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# A Distance Function Model with Good and Bad Outputs

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**Abstract**

We present an approach that pursues an adequate representation of product transformation possibilities for a technology generating, in addition to marketed (good) products, some environmentally detrimental non-marketed byproducts (bad outputs). As the shadow price of a non-marketed output depends on its marginal transformation rates with marketed outputs, representation of technological relationships between different groups of outputs deserves a particular attention. We model the technology by using two functions: an input distance function describing technically feasible input-output combinations, and a hedonic output function capturing relationships among good and bad outputs. This procedure offers more appropriate consideration and modeling of the interactions between different groups of outputs. An empirical application of the approach to the case of Dutch dairy farms[1] demonstrates the complexity of interactions between outputs and the value of more elaborate representations of production possibilities. The analysis indicates that nitrogen surplus abatement costs vary widely among Dutch dairy farms and that these costs have increased substantially over time.

*Keywords*: Input distance function, Hedonic aggregate output function, Shadow price, Dairy farms

## 1. Introduction

The distance function approach offers a flexible method for modeling a production technology involving multiple outputs and multiple inputs. This explains the recent growing interest in applying the distance function approach to empirical problems concerned with multiple output production technologies, especially in the context of undesirable (pollutant) outputs.

Early applications were based on the Färe et al.'s (1985) hyperbolic distance function that allowed for an equiproportional expansion of outputs and contraction of inputs, an approach that was extended to the case of undesirable outputs by Färe et al. (1989). Cuesta et al. (2009) suggest an enhanced formulation of the hyperbolic distance function, which represents the proportion by which good outputs can be expanded, and bad outputs and inputs can be contracted in a multiplicative manner.

Shephard's (1953, 1970) distance functions, which are defined in terms of radial expansion of outputs (or inputs) to the frontier, have been applied to the analysis of technologies with undesirable outputs in several studies (e.g. Färe et al (1993), Coggins and Swinton (1996), Hailu and Veeman (2000)). More recently, directional distance functions (Chambers et al., 1996), which are defined in terms of the translation of a point (corresponding to e.g. an output combination) to the frontier along a specified vector (rather than radially), have gained popularity. Shephard's radial distance functions are special cases of the directional distance function (Färe et al., 2005), as the former can be obtained by defining the vector in the latter to be specific for each observation. Directional distance functions maintain the same vector of translation for all observations and thus consistently

[1] The data used in the present study stem from the Dutch FADN system as collected by the Dutch Agricultural Economics Research Institute (LEI). The Centre of Economic Information (CEI) has provided access to these data. Results shown are and remain entirely the responsibility of the authors; neither they present LEI/CEI views nor constitute official statistics.

apply the same numeraire or yardstick to the whole sample (Hailu and Chambers, 2012). Chung et al. (1997) were the first to apply the directional distance function to model the joint production of bad and good outputs.

The superiority of the directional distance approach in comparison to the hyperbolic distance function lies in its ability to model non-proportional changes in outputs (and inputs) and allowing some outputs to be expanded, while others to be contracted in any chosen direction (Chambers et al., 1996; Färe et al., 2005). The choice of the direction vector (g) is left to the researcher. A common choice of direction is the unit vector (with negative signs on bads). This choice implies that producers have possibilities for extending good outputs while simultaneously contracting negative outputs as stated by the translation property (Färe et al., 2005), that is:

$$\vec{D}_O(x, y + \alpha g_y, b - \alpha g_b; g_y, -g_b) = \vec{D}_O(x, y, b; g_y, -g_b) - \alpha, \quad \alpha \in \Re,$$

where $\vec{D}_O$ is a directional output distance function, defined as

$$\vec{D}_O(x, y, b; g_y, -g_b) = \max\{\alpha : (y + \alpha g_y, b - \alpha g_b) \in P(x)\},$$

$g = (g_y, g_b)$ is a directional vector, $P(x)$ stands for the output set, which is the set of good and bad outputs $(y, b)$ that can be jointly produced from the input vector $x$, i.e.,

$$P(x) = \{(y, b): x \text{ can produce } (y, b)\}.$$

The choice of direction is commonly seen as a non-trivial problem. Furthermore, whether results are sensitive to the direction choice is an issue. For example, Vardanyan and Noh (2006) obtained shadow price estimates that were sensitive to this choice. By employing a directional vector (1,-1) Färe et al. (2005)[2] obtain bad output shadow price estimates that diverge substantially between stochastic and deterministic parametric model formulations.[3]

Recently, Forsund (2009) and Murty et al. (2012) raised further issues related to modeling a technology involving production of detrimental byproducts. These authors recommend separating the technology for production of good outputs from the pollution-generating technology. Fernandez et al. (2002)[4] proceeded in the same manner, but assumed that the two technologies were separable and rather restrictive. In our approach, we use a single technology specification, but allow good and bad outputs to be related via a hedonic function, which is imbedded in the production technology. Therefore, we model the technology by using two functions: one to describe interactions between different products within producible output sets – a hedonic aggregate output function, and another to represent

[2] To the best of our knowledge, no other study has estimated the directional distance function with both types of outputs (good and bad ones) in a stochastic formulation using non-Bayesian methods.

