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A Note on Optimal-Revenue Tariffs and the Uniform Tariff

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Quite some time ago, Vanek developed the notion of an optimal-revenue tariff. Musgrave and Corden (1974) had early noted the tendency for a country's fiscal dependence on trade taxes to be inversely related to its level of development. Lewis' and Hinrichs' research gave empirical content to this relationship. Despite the passage of almost two decades import tariffs remain an important revenue source for many developing countries. And although he finds the relationship weakening through time, Greenaway's empirical analysis reconfirms the relationship between trade taxes and level of development.

Optimal tariff design and reform are, therefore, very real issues for many developing economies. Reform via tariff abolition is often infeasible because developing countries often lack the infrastructure to collect domestic direct or indirect taxes. Thus simple application of optimal-taxation principles may be inappropriate. But such principles should not be ignored. And because these principles imply that uniform tax structures are not usually optimal, arguments that have been made for tariff uniformity should be analyzed carefully.

A common argument for tariff uniformity is that it reduces production distortions in import-competing industries (Harberger). Another argument is its simplicity which should not be underestimated, especially in economies with poorly developed infrastructures. Corden (1958) succinctly summarized the simplicity argument by characterizing uniformity as a means for "cutting through the jungle with the price mechanism."

In evaluating tariff uniformity, however, one must remember that a large component of many developing countries' imports are intermediate goods

intended not for final consumption but instead as inputs in domestic production. Rodrik and Lopez report that in some developing countries as much as 80 percent of total imports are intermediate goods. Thus tariffs on intermediates often degenerate to input taxes which necessarily imply production inefficiencies.

This note deduces sufficient conditions for a small country's net expenditure function to imply that the optimal-revenue tariff structure should be uniform. These conditions are related to, but still differ from, Deaton's sufficient conditions for a uniform Diamond-Mirrlees optimal tax structure. Where the latter require quasi-separability of the representative individual's expenditure function, the former require quasi-separability of the net expenditure function in a sense to be made precise below.

The Model

Assume a small open economy which takes prices of all traded commodities as given. The economy's technology is characterized by the production possibilities set

$$Y = \{(x,y,z):(x,z) \text{ can produce } y;\ x \in \mathbb{R}^k_+,\ y \in \mathbb{R}^n_+,\ z \in \mathbb{R}^o_+\}.$$

Here y is a vector of final consumption items, z is a vector of nontraded (across international borders) inputs, and x are traded intermediate inputs. The vector y can be further partitioned as

$$y = \{(y_m, y_x, y_n):\ y_m \in \mathbb{R}^{n_m}_+,\ y_x \in \mathbb{R}^{n_x}_+,\ y_n \in \mathbb{R}^{n_n}_+;\ n_m + n_x + n_n = n\}.$$

The subscript m denotes "importables", the subscript x denotes "exportables", and the subscript n denotes "nontraded." Y is a nonempty, closed, and strictly convex set. Y satisfies free disposability of y and free disposability of x and z and is bounded from above for finite z.

Dual to Y is the revenue (gross national product) function

$$R(p,w;\ z) = \max_{y,x} \{py - wx:\ (x,y,z) \in Y\}$$

Here $p \in \mathbb{R}^{n_m}_{++} \times \mathbb{R}^{n_x}_{++} \times \mathbb{R}^{n_n}_{++}$ is a vector of output prices (subscripts on p use the same convention as subscripts on y) and $w \in \mathbb{R}^k_{++}$ is a vector of traded intermediate input prices. R is a positively linearly homogeneous and convex function in p and w. Because Y has been assumed strictly convex R is differentiable and

$$R_j(p,w;\ z) = y_j(p,w;\ z) \qquad j = m,\ x,\ n$$

where $R_j(p,w;\ z)$ is the gradient of R in p_j ($j = m,\ x,\ n$) and $y_j(p,w;\ z)$ is the economy's net output vector of j. Moreover

$$R_w(p,w;\ z) = -x(p,w;\ z)$$

where $x(p,w;\ z)$ is the economy's excess demand (total derived demand less domestic production) vector for the traded intermediate inputs. The formulation of Y is general enough to cover the case where some components of x are produced domestically and some are not. The latter is particularly important when some components of x are high-value capital inputs that are only available from the developed world. Our restriction that $x \in \mathbb{R}^k_+$ implies that the country is always a net importer of intermediates. (Domestic production, which may be zero or positive for any element of x, is less than total utilization.) In what follows R is always assumed to be at least twice differentiable.

