Rational Bubbles and Land Prices

by

John Baffes and Robert G. Chambers

November 2, 1989

Working Paper

89-37
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ABSTRACT

Land-price models explain price behavior based on market fundamentals, i.e., the current price of land as the discounted sum of future net returns. This paper tests whether recent U.S. land price fluctuations can be attributed to a rational bubble.

KEY WORDS: Farmland prices, Rational Bubbles.
Rational Bubbles and Land Prices

Before 1973 U.S. farmland values generally moved with farm income. After 1973, however, farmland values and earnings from land began to diverge: through the 1970s farmland values appeared to be much more than just the capitalized value of quasi-rents from farming (Castle and Hoch 1982). As returns per acre declined farmland price steadily increased (Figure 1). In the 1980s, however, the land market crashed (Figure 1). Although many land-price models exist (e.g., Feldstein (1980); Alston (1986); and Burt (1986)), none completely explains this crash. This paper tests whether this episode is consistent with a bursting "rational bubble".

The next section presents a brief explanation of how a rational bubble might affect agricultural-land prices. An empirical model is developed in the second section, while the third section describes the data, the estimation procedure, and results. The last section presents a summary along with some concluding comments.

I. A "Rational-Bubble", Land-Price Model

Agricultural land's fundamental value is determined by the returns from farming. This section briefly develops the relationship between land's fundamental value and rational bubbles. For a risk-neutral profit maximizing farmer the quasi-rent from farming is

\[
\pi(p, w, L, z) = \max_{x, y} (py - wx: (x, y, L, z) \in T),
\]

where \( p \in \mathbb{R}^m_+ \) and \( w \in \mathbb{R}^n_+ \) are output and input price vectors, \( y \in \mathbb{R}^m \) and \( x \in \mathbb{R}_+^n \) are output and input quantity vectors, \( z \in \mathbb{R}^k \) denotes exogenous variables which may affect profitability, \( L \in \mathbb{R}_+ \) represents the amount of land, and \( T \) is
the farmer's technology set. Under relatively weak restrictions on $T$ (Färe (1988)), $\Pi(p,w,L,z)$ is nonincreasing in $w$; nondecreasing in $p$; convex and positively linearly homogeneous in $w$ and $p$. Assuming $T$ satisfies constant returns to scale in $x$, $y$, $z$, and $L$ (i.e., $(x,y,L,z) \in T$ implies $(\lambda x, \lambda y, \lambda L, \lambda z) \in T$ for $\lambda \in \mathbb{R}^+$) then

$$\Pi(p,w,L,z) = \Pi(p,w,z)L.$$

$\Pi(p,w,z)$ is the per-acre, quasi-rent of land.

Let $\Pi_t$ be $\Pi(p,w,z)$ at time $t$, $r$ the rate of return on a riskless asset and $q_t$ the price of land at time $t$. Further, let $\Omega_t = \{q_{t-1}, \Pi_{t-1}, z_{t-1}, 1 = 0, \ldots, \omega\}$ be the information set common to all agents at time $t$. Then, assuming perfect capital markets, the following arbitrage relationship must hold:

$$\mathbb{E}(q_{t+1} | \Omega_t) - q_t + \mathbb{E}(\Pi_{t+1} | \Omega_t) = r,$$

where the left-hand side gives the rate of return on a dollar invested in farmland. Rearranging (2) gives,

$$q_t = (1+r)^{-1} \mathbb{E}(q_{t+1} + \Pi_{t+1} | \Omega_t).$$

Expression (3) is a first-order expectational difference equation. Assuming that expectations are rational and that a solution exists, the law of iterated expectations implies when (3) is solved forward in time that

$$q_t = \sum_{i=0}^{\infty} (1+r)^{-i} \mathbb{E}(\Pi_{t+1} | \Omega_t) + \lim_{T \to \infty} (1+r)^{-T} \mathbb{E}(q_{t+T+1} | \Omega_t).$$

The first term on the right hand side of (4) is the present value of the expected stream of quasi-rents. Following usual terminology we refer to it as land's fundamental value. The second term is the present value of the
expected value of land in the arbitrary far-off future. From (4) land price equals the fundamental value of land if and only if the following transversality condition holds:

\[ \lim_{T \to \infty} (1+r)^{-T-1} E(q_{t+T+1} | \Omega_t) = 0. \]

