Nitrogen as a Capital Input and Stock Pollutant:  
A Dynamic Analysis of Corn Production and Nitrogen Leaching under Non-Uniform Irrigation

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Abstract
A spatially dynamic programming model of nonuniform irrigation is developed to investigate the nitrogen leaching problem associated with irrigated agriculture. We evaluate the importance of temporal and spatial elements in (i) appropriately modeling the interseasonal corn production problem with nitrogen carry-over and leaching under non-uniform irrigation, and (ii) in adequately evaluating alternative policy instruments for pollution control. Comparisons of the time profiles under spatially variable nitrogen levels arising from nonuniform irrigation are provided along with an evaluation of three different price-based policy instruments for reducing nitrogen leaching.

Keywords:  corn production, dynamic optimization, irrigated agriculture, nitrogen leaching, nonuniform irrigation, policy analysis, spatial analysis

JEL codes:  Q1, Q10, Q15

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Introduction

While irrigated agriculture has lead to large returns in agriculture relative to nonirrigated agriculture, especially in the western parts of the United States, nitrate concentrations in excess of maximum contaminate levels set by the USEPA are common in basins where irrigated agriculture is prominent (Vickner et al. 1998). This is not surprising given that in the more arid and semi-arid regions of the U.S., water is often applied in excess of plant requirements so as to leach the soils of salts. These excess flows percolating below the root zone contain both naturally occurring and applied elements such as nitrogen, and often either leach into underlying groundwater aquifers or enter streams, rivers, or lakes via subsurface drainage. Environmental and health concerns associated with excess nitrogen loadings in the U.S. has lead to a variety of regulations and mandates directed towards irrigated agriculture, including limits on fertilizer usage and nitrate concentrations in groundwater (Taylor et al. 1992; Shortle and Abler 2001).

Some prior research evaluating alternative policy instruments for reducing nitrogen pollution from irrigated agriculture has recognized the fact that nitrogen fertilizer is both a capital input and stock pollutant - i.e., applications today affect future yields and leaching.¹ For example, Johnson et al. (1991) integrate a plant simulator model, a two-state variable intra-seasonal dynamic optimization model, and a farm-level linear programming (LP) model to find the optimal crop mix under nitrate pollution restrictions. They find that the nitrate pollution from irrigated farms in Oregon can be reduced relatively inexpensively via changes in the intra-seasonal timing and application rates of nitrogen and water. More recently, Nkonya and Featherstone (2000) illustrate how accounting for the delayed effects of nitrogen, a stock pollutant, on groundwater contamination in Kansas can result in optimal fertilizer rates substantially less than rates from analyses that overlook this delay.
An aspect of the nitrogen leachate problem and irrigated agriculture that has received relatively scant attention is the spatial element arising from non-uniform irrigation. As noted in Knapp (1992), irrigation water is typically distributed nonuniformly over a field due to spatial variability in soil properties and irrigation technology characteristics. The result of this nonuniformity is spatially dependent crop yields, deep percolation flows, and, for the present application, nitrogen leaching. The importance of spatial considerations is illustrated in Taylor et al. (1992), who link the results from a 25-year simulation of crop production and nitrogen leaching to a LP model and evaluate how farm profits in Oregon are impacted by alternative nitrogen reduction policies. Results suggest that site specific factors, such as soil type, and the array of production possibilities are important factors in influencing policy effectiveness.

Vickner et al. (1998), alternatively, evaluate several policy options for controlling nitrate leaching from corn production in Colorado using an impressive two control variable dynamic model that treats nitrogen as both a capital input and stock pollutant while allowing for spatial variability in irrigation applications. Spatial variability is incorporated into the analysis by defining a fraction of a dimensionless field as over-irrigated, and the remaining fraction as under-irrigated. The quantity of applied water infiltrating into the root zone is a random normally distributed variable whose level relative to a predetermined biological requirement determines the fraction of the field that is under- and over-irrigated. They find that ignoring the direct relationship between level and variability of irrigation application (i.e., location and scale parameters) will understate the impact of any nitrate leaching abatement policy.²

The objectives of this paper, which are intended to both extend and expand upon the existing literature, are three-fold. First, we analyze the impacts of overlooking the dynamic nature of nitrogen fertilizer applications, both as a capital input and stock pollutant, on producer profits
and nitrogen leachate levels under uniform and nonuniform irrigation specifications. The time paths for the optimal decision rules - applied water and applied nitrogen rates - are estimated for both a period-by-period optimization (PP) routine and a present value dynamic optimization (PV) routine, where the former routine’s decision rules overlook the dynamic nature of nitrogen fertilizer today on soil nitrogen and nitrogen leaching in future periods. Soil nitrogen and nitrate leaching profiles are also shown. Second, the importance of irrigation nonuniformity on optimal decision rules as well as their impacts on steady-state soil nitrogen and nitrogen leaching levels is highlighted. The important link here is that if irrigation water is distributed nonuniformly over a field, which is likely the norm and not the exception, yields, soil nitrogen levels, and nitrogen leaching will be spatially variable as well.