A representative societal welfare function $U : \mathbb{R}^n_+ \to \mathbb{R}$ exists. U is non-decreasing and strictly quasi-concave. Dual to U is the expenditure function

$$E(p,u) = \min_c \{p \cdot c : U(c) \geq u\}.$$

$E(p,u)$ is positively linearly homogeneous, nondecreasing and concave in p. $E(p,u)$ is nondecreasing in u. Moreover its gradient in p is the vector of Hicksian compensated demands. Gradient notation here follows the same convention used for R except that $E_u(p,u)$ is the partial derivative of E with respect to u. The intermediate inputs are not used for final domestic consumption.

The net expenditure function is defined

$$N(p,w,u;\ z) \equiv E(p,u) - R(p,w;\ z)$$

$N(p,w,u;\ z)$ is positively linearly homogeneous and concave in prices by the above and nondecreasing in u. Moreover its gradient in prices represents the economy's excess demand for the relevant commodity or intermediate input vector. Hence following the notational conventions developed above

$$N_j = E_j - R_j \qquad j = m,\ x,\ n$$

$$N_w = -R_w.$$

are the vector of excess demands for the jth type commodity and the intermediate inputs, respectively.

To concentrate on the optimal tariff problem we ignore non tariff barriers to trade and assume the government only intervenes through the use of tariffs on y_m and x. There are no export taxes.[1] Let world prices be denoted by p_j^* $(j = m,\ x)$ and w^*. Denote the vectors of specific tariffs on y_m and x by t_m and t_w, respectively. Hence

$$p_m = p_m^* + t_m$$

$$w = w^* + t_w.$$

We start the analysis by assuming that the government has a specific revenue target in the amount T. Thus, the government's actions are constrained by

$$T = t_m N_m + t_w N_w. \tag{1}$$

Let y^G represent the government's purchase of y. Assume that the government is not involved in production so that it does not purchase x. Nongovernment purchases of y are c. Total final good expenditures domestically are then $p(y^G + c)$. Nongovernment income is given by $R(p,w;\ z)$ so that we have the respective budget constraints

$$pc = R(p,w;\ z)$$

$$py^G = t_m N_m + t_w N_w.$$

However, we shall always assume that government preferences are identical to the representative consumer so that its preferences and its consumer preferences are fully reflected by u. This allows us to transform these two budget constraints into one

$$N(p,w,u;\ z) = t_m N_m + t_w N_w. \tag{2}$$

We also presume that the nontraded good markets always clear so

$$N_n = 0. \tag{3}$$

The optimal-revenue tariff problem, therefore, is to maximize u subject to (1), (2), and (3). Formally we seek

$$(t_w, t_m, u, p_n) = \text{argmax}\ \{u : N = t_m N_m + t_w N_w,\ T = t_m N_m + t_w N_w;\ N_n = 0\} \tag{4}$$

Uniformity and the Optimal-Revenue Tariff

The Lagrangian for expression (4) is

$$L = u + \mu(N - t_m N_m - t_w N_w) + \lambda(T - t_m N_m - t_w N_w) + \gamma N_n \tag{5}$$

where $\mu \in \mathbb{R}$, $\lambda \in \mathbb{R}$, and $\gamma \in \mathbb{R}^{n_n}$ are Lagrange multipliers. The first-order conditions for (5) do not generally provide either necessary or sufficient conditions for an optimum (Mirrlees, 1987). However, it is standard practice in the optimal tax literature to presume that they do. Moreover, empirical analyses of optimal tax structures in developing economies is almost wholly

based on the first-order conditions for the analogue of (5) (see e.g., Newbery and Stern and the empirical studies contained therein). We adopt the standard approach. However, like the results of Deaton, our conditions are most appropriately interpreted as sufficient conditions for conventional formulae used to calculate optimal-revenue tariffs to imply uniformity.

Deaton has shown for the optimal-tax problem that a sufficient condition for optimal taxation to be uniform is that the expenditure function E(p,u) be quasi-separable. That result is predicated on the efficient production characteristic of the Diamond-Mirrlees optimal tax. Hence, all taxes are commodity taxes and only the structure of preferences affects the degree of uniformity of the optimal tax. Here, by definition, the tariff structure must involve production inefficiencies. It is in fact in an attempt to alleviate some of these inefficiencies within the import-competing industry that the uniform tariff is usually recommended. Because tariff uniformity means that all imports face the same tariff rate (and hence uniform effective protection) their marginal price relations are not disturbed. Hence, marginal price distortions between importables have been removed and production efficiency should be improved in that sector. Suppose, however, that moving to tariff uniformity requires raising a tariff on an imported intermediate used solely in the production of exportables. The move toward uniformity now engenders an inefficiency in exportable production which must be traded off against the efficiency gain in importable production. There is no reason to expect uniformity to be optimal generally.

