HOW SHOULD WE VALUE AGRICULTURAL INSURANCE CONTRACTS?

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Valuation of agricultural insurance contracts is an important issue in agricultural finance. More accurate valuation methods can help make private sector involvement in the agricultural insurance industry more efficient by diffusing information and reducing excess profits or losses on insurance programs. Perhaps more importantly, if the government is heavily involved in offering and supporting agricultural insurance, as in the U.S., better insurance valuation methods can help administrators make more economically sound choices when they set premiums for insurance contracts.

There are several existing approaches to valuing agricultural insurance contracts. The oldest, and probably still the most commonly used method in practical applications, is to value premiums at the present value of the expected indemnity on the insurance contract. This present value method is flexible and fairly straightforward computationally but requires a somewhat arbitrary assumption about what discount rate to use in order to compensate insurers appropriately for taking on the indemnity risk. More recently, option pricing models from the finance literature have been proposed as a way of overcoming this weakness and pricing the risks borne by agricultural insurance providers (Turvey and Amanor-Boadu 1989; Turvey 1992; Stokes, Nayda and English 1997; Yin and Turvey 2003; Stokes and Turvey 2003). The option pricing approach is motivated by the fact that the indemnity on an insurance contract is essentially equivalent to the payoff on a suitably defined put option, and so financial option valuation methods can be applied to the valuation of insurance contracts. The basic Black-Scholes option pricing model has been applied to agricultural insurance valuation (Turvey and Amanor-Boadu 1989; Turvey 1992), and more recently the arbitrage-based derivative asset pricing models for non-traded goods has been suggested as a better alternative (Yin and Turvey 2003; Stokes and Turvey 2003). A final approach that has been used to price indemnity risk in agricultural insurance models is the general equilibrium representative agent model of Lucas (see Cao and Wei 2004; and Richards, Manfredo and Sanders 2004). In this model insurance is priced to include an equilibrium risk premium that is just sufficient to ensure that no insurance will be purchased given the risk aversion level of the representative agent, usually assuming constant relative risk aversion preferences and log-normally distributed risks.

In this paper we argue that all of these existing methods for valuing agricultural insurance contracts have problems. The present value approach is not a true equilibrium model because the discount rate (equilibrium price of risk) is an arbitrary and undefined

parameter. Arbitrage based option pricing models and the Lucas representative agent model are true equilibrium models but rely on the assumption of complete markets. That is, these models assume either implicitly or explicitly that the insurance contract is a redundant asset whose returns can be replicated with a portfolio of other already existing financial assets, or in the case of the Lucas model that there will be no trade on insurance contracts even if they are engineered and introduced. For a variety of reasons that will be explained further below, we believe this complete markets assumption is particularly unsuitable for pricing agricultural insurance. We also argue that to make any real progress in developing improved valuation models for agricultural insurance there is a need to explicitly account for the fact that risk markets in agriculture are inherently incomplete, and that any agricultural insurance contract being valued will alter the non-diversifiable risk profile of a significant number of agents.

This paper has three main objectives. The first is to critique the major approaches being used currently to value agricultural insurance contracts, highlighting their strengths as well as their limitations and weaknesses. The second objective is to provide an example of an alternative valuation method that may help overcome some of the problems with existing methods by accounting explicitly for the incompleteness of the market structure for agricultural risks. The third objective is to present results from a simulation study designed to highlight how significant agricultural insurance pricing errors can be in various situations if the wrong valuation method is applied. The paper proceeds by addressing each of these objectives in turn.

A Critique of Commonly Used Methods For Pricing Agricultural Insurance

Several methods are currently available to price agricultural insurance contracts. In this section of the paper we outline some of the most frequently used methods and discuss their advantages and disadvantages. To keep things simple and concrete we focus the discussion on crop revenue insurance and begin with an outline of a basic crop revenue insurance model.

Let time be indexed by t and let:

Y_t = an insurable crop revenue index that is realized and observed at harvest period t when farm revenues are realized and any insurance indemnities are paid;

$h(Y_t)$ = the density function for Y_t conditional on information available at $t-1$;

G_t = the guaranteed level of the crop revenue index under the insurance contract;

$P_{t-1}(G_t)$ = the planting period value of an insurance contract written on Y_t with guarantee level G_t;

r_t = the risk free rate of interest from $t-1$ to t;

$\beta_t = 1/(1+r_t)$ is a the risk free discount factor corresponding to r_t;

γ_t = a loading factor reflecting a risk premium and insurance transaction costs from $t-1$ to t; and

E_t = expectation conditional on information available at time t.

The crop revenue insurance contract is a contingent claim that pays out at harvest time t the difference between a guaranteed level of the revenue index, G_t, and the realized harvest value of the revenue index, Y_t, but only when Y_t falls below G_t. The insurance indemnity can therefore be written as $\max(G_t - Y_t, 0)$ while the premium value is $P_{t-1}(G_t)$. We assume that the probability distribution $h(Y_t)$ is known by all participants and that insurers can observe farmer effort.[1] We now outline and discuss existing models for computing $P_{t-1}(G_t)$.

Present Value Models

One of the oldest, simplest, and still most commonly used methods for valuing insurance contracts is to price them at the present value of the expected indemnity on the contract. This implies:

$$\begin{aligned} P_{t-1}(G_t) &= (1+\gamma_t)\beta_t E_{t-1}\left[\max(G_t - Y_t, 0)\right] \\ &= (1+\gamma_t)\beta_t \int_0^{G_t} (G_t - Y_t) h(Y_t) dY_t. \end{aligned} \tag{1}$$

Notice that β_t discounts the expected indemnity back to the planting period at the risk-free rate, and γ_t is a loading factor that allows an additional return to insurers that compensates them for taking on any non-diversifiable risk incurred by holding the insurance contract and/or for the transactions costs of selling contracts and processing claims.[2] While γ_t is usually positive or zero it is possible that $\gamma_t < 0$ which would imply

[1] Alternatively, think of Y_t as area revenue so that individual farms have no control over the index.

[2] Some of the insurance literature models are static and so β_t is set to one. However, it would be inappropriate to ignore discounting in the case of agricultural insurance because of the significant time lapse often observed between taking out the insurance and paying the premium at planting time and receiving any indemnity at harvest.

that the premium does not cover the present value of expected indemnities (the insurance is being subsidized). This would normally occur only when the government is operating the insurance program or subsidizing private sector participation. Of course, if we set $\gamma_t = 0$ (no loading factor) then (1) gives the actuarially fair insurance contract value.

The present value model (1) has some significant advantages for pricing agricultural insurance. Perhaps the most important is that it is extremely flexible because the formula is straightforward to compute numerically for just about any specification of the underlying probability distribution $h(Y_t)$. The approach can therefore accommodate a wide range of probability distributions for the insurable revenue index, depending on what the data actually suggest about the distribution of Y_t. Furthermore, the actuarially fair present value formula (1) with $\gamma_t = 0$ can be rationalized as an equilibrium insurance premium in a competitive insurance market where insurers are risk-neutral and incur zero transaction costs (see Rothschild and Stiglitz 1976). In this case, competition under risk neutrality drives the insurance value to the actuarially fair level. Hence, under the special assumption that $\gamma_t = 0$ the present value formula (1) prices insurance contracts as if they were determined in a competitive equilibrium of risk-neutral insurers who are not subject to transaction costs.