If (5) fails, the fundamental value model is not valid and (3) admits a family of solutions. Following Blanchard and Fischer (1989), let

\[ q_t^* = \sum_{i=1}^{\infty} (1+r)^{-i-1} E(\Pi_{t+i} | \Omega_t) \]

and

\[ b_t = \lim_{T \to \infty} (1+r)^{-T-1} E(q_{t+T+1} | \Omega_t). \]

Then a possible solution to (3) becomes,

\[ q_t = q_t^* + b_t. \]

So long as \( b_t \) satisfies

\[ E(b_{t+1} | \Omega_t) = (1+r)b_t. \]

expression (6) represents a general solution to (3). The \( b_t \) term in (6) is the source of departures from the fundamental-value relation. Under the bubble hypothesis \( b_t \) is strictly positive so long as land is freely disposable (Blanchard and Watson). If land prices exhibit excessive volatility in the present model, the cause is \( b_t \) which we refer to following normal conventions as the bubble term.

A clear distinction should be drawn between rational bubbles and excessive returns to farmland. Rational bubbles result from the expectations regarding land prices. The rationale behind their existence is that they will affect the price of land because individuals believe so; they are a kind of self-fulfilling expectations. Excess returns, on the other hand, imply that other factors, not included in the present model, affect land prices and they lead to a divergence between \( q_t \) and \( q_t^* \). Possible explanations for excess returns abound (imperfect capital markets, the presence of risk aversion,
etc). Our primary focus is on detecting rational bubbles. However, in cases where no rational bubble is present we also examine the possibility of excess returns.

II. An Empirical Model

Rewrite (6) as,

\[ q_t = \sum_{i=0}^{\infty} (1+r)^{-i} E(\Pi_{t+1} | \Omega_t) + b_t. \]

If expectations are rational and the fundamental-value completely describes \( q_t \) (b does not exist), the stochastic behavior of \( q_t \) is determined by that of \( \Pi_t \) (subject to a minor restriction on the covariance structure of \( \pi_t \)). So if \( \Pi_t \) is integrated of order \( d \), denoted \( \Pi_t \) is I(d), \( q_t \) should also be I(d).

Suppose, however, that \( \Pi_t \) is I(0) but \( q_t \) is I(1). Because \( \Pi_t \) is stationary but \( q_t \) is explosive, then under the bubble hypothesis a term like \( b_t \) which is explosive must exist. One can then conclude that rational bubbles are present. In general if \( \Pi_t \) is I(d), but \( q_t \) is I(b) with \( b > d \), rational bubbles are present. Formally, this test follows from Proposition 2 in Hamilton and Whiteman.¹

Suppose that \( q_t \) and \( \Pi_t \) have the same order of integration, say I(1). Can anything be concluded about the presence or absence of bubbles? Here explosive behavior by \( q_t \) does not necessarily imply the presence of a bubble because \( \Pi_t \) is also explosive. Proposition 2 of Hamilton and Whiteman is no longer approach. An approach to test for bubbles in this case is to calculate the spread between the price of land and the quasi-rent divided by \( r \) of

\[ S_t = q_t - r^{-1}\Pi_t, \]

and testing for cointegration between \( q_t \) and \( \Pi_t \) where \( r^{-1} \) is the cointegration
parameter. Because $S_t$ is now a linear combination of I(1) variables, normally it would be expected to be I(1). But if the fundamental-value formulation is correct the opportunity cost of a dollar invested in land and quasi-rent converge in the long run implying that $S_t$ is I(0). If $S_t$ is not I(0), the postulated relationship between $q_t$ and $H_t$ does not exist. Any explosive behavior of $S_t$ again must be due to a term like $b_t$ implying that bubbles are present.

These test results are summarized by **Bubble Test Procedure**: Let $q_t$ be I(b), $H_t$ be I(d) with $b > d > 0$ or $b = d > 0$ but $S_t$ is not I(0). Then under the current assumptions rational bubbles are present in the data.

Now suppose that $S_t$ is I(0), either because prices and quasi-rent are I(0) or because of cointegration. Rational bubbles are ruled out. But excess returns may still exist. In other words, land prices may still exceed their fundamental value. To test for excess returns when both $q_t$ and $H_t$ are I(1), the price of land is calculated using the full set of restrictions implied by the fundamental-value model. We follow Campbell and Shiller's (1987) methodology. Consider the VAR representation of $q_t$ and $H_t$,