The similarity of the optimal revenue-tariff problem to the optimal tax problem and the analogous nature of optimal-revenue tariff rules and optimal-tax rule suggest, however, that an analogous result to Deaton's should exist

for the optimal revenue-tariff problem. For example, in the case with no nontraded goods Stiglitz and Dasgupta have shown that the optimal tariff should be set so that "the percentage reduction in excess demand should be the same for all commodities" (p. 20), a result which matches almost exactly with the optimality conditions on demand (not excess demand) for the optimal tax problem. In fact, it turns out that a sufficient condition for an optimal-revenue tariff to be uniform is that the net expenditure function have p_m and w separable from p_x and p_n in N. We have (the proof is in an Appendix):

Proposition 1: If $N(p,w,u;\ z) = N^*(\phi(p_m,w,u;\ z),\ p_x,p_n,u;\ z)$ where N^* is positively linearly homogeneous and convex in ϕ, p_x, and p_n, and ϕ is positively linearly homogeneous and convex in p_m and w, the optimal-revenue tariff is uniform.

The economic intuition behind the proposition is fairly transparent. First note that ϕ is so defined (positively linearly homogeneous and convex in p_m and w) that it can be interpreted as a net expenditure function itself. Moreover, N^* is convex and linearly homogeneous in ϕ, p_m and p_n implying that ϕ can be interpreted as a price index. A net expenditure function of the prescribed form then means that technology and preferences are such that an aggregate "import-competing" sector can be defined whose aggregate excess demand only depends upon its aggregate price ϕ, p_x, p_n, u, and z. The aggregate excess demand for the import-competing sector now can be taken, assuming differentiability, as $\frac{\partial N^*}{\partial \phi}(\phi,p_x,p_n,u;\ z)$ by the Hotelling-Shephard lemma. For this form the optimal-revenue tariff problem reduces to one of choosing a level of protection for the "import-competing" sector and not one of how to protect individual import items.

Optimal-Revenue Tariffs to Finance the Purchase of Public Goods

The quasi-separability result is robust and extends to modifications of the basic model. As an example, Feehan recently derived the characteristics of the optimal-revenue tariff for the situation where the tariff revenue is only used to finance a nontraded public good. This section extends the result to that case.

For simplicity assume $y_n \in \mathbb{R}_+$: the nontraded good is now a public good not purchased by consumers. Define the public-good expenditure function by

$$E^P(p_x, p_m, u;\ c_n) = \min\ \{p_m c_m + p_x c_x\ :\ u(c_x, c_m, c_n) \geq u\}.$$

Because consumers do not purchase c_n its market price is irrelevant to them.

The public good is produced commercially, however. Therefore, the government must purchase the public good from private vendors. And because we have a market for the public good, $R(p,w;\ z)$ remains the appropriate revenue function. Define the public-good net expenditure function as

$$N^P(p,w,u;\ z,c_n) = E^P(p_x, p_m, u;\ c_n) - R(p,w;\ z).$$

Because the public good is nontraded, $c_n = R_n(p,w;\ z)$. If the public-good market is to clear, N^P must be evaluated at $N^P(p,w,u;\ z,R_n(p,w;\ z))$.

Recalling that all tariff revenue goes to public-good expenditure yields the following reformulation of the optimal-revenue tariff problem:

$$\max\ \{u\ :\ N^P(p,w,u;\ z,R_n) = 0;\ p_n R_n = t_m N^P_m + t_w N^P_w\}$$

where, for example, N^P_m is the gradient of N^P with respect to p_m holding R_n constant. Thus (see the Appendix for a proof):

Proposition 2: If $N^P(p,w,u;\ z,c_n) = G(\theta(p_m,w,u;\ z,c_n),\ p_x,p_n,u;\ z,c_n)$ where θ is positively linearly homogeneous and convex in p_m and w and G is

positively linearly homogeneous and convex in θ, p_x, and p_n, the optimal-revenue tariff is uniform.

Summary

Vanek's optimal-revenue tariff has been reconsidered to deduce where a uniform revenue tariff structure is optimal. A uniform revenue-tariff structure is optimal if technology and preferences are such that all import-competing activities can be aggregated into a single sector whose aggregate excess demand only depends upon the aggregate sector price (which in turn depends only on prices in the sector) and prices outside the sector. If the net expenditure function has the prices of import competing activities separable from other prices this condition is satisfied.