Nevertheless, the present value approach (1) has some fairly obvious weaknesses as well. Most importantly, the loading factor γ_t is a free parameter that must be specified exogenously and somewhat arbitrarily. The loading factor will depend on the equilibrium price of any non-diversifiable risk that must be taken on when holding the insurance contract, and on the magnitude of insurer costs. Without further information on the size of these factors there is no way to pin down the value of γ_t. Put another way, the actuarially fair insurance market equilibrium model arising from setting $\gamma_t = 0$ is highly unrealistic because agricultural insurance contracts typically entail non-diversifiable risk that must be priced (e.g. Chambers 1989; Skees and Reed 1986; Miranda and Glauber 1997; Duncan and Myers 2000). Therefore, equation (1) does not, in general, provide a well-defined value for the insurance premium which is consistent with a realistic model of market equilibrium.

It will be useful for comparisons that follow to derive the present value formula for the special case of a lognormally distributed revenue index. Under the lognormality assumption, the integral in (1) can be evaluated to give (see the Appendix of Rubinstein,

1976):

$$(2) \qquad P_{t-1}(G_t) = (1+\gamma_t)\beta_t \left[G_t N\left(\frac{g_t - \mu_t}{\sigma_t}\right) - e^{\mu_t + 0.5\sigma_t^2} N\left(\frac{g_t - \mu_t - \sigma_t^2}{\sigma_t}\right)\right]$$

where $N(.)$ is the cumulative distribution function for the standard normal, $g_t = log(G_t)$, and $\mu_t = E_{t-1}(y_t)$ and $\sigma_t^2 = Var_{t-1}(y_t)$ are the mean and variance of $y_t = log(Y_t)$, conditional on information available at planting time. This formula has the advantage that it is easy to compute numerically without resorting to numerical integration or Monte Carlo methods because $N(.)$ is already compiled and available in most computational software programs. However, while $\{\beta_t, G_t, \mu_t, \sigma_t\}$ are observable or can be estimated from past data, the problem of indeterminancy with respect to γ_t remains.

Despite the fact that the present value model (1) has a free parameter γ_t and is not generally consistent with a realistic model of insurance market equilibrium, this approach continues to be used in many applied studies trying to value agricultural insurance contracts. It is used because it is straightforward computationally and allows for a lot of flexibility in the form of the underlying probability distribution. In such cases a value for γ_t is assigned arbitrarily and sometimes sensitivity analysis is done to determine how the insurance premium may change with different assumed loading factors. However, dissatisfaction with having to define γ_t arbitrarily has led to a search for alternative insurance valuation models that do not have this free parameter feature.

The Black-Scholes Model

A European put option gives the buyer the right, but not the obligation, to sell an underlying asset at a pre-specified strike price and future maturity date. As such, the contingent payoffs embodied in a put option replicate the payoffs under an insurance scheme, where the insurance guarantee level is the strike price and the insurable revenue index plays the role of the underlying asset price. This insight has led to several applications of the Black-Scholes put option pricing model to value agricultural insurance contracts (e.g. Turvey and Amanor-Boadu 1989; and Turvey 1992).

The advantage of the Black-Scholes model is that it is a fully articulated equilibrium asset pricing model, and so it will presumably value options (and hopefully insurance contracts) at what their equilibrium value would be in competitive financial markets. The equilibrium notion behind the Black-Scholes model is that if the underlying asset

the option is written on can be traded continuously on liquid security markets, then it is possible to construct a continuously adjusted portfolio consisting of the underlying asset and a risk free bond that exactly replicates the payoff on the option. Then in competitive equilibrium there should be no pure arbitrage possibilities and so the value of the replicating portfolio should exactly equal the value of the put. Imposing this restriction, assuming that the underlying asset price follows a geometric Brownian motion (so that the price innovation between any two discrete points in time is lognormally distributed), and assuming there are no transaction costs to trading, leads to the well-known Black-Scholes formula for the value of a put option written at time $t = 0$ with a maturity date of $t = 1$ (Black and Scholes 1973):

$$(3) \quad P_{t-1}(G_t) = \beta_t G_t N\left(\frac{g_t - y_{t-1} - r_t + 0.5\sigma_t^2}{\sigma_t}\right) - Y_{t-1} N\left(\frac{g_t - y_{t-1} - r_t - 0.5\sigma_t^2}{\sigma_t}\right).$$

It is interesting to note that the present value formula under lognormality (2) will be equivalent to the Black-Scholes formula (3) as long as $\gamma_t = 0$ and $\mu_t = y_{t-1} + r_t - 0.5\sigma_t^2$. The first condition implies that there are no risk premia or transaction costs, while the second implies that the underlying asset price grows at the risk-free rate of interest r_t.[3] This is just the well-known "risk neutral valuation" result that the Black-Scholes formula can be derived by assuming equilibrium in an economy with risk-neutral investors and no transaction costs, so that all assets are expected to earn the risk-free rate of return. Then the option value can be obtained by discounting the expected value of the option at maturity back to the present at the risk-free rate, and then imposing the restriction that the return on holding the underlying asset also is equal to the risk free rate. It is important to note that risk neutral valuation does not actually require the true observed growth rate in the underlying asset value to be equal to the risk-free rate. The underlying asset may follow any growth rate (consistent with lognormality) but the equilibrium option value is priced as if the return on the underlying asset equals the risk-free rate. Hence, risk-neutral valuation is just a simple means of deriving the Black-Scholes formula.

Despite the fact that the Black-Scholes model is a fully articulated equilibrium asset pricing model, has no free parameters, and is easy to compute, there are several assumptions underlying the model that make it a very questionable approach to valuing agricultural insurance. First of all, lognormality may not be an appropriate distributional

[3] To see this, note that under lognormality $E_{t-1}(Y_t/Y_{t-1}) = e^{\mu_t + 0.5\sigma_t^2 - y_{t-1}}$. Setting this equal to e^{r_t} and taking logarithms implies that $\mu_t = y_{t-1} + r_t - 0.5\sigma_t^2$.

assumption for the insurable revenue index used in the insurance scheme. It is possible to extend the original Black-Scholes formula to account for non-lognormality (e.g. Merton 1973; and Rubinstein 1994), though this typically requires numerical integration or Monte Carlo methods for computation. The simple and commonly used Black-Scholes formula (3) would, of course, not be appropriate in this case.

Second, and more importantly, the Black-Scholes model prices options (insurance contracts) as if their value is being determined in a liquid market where agents are holding the options for investment purposes and can either buy or sell them costlessly in a liquid secondary market. These assumptions may be reasonable when the model is being used to price liquid exchange-traded options written on underlying assets that themselves can be traded freely on liquid financial markets. But this is not the case for agricultural insurance. Agricultural insurance must be tailored to the needs of local areas, or even individual farms, and so the costs of engineering these contracts is much greater than that of standardized financial options. Furthermore, insurance contracts are issued by insurers and purchased by farmers, and there is generally no secondary market for trading these contracts. Hence, it is not at all clear that pricing insurance contracts "as if" frictionless secondary markets exist, as in the Black-Scholes approach, can lead to reasonable models for agricultural insurance valuation.

Third, and perhaps most importantly, the assumption that the underlying index the option is written on is the price of an asset that can be continuously traded on liquid financial markets is fundamental to the notion of equilibrium embodied in the Black-Scholes formula. Agricultural revenue insurance is written on an individual farm crop revenue index (individual farm-based insurance) or an area revenue index (area-based insurance). Not only are these not the prices of continuously tradeable assets, but because of the seasonality of harvests these indices take on a value of zero at all time periods except the harvest period, so that the standard geometric Brownian motion assumption for the underlying index must be violated. Clearly, crop revenues are determined seasonally by technology, management, and the supply and demand for the underlying crops, not by investors holding assets for investment purposes. If an option (insurance contract) is written on an index which is not the price of an asset held for investment purposes and traded on liquid financial markets, as is surely the case with agricultural insurance, then the equilibrium notion underlying the Black-Scholes formula (3) breaks down.