Given that our results indicate that steady-state conditions are reached relatively quickly, and independent of initial soil nitrogen levels, we evaluate and compare the effectiveness of various policy instruments for reducing nitrate leaching under both optimization routines and irrigation uniformity assumptions. Optimal decision rules under baseline conditions that represent no regulatory action are compared to those under (i) a charge on nitrate emissions, (ii) a charge on nitrogen fertilizer applications, and (iii) a charge on water input applications. Results suggest that overlooking the dynamic elements of a capital input and stock pollutant can lead to decision rules characterized by lower levels of applied nitrogen and higher levels of applied water than would be optimal under an analysis that considers such elements. The consequences of these oversights include higher nitrogen leaching levels and a lower level of annual net benefit relative to optimal values estimated from a dynamic optimization framework. More importantly, though, is the treatment of irrigation system nonuniformity. Results suggest that overlooking the spatial variability in nitrogen leaching arising from nonuniform irrigation can lead to substantial
underestimates of the optimal input levels and nitrogen leaching. From a policy perspective, underestimating input or emissions levels result in poorly designed price-based instruments.

While our analysis focuses on a single crop and irrigation system, we both extend and depart from the existing literature in a number of directions. First, and similar to Vickner et al. (1998), we include two control variables, but incorporate nonuniformity in a manner closer to the approach found in literature focusing on the economics of irrigated agriculture and salinity management (Vickner et al. 1998; Knapp 1992). Additionally, by including both water and nitrogen applications as control variables, we account for the potential negative impacts of over-irrigation on soil nitrogen and, consequently, yield. While this relationship corresponds to observed data from the experimental plots (Tanji et al. 1979), it deviates from the assumptions maintained in prior research (e.g., Vickner et al. 1998, p. 402; Taylor, Adams, and Miller 1992, p. 175). Second, we use data from a two year plot-level field experiment where specific information on after-harvest nitrate leaching, average annual nitrogen concentration in the soil, and nitrogen uptake by the plant was provided under different fertilizer rates and irrigation applications (Tanji et al. 1979). Nonlinear least squares using theoretically justified functional specifications derived from the neural net literature was used to fit response surfaces to yield, uptake, leaching, and other sources of inorganic nitrogen loss (Gershenfeld 1999). This is one of the few studies, if not the first study, the authors are aware of that fit response functions to plot level data on nitrogen uptake, soil nitrogen, and nitrogen leaching. Third, we extend the literature by developing a field level model which considers nonuniform water applications which then drive spatial variability in the various nitrogen variables (e.g., soil nitrogen, leaching, uptake, and carryover).
Model

Consider irrigating a corn field using a particular irrigation technology over a finite horizon. At any particular point in the field, the amount of water that infiltrates into the root zone at time $t$ is given by

$$w_t[\beta] = \beta \bar{w}_i,$$

where $\bar{w}_i$ is the annual field average applied water depth, and $\beta \in [0, \infty]$ is the water infiltration coefficient. Hence, the annual amount of water infiltrating at any particular point on the field is some positive fraction of the average annual applied water depth for the field. Similar to the specifications in much of the research incorporating nonuniform irrigation systems (e.g., Feinerman, Letey, and Vaux 1983; Letey, Vaux, and Feinerman 1984; Dinar, Letey, and Knapp 1985), we assume that $\beta$ is spatially distributed over the field according to a log normal density function, $f(\beta)$, where $\text{E}(\beta) = 1$, and the standard deviation, $\sigma(\beta)$, varies by irrigation system. For our analysis, irrigation costs and nonuniformity are consistent with a furrow ½ mile system, $\sigma(\beta) = 0.3$ (University of California Committee of Consultants 1988). The standard deviation is a measure of the Christensen Uniformity Coefficient (CUC), and represents a measure of dispersion of applied water over the field and is calculated as one minus the average of the absolute percentage deviations of water from the mean (Knapp 1992).

Yield at any point within the field is a function of two inputs - infiltrated water and the level of nitrogen in the soil. If irrigation water is distributed nonuniformly over a field, yields, soil nitrogen levels, and nitrogen leaching will be spatially variable as well. The yield function, along with a system of nitrogen-related functions and relationships, at any point over the field can then be defined as
\[y_i[\beta] = \bar{y}_{\text{max}} \left( \frac{1}{1 + (25 + w_i[\beta])^{c_1}} \right) \left( \frac{1}{1 + (c_3 n_i^u[\beta])^{c_2}} \right)\]

(3) \[n_i^u[\beta] = n_{\text{max}}^u \left( \frac{1}{1 + [c_n (25 + w_i[\beta])]^{c_4}} \right) \left( \frac{1}{1 + (c_r n_i^r[\beta])^{c_5}} \right)\]

