OPTIMAL-REVENUE TARIFFS AND THE UNIFORM TARIFF

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For small countries, import tariffs distort domestic production patterns resulting in a deadweight loss. And yet tariffs are often an irreplaceable source of government revenue. In fact, Greenaway's (1984) empirical analysis suggests that the less developed the country, the more important tariffs are as a source of revenue. These countries' revenue needs make a complete tariff abolition in the developing world unlikely no matter how convincingly economists demonstrate the benefits of free trade.

With complete tariff abolition seemingly infeasible politically, the tariff debate more appropriately focuses on tariff design and reform. Faced with a complicated Australian system of tariff and nontariff trade barriers, Corden (1958) early advocated a uniform tariff structure as a means for "cutting through the jungle with the price mechanism." The desirability of a uniform tariff structure continues to be debated (Harberger, 1988). One particularly contentious issue is the desirability of achieving uniformity by imposing new tariffs on imported intermediates. Tariffs on imported intermediates engender production inefficiencies. But general optimal-taxation principles (Mirrlees, 1987) indicate that production efficiency should be preserved.

Unfortunately what optimal tax theory prescribes and what can be achieved are not always the same thing. As Deaton (1981) points out, calculating accurate optimal tax rates empirically is probably beyond current econometric capabilities. And, in any case, domestic commodity taxation may be impossible because many developing countries may not have the machinery (or cannot acquire it at a reasonable cost) to collect taxes. Consequently, tariffs are often the only feasible way to raise government
revenues. (See Corden (1971, 1974) and Shalizi and Squire (1986)).

Any case for or against a uniform tariff structure, therefore, should be judged squarely within the context where it makes the most sense: a world where there is extremely limited flexibility to use other tax instruments. The optimality of a uniform tariff structure is not a Ramsey-Diamond-Mirrlees optimal tax problem but rather an optimal-revenue tariff problem in the sense of Vanek (1971). This paper explores whether an optimal-revenue tariff structure should be uniform when imported intermediate goods exist. That this question is relevant is best attested by the fact that the World Bank frequently recommends a move toward uniformity even when this implies raising tariffs on intermediates (Shalizi and Squire).

Three separate cases are considered: the optimal-revenue tariff for a small economy with importable, exportable, and intermediate goods; the optimal-revenue tariff for a small economy with an importable, exportable, intermediate, and nontraded final goods; and the optimal-revenue tariff for a small open economy where all tariff revenue goes to purchase a nontraded public good. Sufficient conditions for the optimality of a uniform tariff structure are deduced in each case. These involve the quasi-separability of various forms of the net expenditure function.

The Model

Assume a small open economy which takes prices of all traded commodities as given.

The Technology

The economy's technology is characterized by the production possibilities set

\[ Y = \{(z,y) : z \text{ can produce } y\}. \]
Here \( z \in \mathbb{R}^n \) is a vector of nontraded (across international borders) factors of production which are in fixed supply domestically, \( y \in \mathbb{R}^4 \) is a vector of net outputs including an importable final good \( (y_m) \), an exportable final good \( (y_x) \), a nontraded final good \( (y_n) \), and an importable intermediate good \( (y_i) \). If, as assumed below, the country is a net importer of the intermediate good, \( y_i < 0 \). \( Y \) is nonempty, closed and convex. It satisfies free disposability of \( y \) and free disposability of \( z \) (Chambers, 1988) and is bounded from above for every finite \( z \).