[3] Shadow price estimates for the deterministic approach were on average approximately 14 and 15 times higher than those from the stochastic model for 1993 and 1997, respectively, as reported by the authors in Table 4 (Färe et al., 2005, p. 486).

[4] Fernandez et al. (2002) notice that their model may produce different results depending on the choice of the output aggregator function. In their study, they employ an aggregator function with a constant elasticity of transformation, which is close to the Powell and Gruen's (1968) specification. Additionally, Fernandez et al. employ a Cobb-Douglas production function to specify both frontiers.

production possibilities as all technically feasible input-output combinations – an input distance function (IDF). Under this technique, the output vector in the IDF is a bundle of good and bad outputs whose relationship is captured by the hedonic function.

For our empirical analysis, we use data from Dutch dairy farms, which, in addition to good outputs such as milk, other livestock products and crops, generate nitrogen surpluses. The relationships among outputs are rather complex in this setting. Whereas nitrogen surpluses have a tendency to increase with higher levels of dairy and livestock production, they might decrease for farms with a higher involvement in crop production, as the latter absorbs manure generated in dairy and livestock production. In addition, although the crops produced for feeding farm livestock are complements to dairy products, crop production for sale is certainly a substitute for dairy and livestock production. In our approach, we formulate a hedonic output function, which links the two good outputs (dairy and livestock products and crop products for sale) and the bad output (nitrogen surplus).

Our empirical application exemplifies a need in a more elaborate approach for modeling interactions between single categories of outputs, in this particular case, milk and livestock products, crop production outputs, and nitrogen surplus. Our findings suggest that a higher level of specialization in crop production helps Dutch dairy farms to reduce the marginal rate of substitution between livestock products and nitrogen surplus and in this way to damp nitrogen abatement costs. In addition, our study results reveal that Dutch dairy farms have relatively limited options for efficient nitrogen abatement and further cuts in nitrogen surplus can be achieved only at relatively high costs.

The remainder of the paper is structured as follows: In the next section, we present our input distance formulation incorporating a hedonic output function. In section 3, we discuss the data used in our empirical analysis. We present and discuss the estimation results of the IDF and hedonic output function parameters in section 4. The last section concludes.

## 2. Methodology

### *2.1. Input Distance Function with Hedonic Aggregate Output Formulation*

The technology with a vector of good outputs (y), bad outputs (b), and inputs (x) can be expressed as $T(y,b,x)=1$, where $T(\cdot)$ is the transformation function. The treatment of bad (inputs/outputs) has been controversial (Murty et al., 2012). If bad outputs are by-products, then y and b are not substitutable, and therefore, given x, it is not possible to reduce b without reducing y (in the absence of abatement and inefficiency). Murty et al. (2012) advocate for separating the technology of good output production from the pollution-generating technology (technologies). We model the relationship between y and b by defining a hedonic function that can be viewed as an aggregator of outputs $h(b,y)=\tilde{y}$. The hedonic function represents the farms' aggregate output and explicitly captures the relationship among good and bad outputs. Note that we are not using the hedonic function as a pollution-generation technology as in Fernandez et al. (2002), Murty et al. (2012) and Kumbhakar and Tsionas (2013). In the present paper the h(b,y) function is an aggregator of bad and good outputs. The functional form chosen on h(b,y) determines the type of relationship imposed on good and bad outputs. Färe et al. (2005) discussed the issue of a meaningful representation of the relationship between b and y, but did not pursue it formally in their modelling approach. Here we do this

by imposing a specific parametric form on the h(b,y) function. For example, if a translog specification[5] is used, the aggregate output function becomes:

$$\ln h(\boldsymbol{b},\boldsymbol{y}) = \sum_{s=1}^{S} \theta_s \ln b_s + \sum_{m=1}^{M} \delta_m \ln y_m + \frac{1}{2}\sum_{s=1}^{S}\sum_{p=1}^{S} \theta_{sp} \ln b_s \ln b_p + \frac{1}{2}\sum_{m=1}^{M}\sum_{l=1}^{M} \delta_{ml} \ln y_m \ln y_l + \frac{1}{2}\sum_{s=1}^{S}\sum_{m=1}^{M} \varphi_{sm} \ln b_s \ln y_m, \quad (\mathbf{1})$$

where h(b,y) is the hedonic aggregate output function, y is the 1×M vector of good outputs, and b is the 1×S vector of bad outputs. The translog specification can be restricted to incorporate certain structures of production possibilities set, e.g., the positive relationship between good and bad outputs.