(4) \[n_i^r[\beta] = n_i^0[\beta] + n_i^u[\beta] - n_i^r[\beta]\]

(5) \[n_i^e[\beta] = \frac{\left( c_n (n_i^0[\beta] + n_i^u[\beta]) \right)}{1 + e^{-c_9 (w_i[\beta] - s_{90})}}\]

(6) \[n_i^c[\beta] = c_{11} + c_{12} (n_i^0[\beta] + n_i^0[\beta]) + c_{13} (n_i^u[\beta] + n_i^0[\beta])^2 + c_{13} (25 + w_i[\beta])\]

(7) \[n_{i,\text{ini}}[\beta] = n_i^0[\beta] - n_i^u[\beta] - n_i^r[\beta]\]

(8) \[n_i^0[\beta] = n_i^0\]

(9) \[n_i^u[\beta] = n_i^u\]

Parameters \(c_1\) though \(c_{13}\) are estimated parameters and will be discussed in more detail below. In equation (2), yield is specified as a function of maximum potential yield, \(\bar{y}_{\text{max}}\), infiltrated water, and plant uptake of nitrogen, \(n_i^u[\beta]\). Equation (3) specifies nitrogen uptake as a function of maximum potential plant uptake, \(n_{\text{max}}^u\), infiltrated water, and the level of nitrogen in the soil at that point, \(n_i^r[\beta]\). Equation (4) specifies soil nitrogen as a function of initial nitrogen at the beginning of the season, \(n_i^0[\beta]\), applied nitrogen, \(n_i^u[\beta]\), less nitrogen leaching from the soil, \(n_i^r[\beta]\). Equation (5) specifies nitrogen leaching as a function of initial soil nitrogen, along with applied nitrogen and infiltrated water. Nitrogen losses from such factors as denitrification and volatilization, \(n_i^r[\beta]\), is defined in equation (6) and specified as a function of initial soil nitrogen, applied nitrogen, and infiltrated water. Finally, equation (7) specifies the equation of
motion, and consists of what is typically referred to as carry over nitrogen (Segarra et al. 1989). Carryover nitrogen, $n_{i+1}^o [\beta]$, is calculated by mass balance - initial and applied nitrogen less that taken up by the plant or lost via leaching or through volatilization or denitrification. Equation (8) specifies initial field nitrogen in period 1 to be a constant at any point in the field while equation (9) suggests that nitrogen is applied uniformly across the field.

Equations (2) through (6) represent a unique system of equations that were estimated using nonlinear least squares$^4$ and which differ quite dramatically in a number of dimensions from the functions typically used in the literature. First, we adapt a hierarchical approach: component functions are estimated for yield, N-uptake, N-emissions, and other sources/sinks for inorganic N. This approach allows for yield-depressing effects, utilizes well-behaved functions in each case, and avoids the difficulties attendant to polynomials. We also borrow the idea of “activation” functions from the neural net literature (Gershenfeld 1999). These functions have theoretical justification, effectively bound the yield-enhancing range of water and nitrogen uptake while still allowing for a degree of substitution, have very desirable out-of-sample characteristics, and fit the data extremely well (e.g., $R^2$ measures above 0.99).

The plant-level production system was estimated for corn using a very unique and extensive data set (Tanji et al. 1979; Pang, Letey, and Wu 1997). This data set is very rich by providing direct observations on yield, uptake, and soil inorganic N, as well as computed levels of leachate N and mineralized N. This is one of the few, if not the only, analysis the authors are aware of that provides nitrogen leaching estimates from field experiments. These observations allow us the unique opportunity to estimate nitrogen leaching functions. The experimental data consisted of two years of corn field trials at a University of California-Davis site for the period October 1974 through September 1976.$^5$ The tests included varying nitrogen and water applications rates
on corn plots while measuring the resulting yields, residual inorganic N after harvest, and average annual N concentration in the soil solution; N leaching rates and mineralization of organic soil nitrogen were also computed.

Regarding the yield function specified in equation (2), previous research in irrigated agriculture and nitrogen economics has relied heavily on polynomials (Hexum and Heady 1978) and von-Liebig functions (Paris 1992). Polynomials often imply too much opportunity for substitution (Paris 1992), may have a very poor fit to derivatives within the data set despite high $R^2$ values, and exhibit very aberrant behavior outside the data. While Paris (1992) makes a strong case that data supports at least some von-Liebig behavior, data also exhibits declining yield in some instances, the result implying too little opportunity for substitution. Furthermore, recent formal analysis by Berck, Geoghegan, and Stohs (2000) suggests divergences from von-Liebig behavior.

