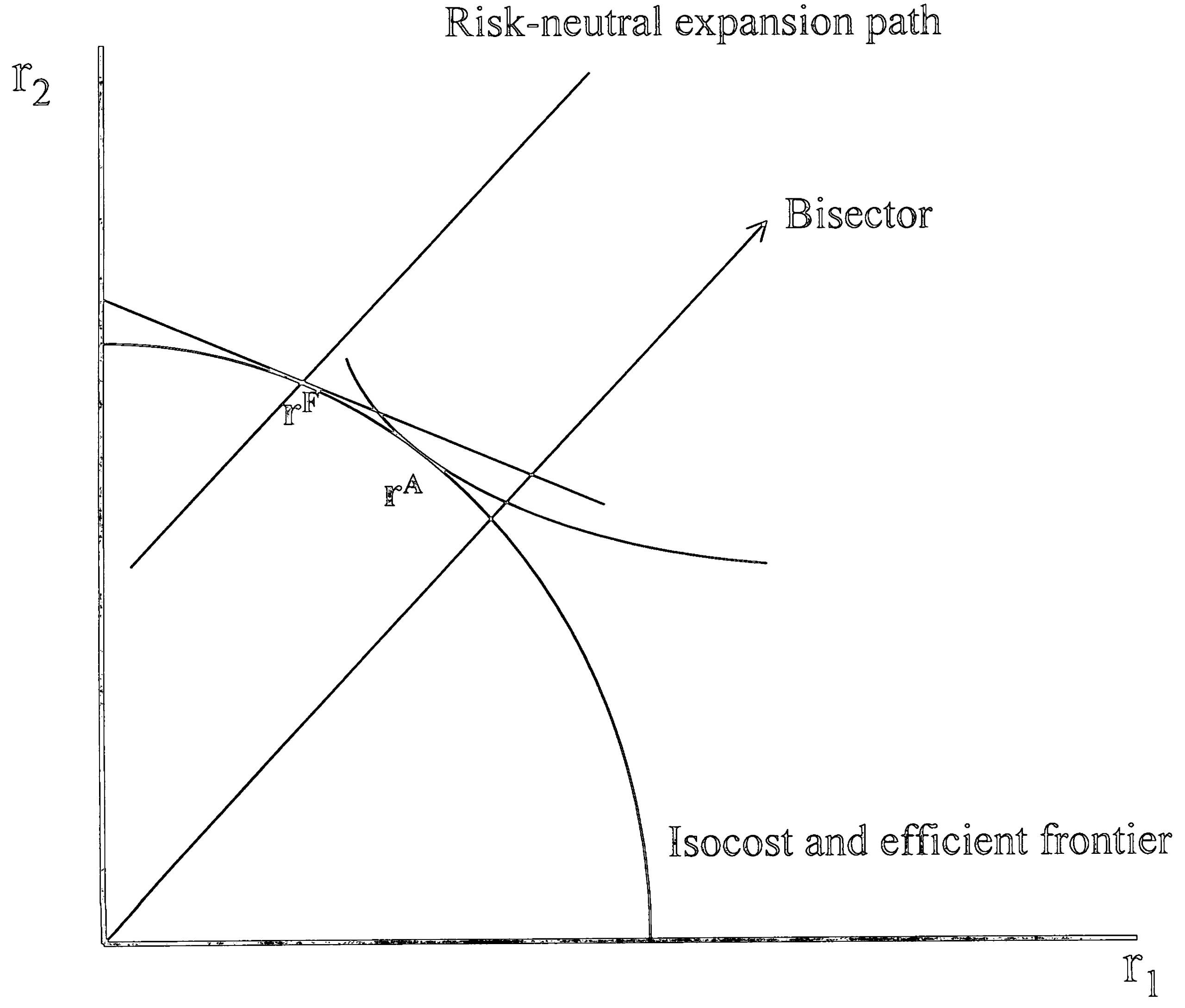
Risk-neutral expansion path
r_2
Bisector
r^F
r^A
Isocost and efficient frontier
r_1

Figure 7: Constant absolute riskiness and insurance