Empirical Tests of the Refutable Implications of Expected Utility Maximization under Risk

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Abstract:

The curvature properties of the indirect utility function imply a set of refutable implications in the form of comparative static results and symmetric relations for the competitive firm operating under uncertainty. These hypotheses, first derived and empirically tested under output price uncertainty by Saha and Shumway (1998), are extended in this paper to the more general case of both price and quantity uncertainty and result in an important theoretical finding. Empirical tests using a panel of state-level observations fail to reject most refutable hypotheses under output price and output quantity risk, but symmetry conditions implied by a twice-continuously-differentiable indirect utility function are rejected. Two restrictive risk preference hypotheses are also rejected. At individual observations, data were consistent with most of the hypotheses implied by individual states acting as though they were expected utility-maximizing firms.

Key words: indirect utility function, refutable implications, risk and uncertainty

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I. Introduction

Because of the long time periods between commitment of resources and generation of marketable output in production agriculture, a high level of uncertainty is associated with many production decisions. Consequently, economists concerned about decision making in production agriculture have had a long history of considering the impact of risk and uncertainty.

Building on the early work of Sandmo (1971) and Batra and Ullah (1974), who developed the theory of the competitive firm under output price uncertainty, agricultural economists have examined firm operations under various sources of uncertainty. The pioneering work of Pope (1980) derived various testable hypotheses expressed in symmetry and homogeneity results under constant absolute risk aversion and price uncertainty. His symmetry results proved simple enough for empirical application under certain classes of utility functions (Antonovitz and Roe, 1986). Chavas and Pope (1985) extended Pope's work by examining price uncertainty within a general risk preference framework which facilitated empirical tests of firm behavior under the expected utility hypothesis. Paris (1988), Paris, Caputo and Holloway (1993), and Adrangi and Raffiee (1999) derived testable implications within a comparative statics framework for the competitive firm operating under price uncertainty. Saha and Shumway (1998) derived refutable implications from the first-order and second-order curvature properties of the dual function under output price uncertainty and empirically tested each postulate for Kansas wheat producers.

The purpose of this paper is to: (a) extend the previous theoretical work by careful derivation of refutable and testable implications of the indirect utility function under both output price and quantity risk, (b) demonstrate that one previously maintained hypothesis is not a necessary condition for the derived implications, and (c) empirically test the derived implications as well as a set of hypotheses about the nature of risk aversion practiced by producers.

The remainder of this paper is organized as follows. Section II gives a brief overview of the behavioral theory implied by the curvature properties of the indirect utility function and derives a set of testable hypotheses. An empirical application follows along with a description of the data. Conclusions are presented in the last section.

II. The Theoretical Model

Traditionally, the introduction of price uncertainty into the theory of the competitive firm has been approached within an expected utility framework. The seminal works of Arrow (1965) and Pratt (1964) defined preferences of expected utility-maximizing decision makers over final wealth. Despite their unambiguous reference to final wealth, much of the analysis of risk taking behavior of agricultural producers, beginning with Sandmo (1971), has used profit rather than wealth as the argument of utility (Meyer and Meyer, 1998). Profit is the appropriate argument only if sources of wealth other than profit are nonrandom and held fixed. Since we do not wish to impose nonrandom constraints on other sources of wealth, we use wealth as the argument of utility in the following theoretical model. Therefore, the firm is assumed to maximize its expected utility of random wealth.

Following Feder (1977) and Saha and Shumway (1998), we assume that a competitive firm's random wealth $\tilde{W}$ can be structured as a nonrandom part $Z(\cdot)$, a random component $S(\cdot)$, and nonrandom initial (beginning of period) wealth endowment I:

(1) $$\tilde{W} = Z(\boldsymbol{x};\beta,\cdot) + S(\boldsymbol{x};\tilde{\varepsilon};\cdot) + \mathrm{I},$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)'$ is an $n \times 1$ vector of decision variables, $\tilde{\varepsilon}$ is a random variable vector, β is a parameter vector, and · denotes the additional parameters concealed in $Z(\cdot)$ and $S(\cdot)$. The parameters, β, only enter the nonrandom part of wealth, $Z(\cdot)$, but not the random part $S(\cdot)$. Although we later demonstrate that it is unnecessary for our refutable implications to hold under output price and output quantity risk, we initially maintain the standard expectation:

(2) $$\mathrm{E}[S(\boldsymbol{x};\tilde{\varepsilon};\cdot)] = 0,$$

where E denotes the expectation operator.

Conditional on twice-differentiable functions of Z and S, the expectation of random wealth defined by (1) and (2) can be written as:

(3) $$\begin{aligned} \bar{W} &= \mathrm{E}(\tilde{W}) \\ &= Z(\boldsymbol{x};\beta,\cdot) + \mathrm{I} + \mathrm{E}[S(\boldsymbol{x};\tilde{\varepsilon};\cdot)] \\ &= Z(\boldsymbol{x};\beta,\cdot) + \mathrm{I}. \end{aligned}$$

Refutable Implications of the Indirect Utility Function

For a competitive firm whose objective is to maximize the expected utility of random wealth specified by (1), the indirect utility function is defined by:

(4) $V(\beta; \mathrm{I}, \cdot) = Max\{\mathrm{E}[U(Z(\boldsymbol{x}; \beta, \cdot) + S(\boldsymbol{x}; \tilde{\varepsilon}; \cdot) + \mathrm{I})]\}$,

where $U(\cdot)$ represents the von Neumann Morgenstern utility function, which is increasing in wealth, therefore is increasing in nonrandom part of wealth, $Z(\boldsymbol{x}; \beta, \cdot)$. Let $\boldsymbol{x}^*(\beta, \mathrm{I}, \cdot)$ denote the optimal input variables which are determined by (4). Under the assumptions of (1) and (2), the indirect utility function defined by (4) implies the following propositions (Saha and Shumway, 1998):

Proposition 1: The indirect utility function defined by (1) has the following first-order curvature properties:

(i) Increasing in I,

(ii) Increasing (decreasing) in β if Z is increasing (decreasing) in β.