The relationship between yield, N-uptake, N-leaching, and carryover nitrogen as a function of applied water, $w$, and soil nitrogen, $n$, are presented in figure 1, plots (a)-(d). The results, while mostly consistent with prior irrigated agricultural economics research, differ in the consequences associated with the interaction between soil nitrogen and applied water at what might be considered the upper levels of each. For example, in figure 1.a, for lower levels of soil nitrogen, excessive water application rates can result in decreased yields; reason being the additional water leaches the nitrogen out of the soil which ultimately impacts yield. This is illustrated in the amount of nitrogen available for plant uptake in figure 1.b for water application rates exceeding 50 cm. Figure 1.c illustrates that more nitrogen is leached out of the soil with excessive water application rates and thus less is also available as carryover into the next period (figure 1.d).
Field-level relationships for yield and nitrogen emissions can now be defined as:

\[
\bar{y}_t = \int_0^\infty y_t[\beta] f[\beta] \, d\beta
\]

(10)

\[
\bar{n}_t^e = \int_0^\infty n_t^e[\beta] f[\beta] \, d\beta
\]

(11)

Two control variables - applied water, \(\bar{w}_t\) (cm/yr), and applied nitrogen, \(\bar{n}_t^a\) (kg/ha) - are used and assumed to be chosen by a producer to maximize present value net benefits subject to production constraints, irrigation uniformity, and initial soil nitrogen conditions (i.e., the state variable). The problem can be represented as discrete-time, dynamic optimization problem:

\[
\text{Max } \pi = \sum_{t=0}^{T} [p_y \bar{y}_t - p_w \bar{w}_t - p_a \bar{n}_t^a - \kappa - p_e \bar{n}_t^e] (1 + r)^{-t}
\]

(12)

where the objective function in equation (16) represents present value net benefits to land and management ($/ha); \(t\) represents years where \(T\) is the planning horizon; \(r\) is the discount rate; \(p_y\), \(p_w\), and \(p_n\) are the unit prices of corn ($/ton), water ($/cm), and nitrogen ($/kg), respectively; \(\bar{y}_t\) is the corn yield (tons/ha); \(\kappa\) represents non-water and non-nitrogen related fixed costs associated with the cropping system; \(p_e\) represents the unit cost of nitrogen leaching ($/kg) when applicable; and \(\bar{n}_t^e\) is a variable representing nitrogen emissions/leaching (kg/ha).

Market prices and production cost data for corn production are derived from University of California Cooperative Extension Service Corn Crop Budgets for the Sacramento Valley (UCCE 2003), the location of the field experiment site. Production costs include all production costs (seed, land preparation, machinery, fertilizer, harvest, etc.) except those associated with water, nitrogen fertilizer, irrigation system, land and management, and cash overhead. Irrigation system data are generally from Posnikoff and Knapp (1997) with adjustment for inflation. Combined,
amortized nonwater production costs are estimated at $673 per acre yr\(^{-1}\). Baseline nitrogen fertilizer costs are $0.59/kg, while baseline water costs are $0.64 per centimeter. Maximum corn yield is 12.02 tons per acre, with a price of $102.02 per ton. All prices are in 2003 dollars.

The dynamic optimization problem associated with equations (1) to (12) is solved using a nonlinear optimization procedure from the GAMS/CONOPT solver system. The optimization problem is run under two different behavioral regimes - a period-by-period (PP) optimization regime and a present value (PV) optimization routine. The PP routine would be consistent with a producer that does not treat nitrogen as neither a capital input. The PV routine treats the nitrogen fertilizer as both a capital input and stock pollutant, with the present value calculated over a rolling horizon of 30 years, \(T\), assuming a 5% discount rate. Other discount rates were analyzed yet did not affect our results in any qualitative fashion.

For each behavioral regime, the implications of irrigation nonuniformity and its impact on the spatial variability of nitrogen levels (uptake, soil, and leaching) is investigated also by comparing the results from assuming a CUC of 70 (\(\sigma[\beta] = 0.3\)), with those results that overlook any nonuniformity, i.e., \(\sigma[\beta] = 0.6\).

**Time Profiles of Optimal Decision Rules**

We first consider and compare the time profiles of our optimal decision rules under both PP and PV optimization and for uniform and nonuniform irrigation systems. Figure 2 presents the optimal rates for applied nitrogen and water applications, along with the resulting soil nitrogen and leachate rates over a 30 year time horizon. The price of nitrogen emissions is set equal to zero, and initial nitrogen rates are set at 100 kg/hectare. Figure 3 presents a similar analysis, the difference being an emissions price on nitrogen leachate equal to \(\frac{1}{2}\) the fertilizer price. As
shown in figures (2a) through (2d), and figures (3a) through (3d), regardless of behavioral regime (PP vs. PV optimization) or irrigation system uniformity, steady-state levels are achieved quite quickly. This finding is consistent throughout regardless of initial parameter values, behavioral regimes, or irrigation uniformity.