The issue of "nontradeability" of the asset underlying an option (insurance contract) has been given considerable attention in the finance literature (e.g. Hull 1993; Merton 1998) and has also been discussed in the agricultural insurance literature (Stokes, Nayda and English, 1997; Stokes and Nayda, 2003; and Yin and Turvey, 2003). One way to resurrect the Black-Scholes no arbitrage arguments when the underlying index is not the price of a tradeable asset is to assume that, even though the underlying index itself is not the price of a tradeable asset, there exist a complete set of continuously tradeable assets that allow agents to construct a portfolio whose risk tracks or spans the uncertainty in the nontradeable index (Dixit and Pindyck 1994; Merton 1998). This spanning portfolio is often called the "spanning asset".

Given the existence of a spanning asset we can construct a (tradeable) portfolio consisting of the spanning asset and a risk-free bond that can be continuously adjusted to exactly replicate the return on the option (insurance contract). Or if the return on the option cannot be exactly replicated (i.e. there is a "tracking error" between the return on the tradeable portfolio and that on the option), the "tracking error" component is priced at zero under a complete markets assumption because idiosyncratic risks are not valued in complete market economies (Merton 1998). Hence, in equilibrium the no arbitrage condition implies that the value of the portfolio and the value of the option (insurance contract) must be equal. This equilibrium condition, combined with the standard assumption that the underlying revenue index evolves continuously as a geometric Brownian motion, leads to the following formula for the insurance value at time $t-1$ (Hull 1993; and Merton 1973):[4]

$$(4) \qquad V_{t-1}(G_t) = \beta_t \left[G_t N\left(\frac{g_t - \mu_t + \lambda_t \sigma_t}{\sigma_t}\right) - e^{\mu_t + \frac{1}{2}\sigma_t^2 - \lambda_t \sigma_t} N\left(\frac{g_t - \mu_t + \lambda_t \sigma_t - \sigma_t^2}{\sigma_t}\right)\right]$$

where λ_t is the so-called "market price of risk" for the spanning asset, which is a measure of the excess equilibrium expected return of the spanning asset above the risk free rate.

[4]Because crop revenue is seasonal and not observed continuously, the only interpretation that really makes sense for this geometric Brownian motion assumption is that the underlying index represents the expected harvest time revenue conditional on information available at the current t. Then this expectation evolves continuously over time as new information becomes available and, at the harvest date, the expected revenue equals the actual revenue.

It is interesting to note that if the market price of spanning asset risk is zero ($\lambda_t = 0$), which implies that the spanning asset risk is completely diversifiable and so the expected equilibrium rate of return on the spanning asset is just the risk-free rate, then (4) reduces to the simple present value formula (2) with $\gamma_t = 0$. Hence, when the underlying index is not the price of a tradeable asset but its risk can be tracked by a spanning asset whose risk is completely diversifiable, then the insurance contract can be priced according to its discounted expected present value (with discounting occurring at the risk-free rate). Also notice that if the underlying index was the price of a traded asset then the expected equilibrium return on holding this asset would have to equal $(r_t + \lambda_t \sigma_t)$ because the market price of risk for the spanning asset and the market price of risk for the asset whose price is the underlying index must be equal by definition. That is, $E_{t-1}(Y_t/Y_{t-1}) = e^{\mu_t + 0.5\sigma_t^2 - y_{t-1}} = e^{r_t + \lambda_t \sigma_t}$ which implies $\mu_t - \lambda_t \sigma_t = y_{t-1} + r_t - 0.5\sigma_t^2$. Substituting this result into (4) returns the Black-Scholes formula (3) for a tradeable underlying asset (as expected).

The no arbitrage option valuation formula (4) for an option (insurance contract) whose underlying index is not the price of a tradeable asset overcomes one of the key weaknesses of the Black-Scholes formula because it does not require the underlying index to be the price of a tradeable asset. Equation (4) is also computationally straightforward to implement. Nevertheless, (4) is not without problems of its own for pricing agricultural insurance contracts. First, (4) contains a free unobservable parameter in the market price of risk for the spanning asset, λ_t. So to compute an insurance value using (4) then λ_t has to be determined. This is usually done by imposing some equilibrium asset pricing model, such as the Capital Asset Pricing Model (CAPM) or the Arbitrage Pricing Model (APT) (e.g. Yin and Turvey 2003; and Stokes and Turvey 2003). For example, if the price of the spanning asset is determined according to the CAPM, then it can be shown that $\lambda_t = (\rho_t/\sigma_{mt})(\alpha_{mt} - r_t)$, where ρ_t is the correlation coefficient between the spanning asset and the market portfolio returns, σ_{mt} is the standard deviation of the return on the market portfolio, and α_{mt} is the expected return on the market portfolio. In this case, λ can be estimated from data on the underlying index and the market portfolio return, without requiring any data on the spanning asset return. Notice, however, that this approach requires the assumption of an equilibrium asset pricing model for all assets in the economy which is much more restrictive than the simple no-arbitrage argument underlying the Black-Scholes model.

Even if λ_t can be determined using the CAPM or APT, (4) still has problems for pricing agricultural insurance. Clearly, the choice of equilibrium model to use to compute λ_t, and the assumptions used to implement it, will have an influence (perhaps a big influence) over the insurance valuation result. And if these equilibrium models have to be called on anyway to value the market price of risk, then why not just use them to value the insurance contract directly? There may be little real benefit to filtering the valuation through a no arbitrage argument if, say, the CAPM has to be used to value the market price of risk anyway. Also, equation (4) still implies that the options are being priced as if their value is determined in a liquid market where agents are holding the options for investment purposes and can either buy or sell options costlessly in a liquid secondary market. As discussed earlier, these assumptions may be reasonable for pricing liquid exchange-traded financial options but are more problematic for pricing agricultural insurance contracts.

Most importantly, (4) assumes that an appropriate spanning asset (or portfolio) is available. The problem with spanning in the case of crop insurance is that the required contingent claims generally do not exist. It is true that futures and options markets exist for many crops, and state average yield futures and options markets have been available over some periods. But these assets will not allow perfect replication of the risk associated with revenue outcomes for individual farms, or even county areas. Available contingent claims markets just do not appear extensive enough to allow replication in the case of crop insurance revenue indices. Furthermore, there is somewhat of a tautology in the spanning argument because if a spanning portfolio of continuously tradeable contingent claims exist then the insurance contract itself is redundant. That is, the spanning portfolio would itself allow replication of the option (insurance) return, and the transaction costs of trading the contingent claims would likely be lower than the costs of operating an insurance program.[5] So using (4) [or for that matter (3)] to price agricultural insurance implies that the insurance contract is redundant (downside revenue risk can be insured with existing contingent claims and insurance contracts are not required to help complete the market structure (Merton, 1998). This assumption seems particularly untenable in the case of agricultural insurance. Is it really true that farmers can completely insure their revenue risk without the insurance contract? If so, then why has there been so much government policy directed at engineering and innovating new agricultural insur-

[5]This same argument applies equally to the Black-Scholes model for tradeable assets.

ance contracts? If no spanning portfolio is available then the introduction of agricultural insurance contracts will alter the non-diversifiable risk profile of a significant number of investors (farmers) and the arbitrage based option pricing equation for nontradeable assets (4) will no longer be applicable.