Proposition 2: The second-order curvature properties of the indirect utility function indicate:

(i) V quasiconvex in β and I if Z is convex in β,

(ii) V quasiconvex in β and I $\Leftrightarrow \Omega$ symmetric and positive semidefinite (SPSD), where $\Omega \equiv Z_{\beta\beta} + Z_{\beta\boldsymbol{x}}\{\boldsymbol{x}_\beta^* - \boldsymbol{x}_\mathrm{I}^* Z_\beta\}$.[1]

Corollary: Under risk neutrality or CARA, $\boldsymbol{x}_\mathrm{I}^*=0$, and Z convex in β $\Leftrightarrow Z_{\beta\beta} + Z_{\beta\boldsymbol{x}}\boldsymbol{x}_\beta^*$ is SPSD.

Obviously, $V(\beta; \mathrm{I}, \cdot)$ is increasing in I. Proposition 1(ii) indicates that the first-order curvature properties of the indirect utility function corresponding to β can be revealed by the first-order curvature characters of the nonrandom part of wealth $Z(\boldsymbol{x}; \beta, \cdot)$. Proposition 2(i) implies the fundamental second-order curvature properties of the indirect

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[1] The following notation is used throughout this paper: h_x denotes the partial derivative of $h(\cdot)$ with respect to x, h_{xy} represents the Hessian matrix whose ij^{th} element is $\partial^2 h / \partial x_i \partial y_j$, where $h(\cdot)$ is a real-value function of vectors x and y.

utility function which can be explored by observing the properties of the second-order curvature of $Z(\boldsymbol{x}; \beta, \cdot)$. By proposition 2(i), $V(\beta; \mathrm{I}, \cdot)$ is quasi-convex in β if Z is convex in β. This property implies and is implied by the testable postulates contained in proposition 2(ii). In proposition 2(ii), the symmetric and positive semi-definite (SPSD) matrix, Ω, which contains the comparative static and reciprocity results demonstrating the firm behaviors, includes the complete set of the refutable implications for the competitive firm under risk. Most importantly, propositions 1 and 2 do not rely on specific forms of $U(\cdot)$ that would otherwise impose an explicit risk preference (Love and Buccola, 1991; Saha, Shumway and Talpaz, 1994). When combined with the empirically testable curvature properties of $Z(\boldsymbol{x}; \beta, \cdot)$, they allow us to test the behavioral postulates without assuming a specific functional form for the indirect utility function.

These refutable propositions derived by Saha and Shumway (1998) have been empirically tested only under output price uncertainty. One important theoretical contribution of this paper, the importance of which will be explained in the next section, is to demonstrate that the propositions hold even without assumption (2). From the proof in Saha and Shumway (1998), it is obvious that proposition 1 and proposition 2(ii) aren't conditioned on assumption (2), and all that is needed for them to hold is assumption (1). We refer readers to Saha and Shumway (1998) for the details. Before proving that proposition 2(i) holds without assumption (2), we claim the following result.

Claim. The firm's optimization problem defined in (4) is equivalent to a constrained optimization problem where $\boldsymbol{x}$ and $\bar{W}$ are jointly chosen. Defining $\mathbf{k} = \{\boldsymbol{x}, \bar{W}\}$ and $\boldsymbol{\lambda} = \{\beta, \mathrm{I}\}$, then:

(5) $V = \max_{\boldsymbol{x}} \ \mathrm{E}U[Z(\boldsymbol{x};\beta,\cdot) + S(\boldsymbol{x};\tilde{\varepsilon};\cdot) + \mathrm{I}]$

$$\Leftrightarrow V = \max_{\mathbf{k}} \{ \ \mathrm{E}U[\bar{W} + S(\boldsymbol{x};\tilde{\varepsilon};\cdot) - \mathrm{E}(S(\boldsymbol{x};\tilde{\varepsilon};\cdot))] \mid \bar{W} \le Z(\boldsymbol{x};\beta,\cdot) + \mathrm{E}[S(\boldsymbol{x};\tilde{\varepsilon};\cdot)] + \mathrm{I}\}.$$

Proof: First, we demonstrate that the constraint, $\bar{W} \le Z(\boldsymbol{x};\beta,\cdot) + \mathrm{E}[S(\boldsymbol{x};\tilde{\varepsilon};\cdot)] + \mathrm{I}$, will be binding for all optimal values of $\bar{W}$ and $\boldsymbol{x}$. Suppose the constraint is not binding, then there must exist some parameter values $\mathbf{k}^0 = \{\boldsymbol{x}^0, \ \bar{W}^0\}$ and $\boldsymbol{\lambda}^0 = \{\beta^0, \ \mathrm{I}^0\}$ such that $\mathbf{k}^0 = \{\boldsymbol{x}^0, \ \bar{W}^0\}$ and $\boldsymbol{\lambda}^0 = \{\beta^0, \ \mathrm{I}^0\}$ maximize the indirect utility, given by (5), with the following condition

(6) $\bar{W}^0 < Z(\boldsymbol{x}^0;\beta^0,\cdot) + \mathrm{E}[S(\boldsymbol{x}^0;\tilde{\varepsilon};\cdot)] + \mathrm{I}^0$.