Figures (2) and (3) suggest that despite irrigation system uniformity, PP optimization consistently results in lower optimal levels of applied nitrogen and soil nitrogen, and higher levels of applied water, than PV optimization. Interestingly, applied nitrogen in period 1 under PP optimization reflect steady-state nitrogen application rates under PV optimization; yet without valuing nitrogen carry-over from one period to another, PP optimization leads to lower nitrogen application rates thereafter. The only ambiguity that arises, as shown in figures (2d) and (3d), is whether the optimal level of nitrogen leaching under the PP routine is greater than or less than the optimal level under the PV routine. This ambiguity arises because, recalling equation (4), nitrogen emissions depend on both applied nitrogen rates and water application rates. When there is a cost imposed on nitrogen leaching, as incorporated in figure 3d relative to figure 2d, applied water rates drop substantially under PV optimization (relative to PP optimization), resulting in significantly less leaching. This result is not surprising since PP optimization overlooks the dynamic elements of nitrogen as a stock pollutant; hence when nitrogen emissions are priced PP optimality will likely result in nitrogen leaching rates that are greater than those rates estimated under the PV routine.

Figures 2 and 3 also illustrate that analyses that overlook the nonuniformity likely associated with irrigation will underestimate optimal input levels and leaching levels, and overestimate soil nitrogen levels, similar to the results found in Vickner et al. (1998; p. 404). The magnitude of these differences dissipates slightly as the price of nitrogen emissions increase relative to other
system prices. Perhaps the most noticeable artifact arising out of a comparison of these time
profiles is the importance of spatial variability to our estimated steady-state levels of nitrogen
leaching. The large relative differences between the uniform and nonuniform estimates of
nitrogen emissions is being driven by appreciable differences in steady-state water applications
with slight adjustments in applied nitrogen rates. In addition to highlighting the importance of
irrigation nonuniformity in explaining the excessive nitrogen leaching observed in irrigated
agriculture, these results emphasize the significance of incorporating both water and nitrogen as
complementary inputs into the production function in a flexible manner.

While figures 2 and 3 illustrate that the optimal decisions are quick to reach steady-state
values, the sensitivity of these values to initial nitrogen levels are analyzed in table 1, assuming
no price on nitrogen emissions ($p^e = 0$). As indicated, the optimal steady-state values of $N^*, W^*,
E^*, S^*$, and $Q^*$ are independent of the initial nitrogen level, which ranged from 100 to 400 kg/ha.
By accounting for the carry-over affect of nitrogen from one period to another, PV optimization
maintains a higher steady-state level $S^*$ than does PP optimization, resulting in higher $Q^*$ despite
a slightly lower level of $W^*$. Qualitatively, the impact of irrigation nonuniformity on the spatial
variability of soil nitrogen encourages a combination of applied nitrogen and water rates that lead
to considerably more nitrogen emissions than what would likely occur under uniform irrigation.

**Evaluating Model Parameters and Policy Instruments**

Tables 2, 3, and 4 present the optimal steady-state values of $W^*, N^*, S^*, Q^*, E^*$, and annual net
benefits ($ANB^*$) under a variety of model specifications for sensitivity analysis and policy evaluation.
Similar to table 1, each table presents the steady-state levels of these variables for both behavioral
regimes - period-by-period (PP) and present value (PV) optimality - under uniform and nonuniform
irrigation specifications.
Charge on Nitrogen Emissions

Table (2) presents the results associated with a charge on nitrogen emissions ranging from $0.20 to $1.00. The first row of table 2a and 2b presents the baseline results, which will also be used as a benchmark (e.g., the unregulated outcome) against which the impacts of alternative nitrogen reducing strategies can be investigated. The outcome of this charge would be consistent with a first best solution to reducing nitrogen emissions, with the loss in profits reflecting the expenditures the grower incurs on the remaining emissions.

The first noticeable characteristic of these tables is the difference in $N^*$ depending on irrigation uniformity. $N^*$ is at least 4.5 greater under nonuniform irrigation relative to uniform irrigation. These differences are a result of both larger $W^*$ and $N^*$ under the nonuniform system which gives rise to greater leaching and, consequently, a lower steady-state soil nitrogen level, $S^*$. As observed, yields are slightly lower under the nonuniform system, which, when combined with higher input levels of both $N^*$ and $W^*$ result in lower $ANB^*$.

Comparisons across behavioral regimes indicate that applied nitrogen rates are lower under the PP optimality routine, yet applied water rates are higher. The higher soil nitrogen levels under PV optimality more than compensate for the lower water rates in that yields, as well as annual net benefits, are consistently larger. Furthermore, by applying less water, fewer kilograms of nitrogen are leached out of the soil and can carry-over to next period.

Considering an emissions charge on nitrogen emissions, table 2 shows that as the charge is increased, $E^*$ is reduced through a combination of reduced $N^*$ and $W^*$, which in turn lead reductions in $Q^*$ and $ANB^*$. Soil nitrogen levels remain somewhat consistent, except for in the case of PV optimality under nonuniform irrigation were it is observed that the steady-state values of $S^*$ are very sensitive to changes in $N^*$ and $W^*$. Holding irrigation uniformity constant, the optimal steady-state levels of $N^*$ and $W^*$ are slightly lower and higher, respectively, under the
PP optimization routine relative to the PV optimization. Under PV optimization, the combination of maintaining higher soil nitrogen levels by applying less water and more nitrogen relative to the PP optimization results in slightly lower $E^*$ and slightly higher $ANB^*$.