Lucas Representative Agent Equilibrium Models

One final class of models that has been used to value agricultural insurance contracts is the consumption-based Lucas representative agent model (Lucas, 1978). This model prices contingent claims (including insurance) using an equilibrium pricing kernel derived from a well-defined dynamic optimization problem and the imposition of market clearing conditions. The equilibrium pricing kernel takes the form:

$$P_{t-1}(G_t) = \delta_t E_{t-1}\left[\frac{U'(C_t)}{U'(C_{t-1})}\max(G_t - Y_t, 0)\right] \tag{5}$$

where $U(.)$ is a concave von Neumann-Morgenstein utility function, δ is consumer's time preference parameter, C_t is consumption in period t (which depends on realized Y_t). This model is a representative agent model and the utility function is that of the representative agent. Notice that if the representative agent is risk neutral then (5) just reduces to the present value formula (1) (with $\gamma_t = 0$). But if the representative agent is risk averse then $U'(C_t)/U'(C_{t-1})$ represents a risk adjustment factor that compensates investors for taking on the risk of issuing the insurance contracts [in much the same way that the market price of risk adjusts the arbitrage based model (4)].

There are several problems associated with the general equilibrium pricing kernel (5). First of all, whose consumption should be used to compute the marginal utility of consumption? Should it be aggregate consumption in the economy or consumption of farmers or some local agricultural region?[6] There is no reason why it should be farm or regional consumption because all agents in the economy can invest in insurance by issuing insurance contracts or investing in insurance companies. And yet aggregate economy-wide consumption tends to have very little correlation with agricultural crop revenues in the U.S. and so applying (5) using aggregate economy-wide consumption tends to just reduce to the risk neutral present value formula (1). Another problem in computing the marginal utility of consumption is that a specific form must be assumed for the utility

[6]It have been done both ways(see Cao and Wei 2004; and Richards, Manfredo and Sanders 2004).

function. Clearly, insurance pricing results may be sensitive to which utility function and consumption variable are included in the analysis. More importantly, while the equilibrium pricing kernel (5) is not based on arbitrage arguments, and so does not require spanning, it is a single good representative agent exchange model in which no trades of any commodity or asset occur in equilibrium (no trades being necessary because all agents are identical). Hence the model generates equilibrium pricing kernels for contingent claims but the claims are redundant since they are never traded in equilibrium. Put another way, the Lucas general equilibrium asset pricing model is implicitly a complete markets model and so prices insurance contracts assuming they are redundant assets. We have already argued above that this complete markets assumption is likely to be very unreasonable in the case of crop insurance. It would appear that the introduction of crop insurance will alter the non-diversifiable risk profile of a significant number of investors (i.e. farmers) and so any model that prices agricultural insurance as a redundant asset is unlikely to give appropriate answers to the valuation question.

Summary

We have examined several existing methods for pricing agricultural insurance contracts and found them wanting to various degrees. The present value model is simple and flexible but has a free parameter and is not consistent with a reasonable model of market equilibrium in the insurance market. The Black-Scholes model is a fully articulated equilibrium model with no free parameters but requires the insurance contract to be a redundant asset and the underlying index for the insurance contract to be the price of a tradeable asset. Neither of these assumption is very palatable in the case of agricultural insurance. The arbitrage based model for claims on non-tradeable assets overcomes some of the weaknesses of the Black-Scholes model but has a free parameter in the market price of risk, and still assumes that the insurance contract is a redundant asset (via spanning). The general equilibrium Lucas model does not rely on spanning and arbitrage but is sensitive to required assumptions about utility and consumption, and is implicitly a complete markets model so that the insurance contract is again priced as if it were a redundant asset. Indeed, all of these models (except for the present value model) price insurance contracts as if they were freely traded on liquid secondary markets and either implicitly or explicitly price insurance as if it were a redundant asset in

a complete markets model. We view these as significant weaknesses in the context of agricultural insurance valuation and in the next section we begin the task of developing an insurance valuation model that relaxes the assumption of complete markets and recognizes that agricultural insurance contracts are issued by investors (or the government) but held by farmers, and that they cannot usually be traded on liquid secondary markets.

Incomplete Markets Models For Agricultural Insurance Valuation

We have argued above that complete market models of insurance valuation, which price insurance as a redundant asset are not completely appropriate for most agricultural insurance valuation problems. In this section we introduce two alternative models that allow for incomplete markets. Both models are based on a discrete-time Lucas exchange economy with an infinite horizon and a single nondurable consumption good serving as numeraire. However, we allow for agent heterogeneity and uninsurable background risk so that insurance valuation occurs in an incomplete markets environment. We begin with a benchmark model in which insurance contracts can be engineered and traded costlessly by heterogenous agents operating in a competitive market. The heterogenous agent assumption distinguishes this model from the Lucas representative agent model and allows insurance to be a non-redundant asset which helps complete the market structure. Second we extend the benchmark model by assuming that insurance contracts are costly to engineer and administer so that they cannot be freely traded among individuals, (i.e., there is no liquid secondary market for trading insurance). This creates a role for insurance companies to act as market intermediaries between farmers and the capital markets. The second model is more consistent with the way agricultural insurance markets work in practice.

A Model With Liquid Secondary Market For Insurance

In the model with liquid secondary market, markets are not complete (there is uninsurable background risk), and agricultural insurance is not a redundant asset. However, there is a liquid secondary market in which insurance contracts can be traded costlessly among individuals, with no restrictions on who insures and who are the insurers (i.e., contracts can be bought or sold by any agent in the economy). This model is characterized by

Assumptions 1.1-1.4.

Assumption 1.1. There are n agents indexed $i = 1, \ldots, n$ who maximize the same objective function $E_0 \left[\sum_{t=0}^{\infty} \delta^t U(C_{it})\right]$, but are heterogeneous in their endowment of uninsurable random "wage" income Z_{it} and insurable random "farm" income Y_{it} realized at each period.[7] Note that Y_{it} can be zero which would indicate that agent i has no insurable income (and is not a farmer).

Assumption 1.2. There are $n + m + 1$ assets in the economy. These include n insurance contracts written on agent i's insurable income Y_{it} for $i = 1, \ldots, n$, a risk-free bond with exogenously given fixed gross return $R_t = (1 + r_t)$, and m stocks indexed by j. The insurance contract written on agent i's insurable income at time t, Y_{it}, has a period $t-1$ price of $P_{i,t-1}$ at $t-1$ and a random current value of $V_{it} = \max(G_{it} - Y_{it}, 0)$, where G_{it} is the insurance guarantee on Y_{it}. Let $P_{t-1} = (P_{1,t-1}, \ldots, P_{n,t-1})$ and $V_t = (V_{1t}, \ldots, V_{nt})$ denote n-dimensional vectors of $t-1$ and t-period insurance values. At time t, stock j pays a net dividend of D_{jt} and has ex-dividend price P^s_{jt}. Let $D_t = (D_{1t}, \ldots, D_{mt})$ and $P^s_t = (P^s_{1t}, \ldots, P^s_{mt})$ denote m-dimensional vectors of the dividend and price processes.

Assumption 1.3. In equilibrium, the bond and all insurance contracts are assumed to be in zero net supply, and each stock is assumed to be in fixed positive supply which is normalized to be one.[8] Then let $D^s_t = \frac{1}{n} \sum_{j=1}^{m} D_{jt}$ denote the aggregate dividends per capita.

Assumption 1.4. For tractability and consistent with distributional assumptions in most other non-arbitrage base option valuation models, assume constant relative risk aversion preferences (CRRA) and that (C_{it}, Y_{kt}) and (D^s_t, Y_{kt}) are joint lognormally distributed respectively for all i and k.[9] Also assume that insurable and uninsurable risks are uncorrelated, $Cov_{t-1}(Z_{it}, Y_{kt}) = 0$ for all i and k.