Therefore, there exists some $\bar{W}' > \bar{W}^0$ such that

(7) $\bar{W}' = \mathrm{E}W' = Z(\boldsymbol{x}^0;\beta^0,\cdot) + \mathrm{I}^0 + \mathrm{E}S(\boldsymbol{x}^0;\tilde{\varepsilon};\cdot)$,

which implies $\{\boldsymbol{x}^0, \ \bar{W}'\}$ is feasible.

Since the utility function is increasing in wealth, we have

(8) $\mathrm{E}U(\bar{W}' + S(\boldsymbol{x}^0;\tilde{\varepsilon};\cdot) - \mathrm{E}[S(\boldsymbol{x}^0;\tilde{\varepsilon};\cdot)]) > \mathrm{E}U(\bar{W}^0 + S(\boldsymbol{x}^0;\tilde{\varepsilon};\cdot) - \mathrm{E}[S(\boldsymbol{x}^0;\tilde{\varepsilon};\cdot)])$,

which contradicts the fact that $\mathbf{k}^0 = \{\boldsymbol{x}^0, \ \bar{W}^0\}$ and $\boldsymbol{\lambda}^0 = \{\beta^0, \ \mathrm{I}^0\}$ maximize the indirect utility. Thus, the constraint is binding for all optimal values of $\mathbf{k}$ and $\boldsymbol{\lambda}$, and the claim is proved by substituting the binding constraint $\bar{W} = \mathrm{E}W = Z(\boldsymbol{x};\beta,\cdot) + \mathrm{I} + \mathrm{E}S(\boldsymbol{x};\tilde{\varepsilon};\cdot)$ into (5).

With claim 1 proven, we can now prove that proposition 2(i) is implied by assumption (1). Let $H(\mathrm{k},\boldsymbol{\lambda}) = \bar{W} - \mathrm{Z}(\boldsymbol{x};\beta,\cdot) - \mathrm{E}S(\boldsymbol{x};\tilde{\varepsilon};\cdot) - \mathrm{I}$, which is non-positive. Then (5) is equivalent to the following expression:

(9) $V(\mathbf{k},\cdot) = \max_{\mathbf{k}} \{ \ \mathrm{E}U[\bar{W} + S(\boldsymbol{x};\tilde{\varepsilon};\cdot) - \mathrm{E}(S(\boldsymbol{x};\tilde{\varepsilon};\cdot))] \mid H(\mathbf{k},\boldsymbol{\lambda}) \le 0\}$.

If $Z(\boldsymbol{x};\beta,\cdot)$ is convex in β, $Z_{\beta\beta} \geq 0$ and $-Z_{\beta\beta} \leq 0$. The Hessian matrix of $H(\mathbf{k},\boldsymbol{\lambda})$ with respect to β and I is

(10) $$D = \begin{bmatrix} \frac{\partial^2 H}{\partial \beta^2} & \frac{\partial^2 H}{\partial \beta \partial \mathrm{I}} \\ \frac{\partial^2 H}{\partial \mathrm{I} \partial \beta} & \frac{\partial^2 H}{\partial \mathrm{I}^2} \end{bmatrix} = \begin{bmatrix} -Z_{\beta\beta} & 0 \\ 0 & 0 \end{bmatrix}.$$

Let $\boldsymbol{\lambda}'$, $\boldsymbol{\lambda}''$ and $\bar{\boldsymbol{\lambda}}$ be any feasible vectors such that $\bar{\boldsymbol{\lambda}} = t\boldsymbol{\lambda}' + (1\text{-}t)\,\boldsymbol{\lambda}''$, $0 \leq t \leq 1$, and $\bar{\mathrm{k}}$ denotes the optimal vector corresponding to $\bar{\boldsymbol{\lambda}}$. Under the conditions $-Z_{\beta\beta} \leq 0$ and $|D| = 0$, D is negative semi-definite, which implies $H(\mathbf{k},\boldsymbol{\lambda})$ is quasiconcave in $\boldsymbol{\lambda} = (\beta, \mathrm{I})$. Therefore, the following inequality holds:

(11) $$\min\{H(\bar{\mathbf{k}},\boldsymbol{\lambda}'),\ H(\bar{\mathbf{k}},\boldsymbol{\lambda}'')\} \leq H(\bar{\mathbf{k}},\bar{\boldsymbol{\lambda}}) \leq 0,$$

which is sufficient to ensure that either $H(\bar{\mathbf{k}},\boldsymbol{\lambda}') \leq 0$ or $H(\bar{\mathbf{k}},\boldsymbol{\lambda}'') \leq 0$ or both. Therefore,

(12) $$V(\bar{\boldsymbol{\lambda}},\cdot) \leq \max\{V(\boldsymbol{\lambda}',\cdot),\ V(\boldsymbol{\lambda}'',\cdot)\}.$$

By definition, the inequality in (12) implies that $V(\cdot)$ is quasiconvex in $\boldsymbol{\lambda}$.