Focusing on the last row of table 1a and 1b illustrates that the greatest reductions from increases in the emissions charge is in water applications. For instance, to achieve a 58% reduction in $E^*$ under the PV specification (table 1a, nonuniform irrigation), $W^*$ was reduced by 29% while $N^*$ was reduced by only 8%. Alternatively, under the uniform irrigation scheme and PV optimality, a 2% reduction in $E^*$ is achieved through reducing $W^*$ by 6%, yet $N^*$ by only 1%. Based on these results, the efficient approach to minimizing the impacts of the emissions charge is to reduce applied water rates by a greater percentage than applied nitrogen rates, the effect being less nitrogen leaching through the soil and more nitrogen remaining on the field.

**Nitrogen Input Charge**

Table 3 evaluates the sensitivity of the steady-state values to changes in the price of fertilizer from 10% to 50% of the baseline price of nitrogen fertilizer. The increase in fertilizer price could be considered equivalent to evaluating the consequences of a fertilizer charge in reducing N leaching. For an equal percentage increase in the nitrogen input charge, the steady-state solutions under PP optimality consistently lead to lower values of $N^*$, and higher values of $W^*$, than the steady state solutions under PV optimality. Under PP optimality, the combination of these input levels result in values of $S^*$, $Q^*$, and $ANB^*$ that are also lower than those values estimated from a model that considers the nitrogen fertilizer dynamics. The higher water rates lead to more leaching and thus $E^*$ is slightly larger under the period-by-period routine relative to the present value routine.
Greater differences arise, though, across the uniform and nonuniform irrigation specification, where steady-state values under the uniform assumption significantly underestimate optimal levels of $W^*$ and $N^*$, while overestimating $S^*$ and $ANB^*$, relative to the nonuniform case. Similar to previous tables, overlooking the nonuniformity in irrigation application and the resulting impacts on the spatial variability in soil nitrogen levels over time, leads to a gross underestimate of the resulting nitrogen leaching, as evidenced by comparing $E^*$ under the uniform and nonuniform specifications.

Regardless of behavioral regime or spatial assumption, the qualitative responses to the nitrogen input charge conform to theory - an increase in the price of $N$ fertilizer decreases the amount of nitrogen applied as well as nitrogen emissions. Large differences arise, though, when compared with the solutions under the nitrogen emissions charge (table 2). As shown in table 3a and 3b, the nitrogen input charge imposes a much greater percentage loss in ANB yet for a substantially less percentage reduction in nitrogen emissions relative to the emissions charge. For instance, comparing $E^*$ and $ANB^*$ under PV optimality and assuming a nonuniform irrigation system, a 16% reduction in emissions under the nitrogen input charge (table 3) results in a 35% reduction in $ANB^*$, yet under the nitrogen emission charge (table 2), a 58% reduction in $E^*$ results in a 13% reduction in $ANB^*$. The difference arises, and this is true for each of the specifications in tables 2 and 3, due to an overemphasis on reduction on the nitrogen input relative to the water input. The optimal strategy, as shown in table 2, is to decrease water applications by a greater percentage than nitrogen applications, the result being less nitrogen leaching out of the soil this period thereby leaving more nitrogen on the field for plant uptake in the current and future periods. The largest inefficiencies from the $N$ input charge seem to arise under the nonuniform case suggesting that models which overlook the spatial variability of
fertilizer usage, storage and leaching arising from nonuniform irrigation will grossly overestimate the ability of an input charge on nitrogen to achieve reductions in nitrogen leaching.

Water Input Charge

Table 4 analyzes the sensitivity of the steady-state levels of inputs and outputs to changes in the current price of irrigation water. Similar to table 3, input prices are increased from 10 to 50% of the baseline input rate. These changes could be considered equivalent to evaluating the consequences of a water charge on nitrogen leaching. Consistent with the previous figures and tables, the steady-state levels of $N^*$, $S^*$, $Q^*$, and $ANB^*$ are lower, and $W^*$ and $E^*$ are higher, under PP optimality relative to PV optimality. Again, greater differences arise across the uniform and nonuniform irrigation specification, where steady-state values under the uniform assumption significantly underestimate optimal levels of $W^*$, $N^*$, and $E^*$, while overestimating $S^*$, $Q^*$, and $ANB^*$ relative to the nonuniform case.