Under these assumptions an equilibrium insurance valuation formula can be derived

[7] We will refer to Z_{it} as "wage" income and Y_{it} as "farm" income but notice that the key distinction is that Y_{it} is insurable while Z_{it} is not. We can easily think of Z_{it} as including other non-insurable components of farm income and not restrict it to just off-farm income.

[8] The assumption on stocks follows from Constantinides and Duffie (1996).

[9] Models that feature uninsurable background risk cannot be preference free and so some assumption about preferences has to be made. CRRA is a natural and common choice (see Constantinides and Duffie, 1996). Similarly, lognormality is a common assumption in both the option pricing and insurance literatures (see Rubinstein, 1976). The joint lognormally of (C_{it}, Y_{kt}) and (D^s_t, Y_{kt}) can be justified in that there is no restriction on the distribution of Z_{it}.

as given in the following proposition.

Proposition 1. Under Assumptions 1.1-1.4, the equilibrium price of an insurance contract written on agent k's insurable income Y_{kt} is

(6)
$$P_{k,t-1}(G_{kt}) = \beta_t \left[G_{kt} N\left(\frac{g_{kt} - \mu_{kt} + \eta_{kt}}{\sigma_{kt}} \right) - e^{\mu_{kt} + \frac{1}{2}\sigma_{kt}^2 - \eta_{kt}} N\left(\frac{g_{kt} - \mu_{kt} + \eta_{kt}}{\sigma_{kt}} - \sigma_{kt} \right) \right]$$

where

$$\eta_{kt} = \alpha \left[\frac{E_{t-1}(D_t^s) Cov_{t-1}(d_t^s, y_{kt}) + \frac{1}{n}\sum_{i=1}^n E_{t-1}(Y_{it}) Cov_{t-1}(y_{it}, y_{kt})}{\frac{1}{n}\sum_{i=1}^n E_{t-1}(C_{it})} \right], \tag{7}$$

α is the CRRA parameter, $N(.)$ is the cumulative distribution function for the standard normal, g_{kt}, y_{kt} and d_t^s are natural logarithms of G_{kt}, Y_{kt} and D_t^s respectively, $\mu_{kt} = E_{t-1}(y_{kt})$, and $\sigma_{kt}^2 = Var_{t-1}(y_{kt})$.

Proof. See appendix.

The only real difficulty in making this formula operational is finding an appropriate value of η_{kt}. From (7) we see that η_{kt} depends on the CRRA parameter α and the joint distribution of the insurable yield indices Y_{it} and aggregate stock market dividends per capita D_t^s, weighted by the proportion of aggregate consumption per capita accounted for by aggregate dividends per capita and by aggregate insurable farm income per capita. Obtaining an appropriate value for α is difficult and sensitivity analysis might be used to investigate the sensitivity of equilibrium insurance values to different α values. However, remaining aggregate consumption, dividend, and insurable farm revenue parameters could be estimated for observable data.

It is interesting to note that the liquid market equilibrium insurance pricing formula (6) takes exactly the same form as the arbitrage-based option pricing formula (4) for nontradeable assets, with η_t in (6) playing the role of the market price of risk term $\lambda_t \sigma_t$ in (4). This highlights that these two models provide very similar results except that the equilibrium price of non-diversifiable risk is priced in a different way in the two models. Nevertheless, for any CRRA parameter α used in (6) there is clearly a corresponding value for the market price of risk λ_t that will make the two formulas give the exact same equilibrium insurance price. This highlights that our liquid market model is just an alternative way of pricing the non-diversifiable risk inherent in holding the insurance contract – one that allows for uninsurable background risk, incomplete markets and does not require insurance to be a redundant asset.

In the above model with liquid secondary market, insurance contracts can be traded costlessly among individuals, and all individuals can choose the number of contracts to buy and sell, irrespective of whether they receive insurable farm income or not. However, there is no liquid secondary market for trading actual agricultural insurance contracts (only farmers can buy contracts and, once purchased, these cannot be resold). Moreover, farmers are usually allowed to buy only one insurance contract for the same insured risks out of a concern for moral hazard. The model in this section modifies these two implausible assumptions imposed in the model with liquid secondary market and creates a role for insurance companies to act as market intermediaries between farmers and the capital markets. However, some more restrictive technical assumptions are also imposed for tractability. Assumptions 2.1-2.4 characterize this model.

Assumption 2.1. There are n agents indexed $i = 1, \ldots, n$ who maximize the same objective function $E_0 \left[\sum_{t=0}^{\infty} \delta^t U(C_{it})\right]$. Among them, there are n_1 identical farmers indexed $1, \ldots, n_1$ who have insurable farm income Y_{it} and uninsurable income Z_{it}, and $n_2 = n - n_1$ heterogenous wage-earners indexed $n_1 + 1, \ldots, n$ who have heterogenous uninsurable wage income Z_{it} but no insurable income.[10] There is a single insurer in the market who is regulated by the government to ensure competitive behavior. The insurer works as a broker. By signing insurance contracts with individual farmers, pooling the risky assets (insurance contracts) together, and trading the repackaged asset in the capital market with both wage-earners and farmers.

Assumption 2.2. There are $m+2$ traded assets in the economy, consisting of a risk-free bond with exogenously given fixed gross return $R_t = 1 + r_t$, m stocks indexed by j, and a repackaged insurance asset with a random gross return R^h. Similar to Assumption 1.2, let $D_t = (D_{1t}, \ldots, D_{mt})$ and $P_t^s = (P_{1t}^s, \ldots, P_{mt}^s)$ denote the m-dimensional dividend and price processes. The insurance contract written on farmer k's income can only be traded between farmer k and the insurer. Each farmer is allowed to buy only one insurance contract. The farmer chooses a guarantee level G_{it} for a given price formula $P_{t-1}(G_{it})$ at time $t-1$, and receives a payoff $V_{it} = \max(0, G_{it} - Y_{it})$ at t.

Assumption 2.3. In equilibrium, the bond is assumed to be in zero net supply, each

[10] Note that the realized income Y_{it} is not identical across i, but the distribution of Y_{it} is identical for all farmers.

stock is assumed to be in fixed positive supply which is normalized to one, and the insurer makes zero economic profit (as a broker, the insurer does not bear risk by assumption).

Assumption 2.4. In addition to all of the assumptions in 1.4, $(C_{it},\ R_t^h)$, $(D_t^s,\ R_t^h)$ and $(Y_{it},\ R_t^h)$ are joint lognormally distributed, respectively.

Under these assumptions an equilibrium insurance valuation formula can be derived as given in the following proposition.

Proposition 2. Under Assumptions 2.1-2.4, the equilibrium price of an insurance contract written on a representative farmer's insurable income Y_{kt} is

$$P_{k,t-1}(G_{kt}) = \beta_t e^{\Phi(G_{kt})\eta_{kt}} \left[G_{kt} N\left(\frac{g_{kt}-\mu_t}{\sigma_t}\right) - e^{\mu_t + \frac{1}{2}\sigma_t^2} N\left(\frac{g_{kt}-\mu_t}{\sigma_{kt}} - \sigma_{kt}\right)\right] \tag{8}$$

where

$$\Phi(G_{kt}) = \frac{e^{(\mu_{kt}+\frac{1}{2}\sigma_{kt}^2)} \cdot N\left(\frac{g_{kt}-\mu_{kt}}{\sigma_{kt}} - \sigma_{kt}\right)}{G_{kt} N\left(\frac{g_{kt}-\mu_{kt}}{\sigma_{kt}}\right) - e^{(\mu_{kt}+\frac{1}{2}\sigma_{kt}^2)} N\left(\frac{g_{kt}-\mu_{kt}}{\sigma_{kt}} - \sigma_{kt}\right)}. \tag{9}$$

Proof. See appendix.