The requisite separability of the net expenditure function is, of course, an empirical issue. Only explicit econometric tests can determine whether a country's net expenditure function belongs to this particular class of separable structures. At present little direct evidence on this issue exists. This condition, however, is reminiscent of similar separability restrictions on cost, profit, and expenditure functions which have been the subject of intensive investigation in the empirical demand and production literature. Even the most casual perusal of those literatures should convince the reader that little, a priori, reason exists to suspect that the underlying technologies or preferences will satisfy these separability requirements. Lacking evidence to the contrary, therefore, one must conclude that efficiency-based arguments for the optimality of uniform revenue tariff structures remain to be made in the context of most developing economies. If this evidence is not forthcoming, the case for or against a uniform tariff structure must explicitly account for the trade off between the gains from

uniformity associated with administrative simplicity and the lower efficiency generated by a uniform tariff structure that diverges from an appropriately diverse optimal revenue tariff structure.

Appendix

Proof of Proposition 1:

Define $\tau_{mi} \equiv t_{mi}/p_{mi}$ $(i = 1,\ldots,n_m)$ and $\tau_{w_i} \equiv t_{w_i}/w_i$ $(i = 1,\ldots,k)$. Then let $\tau_m p_m \equiv (\tau_{m1}p_{m1},\ldots,\tau_{mn_m}p_{mn_m})$ and $\tau_w w \equiv (\tau_{w1}w_1,\ldots,\tau_{wk}w_k)$. Under the symmetry and homogeneity properties of $N(p,w,u;\ z)$ (Dixit and Norman) the first-order conditions for t_m and t_w require

$$-(\mu + \lambda)[\varepsilon_{mm}(\tau_m p_m)' + \varepsilon_{mw}(\tau_w w)'] + \varepsilon_{mn}\gamma' - \lambda e_{n_m} = 0$$

$$-(\mu + \lambda)[\varepsilon_{wm}(\tau_m p_m)' + \varepsilon_{ww}(\tau_w w)'] + \varepsilon_{wm}\gamma' - \lambda e_k = 0$$

where ε_{ij} is the matrix of excess demand elasticities for commodity group y_i (or x) with respect to price vector p_j (or w), and $e_j = (1,\ldots,1)$ (there are j elements in e_j). A uniform tariff structure requires a uniform *ad valorem* tariff of $(1 + \delta)$ so that $p_m = (1 + \delta)\ p_m{}^*$ and $w = (1 + \delta)w^*$ implying $t_m = \delta p_m{}^*$ and $t_w = \delta w^*$. But this means that $\tau_{m_i} = \tau_{w_i} = \delta$. So the first-order conditions become

$$-\delta(\mu + \lambda)[\varepsilon_{mm}p_m' + \delta_{mw}w'] + \varepsilon_{mn}\gamma' - \lambda e_{n_m} = 0$$

$$-\delta(\mu + \lambda)[\varepsilon_{wm}p_m' + \varepsilon_{ww}w'] + \varepsilon_{wn}\gamma' - \lambda e_k = 0.$$

Exploiting the homogeneity properties of excess demands then gives

$$\delta(\mu + \lambda)[\varepsilon_{mx}p_x' + \varepsilon_{mn}p_n'] + \varepsilon_{mn}\gamma' - \lambda e_{n_m} = 0$$

$$\delta(\mu + \lambda)[\varepsilon_{wx}p_x' + \varepsilon_{wn}p_n'] + \varepsilon_{wn}\gamma' - \lambda e_k = 0$$

These conditions must be satisfied for all prices, all ranges of elasticities, and all μ, λ, and γ if a uniform rate of δ is to be optimal. Notice that quasi-separability of $N(p,w,u;\ z)$ implies that each row of ε_{mx} and ε_{wx} are identical while each row of ε_{mn} and ε_{wn} are identical with typical

elements ξ_j and ρ_j, respectively. So for this class of net expenditure functions this system of equations degenerates to one equation of the form

$$\delta(\mu + \lambda)\left(\sum_{j=1}^{m_x} \xi_j p_{x_j} + \sum_{j=1}^{m_n} \rho_j p_{n_j}\right) + \sum_{j=1}^{m_n} \gamma_j \rho_j = \lambda$$

When combined with the first-order conditions for u, μ, λ, and γ this gives a system of $m_n + 4$ equation to be solved for u, δ, μ, λ, and γ. The uniform δ isolated then satisfies the first-order conditions and is thus optimal.