Testable Hypotheses

Consider a firm's production function that has the following general form:

(13) $$\tilde{\boldsymbol{Y}} = f(\boldsymbol{x}) + \varepsilon_Y,$$

and random price denoted by:

(14) $$\tilde{\boldsymbol{P}} = \bar{\boldsymbol{P}} + \varepsilon_P,$$

where $\tilde{\boldsymbol{Y}}$ is random output quantity; $f(\boldsymbol{x})$, a function of input vectors $\boldsymbol{x}$, is called the mean output function; $\tilde{\boldsymbol{P}}$ denotes random price; $\bar{\boldsymbol{P}}$ is the mean of price; ε_Y and ε_P are stochastic terms which represent random production shock and random price shock respectively; $\mathrm{E}(\varepsilon_Y) = 0$ and $\mathrm{E}(\varepsilon_P) = 0$. Letting $\mathbf{r} = \{r_1, \ldots, r_n\}'$ be the price vector of inputs, random wealth under output price and output quantity uncertainty will be:

(15) $$\tilde{W} = \tilde{\boldsymbol{P}} \cdot \boldsymbol{Y} - \mathbf{r} \cdot \boldsymbol{x} + \mathrm{I} = \overline{\boldsymbol{P}} f(\boldsymbol{x}) + \overline{\boldsymbol{P}} \cdot \varepsilon_Y + \varepsilon_P \cdot f(\boldsymbol{x}) + \varepsilon_P \cdot \varepsilon_Y - \mathbf{r} \cdot \boldsymbol{x} + \mathrm{I}.$$

In terms of the notation in the preceding section, **r** corresponds to β, the nonrandom part of wealth is:

(16) $$Z(\boldsymbol{x}; \mathbf{r}, \cdot) = \overline{\boldsymbol{P}} \cdot f(\boldsymbol{x}) - \mathbf{r} \cdot \boldsymbol{x} + \mathrm{I},$$

and the random component of wealth is:

(17) $$S(\boldsymbol{x}; \tilde{\varepsilon}; \cdot) = \overline{\boldsymbol{P}} \cdot \varepsilon_Y + \varepsilon_P \cdot f(\boldsymbol{x}) + \varepsilon_{\mathrm{P}} \cdot \varepsilon_Y.$$

Therefore, $\mathrm{E}[S(\boldsymbol{x}; \tilde{\varepsilon}; \cdot)] = \mathrm{E}[\overline{\boldsymbol{P}} \cdot \varepsilon_Y + \varepsilon_P \cdot f(\boldsymbol{x}) + \varepsilon_P \cdot \varepsilon_Y] = \mathrm{E}(\varepsilon_P \cdot \varepsilon_Y)$. Under the assumption of no correlation between output prices and quantities, $\mathrm{E}(\varepsilon_P \cdot \varepsilon_Y) = 0$ and thus $\mathrm{E}[S(\boldsymbol{x}; \tilde{\varepsilon}; \cdot)] = 0,$ which is consistent with assumption (2).

For an individual firm operating in a competitive market, $\mathrm{E}(\varepsilon_P \cdot \varepsilon_Y) = 0$ because the firm's decisions cannot affect the general equilibrium of the market. However, much empirical analysis, including ours, uses data for aggregates of firms. Sometimes that is for convenience and other times it is necessary because essential firm-level data don't exist. Even though the decisions of individual price-taking firms can't affect the market equilibrium, the collective decisions of many firms can. Thus, since we have demonstrated that assumption (2) is unnecessary for any of the previous implications to hold, it is clear that we can make use of aggregate data, if necessary, to conduct empirical tests of both propositions.

With random wealth under output price and output quantity uncertainty defined as in equations (15), (16) and (17), the indirect utility function becomes:

(18) $$V(\mathbf{r};\mathrm{I},\cdot) = Max\{\mathrm{E}[U(Z(\boldsymbol{x};\mathbf{r},\cdot) + S(\boldsymbol{x};\tilde{\varepsilon};\cdot) + \mathrm{I})]\}.$$

By proposition 1(ii), the firm's indirect utility function, V(**r**; I, ·), is decreasing in **r** since the firm's expected profit, i.e., a nonrandom portion of wealth, decreases in **r**. Applying the envelope theorem to (16), proposition 1(ii) can thus be translated to the following:

(19) $$V_{\mathbf{r}} \overset{S}{=} Z_{\mathbf{r}} = -\boldsymbol{x}^* < 0,$$

where $\overset{S}{=}$ denotes 'same sign as'. The result in (19) is the first-order curvature property of the indirect utility function. It indicates that, as input prices increase, the terminal wealth of the producer diminishes and leads to a decrease in the utility of final wealth. By again applying the envelope theorem, $Z_{\mathbf{rr}} = -\boldsymbol{x}_{\mathbf{r}}^*$ and $Z_{\mathbf{r}\boldsymbol{x}}$ is a negative identity matrix. Thus, we have:

(20) $$\begin{aligned}\Omega &\equiv Z_{\mathbf{rr}} + Z_{\mathbf{r}\boldsymbol{x}}\{\boldsymbol{x}_{\mathbf{r}}^* - \boldsymbol{x}_{\mathrm{I}}^* Z_{\mathbf{r}}\} \\ &= \boldsymbol{x}_{\mathrm{I}}^* Z_{\mathbf{r}} - 2\boldsymbol{x}_{\mathbf{r}}^* \\ &= -(\boldsymbol{x}_{\mathrm{I}}^*\boldsymbol{x}^* + 2\boldsymbol{x}_{\mathbf{r}}^*),\end{aligned}$$

since $Z_{\mathbf{r}} = -\boldsymbol{x}^*$. Using this result, the second-order curvature result of proposition 2(ii) translates to:

(21a) $V(\mathbf{r};\ \mathrm{I}, \cdot)$ quasiconvex in **r** and I $\Leftrightarrow \Omega \equiv -(\boldsymbol{x}_{\mathrm{I}}^*\boldsymbol{x}^* + 2\boldsymbol{x}_{\mathbf{r}}^*)$ is SPSD,

which implies the following matrix is symmetric negative semidefinite:

(21b) $$\Psi = \boldsymbol{x}_{\mathrm{I}}^*\boldsymbol{x}^* + 2\boldsymbol{x}_{\mathbf{r}}^*.$$

Specifically, when there are three input variables, (21b) can be rewritten as:

$$\text{(21c)} \qquad \begin{bmatrix} 2\cdot x_{1r1}^{*}+x_{1I}^{*}\cdot x_{1}^{*} & 2\cdot x_{1r2}^{*}+x_{1I}^{*}\cdot x_{2}^{*} & 2\cdot x_{1r3}^{*}+x_{1I}^{*}\cdot x_{3}^{*} \\ 2\cdot x_{2r1}^{*}+x_{2I}^{*}\cdot x_{1}^{*} & 2\cdot x_{2r2}^{*}+x_{2I}^{*}\cdot x_{2}^{*} & 2\cdot x_{2r3}^{*}+x_{2I}^{*}\cdot x_{3}^{*} \\ 2\cdot x_{3r1}^{*}+x_{3I}^{*}\cdot x_{1}^{*} & 2\cdot x_{3r2}^{*}+x_{3I}^{*}\cdot x_{2}^{*} & 2\cdot x_{3r3}^{*}+x_{3I}^{*}\cdot x_{3}^{*} \end{bmatrix}.$$

Equations (19) and (21a)-(21c) reveal that the propositions imply a set of testable hypotheses associated with the input responses of the firm operating under output price and output quantity uncertainty. Therefore, the propositions implied by the indirect utility function can be empirically tested by imposing parameter restrictions on a firm's demand equations.

III. Empirical Application

The Data Set

Because we lack essential data to conduct tests of these propositions for a broad cross-section of individual U.S. firms, the above methodology was applied to annual state-level data for the period, 1960-1999.[2] The major data source was the ERS annual agricultural output and input series for each of the contiguous 48 states for the period 1960-1999 (Ball, 2002). This high-quality aggregate data set includes a comprehensive inventory of agricultural output and input prices and quantities compiled using theoretically and empirically sound procedures consistent with a gross output model of production (see Ball et al., 1999, for details). The data set includes three output groups (crops, livestock, and secondary outputs) and four input groups (materials, capital, labor,

[2] The theory of the expected utility maximization applies to the individual, in this case the individual firm. Although tests of utility maximization have not been reported for state-level data, Lim and Shumway (1992) failed to reject the hypothesis that each of the states acted as though they were profit-maximizing firms. They used nonparametric testing procedures on annual data for the period 1956-1982, which overlaps with the first 23 years of our data period.

and land). Initial stock of wealth I was proxied by equity, or "net worth", which measures farm business assets minus farm business debt. These data for each state were taken from the *Farm Balance Sheets* (USDA/ERS).

Lagged output prices were used as proxies for expected output prices. Lagged equity was used as a proxy for initial (beginning period) wealth. To partially mitigate the effects of trending and autocorrelated data, expected output prices, equity, and current input prices were normalized by the price of land. To reduce heteroskedasticity and to permit estimation of identical non-intercept coefficients for all states in the panel data set, input quantities and normalized equity were scaled by the quantity of land.[3]

Econometric Model

Without maintaining any additional hypotheses about the input demand equations, we used a quadratic (second-order Taylor-series expansion) functional form to approximate the input demand framework. Input demand equations for materials/land, capital/land, and labor/land were each estimated as a fixed-effects panel data model:

$$(22) \qquad x_j = \mathbf{d}\alpha_j + \mathbf{z}\phi_j + 0.5\mathbf{z}\Gamma_j\mathbf{z}' + \delta_{1j}t + 0.5\delta_{2j}t^2 + e_j, \;\; j = 1,2,3$$

where x_j is the quantity of the j^{th} input measured as input per unit of land; **d** is the vector of state dummy variables; the vector $\mathbf{z} = \{p_1, p_2, p_3, r_1, r_2, r_3, I\}$ contains lagged output prices p_i (for crops, livestock, and secondary outputs), current input prices r_j (for materials, capital, and labor), and lagged farm equity per unit of land I, each normalized by the price of land; the quadratic form of the time variable t=1, . . ., 40, is used as a proxy for technological innovations; the error term is denoted by e_j; parameters to be estimated are the vectors α_j, ϕ_j, Γ_j, and the scalars δ_{1j}, δ_{2j}.

[3] Significant (5% level) groupwise heteroskadasticity was still found in the scaled data.

For each individual equation in the demand system specified by (22), fixed effects across cross-sectional observations were considered. So that all refutable implications under output price and output quantity risk contained in (19) and (21a)-(21c) could be tested, no restrictions were imposed on the estimated parameters across the equations.