Again, the only real difficulty in making this formula operational is finding an appropriate value for η_{kt}, which can be characterized and estimated as described above under the discussion of the model with liquid secondary market (6).

In this case it is interesting to note that the broker model (8) takes exactly the same form as the present value model (2), with $e^{\Phi(G_{kt})\eta_{kt}}$ playing the role of the loading factor $(1+\gamma_t)$. Clearly, setting the loading factor to $\gamma_t = e^{\Phi(G_{kt})\eta_{kt}} - 1$ then these two formulas give equivalent results. This highlights the fact that the broker model provides a way of pinning down the equilibrium price of risk, thereby defining the equilibrium value of the free loading factor parameter in the present value model.

Discussion

An important property of the two price formulae (6) and (8) is that they both are increasing functions of η_{kt}. This is obvious for (8) because $\Phi(G_{kt}) > 0$. To see this point in (6), notice that (6) is equivalent to

$$P_{k,t-1}(G_{kt}) = \beta_t E_{t-1}\left[\max(G_{kt} - e^{y_{kt} - \frac{1}{2}\sigma_{kt}^2 - \eta_{kt}}, 0)\right] \tag{10}$$

We then interpret η_{kt} as market compensation for the risk related to holding agricultural

insurance contracts, so that as it gets higher so does the equilibrium price of the insurance contract.

Some examples will highlight how η_{kt} is determined and how insurance prices depend on η_{kt} in the two formulae we propose. Time and individual indexes are omitted when there is no confusion.

First, note that if $Cov(y_i, y_k) = Cov(d^s, y_k) = 0$ for all $i \neq k$ (i.e. each agent's insurable income is uncorrelated with that of other agents and the aggregate dividend, so that individual risks are fully diversifiable), then $\eta_t = 0$ and the valuation formulae (6) and (8) both revert to a simple net present value (NPV) calculation [equation (2) with γ_t set to zero].

Second, assume that $Cov(d^s, y_k) = 0$ for all $i \neq k$ (insurable risk k is uncorrelated with the aggregate dividend). Furthermore, let $Cov(y_i, y_k) = \sigma^2_{y_k}$ for $i = k$ and $\rho\sigma^2_{y_k}$ for all $i \neq k$, where ρ is the correlation coefficient between y_i and y_k ($i \neq k$).[11] Then (7) becomes

$$\eta = \alpha \left(\frac{\rho \sigma_y^2 \bar{Y}}{\bar{C}} \right)$$

where $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is average per capita insurable income and $\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i$ is average per capita total consumption. Note that for countries with small proportions of total consumption coming from agricultural income, then $\bar{Y}$ will be small relative to $\bar{C}$ and η will be small.

Third, for countries where agriculture-related industry is a major sector in the national economy, we would expect $Cov(d^s, y_k) > 0$ and agricultural income to account for a large proportion of total income (the latter is especially true for less developed countries). In this case, η_k will be large. However, for countries where agriculture is a small sector (e.g. the U.S.), both $Cov(d^s, y_k)$ and $\bar{Y}/\bar{C}$ will be small, and η_k will consequently be expected to be small.

Simulation

In this section, we compare (6), (8), and the net present value (NPV) formula [(2) with loading factor γ_t set to zero] to highlight how different answers can be provided by the

[11] This implies that $\sigma^2_{y_i} = \sigma^2_{y_k}$ for all i (insurable incomes have the same variance and a constant correlation coefficient ρ).

Table 1: Model parameters summary

Parameter	Notation	Figure 1 Value	Figure 2 Range
α	CRRA parameter	10	$[0, 40]$
δ	Risk-free discount factor	0.96	$[.90, .99]$
CV_Y	$= \frac{\sigma_Y}{\bar{Y}}$, coefficient of variation for Y	0.4	$[.1, .6]$
CV_{D^s}	$= \frac{\sigma_{D^s}}{\bar{D^s}}$, coefficient of variation for D^s	0.5	$[.2, .8]$
$\rho(Y_i, Y_k)$	Correlation coefficient of Y_i and Y_k, $i \neq k$	0.6	$[.2, .8]$
$\rho(Y, D^s)$	Correlation coefficient of Y and D^s, $i \neq k$	0.6	$[.2, .8]$
$\frac{\bar{Y}}{\bar{C}}$	% of per capita farmer income in total income	0.25	$[.02, .6]$
$\frac{\bar{D^s}}{\bar{C}}$	% of per capita dividend in total income	0.5	$[.2, .8]$
η	Market compensation of risk	0.33	$[0, 1]$

different valuation models in different situations. To make the models comparable, we assume that farmers are identical. The parameters in the pricing formulae (6) and (8) are calibrated by assigning some reasonable values to the parameters to see how the formulae work. For convenience, we denote P_1 as the NPV formula, P_2 as the liquid market formula (6), and P_3 as the broke formula (8).

The equilibrium risk factor η in Propositions 1 and 2 can be approximately expressed as:

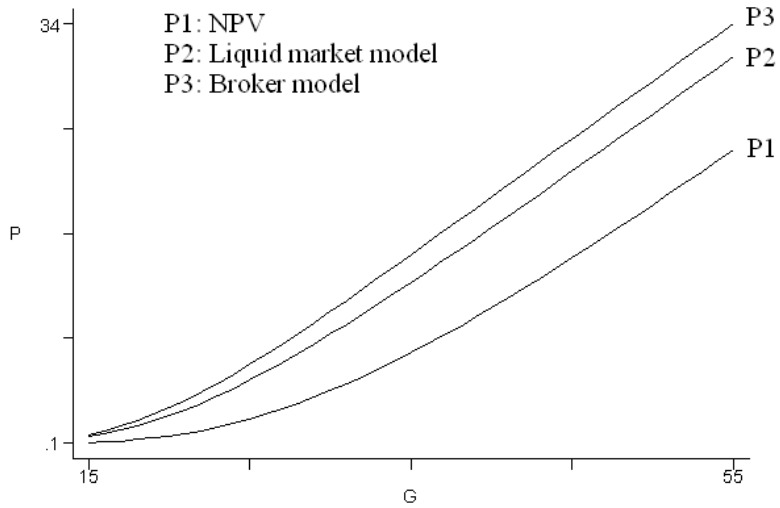
$$\eta = \alpha \left[\frac{\bar{D}}{\bar{C}} \cdot \rho(D^s, Y) \cdot CV_{D^s} \cdot CV_Y + \frac{\bar{Y}}{\bar{C}} \cdot \rho(Y_i, Y_k) \cdot CV_Y^2 \right] \tag{11}$$

where the parameter definitions are listed in Table 1. Equation (11) suggests that the market compensation for risk, η, will be large in an economy where agriculture is a major sector, where farmers' income accounts for a large percentage in aggregate consumption, and where farmer incomes are variable and highly correlated.

Figure 1 shows how the three pricing formulae perform at different guarantee levels and a single set of parameters given as in the third column of Table 1. In the simulations, $\bar{Y}$ is set at 30. At $G = \bar{Y}$, P_2 is 106% and P_3 is 148% larger than P_1. In other words, if we consider the broker model with no liquid secondary market as the true model, then using the actuarially fair NPV model tends to underestimate insurance price by 60%, and using the liquid market model underestimates it by 17%.