\begin{equation}
\mathbf{x}_t = A\mathbf{x}_{t-1} + \mathbf{v}_t
\end{equation}

where $\mathbf{x}_t$ contains current and lagged values of $q_t$ and $H_t$ and $\mathbf{v}_t$ is white noise. Expression (9) implies that $E(\mathbf{x}_{t+i}|\Psi_t) = \Lambda^i \mathbf{x}_t$, for all $i$ where $\Psi_t \equiv (q_{t-1}, H_{t-1}, \ldots, 0)$ denotes the limited information set, subset of $\Omega_t$. Projecting (4) onto $\Psi_t$ gives

\begin{equation}
q_t = \sum_{i=1}^{\infty} (1+r)^{-i} e' \Lambda^i \mathbf{x}_t.
\end{equation}

e is a (1 X 2p) vector with unity in the first element and zeros elsewhere. Taking the infinite sum,
\[ E(q_t^* | \psi_t) = q'_t = e'(1+r)^{-1}A(I - (1+r)^{-1}A)^{-1}x_t. \]

$q'_t$ is the price of land given the restrictions imposed by the fundamental-value model while $I$ is an identity matrix of dimension $2p$. To test for excess returns $q'_t$ is regressed on $q_t$; if excess returns are absent that regression produces a zero intercept term and a unity slope coefficient. Lastly, if $S_t$ is $I(0)$ because both $q_t$ and $\Pi_t$ are $I(d)$ and cointegrated, excess returns can be detected by following a similar methodology using $S_t$ and $\Delta^k q_t$ or $S_t$ and $\Delta^k \Pi_t$ in the VAR system. (Here $\Delta^k$ is the $k^{th}$ differences operator.) These two $I(0)$ variables fully summarize the bivariate history of $q_t$ and $\Pi_t$.

III. Data, Estimation, and Results

Data used in this study are for the 48 contiguous U.S. states and cover the 1950-86 time period. Land prices were obtained from Farm Real Estate: Historical Series Data, 1950-1985; net returns to land were obtained from Farm Income Data: A Historical Perspective, 1950-1984. Both series were adjusted to 1982 dollars using the GNP implicit price deflator. To conserve space results are only presented for representative states from six different regions. The six states are: New York, Georgia, Ohio, Kansas, Texas, and California. Results on all 48 states are available from the authors on request.

Before analyzing the results, several practical issues must be resolved. The first concerns the role of the time trend. Inclusion or exclusion of a time-trend term can affect even qualitative econometric results. For example, Kang (1985) shows that inclusion of a time trend can change the outcome of causality tests. Therefore, prices and quasi-rent were regressed against
time. For all 48 states a significant positive effect of time on price and a significant negative effect on quasi-rent was found. Because of this strong relationship all stationarity tests were carried out both with and without a time trend.

Measuring $r$ is the next concern. For the stationarity tests $r$ is calculated as $\beta^{-1}$ from: $q_t = \beta \pi_t + \epsilon_t$. This is the cointegration regression when both $q_t$ and $\pi_t$ are $I(d)$, $d > 0$. To confirm the validity of this estimate of $r$, two other methods were also used; first, define $r = \Pi_t / \Pi_t$, where $\Pi_t = T^{-1} \sum_{i=1}^{T} q_t$ and $\Pi_t = T^{-1} \sum_{i=1}^{T} \pi_t$; and second, following West (1987), estimate (3). This is done by adding and subtracting $(1+r)^{-1} (q_{t+1} + \pi_{t+1})$ from the right hand side of (3) leaving

$$q_t = (1+r)^{-1} (q_{t+1} + \pi_{t+1}) + \epsilon_{t+1},$$

where $\epsilon_{t+1} = (1+r)^{-1} [E(q_{t+1} | \Omega_t) + E(\pi_{t+1} | \Omega_t) - q_{t+1} - \pi_{t+1}]$ denotes the forecast error. An instrumental variables estimator yields $\hat{\Pi}_{t+1}$ and $\hat{q}_{t+1}$ using the information set $\Psi_t$. A consistent estimate of $r$ can be obtained by regressing $q_t$ on $\hat{\Pi}_{t+1}$ and $\hat{q}_{t+1}$. Although $r$ and the $S_t$ are only needed to test for cases where both $q_t$ and $\pi_t$ are $I(d)$, they are of interest on their own right. Therefore, $r$ and $S_t$ were calculated for all states regardless of the order of integration. Table 1 reports estimates of $r$ for the six states obtained using all three methods. In general, the estimates of $r$ are fairly insensitive with respect to the method of calculation even though there are larger differences between states.