Empirical Results

We first tested for a 1st-order autoregressive (AR(1)) process in the error terms for each input demand equation defined in (22). Evidence of an AR(1) process was found in each equation with Durbin-Watson test statistics of 0.311, 0.317, and 0.674, respectively, for the materials, capital, and labor input demand equations. Subject to the assumption that the autoregressive coefficients (rho) within a demand equation were identical across states, estimates of rho for the three input demand equations were 0.971, 0.923, and 0.870, respectively. The data were transformed for 1st-order autocorrelation and used in a seemingly unrelated regression (SUR) estimation of the system of three input demand equations. [4] Since each equation had the same regressors and no across-equation restrictions were imposed, the SUR parameter estimates were identical to OLS estimates. The SUR estimation procedure was used to permit across-equation tests to be conducted, as required for proposition 2.

The estimates of the input demand equations are reported in Table 1. The R^2 values for the three equations in (22) were 0.834, 0.542, and 0.791 respectively. For the materials input demand equation, 49 of all 85 estimated coefficients and 13 of the 37 coefficients on variables other than state intercept dummies were significant at the 5% level. The corresponding numbers of significant coefficients were 65 and 18 for the

[4] Although evidence was found that significant heteroskedasticity exists in these data across states, we were unable to transform the data for heteroskedasticity because we had more cross-sectional units than time periods.

capital demand equation and 62 and 16 for the labor demand equation. Thus, 1/2 to 3/4 of all estimated coefficients were statistically significant at the 5% level. That included most of the coefficients on state dummy variables. A little more than 1/3 to 1/2 of the estimated coefficients on other variables were statistically significant.

Hypothesis tests of the propositions and corollary were conducted on the estimated parameters at the data means. These results, as well as a tabulation of predicted values consistent with the hypotheses at each observation, are presented in Table 2. Proposition 1 was examined by testing whether each of the three predicted input demands in equation (22) was positive. These test results are listed as propositions 1.1-1.3 in Table 2. The null hypothesis of a zero input demand level was rejected in favor of positive predicted input demands at the data means for each input at a 1% significant level. In addition, nearly all the predicted input quantities are strictly positive at individual observations. Among 1872 observations, only 11 predicted capital quantities and one predicted labor quantity were found to violate the first-order curvature properties.

The second proposition that $\Omega \equiv -(\boldsymbol{x}_{\mathrm{I}}^{*}\boldsymbol{x}^{*} + 2\boldsymbol{x}_{\mathbf{r}}^{*})$ is symmetric positive semidefinite was tested by the equivalent specification that $\Psi = \boldsymbol{x}_{\mathrm{I}}^{*}\boldsymbol{x}^{*} + 2\boldsymbol{x}_{\mathbf{r}}^{*}$ is symmetric negative semidefinite. To test this proposition, three individual tests (tests 2.1-2.3 in Table 2) were conducted for negative semidefiniteness and a joint test (test 3 in Table 2) for symmetry. The tests for negative semidefiniteness involved tests that all the leading principal minors of Ψ alternative in signs, starting with a nonpositive first leading principal minor, i.e., the first diagonal element. None of the refutable behavioral hypotheses implied by second-order curvature properties of the indirect utility function was rejected at the data means. Although both the second leading principal minor (test

2.2) and the determinant (test 2.3) of Ψ had unexpected signs at the data means, they were not significantly different from zero at the 5% level of significance. Considerably more evidence of second-order curvature violations was found at individual observations than of first-order condition violations, but they didn't excess 25% of the observations for any of the tests.

The test results for symmetry of Ψ are presented in test 3 in Table 2. The three symmetric restrictions were rejected at the 5% significance level by the joint test conducted at data means. Thus, the hypothesis implied by proposition 2 that Ω is symmetric positive semidefinite is statistically rejected at this data point. Whether rejection of symmetry constitutes a rejection of the hypothesis that the collection of firms in each state act as though they were a single expected utility-maximizing firm, or whether it simply implies that the indirect utility function is not twice continuously differentiable at the data means is ambiguous from these test results. Unfortunately, we are unable to resolve the ambiguity in this paper.

Decision making consistent with constant absolute risk aversion or risk neutrality implies three restrictions on input demand responses. The result (test 4 in Table 2) indicates that these restrictions were rejected by the joint test at the data means at the 5% significance level.

Our results using state-level aggregates were similar in a number of respects to Saha and Shumway's (1998) findings about output price risk for Kansans wheat farmers. However, we found less support in the aggregate data than they found in the firm-level data for symmetry of the indirect utility function. Our conclusions about first-order curvature properties and the nature of producers' risk preference were the same as theirs.

The extant literature has not reached a consensus regarding the nature of farmers' risk preferences (Goodwin and Mishra, 2002), but a few have found empirical support for the hypothesis of constant absolute risk aversion (CARA). Among those are the work of Park and Antonovitz (1992a, 1992b) who failed to reject CARA for California feedlots.

IV Conclusions

This study has extended the Saha and Shumway (1998) model of a competitive firm operating under output price risk to a firm operating under both output price and output quantity risk. One important theoretical contribution to the previous literature is that the refutable propositions implied by the indirect utility function are shown to hold without one of the previously maintained hypotheses. Therefore, the only conditions required for the propositions to hold are: (a) random wealth can be structured as three parts – a nonrandom part of profit, a random part of profit, and nonrandom initial wealth, and (b) there exists an optimal input vector that maximizes the expected utility function. Both are common assumptions in the firm theory under uncertainty. Without requiring the previously imposed assumption that the expectation of the random part of profit is zero, the propositions can be empirically applied to varied market structures by permitting tests when there is a nonzero correlation between the error terms of random output price and random output quantity.