Entrepreneurs are rational profit maximizers so that the technology is fully characterized by the revenue (gross national product) function dual to \( Y \)

\[
R(p, z) = \text{Max} \{ py : (x,y) \in Y \}.
\]

Here \( p \in \mathbb{R}^4_{++} \) is a vector of net output prices. Subscripts on \( p \) follow the same convention as subscripts on \( y \). \( R(p, z) \) is a positively linearly homogeneous and convex function of \( p \). It is also nondecreasing and concave in the elements of \( x \). If \( Y \) is conical (satisfies constant returns to scale), \( R \) is positively linearly homogeneous in \( x \). Finally, if an unique solution to the revenue maximization problem exists \( R \) is differentiable and satisfies the Samuelson-McFadden lemma (Chambers, 1988):

\[
R_j(p, z) = y_j(p, z) \quad j = m,x,n,i
\]

where \( R_j(p, z) \) is the partial derivative of the revenue function with respect to \( p_j \) and \( y_j(p, z) \) is the economy's net output of \( j \). In what follows \( R \) is assumed to be at least twice differentiable.
Preferences

Individual preferences are quasi-homothetic so that a societal expenditure function \( E(p, u) \) exists (Gorman, 1953; Muellbauer, 1976). Here \( u \) is the level of societal welfare and \( p \in \mathbb{R}^3_+ \) (with a slight abuse of notation) is a three dimensional vector containing the price of the exportable, the price of the importable, and the price of the nontraded final good. (This specification is changed when a nontraded public good is introduced.) The intermediate traded good is not a consumption item.

\( E(p, u) \) is a positively linearly homogeneous, nondecreasing, and concave function of prices and is assumed to satisfy Shephard's lemma so that its derivatives with respect to commodity prices are the Hicksian compensated demands. \( E(p, u) \) is also nondecreasing in \( u \). Partial derivatives follow the same notational conventions used for the revenue function except that \( E_u(p, u) \) represents the partial derivative of \( E \) with respect to utility. By usual results from consumer theory it is the reciprocal of the marginal utility of income. \( E(p, u) \) is at least twice differentiable.

The net expenditure function \( N(p, u, z) \) is defined

\[
N(p, u, z) = E(p, u) - R(p, z).
\]

\( N(p, u, z) \) is positively linearly homogeneous and concave in prices, nondecreasing in \( u \), and nonincreasing in \( z \).

Government Intervention

The government only intervenes through the price mechanism. The price of good \( j \) in the world market is denoted by \( p_j^* \) \((j = m, x, i)\) and the specific tariff for good \( j \) is denoted by \( t_j \) \((j = m, i)\). Hence, domestic prices are related to international prices according to

\[
p_j = p_j^* + t_j \quad (j = m, i).
\]
The ad valorem tariff rate \( \frac{t_j}{p_j^*} \) is not used in what follows. However, the tariff expressed as a percentage of the domestic price is used frequently so that it is convenient to introduce the notation

\[
\tau_j = \frac{t_j}{p_j}.
\]

Notice, in particular, that

\[
\frac{\tau_j}{(1 - \tau_j)} = \frac{t_j}{p_j^*}.
\]

If ad valorem rates across commodities are constant so are the \( \tau_j \).

Throughout, it is assumed that there are no export taxes.

**Equilibrium**

To start assume that either the government has the same preferences as consumers or that it redistributes tariff income in a lump-sum fashion to consumers. (This assumption is modified when the public good is introduced.) In this case the equilibrium conditions are

\[
E(p,u) = R(p,z) + t \sum_m (E_m - R_m) - t \sum_i R_i
\]

\( E_n = R_n \).

The first of these equations says that the country meets its budget constraint: consumer expenditures on commodities equals producer income plus tariff revenue. The second equation is the market-clearing condition for the nontraded goods market. These equations determine the equilibrium levels of \( u \) and \( p_n \).

**The Optimal-Revenue Tariff -- No Nontraded Final Goods**

The government's problem is to choose a tariff structure which simultaneously meets the material balance condition and achieves a given tariff revenue of, say, \( T \), while making welfare as large as possible. Formally,

\[
\text{Max } \{ u: E(p,u) = R(p,z) + t \sum_m (E_m - R_m) - t \sum_i R_i, \ T = t \sum_m (E_m - R_m) - t \sum_i R_i \}.
\]
Here the controls are $u_m$, $t_m$, and $t_1$. The corresponding Lagrangian expression is

\begin{equation}
L = u + \mu \left( E - R - t_m (E_m - R_m) + t_{11} R_{11} \right) + \lambda \left( T - t_m (E_m - R_m) + t_{11} R_{11} \right)
\end{equation}

where $\mu$ and $\lambda$ are Lagrangian multipliers.