Figure 2 shows how the three pricing formulae perform at different values of market compensation for risk η and a fixed guarantee level $G = \bar{Y} = 30$. The reasonable ranges

Figure 1: Insurance prices versus guarantee levels

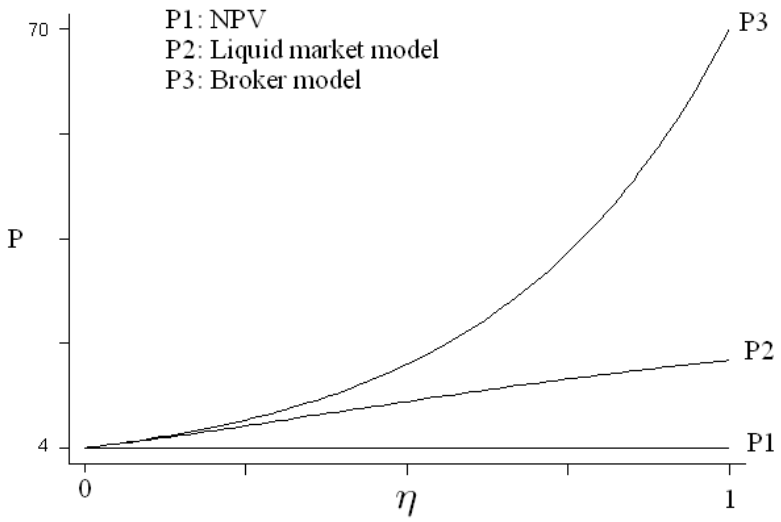


for the parameters in (11) are listed in the fourth column of Table 1. Overall, a reasonable range for η would be $[0, 1]$. The more industrialized the economy is, the smaller η would be. When $\eta = 0$, the three formulae give the same insurance price. When $\eta > 0$, $P_3 > P_2 > P_1$. However, their differences are small for small values of η. As examples, when $\eta < 0.01$, P_3 is less than 0.1% larger than P_2 and is less than 3% larger than P_1; When $\eta < 0.07$, P_3 is less than 1% larger than P_2. The differences among the three formulae grow larger as η increases.

Conclusions

In this paper, we argue that existing option pricing models and the Lucas representative agent model for pricing agricultural insurance may be misleading because they all assume complete markets which is unsuitable for most agricultural insurance applications. We propose two incomplete markets models and derive two corresponding insurance pricing formulae under the usual assumptions of CRRA preferences and lognormally distributed random variables. Our major contributions lie in the broker model with no liquid secondary market, which appears more consistent with the way agricultural insurance markets actually operate. In this model, two types of market incompleteness

Figure 2: Insurance prices versus η, the market compensation of risk



are incorporated. The first is that insurance is not a redundant asset and helps to complete the market structure. The second and more important type is nonexistence of liquid secondary markets for trading agriculture insurance contracts. We also make a contribution by modelling a role for an insurance company to work as an intermediary between farmers and the capital market in constructing insurance valuations. Another contribution is that the pricing formula derived by this model justifies and helps to pin down the unknown loading parameter in the present value formula, thereby pricing the non-diversifiable risk embodied in holding agricultural insurance contracts. Simulation results suggest, for reasonable parameter values, the model that incorporates a higher level of market incompleteness derives a higher equilibrium insurance price than the model that incorporates a lower level of market incompleteness. Furthermore, the effect of non-existence of liquid secondary market on insurance price is not negligible unless the market compensation of risk, η, is very small ($> 1\%$ if $\eta > 0.07$).

Appendix

Proof of Proposition 1

Agent i's objective is

$$\max_{\{C_{it}\}} E_0 \left[\sum_{t=0}^{\infty} \delta^t U(C_{it}) \right] \tag{12}$$

The budget constraint is

$$C_{it} = Z_{it} + Y_{it} + b_{it}R_t + x_{i,t-1}V_t + \theta_{i,t-1}(P_t^s + d_t) - b_{i,t+1} - x_{it}P_t - \theta_{it}P_t^s \tag{13}$$

where, b_{it} is the agent's investment in the bond, $x_{it} = (x_{ikt} : k = 1, \ldots, n)$ is agent i's portfolio of numbers of insurance contracts purchased (sold if negative), and $\theta_{it} = (\theta_{ijt} : j = 1, \ldots, m)$ is agent i's portfolio of shares of stocks purchased (sold if negative). Note that short selling of stocks and insurance contracts is allowed, as is borrowing or lending at the risk-free rate.

Substituting (13) into (12) and equating the derivatives of (12) with respect to $x_{i,t-1}$ and b_{it} to zeroes, we derive the first order conditions for solving agent i's problem in choosing $x_{i,t-1}$ and b_{it} :[12]

$$-U'(C_{i,t-1}) + \delta_t R_t E_{t-1}[U'(C_{it})] = 0 \tag{14}$$

$$-U'(C_{i,t-1})P_{t-1} + \delta_t E_{t-1}[U'(C_{it})V_t] = 0 \tag{15}$$

(14) and (15) together imply

$$E_{t-1}\left[U'(C_{it})(V_{kt} - P_{k,t-1}R_t)\right] = 0, \quad k = 1, \ldots, n \tag{16}$$

which can be written using the CRRA assumption as:

$$E_{t-1}\left\{C_{it}^{-\alpha}\left[max(G_{kt} - Y_{kt}, 0) - P_{k,t-1}R_t\right]\right\} = 0, \quad k = 1, \ldots, n$$

or

$$\int_{-\infty}^{\infty}\int_{-\infty}^{g_{kt}} e^{-\alpha c_{it}}\left[G_{kt} - e^{y_{kt}} - P_{k,t-1}R_t\right] f(c_{it}, y_{kt}) dy_{kt} dc_{it} = 0 \tag{17}$$

[12] Second order conditions are satisfied by the concavity of U.

where lower case letters denote natural logarithms and $f(c_{it}, y_{kt})$ is the joint probability density function (PDF) of c_{it} and y_{kt} conditional on information available at $t-1$. From results in the appendix of Rubinstein, the integral in (17) can be evaluated and solved to give:

(18)

$$P_{k,t-1} = \beta_t \left[G_{kt} N\left(\frac{g_{kt} - \mu_{kt} + \alpha\sigma_{c_{it}y_{kt}}}{\sigma_{kt}} \right) - e^{\mu_{kt} + \frac{1}{2}\sigma_{kt}^2 - \alpha\sigma_{c_{it}y_{kt}}} N\left(\frac{g_{kt} - \mu_{kt} + \alpha\sigma_{c_{it}y_{kt}}}{\sigma_{kt}} - \sigma_{kt} \right) \right]$$

where $\sigma_{c_{it}y_{kt}} = Cov_{t-1}(c_{it}, y_{kt})$.

In equilibrium, (18) must hold for every i. Therefore, if all individuals have identical beliefs about probability distributions, and identical CRRA parameters α, then (18) implies that in equilibrium $\sigma_{c_{it}y_{kt}}$ must be equal across all i. Thus, an equilibrium expression for the value of insurance contract k can be written as equation (6), where $\eta_{kt} = \alpha\sigma_{c_{it}y_{kt}}$ which is constant across individuals (in equilibrium). It remains to show how η_{kt} might be computed.