Regardless of behavioral regime or spatial assumption, the qualitative responses to the water input charge conform to theory - an increase in the price of water decreases water applications and nitrogen emissions. Furthermore, given the complementary relationship between $W^*$ and $N^*$, $N^*$ decreases with increases in the water charge. As shown in table 4, a 50% charge on water decreases emissions and annual net benefits between 28% to 38%, and 10% to 15%, respectively, depending on behavioral regime and irrigation uniformity. Notice though that while the charge has a very limited impact on $N^*$ across specifications, the difference in its impact on applied water rates is substantial depending on irrigation uniformity. For instance, a 50% increase in the price of water due to a charge leads to a mere 3% reduction in $W^*$ under a uniform assumption, but a 20% reduction in $W^*$ under the nonuniform assumption.
The potential efficiency of a water charge can be ascertained by comparing the results from table 4 with table 2 (emissions charge) and table 3 (nitrogen input charge). Comparisons with table 2 illustrate the inefficiency associated with the water charge. For instance, under PV optimality and nonuniform irrigation, a nitrogen emission charge can achieve a near 60% reduction in $E^*$ for a 13% reduction in $ANB^*$, while a 50% charge on water inputs achieves a 38% reduction in $E^*$ for a 14% reduction in $ANB^*$. In other words, for approximately the same loss in $ANB^*$, the emission charge achieves a reduction in $E^*$ that is 30% greater than achieved under the water charge. Similar results occur under alternative specifications. While applied water rates are reduced in a manner consistent with the emission charge, there is too little reduction in $N^*$ under the water charge relative to the emissions charge, the result being higher soil nitrogen levels. For any given level of $W^*$, higher soil nitrogen levels mean greater nitrogen leaching rates.

While the inefficiencies with the water charge may seem large, they are significantly less than those associated with the nitrogen charge. For example, under the PV optimality and nonuniform specification, the water charge achieves a 38% reduction in $E^*$ for a 14% reduction in $ANB^*$; alternatively, the nitrogen input charge achieves a much more modest 16% reduction in $E^*$ but for a 35% reduction in $ANB^*$. The difference lies in the fact that water applications are a crucial component to managing nitrogen leaching and under the $N$ input charge, there is very little reduction in water application rates relative to nitrogen application rates.

Conclusions

It is estimated that each year in the United States fertilizers add 8 billions pounds more nitrogen than are taken up by the plants on the field (Environmental Working Group 2005). Much of this
nitrogen, when transported off the field into nearby streams and lakes or when leached through the soil horizon in excessive amounts into groundwater, becomes pollutants and harmful to human health and the environment. A survey of nearly 200,000 water sampling records found that more than 2 million people nationwide drank water from systems in violation of federal nitrate standards. In California alone, where between 10 to 15% of California’s water supply wells exceed federal nitrate standards, more than 380,000 people have been reported to drink water in violation of these standards.

The results of research suggest two factors that may give rise to exorbitant nitrogen emissions from irrigated agriculture: (i) a decision making process that overlooks the dynamic elements of nitrogen both as a capital input and stock pollutant, and (ii) nonuniformity in irrigation applications that give rise to spatial variability in field nitrogen levels. The consequences of the first factor - period by period optimization - are lower nitrogen application rates and higher water application rates than would be optimal for maximizing present value net benefits. These higher water application rates leach additional nitrogen out of the soil leaving less for carryover into future periods. As shown, lower soil nitrogen levels translate into less plant uptake of nitrogen and, consequently, lower yields. From a research perspective, our results illustrate the potential consequences of overlooking the dynamic elements of this problem - i.e., lower levels of $N^*$, $S^*$, $Q^*$, and $ANB^*$, and higher levels of $W^*$ and $E^*$.

The consequences of overlooking the second factor - nonuniform irrigation applications - are shown to be much more severe. Overlooking irrigation nonuniformity and its resulting impact on the spatial variability in soil nitrogen levels leads to a gross underestimate of the resulting nitrogen leaching, as evidenced by comparing $E^*$ under the uniform and nonuniform specifications above. These gross underestimates occur not only because the estimated steady-
state value of both inputs under the uniform specification are lower than the optimal values from
the nonuniform specification, but also because water application rates are underestimated by a
greater percentage than nitrogen application rates, the result being lower predicted \( N \) emissions.

From a policy perspective, a nitrogen emissions charge of \$1\ that reduces steady-state annual
net benefits by approximately 13\% is shown to reduce nitrogen emissions by over 55\%. For the
same financial burden on the grower, a water input charge that increases the price of water by
50\% would result in a reduction in nitrogen emissions by only 38\%. By far the least efficient
charge evaluated was the nitrogen input charge. A charge that increased the price of nitrogen
fertilizer by 50\%, for instance, would lead to a substantial reduction in annual net benefits by
35\%, yet achieve only a 16\% reduction in nitrogen emissions. A comparison of the outcomes
under the two input charges relative to the nitrogen emission charge highlights the importance of
reducing water application rates by a greater percentage than nitrogen application rates to
achieve the efficient level of emissions. Furthermore, comparisons across behavioral
specification and irrigation uniformity assumptions stresses the importance of treating nitrogen
as both a capital input and stock pollutant with spatial variability; failure to do so might lead to
policy recommendations that deviate far from what might be efficient.