The first-order conditions for (1) do not generally provide either necessary or sufficient conditions for an optimum (Mirrlees, 1987). However, we follow most of the literature on this subject and presume that these first-order conditions are necessary for an optimum. Thus, as with Deaton (1981), our results are most appropriately interpreted as sufficient conditions for conventional formulae used to calculate optimal revenue-raising tariff rates to imply a uniform tariff structure.

The first-order conditions for (1) are:

\begin{equation}
\frac{\partial L}{\partial u} = 1 + \mu \left[ E_u - t_m E_{mu} \right] - \lambda t_m E_{mu} = 0
\end{equation}

\begin{equation}
\frac{\partial L}{\partial t_m} = \mu \left[ -t_m (E_m - R_m) + t_{11} R_{11} \right] + \lambda \left[ -(E_m - R_m) - t_m (E_m - R_m) + t_{11} R_{11} \right]
\end{equation}

\begin{equation}
\frac{\partial L}{\partial t_1} = \mu \left[ t_m R_{m} + t_{11} R_{11} \right] + \lambda \left[ t_m R_{m} + R_{1} + t_{11} R_{11} \right] = 0
\end{equation}

After a little manipulation the first-order conditions for $t_m$ and $t_1$ can be written

\begin{equation}
\frac{\partial L}{\partial t_1} = \mu \left( \tau_{mp} R_{m1} + \tau_{p} R_{11} \right) + \lambda \left( \tau_{mp} R_{m1} + \tau_{p} R_{11} + R_{1} \right) = 0
\end{equation}

\begin{equation}
\frac{\partial L}{\partial t_m} = \mu \left( -\tau_{mp} (E_m - R_m) + \tau_{p} R_{11} \right) + \lambda \left( -(E_m - R_m) - \tau_{mp} (E_m - R_m) + \tau_{p} R_{11} \right) = 0.
\end{equation}
Because our main interest is in deducing cases where (2) are consistent with a uniform tariff we shall not stop here to develop the intuitive content of (2). This would constitute a repetition of fairly well-known optimal tax results as applied to the optimal-revenue tariff problem. Interested readers should refer to either Feehan (1988), Heady and Mitra (1987), or Devarajan et al. (1986) for a good discussion.

For a uniform tariff structure \((\tau_1 = \tau_m = \tau)\) to be optimal the expressions in (2) must equal zero when evaluated at \(\tau\). Using the symmetry \((E_{ij} = E_{ji}; R_{ij} = R_{ji})\) and homogeneity properties of revenue and cost functions then gives

\[
\begin{align*}
\frac{p_x R_{1x}}{-R_1} &= -\frac{\lambda}{(\mu \tau + \lambda \tau)} \\
\frac{p_x (E_{mx} - R_{mx})}{(E_m - R_m)} &= \frac{\lambda}{(\mu \tau + \lambda \tau)}
\end{align*}
\]

A uniform tariff structure is optimal if, when evaluated at the uniform rate \(\tau\), the elasticity of demand for intermediates with respect to the price of exportables equals the export-price elasticity of excess demand for the importable final commodity. Although, equations (3) can be satisfied for some preferences and technologies, they will not generally hold for all families of preferences and technologies. To see why consider the simple example where both of these elasticities are constant but different numbers. Equations (6) can never be satisfied in this case.