Moreover, a set of testable hypotheses associated with input responses under multiple sources of risk were derived from these propositions, and empirically tested for aggregates of firms under both output price and output quantity risk. This is the first study using an aggregate state-level panel data to empirically test for utility-maximizing behavior by considering each aggregate as though it were an expected utility-maximizing firm. Aggregate agricultural production data for these states have previously been found to approximate nonparametric conditions for consistent behavior with this hypothesis.

Parametric findings from this study show that the behavioral postulates implied by the first-order curvature properties of the indirect utility function could not be rejected at the data means, and the data at nearly all individual observations were consistent with these properties. The second-order curvature properties were also not rejected at the data means, but up to 25% of the observations were inconsistent with the hypotheses. However, the symmetry property implied by a twice continuously differentiable indirect utility function was soundly rejected at the data means. The empirical evidence also failed to support ad hoc risk preference assumptions of either risk neutrality or constant absolute risk aversion.

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Table 1. Parameter Estimates for the Input Demand Equations

Variable[a]	Material/Land Equation (x_1)		Capital/Land Equation (x_2)		Labor/Land Equation (x_3)	
	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]
d1	0.218***	0.032	0.132***	0.014	0.359***	0.072
d2	0.091***	0.032	0.087***	0.014	0.239***	0.074
d3	0.035	0.031	0.028**	0.014	0.134*	0.072
d4	0.239***	0.030	0.132***	0.013	0.730***	0.067
d5	0.047	0.031	0.061***	0.014	0.162**	0.074
d6	0.170***	0.031	0.284***	0.014	1.068***	0.069
d7	0.739***	0.030	0.301***	0.013	0.722***	0.067
d8	0.112***	0.031	0.075***	0.014	0.413***	0.068
d9	0.235***	0.031	0.156***	0.014	0.435***	0.069
d10	0.114***	0.031	0.171***	0.014	0.362***	0.069
d11	0.070**	0.031	0.089***	0.014	0.266***	0.072
d12	0.092***	0.031	0.180***	0.014	0.310***	0.068
d13	0.138***	0.031	0.230***	0.014	0.454***	0.068
d14	0.075**	0.031	0.093***	0.014	0.230***	0.071
d15	0.091***	0.031	0.157***	0.014	0.435***	0.069
d16	0.086***	0.031	0.102***	0.014	0.278***	0.069
d17	0.125***	0.031	0.259***	0.014	1.070***	0.069
d18	0.309***	0.031	0.288***	0.013	0.755***	0.067
d19	0.146***	0.034	0.252***	0.015	0.730***	0.077
d20	0.200***	0.031	0.307***	0.014	0.786***	0.069
d21	0.178***	0.031	0.227***	0.014	0.567***	0.069
d22	0.115***	0.031	0.165***	0.014	0.465***	0.069
d23	0.131***	0.031	0.099***	0.014	0.273***	0.071
d24	-0.026	0.033	0.043***	0.015	0.159**	0.079

Table 1 (continued)

Variable[a]	Material/Land Equation (x_1)		Capital/Land Equation (x_2)		Labor/Land Equation (x_3)	
	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]
d25	0.191***	0.031	0.158***	0.014	0.516***	0.069
d26	0.018	0.033	0.076***	0.015	0.198***	0.077
d27	0.127***	0.031	0.107***	0.014	0.295***	0.070
d28	0.086***	0.031	0.202***	0.014	0.716***	0.070
d29	0.156***	0.031	0.529***	0.014	1.234***	0.067
d30	-0.011	0.033	0.039***	0.015	0.158**	0.079
d31	-0.041	0.037	0.033**	0.017	0.092	0.086
d32	0.184***	0.031	0.283***	0.014	0.757***	0.070
d33	0.145***	0.031	0.290***	0.014	0.650***	0.068
d34	0.041	0.031	0.068***	0.014	0.237***	0.071
d35	0.125***	0.031	0.100***	0.014	0.369***	0.070
d36	0.221***	0.031	0.305***	0.014	1.000***	0.069
d37	0.097***	0.031	0.302***	0.014	1.012***	0.070
d38	0.158***	0.031	0.177***	0.014	0.518***	0.069
d39	0.033	0.031	0.079***	0.014	0.198***	0.074
d40	0.065**	0.031	0.122***	0.014	0.361***	0.071
d41	0.022	0.031	0.054***	0.014	0.165**	0.072
d42	0.015	0.032	0.056***	0.014	0.171**	0.075
d43	0.008	0.031	0.128***	0.014	0.347***	0.070
d44	0.110***	0.033	0.155***	0.015	0.493***	0.074
d45	0.121***	0.031	0.139***	0.014	0.469***	0.069
d46	0.249***	0.031	0.347***	0.014	0.993***	0.070
d47	0.0714**	0.031	0.135***	0.014	0.435***	0.071
d48	0.003	0.031	0.047***	0.014	0.157**	0.075

Table 1 (continued)