It might be considered unrealistic to consider an analysis of alternative policy instruments in
the context of a single field, a single crop, and a comparison of two assumptions associated with
the uniformity of a single irrigation system; yet such a simplification provides for a better
understanding of the implications of overlooking both the dynamic and spatial elements
associated with nitrogen leaching from irrigated agriculture. Without an appreciation of how
such oversight or simplification can impact estimates of the optimal levels of nitrogen and
fertilizer applications for a single crop and irrigation system, policy recommendations based on
results from larger scale models that, while perhaps including more crops or irrigation systems, overlook these elements may be seriously flawed.

Finally, our results highlight the importance of including both water and nitrogen inputs as control variables in analyses intended to evaluate the impact of alternative policy instruments to control nitrogen emissions. Using a unique data set that contained information and data on field level nitrogen emissions, nitrogen leaching, and carryover nitrogen from corn production, a unique system of responses functions whose flexibility and continuity were consistent with real world observations.
Endnotes

1 Segarra et al. (1989) was one of the first to analyze nitrogen fertilizer as a capital input in irrigated agriculture and showed that period-by-period optimization that ignore nitrogen carry-over lead to suboptimal nitrogen application levels for cotton production in Texas.

2 Chiao and Gillingham (1989) also combine a dynamic model of fertilizer applications and nonuniformity in fertilizer application to evaluate the value of increasing irrigation uniformity and the cost of being wrong. Their focus is the use of phosphorous in New Zealand and do not address the stock pollutant aspect of this problem.

3 Our analysis does not consider risk averse attitudes of growers who might confront stochastic events or uncertainty surrounding irrigation uniformity. Previous studies that investigate these issues include Lambert (1990), who stresses the importance of accounting for risk aversion in the presence of price and yield uncertainty in efforts to control the use of nitrogen fertilizer on multiple crops in Arizona, and Choi and Feinerman (1995), who evaluate the relative attractiveness of chargees versus quotas in the presence of risk adverse attitudes and focus on nitrogen leaching from wheat farmers in Israel, and Teague, Bernardo, and Mapp (1995) who use a farm-level model to evaluate income/environmental risk tradeoffs.

4 We use a third party global solver within the Mathematica software to estimate these functions.

5 While the field trials are nearly 20 years old, we see no reason why this would affect our analysis or conclusions.

6 Nonwater production costs, included irrigation system capital costs, are assumed constant regardless of irrigation system uniformity to better illustrate the importance of spatial variability.
References


Table 1. Baseline Steady-State Values by Initial Soil Nitrogen Level

(a.) Present Value (PV) Optimization

<table>
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<th></th>
<th></th>
<th></th>
<th>Nonuniform</th>
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(b.) Period-by-period (PP) Optimization

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W—applied water (cm); N—applied nitrogen (kg/ha); S—soil nitrogen (kg/ha); Q—yield (tons/ha); E—nitrogen emissions (kg/ha)
Table 2. Optimal Steady-state Values under a Nitrogen Emissions Charge

(a.) Present Value (PV) Optimization

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(b.) Period-by-period (PP) Optimization

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% Change from Baseline:  6%  3%  -  -  58%  2%  25%  9%  -  -  50%  13%

$W$~applied water (cm); $N$~applied nitrogen (kg/ha); $S$~soil nitrogen (kg/ha); $Q$~yield (tons/ha); $E$~nitrogen emissions (kg/ha); ANB~annualized net benefits
Table 3. Optimal Steady-state Values under a Nitrogen Input Charge

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(b.) Period-by-period (PP) Optimization

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$W$—applied water (cm); $N$—applied nitrogen (kg/ha); $S$—soil nitrogen (kg/ha); $Q$—yield (tons/ha); $E$—nitrogen emissions (kg/ha); $ANB$—annualized net benefits
Table 4. Optimal Steady-state Values under a Water Input Charge

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(b.) Period-by-period (PP) Optimization

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<td>10%</td>
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<td>170</td>
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<td>$183$</td>
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<td>189</td>
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<td>% Change from Baseline</td>
<td>3%</td>
<td>1%</td>
<td>-</td>
<td>-</td>
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<td>4%</td>
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<td>32%</td>
<td>15%</td>
</tr>
</tbody>
</table>

$W$~applied water (cm); $N$~applied nitrogen (kg/ha); $S$~soil nitrogen (kg/ha); $Q$~yield (tons/ha); $E$~nitrogen emissions (kg/ha); ANB~annualized net benefits
Figure 1. Production Relations for Plant-Level Water-Nitrogen Product Functions
Figure 2. Period by Period (PP) vs. Present Value (PV) Optimization under Uniform (U) and Nonuniform (NU) Irrigation ($P_c = 0$).
Figure 3. Period by Period (PP) vs. Present Value (PV) Optimization under Uniform (U) and Nonuniform (NU) Irrigation ($P_c = 0.247$).