At least one family of net expenditure functions exists for which (6) is always satisfied: those having the prices of all importables (the intermediate and the importable final good) quasi-separable from the price of exportables

\[
N(p, u, z) = N^*\left(\phi(p_m, p_i, u, z), u, z, p_x\right).
\]
The reader can easily verify computationally that net expenditure functions of this type always satisfy (3) regardless of price and fixed input levels. The common elasticity of excess demand for importables and intermediates with respect to $p_x$ is

$$N_{\phi x}^* \frac{p_x}{N_{\phi x}^*}.$$

To see why (4) is implied by (3) notice first that (3) is satisfied for all prices if

$$-\frac{R_{Ix}}{R_i} - \frac{p_x (E_{mx} - R_{mx})}{(E_m - R_m)} = 0.$$

So long as excess demand for both intermediates and importables is nonzero, this last expression implies the following condition is always satisfied

$$p_x \frac{\partial}{\partial p_x} \left( \frac{-R_i}{(E_m - R_m)} \right) = 0.$$

Shephard's lemma and the Samuelson-McFadden lemma together with this last expression imply the separability of $p_m$ and $p_i$ from $p_x$ in $N(p,u,z)$.

Expression (7) represents the natural extension of Deaton's (1981) result on quasi-separable expenditure functions for uniform commodity taxation to the current case. Because $N(p,u,z)$ reflects both the technology and preferences the optimality of uniform tariffs involves different restrictions than Deaton's result. Both preferences and the technology are involved and not just preferences as is true for optimal commodity taxes.

To illustrate the need for assumptions on both the structure of preferences and the technology to insure the optimality of uniform tariffs consider the special case where the expenditure function is of the homothetic Cobb-Douglas form

$$E(p,u) = H(u)p_x^{\alpha}p_m^{1-\alpha}$$

$\alpha > 0$. 
The reader can easily verify that the associated $N(p,u,z)$ is not quasi-separable for arbitrary $R$ structures. But the Cobb-Douglas in (5) and its extension to an arbitrary number of commodities is additively separable and thus consistent with quasi-separability of consumer preferences. Hence, quasi-separability of $E(p,u)$ is not sufficient to guarantee the optimality of a uniform tariff. Because tariffs impinge on both producers and consumers, restrictions on $Y$ are needed as well.

The reader is also cautioned that (4) places a restriction on Hicks-compensated excess demands and not upon the Marshallian excess demands themselves. In particular, (4) requires more than that the ratio of uncompensated excess demands for importables are independent of the price of exportables. Finally, it must be emphasized again that (4) has been derived as a sufficient condition for the optimality of a uniform tariff structure and not as a necessary condition. Hence, departures from (4) which are consistent at least locally with a uniform tariff can, in principle, occur. However, it would seem clear intuitively that forms departing from (4) will not be consistent with having uniform tariffs optimal for a wide range of prices.

The Optimal-Revenue Tariff -- Nontraded Final Goods Present

The results obtained without nontraded final goods and their similarity to Deaton's results suggests that generally a uniform tariff structure is optimal when the ratios of compensated excess demands for the importable final and intermediate goods are independent of $p_x$ and $p_n$. Both Deaton's result and result (4) require that the ratio of compensated demands for taxed (tariffed) commodities be independent of the prices of nontaxed (nontariffed) commodities. Ramsey tax rules are frequently interpreted in
terms of complementarity with a nontaxed good (Harberger). Goods which are relatively close complements with the nontaxed goods should be taxed at a higher rate than those which are not. Hence, by extension the optimality of uniform tariffs hinges upon net substitution relationships between importables and exportables and intermediates and exportables. If both these substitution relationships are the same (quasi-separability insures this holds) tariff uniformity is optimal. On the other hand, if the excess demand ratios for importables and intermediates are sensitive to the prices of nontaxed commodities (nontraded final goods and exportables), moving to tariff uniformity institutionalizes a wedge between the price of nontaxed commodities and the intermediate good. In general this is inefficient.

The theory of the second best indicates that the marginal efficiency losses arising thereby should be balanced against the marginal tax revenue gains that result from the manipulation of this wedge. A priori there is no reason why these gains and losses should balance where all ad valorem rates are equal.