The last issue is the actual stationarity test. The most popular tests are: the Dickey-Fuller (DF), the augmenting Dickey-Fuller (ADF), and the Durbin-Watson (DW). The DF test is based on the regression: $\Delta X_t = \mu + \beta X_{t-1} + \epsilon_t$, where $X_t$ is the variable being tested. The null hypothesis ($H_0$) is: $X_t$
is not I(0). $H_0$ is rejected if the estimate of $\beta$ is negative and significantly different from zero. The ADF test is based on a similar regression:

$$\Delta X_t = \mu + \beta X_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta X_{t-1} + \varepsilon_t,$$

where $p$ is selected so that $\varepsilon_t$ is white noise. Again $H_0$ is rejected if the estimate of $\beta$ is negative and significantly different from zero. The Akaike information criterion is used to determine $p$. Finally, the DW test is the Durbin-Watson statistic from the following regression: $X_t = \mu + \varepsilon_t$. Engle and Granger (1987) report critical values for these tests derived through Monte-Carlo experiments.

Table 2 reports the stationarity tests for the various states. Overall, the results indicate some sensitivity to the type of test and to whether a time trend was included in the regressions. Here, only results without time trend are reported.

Land prices typically are either I(1) or I(2). Returns to land, on the other hand, are typically either I(0) or I(1). Generally a higher order of differencing is required to achieve stationarity for land prices than returns to land. Therefore, we conclude that the empirical evidence supports the hypothesis of rational bubbles in the U.S. land market.

Specifically in terms of Table 2: using the ADF test indicates that New York land prices are I(2) while return to land are I(1). Our bubble test procedure indicates the presence of bubbles. However, both the DF and DW tests suggest that both land prices and returns to land are I(1). These tests do not imply the presence of rational bubbles, nor do they imply the absence of rational bubbles. We next tested for stationarity of the New York spread $S_t$ and determined that $S_t$ is I(1) which again implies the presence of bubbles.

For Kansas, the DW, DF, and ADF all indicate that returns to land are I(0). But the DW, DF, and ADF all imply that Kansas land prices are I(2).
Under the present model we attribute this excess volatility in Kansas land prices to the presence of rational bubbles.

In Georgia, all tests imply that returns to land are I(0). The DW test indicates land prices are I(1) while both the ADF and DF suggest that Georgia land prices are I(2). All these tests therefore indicate excessive volatility in land prices and we do not reject the rational bubble hypothesis.

Returns to land in Texas are also I(0) using all these tests. The DW and DF tests indicate that land prices are I(1) while the ADF suggests land prices are I(2). Again excess volatility is detected and we attribute these to the presence of a rational bubble. Texas is the one state, however, where the inclusion of a time term makes a significant difference in the results. When a time trend is included both $q_t$ and $\pi_t$ are found to be I(0). So the hypothesis of bubbles are rejected. The calculated spread, $S_t$, is also I(0) when the time trend is included. Again the bubble hypothesis must be rejected. As a first step $q_t^1$ was calculated according to (16) and then regressed on $q_t$. The null hypothesis that the resulting slope coefficient equals one was rejected and we conclude that while the Texas land market does not exhibit excessive volatility (with a time trend) excess returns were present over the sample period.

Using the DF and ADF tests indicates that Ohio returns to land are I(1) while Ohio land prices are I(2). These tests, therefore, support the presence of rational bubbles in the Ohio land market.

All tests indicate that California land prices are I(2) while California land returns are I(1). We again conclude that a rational bubble was present in the California land market.
IV. Concluding Comments

This paper develops a methodology to detect rational bubbles in agricultural land markets based on results in Hamilton and Whiteman and Campbell and Shiller. The test was applied to land price data for the 48 contiguous U.S. states. Although some differences exist among states, excessive volatility which we attribute to rational bubbles typified agricultural land price fluctuations during the 1950-1986 period. Land prices virtually always require a higher degree of differencing to induce stationarity than returns.
ENDNOTES

1. However, as Burmeister, Flood, and Garber and Hamilton and Whiteman point out a variety of unobserved component models are consistent with the stochastic behavior attributed to bubbles in the present paper.
Table 1: The Discount Rate for Six States.