Variable[a]	Material/Land Equation (x_1)		Capital/Land Equation (x_2)		Labor/Land Equation (x_3)	
	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]
p_1	-0.048***	0.010	-0.006	0.004	0.032	0.024
p_2	0.0602***	0.012	0.017***	0.006	-0.064**	0.031
p_3	-0.034**	0.021	-0.019**	0.011	0.023	0.062
r_1	0.118***	0.044	0.002	0.022	0.321***	0.119
r_2	-0.046**	0.019	0.003	0.009	0.038	0.051
r_3	0.0002	0.023	-0.044***	0.012	-0.379***	0.072
I	0.003***	0.001	0.0003	0.000	0.002	0.002
p_1^2	0.015 **	0.007	-0.008**	0.003	-0.017	0.018
$p_1 p_2$	-0.005	0.011	0.014***	0.006	0.076**	0.029
$p_1 p_3$	-0.017	0.020	0.008	0.009	0.001	0.046
$p_1 r_1$	0.033	0.039	0.004	0.016	-0.043	0.075
$p_1 r_2$	-0.027	0.016	0.0002	0.007	-0.023	0.034
$p_1 r_3$	0.021	0.025	-0.016	0.011	0.059	0.055
$p_1 I$	-0.001	0.001	-0.003***	0.001	-0.016***	0.003
p_2^2	0.017	0.020	-0.022**	0.009	-0.197***	0.044
$p_2 p_3$	-0.008	0.023	0.017	0.010	0.182***	0.048
$p_2 r_1$	-0.120**	0.048	-0.069***	0.020	-0.162**	0.094
$p_2 r_2$	0.018	0.021	0.002	0.009	0.069	0.045
$p_2 r_3$	0.02806	0.038	0.019	0.017	-0.060	0.086
$p_2 I$	0.0003	0.002	0.001*	0.001	0.009**	0.003
p_3^2	0.077	0.047	0.002	0.019	-0.122	0.087
$p_3 r_1$	0.096	0.094	0.058	0.037	0.074	0.168
$p_3 r_2$	-0.045	0.039	-0.016	0.017	-0.146*	0.080
$p_3 r_3$	-0.014	0.058	0.002	0.027	0.239*	0.131

Table 1 (continued)

Variable[a]	Material/Land Equation (x_1)		Capital/Land Equation (x_2)		Labor/Land Equation (x_3)	
	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]	Estimated coefficient[b]	SE[c]
p_3I	0.0003	0.004	-0.006***	0.002	-0.016**	0.007
r_1^2	-0.337**	0.187	-0.192***	0.067	-0.515*	0.298
$r_1 r_2$	0.059	0.053	0.023	0.021	0.079	0.103
$r_1 r_3$	0.206*	0.115	0.138***	0.044	0.230	0.201
r_1I	0.008	0.006	0.015***	0.002	0.048**	0.009
r_2^2	-0.004	0.032	0.002**	0.014	0.053	0.065
r_2r_3	-0.040	0.041	-0.041**	0.017	-0.180**	0.080
r_2I	0.008***	0.003	-0.001	0.001	0.003	0.005
r_3^2	-0.052	0.067	0.004	0.029	0.185	0.139
r_3I	-0.010**	0.005	-0.001	0.002	-0.029***	0.008
I^2	0.0001	0.000	0.001***	0.0001	0.003***	0.0003
t	-0.004**	0.002	0.003***	0.001	-0.010***	0.004
t^2	0.0003***	0.00008	0.0002***	0.00003	0.0003**	0.0002
R-Square	0.834		0.542		0.791	

[a] Variable codes: p_1 is crop price, p_2 is livestock price, p_3 is secondary output price, r_1 is materials input price, r_2 is capital input price, r_3 is labor input price, I is farm equity, t is the time variable, d1-d48 are state dummy variables.

[b] Parameter estimates marked with *** are significant at the 1% level, ** at the 5% level, and * at the 10% level.

[c] SE is standard error.

Table 2. Hypotheses Test Results

Proposition	Null hypothesis	Test type [a]	Test at Data Means		Rejections among 1,872 Observations
			Statistic	P-value	
1. V is decreasing in **r**					
1.1 V is decreasing in r_1, $\hat{x}_1 > 0$	$\hat{x}_1 = 0$	AN	98.706	0.000	11
1.2. V is decreasing in r_2, $\hat{x}_2 > 0$	$\hat{x}_2 = 0$	AN	9.963	0.000	0
1.3 V is decreasing in r_3, $\hat{x}_3 > 0$	$\hat{x}_3 = 0$	AN	56.521	0.000	1
2. $\Psi = \boldsymbol{x}_{\mathrm{I}}^{*}\boldsymbol{x}^{*} + 2\boldsymbol{x}_{\mathbf{r}}^{*}$ is negative semidefinite					
2.1 1st leading principal minor: $2x^*_{1r_1} + x^*_{1I}\cdot x^*_1 \le 0$	= zero	AN	-2.284	0.022	387
2.2 2nd leading principal minor of $\Psi \ge 0$	= zero	AN	-1.736	0.083	460
2.3 Determinant of $\Psi \le 0$	= zero	AN	0.772	0.440	450
3. Symmetry of Ψ [b]		W	71.770	0.000	--
4. CARA or RN [c]					
$x^*_{1I} = x^*_{2I} = x^*_{3I} = 0$	= zero	W	99.116	0.000	--

[a] AN is asymptotic normal test, and W is Wald chi-squared test.

[b] Test of symmetry involves jointly testing H_0: $2\cdot x^*_{1r_2} + x^*_{1I}\cdot x^*_2 = 2\cdot x^*_{2r_1} + x^*_{2I}\cdot x^*_1$, $2\cdot x^*_{1r_3} + x^*_{1I}\cdot x^*_3 = 2\cdot x^*_{3r_1} + x^*_{3I}\cdot x^*_1$, and $2\cdot x^*_{2r_3} + x^*_{2I}\cdot x^*_3 = 2\cdot x^*_{3r_2} + x^*_{3I}\cdot x^*_2$

[c] CARA is constant absolute risk aversion, and RN is risk neutrality.