To see that this intuition is correct formally, consider the optimal tariff problem as reformulated to account for the presence of nontraded final goods:

$$\text{Max} \left\{ u: E(p,u) = R(p,z) + t_m (E_m - R_m) - t_i R_i; \quad T = t_m (E_m - R_m) - t_i R_i; \quad E_n = R_n \right\}.$$ 

The controls are $t_m, t_i, p_n,$ and $u$. The corresponding Lagrangian expression is
In (6), \( \lambda, \mu, \) and \( \gamma \) are Lagrangian multipliers. After some manipulation, expression (6) yields the following first-order conditions for \( t_m \) and \( t_1 \):

\[
\frac{\partial L}{\partial t_1} = \mu \left( \tau_m p R + \tau_p R \right) + \lambda \left( \tau_m p R + \tau_p R \right) - \gamma R = 0
\]

\[
\frac{\partial L}{\partial t_m} = \mu \left( -\tau_m p (E - R) + \tau_p R \right) + \lambda \left( -\tau_m p (E - R) + \tau_p R \right) - \gamma R = 0.
\]

Under the symmetry and homogeneity properties of the revenue and expenditure functions a uniform-tariff structure \( (\tau_1 = \tau_m = \tau) \) is optimal only if the following condition derived from (7) is satisfied:

\[
(\mu + \lambda \tau) \left( \frac{p R}{R_1} + \frac{p_n R}{R_1} \right) - \gamma R = (\mu + \lambda \tau) \left( \frac{p_x (E - R)}{(E - R)} + \frac{p_n (E - R)}{(E - R)} \right)
\]

\[
+ \frac{\gamma (E - R)}{(E - R)}.
\]

A sufficient condition for (8) to hold is that the ratio of compensated excess demands for the importable and the intermediate be independent of both the price of exportables and the price of nontradable final goods. So the extension of (4) to the case of nontradable final goods is that the net expenditure function should have the price of the importable and the intermediate commodities quasi-separable from the prices of the nontraded and exportable goods. Symbolically,

\[
N(p, u, z) = H(\psi(p_m, p_1, z, u), z, u, p_x, p_n)
\]
The Optimal-Revenue Tariff -- Tariff Revenue used to purchase Nontraded Public Good

Feehan (1988) recently deduced the characteristics of the optimal-revenue tariff for the case where the tariff revenue's sole purpose is to finance the purchase by the government of a nontraded, public good. To study the optimality of a uniform tariff structure in this context, it is necessary to introduce some further notation and definitions which recognize explicitly that the nontraded, public good is not purchased directly by consumers. Let $U: \mathbb{R}^3 \to \mathbb{R}$ denote the community utility function dual to $E(p,u)$. By standard duality results $U$ is quasi-concave and nondecreasing in its arguments.

In what follows, the nontraded good is treated as a public good. Therefore, $E(p,u)$ is inappropriate to our current purposes because it presumes all commodities are freely traded at fixed prices. Define the public-good expenditure function as

$$E(p, u, c) = \min \{ p_c x + p_m c : U(c_x, c_m, c_n) \geq u \}.$$

Here $c_j$ represents consumption of commodity $j$ and (again with an abuse of notation) $p \in \mathbb{R}^2_+$ is the two-dimensional price vector containing the price of the exportable and the price of the importable. Because consumers do not purchase the nontraded public good its price is irrelevant to them.

We continue to assume, however, that the public good is produced commercially in competition with the other net outputs (importables, exportables, and intermediates). Hence, the original revenue function remains appropriate. Because the public good is not traded across borders all of its production must be consumed domestically so that $c_n = R_n(p, z)$. Moreover, because the government uses all its tariff revenue to purchase
the nontraded public good, tariff revenue equals the value of domestic production of the public good,

\[ pR = t \left( E - R \right) - t R. \]

Note that \( E \) now refers to the derivative of the public good expenditure function, \( E(p,u,c) \).