<u>Proof of Proposition 2</u>:

The Lagrangian expression is

$$L = u + \mu N^P + \lambda(p_n R_n - t_m N^P_m - t_w N^P_w).$$

To start notice that the structure of N^P implies that

$$\frac{\partial N^P(p,w,u;\ z,c_n)}{\partial p_n} = -\frac{\partial R(p,w;\ z)}{\partial p_n}$$

recalling that $p_n \in \mathbb{R}_{++}$. Hence

$$N^P_{nm}(p,w,u;\ z,c_n) = -R_{nm}(p,w;\ z)$$

in the obvious notation. Also note that

$$N^P_{nw}(p,w,u;\ z,c_n) = -R_{nw}(p,w;\ z).$$

The first-order condition for t_m and t_w now require under symmetry of $N^P(p,w,u;\ z,\ c_n)$

$$(\mu - \lambda) - \lambda\varepsilon_{mn}p_n - \lambda[(\varepsilon_{mm} - N^P_{mc}\varepsilon_{mn})(\tau_m p_m)' + \varepsilon_{mw}(\tau_w w')] = 0$$

$$(\mu - \lambda) - \lambda\varepsilon_{wn}p_n - \lambda[\varepsilon_{ww}(\tau_w w') + (\varepsilon_{wn} - N^P_{mc}\varepsilon_{mn})(\tau_m p_m)'] = 0$$

where $N^P_{mc} = N^P_{mc}(p,w,u;\ z,c_n)$ and ε_{ij} is the elasticity of commodity vector y_i (or x) with respect to price gradient p_j (or w) evaluated at $(p,w,u;\ z,c_n)$.

Homogeneity of $N^P(p,w,u;\ z,c_n)$ then implies for a uniform δ

$$(\mu - \lambda) - \lambda\varepsilon_{mn}p_n + \delta\lambda[\varepsilon_{mx}p_x' + \varepsilon_{mn}p_n' + N^P_{mc}\varepsilon_{mm}p_m'] = 0$$

$$(\mu - \lambda) - \lambda\varepsilon_{wn}p_n + \delta\lambda[\varepsilon_{wx}p_x' + \varepsilon_{wn}p_n' + N^P_{mc}\varepsilon_{nm}p_m'] = 0$$

which under the quasi-separable form reduces to a single equation as in the proof of Proposition 1. The first-order conditions again reduce to $m_n + 4 = 5$ equations to be solved for the 5 unknowns u, δ, μ, λ, and γ. (Recall again $m_n = 1$.)

References

Corden, W. M. "Import Restrictions and Tariffs: A New Look at Australian Policy." Economic Record 34(1958):331-46.

_____. Trade Policy and Economic Welfare Oxford University Press (New York, 1974).

Deaton, A. "Optimal Taxes and the Structure of Preferences." Econometrica 49(1981):1245-60.

Diamond, P. and J. Mirrlees. "Optimal Taxation and Public Production." American Economic Review 61(1971):8-27 and 261-78.

Feehan, J. P. "Efficient Tariff Financing of Public Goods." Journal of International Economics 25(1988):155-164.

Greenaway, D. "A Statistical Analysis of Fiscal Dependence on Trade Taxes and Economic Development." Public Finance XXXIX(1984):70-89.

Harberger, A. "Reflections on Uniform Taxation" paper presented to the 44th Congress of the International Institute of Public Finance, Istanbul, August 1988.

Hinrichs, H. "Determinants of Government Revenue Shares Among Less Developed Countries." Economic Journal 75(1965):546-56.

Lewis, A. "Government Revenue From Foreign Trade: An International Comparison." Manchester School 31(1963):39-46.

Mirrlees, J. "The Theory of Optimal Taxation." Chapter 24 in Handbook of Mathematical Economics vol. III, eds. K. J. Arrow and M. Intriligator. North-Holland/Elseveir (New York, 1986).

Musgrave, R. Fiscal Systems, New Haven: Yale University Press, 1969.

Newbery, D. and N. Stern (eds.) The Theory of Taxation for Developing Countries. Oxford University Press: New York, 1987.

Rodrik, D. and R. Lopez. "Trade Restrictions with Imported Intermediate Inputs: When Does the Trade Balance Improve." Journal of Development Economics (forthcoming).

Stiglitz, J. and P. Dasgupta. "Benefit-Cost Analysis and Trade Policies." Journal of Political Economy (1974):1-25.

Vanek, J. "Tariffs, Economic Welfare, and Development Potential." Economic Journal 81(1971):904-915.

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Footnote

[1] If export taxes exist the analysis would change especially if one were interested in the optimal choice of all trade taxes. However, the results would continue to apply as derived below with only some minor notational changes if one were interested in the optimal revenue raising tariff given an unchanging export tax structure.