In equilibrium bond and all insurance contracts are assumed to be in zero net supply, and each stock is assumed to be in fixed positive supply normalized to one. Therefore, $\sum_{i=1}^{n} x_{it} = 0$, $\sum_{i=1}^{n} b_{it} = 0$, and $\sum_{i=1}^{n} \theta_{it} = 1$ for all t. The budget constraint (13) then becomes:

$$\sum_{i=1}^{n} C_{it} = \sum_{i=1}^{n} Z_{it} + \sum_{i=1}^{n} Y_{it} + nD_t^s \tag{19}$$

This implies

$$\sum_{i=1}^{n} Cov_{t-1}(C_{it}, y_{kt}) = nCov_{t-1}(D_t^s, y_{kt}) + \sum_{i=1}^{n} Cov_{t-1}(Y_{it}, y_{kt}) \tag{20}$$

because $Cov_{t-1}(Z_{it}, y_{kt}) = 0$ by assumption. From results in the appendix of Rubinstein then if c_{it} and y_{kt} are joint normally distributed (C_{it} and Y_{kt} lognormal) then

$$Cov(C_{it}, y_{kt}) = E(C_{it})Cov(c_{it}, y_{kt}) \tag{21}$$

Now substituting (21) into (20) and using the fact that $Cov_{t-1}(c_{it}, y_{kt}) = \eta_{kt}/\alpha$ is constant across i in equilibrium, then (20) becomes:

$$\frac{\eta_{kt}}{\alpha} \sum_{i=1}^{n} E_{t-1}(C_{it}) = nE_{t-1}(D_t^s)(Cov_{t-1}(d_t^s, y_{kt}) + \sum_{i=1}^{n} E_{t-1}(Y_{it})Cov_{t-1}(y_{it}, y_{kt}) \tag{22}$$

Rearranging (22) gives (7).

Proof of Proposition 2

Each agent's objective is represented in (12). But farmers (indexed $1, \ldots, n_1$) and wage-earners (indexed $n_1 + 1, \ldots, n$ have different budget constraints.

Farmers' budget constraints are

$$C_{it} = Z_{it} + Y_{it} + b_{it}R_t - h_{i,t-1}R_t^h + \theta_{i,t-1}(P_t^s + D_t) + V_{it}(G_{it}) - \\ - b_{i,t+1} + h_{it} - \theta_{it}P_t^s - P_t(G_{i,t+1}), \quad i = 1, \ldots, n_1 \tag{23}$$

Wage-earners' budget constraints are

$$C_{it} = Z_{it} + b_{it}R_t - h_{i,t-1}R_t^h + \theta_{i,t-1}(P_t^s + D_t) - b_{i,t+1} + h_{it} - \theta_{it}P_t^s, \quad i = n_1 + 1, \ldots, n \tag{24}$$

where, h_{it} is agent i's investment in the repackaged insurance asset. The other notations are the same as those in the proof of Proposition 1.

The first order conditions for solving agent i's problem in choosing $h_{i,t-1}$ are:[13]

$$E_{t-1}\left[U'(C_{it})(R_t^h - R_t)\right] = 0 \tag{25}$$

Since the farmers are identical, they choose the same guarantee level G_t^0, which satisfies

$$E\left\{U'(C_{it})\left[1_{\{G_t > Y_{it}\}} - P'_{t-1}(G_t)R_t\right]\right\} = 0 \tag{26}$$

where $1_{\{G_t > Y_{it}\}}$ is a random variable which equals one if $G_t > Y_{it}$ and zero otherwise.

The insurer's problem is to choose an insurance price formula $P_{t-1}(G_t)$ to maximize profit (insurers bear no risk). Profit equals zero based on the assumption of competitive behavior. Thus we have the brokerage conditions:

$$\textstyle\sum_{i=1}^{n} h_{i,t-1} = n_1 P_{t-1}(G_t^0) \tag{27}$$

$$\textstyle\sum_{i=1}^{n} h_{i,t-1}R_t^h = \sum_{i=1}^{n_1} V_{it}(G_t^0) \tag{28}$$

(27) and (28) together imply

$$R_t^h = \frac{1}{n_1 P_{t-1}(G_t^0)} \sum_{i=1}^{n_1} V_{it}(G_t^0) \tag{29}$$

[13]Second order conditions are satisfied by the concavity of U.

We further assume C_{it} and R_t^h are joint lognormal distributed. Then (25) becomes:

$$log\left[E_{t-1}(R_t^h)\right] - r_t = \alpha Cov_{t-1}(c_{it}, r_t^h) \;\; i = 1, \ldots, n. \tag{30}$$

Equation (30) implies that $Cov_{t-1}(c_{it}, r_t^h) = \sigma_{c_t r_t^h}$ must be equal across all i. Equations (29) and (30) together imply

$$\begin{aligned} P_{t-1}(G_{kt}) &= \beta_t e^{-\alpha\sigma_{c_t r_t^h}} E_{t-1}(V_{it}) \\ &= \beta_t e^{-\alpha\sigma_{c_t r_t^h}} \left[G_{kt} N\left(\frac{g_{kt}-\mu_{kt}}{\sigma_{kt}}\right) - e^{\mu_{kt}+\frac{1}{2}\sigma_{kt}^2} N\left(\frac{g_{kt}-\mu_{kt}}{\sigma_{kt}} - \sigma_{kt}\right)\right] \end{aligned} \tag{31}$$

It remains to compute $\sigma_{c_t r_t^h}$.

In equilibrium, the bond is in zero net supply and each stock is in fixed positive supply normalized to one. Therefore, $\sum_{i=1}^n b_{it} = 0$, and $\sum_{i=1}^n \theta_{it} = 1$ for all t, and brokerage conditions (27) and (28) hold in equilibrium. The budget constraints (24) and (24) then become (19), which implies

$$\sum_{i=1}^n Cov_{t-1}(C_{it}, r_t^h) = nCov_{t-1}(D_t^s, r_t^h) + \sum_{i=1}^n Cov_{t-1}(Y_{it}, r_t^h) \tag{32}$$

because $Cov_{t-1}(Z_{it}, r_t^h) = 0$ by the assumption that Z_{it} and Y_{kt} are independent conditional on information available at $t-1$. Using the property of joint lognormality, (32) becomes

$$\sigma_{c_t r_t^h} = \frac{nE_{t-1}(D_t^s)Cov_{t-1}(d_t^s, R_t^h) + \sum_{i=1}^n E_{t-1}(Y_{it})Cov_{t-1}(y_{it}, R_t^h)}{E_{t-1}(R_t^h)\sum_{i=1}^n E_{t-1}(C_{it})} \tag{33}$$

and (29) and (33) together imply

$$\sigma_{c_t r_t^h} = \frac{nE_{t-1}(D_t^s)Cov_{t-1}(d_t^s, V_{kt}) + \sum_{i=1}^n E_{t-1}(Y_{it})Cov_{t-1}(y_{it}, V_{kt})}{E_{t-1}(V_{kt})\sum_{i=1}^n E_{t-1}(C_{it})} \tag{34}$$

We then need to compute $Cov_{t-1}(d_t^s, V_{kt})$ and $Cov_{t-1}(y_{it}, V_{kt})$.[14]

$$\begin{aligned} Cov_{t-1}(d_t^s, V_{kt}) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{g} d^s(G - e^y) f(d^s, y) dy d(d^s) - E(d^s)E(V) \\ &= -Cov_{t-1}(d_t^s, y_t) e^{\mu_y + \frac{1}{2}\sigma_y^2} N\left(\frac{g-\mu_y}{\sigma_y} - \sigma_y\right) \end{aligned} \tag{35}$$

where $f(d^s, y)$ is the conditional joint pdf of d_t^s and y_{kt}, and $\sigma_{d^s y} = Cov_{t-1}(d_t^s, y_{kt})$. Similarly,

$$Cov_{t-1}(y_{it}, V_{kt}) = -Cov_{t-1}(y_{it}, y_{kt}) e^{\mu_y + \frac{1}{2}\sigma_y^2} N\left(\frac{g-\mu_y}{\sigma_y} - \sigma_y\right) \tag{36}$$

[14] Time and agent indexes are omitted in the following computation.

(34), (35) and (36) together imply

$$\alpha\sigma_{c_t r_t^h} = -\Phi(G_{kt})\eta_{kt} \tag{37}$$

where η_{kt} is given in (7) and $\Phi(G_{kt})$ is given in (9). Equations (37) and (31) together imply (8).

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