Parallel to the net expenditure function discussed earlier we define the public-good, net expenditure function as

\[ N(p,u,z,c) = E(p,u,c) - R(p,z). \]

With these refinements in hand, it is now possible to state the appropriate optimal-revenue tariff problem after substituting \( R \) for \( c \) in \( E(p,u,c) \) as

\[ \text{Max} \left\{ u: E(p,u,R) = R(p,z); \quad pR = t \left( E - R \right) - t R \right\}. \]

The controls are \( u, t_m, t_i, \) and \( p_n \). The Lagrangian is

\[ L = u + \mu \left[ E(p,u,R) - R(p,z) \right] + \lambda \left[ pR - t \left( E - R \right) + tR \right] \]

where \( \mu \) and \( \lambda \) are Lagrangian multipliers. First-order conditions for \( t_i \) and \( t_m \) require

\[ \frac{\partial L}{\partial t_m} = \mu \left[ E + E R - R \right] \]

\[ \quad + \lambda \left[ pR - (E - R) - t \left( E - R + E R \right) + tR \right] = 0 \]

\[ \frac{\partial L}{\partial t_i} = \mu \left[ E R - R \right] + \lambda \left[ pR - t \left( E - R + E R \right) + R + tR \right] = 0. \]
Here, for example, \( E_n = \frac{\partial E(p,u,c)}{\partial c_n} \). These first-order conditions after a slight manipulation require

\[
\frac{\partial L}{\partial t_m} = \mu \left[ E_{n\,nm} + (E_n - R_m) \right] \\
+ \lambda \left[ p_{n\,nm} - (E_n - R_m) - p_{m\,a\,a} (E_m - R_m) + E_{mn\,nm} + p_{t\,1\,1\,11} \right] = 0
\]

\[
\frac{\partial L}{\partial t_1} = \mu \left[ E_{n\,n1} - R_1 \right] + \lambda \left[ p_{n\,n1} - p_{m\,a\,m} (E_m - R_m) + E_{mn\,n1} + R_1 + p_{t\,1\,1\,11} \right] = 0
\]

Therefore, for a uniform tariff structure \( \tau \) to be optimal the following condition which is derived from the above equations must be fulfilled.

\[
- \mu E_n \frac{R_{n1}}{R_1} + \lambda \tau \left( p_x \frac{R_{1x}}{R_1} + p_{n\,ln} \frac{R_{ln}}{R_1} + p_{m\,mn\,ln} \right) - \lambda p_n \frac{R_{ln}}{R_1}
\]

\[
= \mu E_n \frac{R_{mn}}{E_m - R_m} + \lambda \tau \left( \frac{p_x (E_{mx} - R_{mx})}{E_m - R_m} - \frac{p_{n\,mn}}{E_m - R_m} - \frac{p_{m\,mn\,mn}}{E_m - R_m} \right)
\]

\[
+ \lambda p_n \frac{R_{mn}}{E_m - R_m}
\]

Inspecting this last expression reveals that a set of sufficient conditions for it to hold are that

\[
\frac{E_{mx} - R_{mx}}{E_m - R_m} - \frac{R_{1x}}{R_1} = 0
\]

\[
\frac{R_{mn}}{(E_m - R_m)} + \frac{R_{ln}}{R_1} = 0
\]

The reader can easily verify that the class of public-good net expenditure functions with the price of importables and intermediates quasi-separable from the price of nontradeables and exportables satisfies this last set of
elasticity conditions. Hence, if tariff revenue is devoted solely to the purchase of public goods which are nontraded the family of public-good expenditure functions given by

\[ N(p,u,z,c_n) = G(\sigma(p_m, p_i, c_n, z, u), p_x, p_n, u, z, c_n) \]

generates a uniform optimal-revenue tariff structure.

Conclusion

This paper has investigated the structure of the optimal-revenue tariff when there are imported intermediates and deduced sufficient conditions for the uniform tariff structure to be optimal. These conditions require the compensated, excess-demand elasticities for importables and intermediates with respect to the price of exportables and nontraded goods to be the same. In other words, the ratio of the compensated excess demand for importables to that of intermediates must be independent of the price of nontraded goods and exportables. These elasticity conditions are met for the entire price constellation if the appropriate version of the net expenditure function has the prices of exportables and nontraded goods quasi-separable from the prices of intermediates and importables.
References