<table>
<thead>
<tr>
<th>State</th>
<th>Method A</th>
<th>Method B</th>
<th>Method C</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>0.0953</td>
<td>0.1098</td>
<td>0.1025</td>
</tr>
<tr>
<td>Georgia</td>
<td>0.0869</td>
<td>0.0953</td>
<td>0.1050</td>
</tr>
<tr>
<td>Ohio</td>
<td>0.0548</td>
<td>0.0661</td>
<td>0.0571</td>
</tr>
<tr>
<td>Kansas</td>
<td>0.0559</td>
<td>0.0757</td>
<td>0.0548</td>
</tr>
<tr>
<td>Texas</td>
<td>0.0515</td>
<td>0.0515</td>
<td>0.0582</td>
</tr>
<tr>
<td>California</td>
<td>0.0709</td>
<td>0.0834</td>
<td>0.0893</td>
</tr>
</tbody>
</table>

NOTE: Method A, calculates $r$ from the ratio: $\overline{\pi}_t / \overline{q}_t$. Method B gives the estimate of the cointegration regression. Method C, gives the estimate of $(3')$. 
Table 2: Stationarity Tests for Six States (without time trend).

<table>
<thead>
<tr>
<th>Variable</th>
<th>DW</th>
<th>DF</th>
<th>ADF</th>
<th>DW</th>
<th>DF</th>
<th>ADF</th>
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<tbody>
<tr>
<td></td>
<td>New York</td>
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<td></td>
<td>Kansas</td>
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<td></td>
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<tr>
<td>$q_t$</td>
<td>0.028</td>
<td>-1.27</td>
<td>-1.28</td>
<td>0.088</td>
<td>-1.15</td>
<td>-1.81</td>
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<tr>
<td>$\Pi_t$</td>
<td>0.415</td>
<td>-2.14</td>
<td>-2.16</td>
<td>0.796</td>
<td>-3.20</td>
<td>-2.92</td>
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<tr>
<td>$\Delta q_t$</td>
<td>1.416</td>
<td>-4.25</td>
<td>-2.08</td>
<td>0.653</td>
<td>-2.07</td>
<td>-0.47</td>
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<tr>
<td>$\Delta \Pi_t$</td>
<td>2.230</td>
<td>-6.72</td>
<td>-4.68</td>
<td>2.430</td>
<td>-7.29</td>
<td>-4.32</td>
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<tr>
<td>$\Delta^2 q_t$</td>
<td>3.035</td>
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<td>2.584</td>
<td>-7.86</td>
<td>-5.19</td>
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<tr>
<td>$\Delta^2 \Pi_t$</td>
<td>3.066</td>
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<td>-8.13</td>
<td>3.173</td>
<td>-11.67</td>
<td>-7.20</td>
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<tr>
<td>$S_t$</td>
<td>0.133</td>
<td>-1.46</td>
<td>-1.64</td>
<td>0.415</td>
<td>-2.33</td>
<td>-2.55</td>
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<table>
<thead>
<tr>
<th></th>
<th>Georgia</th>
<th>Texas</th>
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<tr>
<td>$q_t$</td>
<td>0.024</td>
<td>0.060</td>
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<td>$\Pi_t$</td>
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<td>$\Delta q_t$</td>
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<td>$\Delta \Pi_t$</td>
<td>2.891</td>
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<td>-8.09</td>
</tr>
<tr>
<td>$\Delta^2 \Pi_t$</td>
<td>3.245</td>
<td>3.223</td>
<td>-11.88</td>
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<tr>
<td>$S_t$</td>
<td>0.390</td>
<td>0.417</td>
<td>-2.48</td>
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<table>
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<tr>
<th></th>
<th>Ohio</th>
<th>California</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_t$</td>
<td>0.090</td>
<td>0.085</td>
<td>-1.78</td>
</tr>
<tr>
<td>$\Pi_t$</td>
<td>0.568</td>
<td>0.570</td>
<td>-2.33</td>
</tr>
<tr>
<td>$\Delta q_t$</td>
<td>0.779</td>
<td>0.476</td>
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<tr>
<td>$\Delta \Pi_t$</td>
<td>2.556</td>
<td>1.563</td>
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<tr>
<td>$\Delta^2 q_t$</td>
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<tr>
<td>$\Delta^2 \Pi_t$</td>
<td>3.319</td>
<td>2.272</td>
<td>-6.64</td>
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<tr>
<td>$S_t$</td>
<td>0.194</td>
<td>0.320</td>
<td>-2.03</td>
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NOTE: Critical values for cointegration are (relevant for $S_t$, New York): $DW = 0.386$, $DF = -3.37$, $ADF = -3.17$ ($\xi_1$, $\xi_2$, $\xi_3$ statistics in Engle and Granger). Critical Values for Stationarity are: $DW = 0.770$, (from Sargan and Bhargava) and for $DF$ and $ADF$ is $-3.00$ (Dickey, $\tau_r$ statistic). The level of significance is 5%.
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