The hedonic aggregated output function h(b,y) is then incorporated into the transformation function, which is represented by an IDF (which follows from the transformation function by imposing linear homogeneity (in x) on T(h(b,y),x) =1). Considering a flexible functional form for the IDF formulation by using a translog function, we obtain:

$$\ln D_I(x,h(b,y),t) = \alpha_0 + \sum_{j=1}^{J} \alpha_j \ln(x_j) + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{J} \alpha_{jk} \ln(x_j)\ln(x_k) + \beta_h \ln h(b,y) + \frac{1}{2}\beta_{hh} \ln h(b,y) \ln h(b,y) + \sum_{j=2}^{J} \beta_{jh} \ln(x_j) \ln h(b,y) + \omega_t t + \frac{1}{2}\omega_{tt} t^2 + \sum_{j=1}^{J} \omega_{jt} \ln(x_j) t + \omega_{ht} \ln h(b,y) t, \quad (\mathbf{2})$$

where x denotes a vector of J inputs. In (2), h(b,y) represents the aggregate production output. The coefficient on lnh(b,y) is set to unity for identification (i.e., $\beta_h = 1$). The other way to impose identifying restrictions is to make lnh(b,y) homogenous.

By defining $\ln D_I(x,h(b,y),t) = u$, allowing for a stochastic noise and after imposing the homogeneity in inputs[6] property, the input distance function in (2) leads to the following form:

[5] Other functional forms such as Cobb-Douglas or quadratic are feasible as alternatives to a translog specification of $h(b,y)$.

[6] Linear homogeneity (in inputs) property of the input distance function is imposed by normalizing the inputs by $x_1$.

$$-\ln x_1 = \alpha_0 + \sum_{j=2}^{J} \alpha_j \ln(x_j/x_1) + \frac{1}{2}\sum_{j=2}^{J}\sum_{k=2}^{J} \alpha_{jk} \ln(x_j/x_1)\ln(x_j/x_1) + \ln h(b,y)$$
$$+ \frac{1}{2}\beta_{hh}\ln h(b,y)\ln h(b,y) + \sum_{j=2}^{J} \beta_{jh}\ln(x_j/x_1)\ln h(b,y) + \omega_t t + \frac{1}{2}\omega_{tt}t^2$$
$$+ \sum_{j=2}^{J} \omega_{jt}\ln(x_j/x_1)t + \omega_{ht}\ln h(b,y)t - u + v, \qquad (\mathbf{3})$$

where u is the one-sided error term, which is i.i.d. $N^+(0,\sigma_u^2)$, and v is the symmetric error term i.i.d. $N(0,\sigma_v^2)$; and u and v are assumed to be distributed independently.

Instead of using the hedonic function lnh(b,y), one can incorporate y and b directly into the input distance function. In doing so the model becomes more flexible but quite unstructured in the sense that bad and good outputs can have/obtain a relationship that is counter-intuitive. Färe et al. (2005) refer to a relationship between a single good and a single bad output that is concave. But such a relationship has never been used while estimating the technology in the previous literature. Here we first specify the h(b,y) function that can be formulated in such a way that it satisfies the theoretical properties. Then, we treat it as an output aggregator in the input distance function which satisfies the usual properties.

Our translog input distance function has six inputs (i.e., land, labor, capital, livestock, materials, and purchased feed) and a hedonic aggregate output function which itself is a function of two good outputs (dairy and livestock products and crop production output for sale) and one bad output (nitrogen surplus). Equations (1) and (3) can be estimated simultaneously by means of the maximum likelihood (ML) method. Note that only (3) is formulated as a stochastic model.

*2.2. Derivation of Shadow Prices*

Since there are no markets for most environmentally detrimental byproducts, the derivation of their shadow prices is of a particular relevance for designing and targeting environmental policy instruments. Based on our approach, the shadow prices for bad outputs can be derived considering the ratio of the shadow prices for two outputs which should be equal to the marginal rate of transformation between these two outputs (Färe et al., 1993). The latter can be derived as the ratio of the derivatives of the distance function with respect to the bad and good outputs. Färe et al. (1993), Färe and Grosskopf (1998) and Färe et al (2005) show how to derive shadow prices for pollutants in the context of an output distance function. The procedure is similar for an input distance function approach (Hailu and Veeman, 2000).

Considering that one output is a pollutant and another is a traditional marketed output, the shadow price ratio is equal to:

$$-\frac{r_s}{p_m} = \frac{\partial D(x,h(b,y),t)/\partial b_s}{\partial D(x,h(b,y),t)/\partial y_m}, \qquad (\mathbf{4})$$

where $r_s$ and $p_m$ are shadow prices of bad output *s* and good output *m*, respectively.

By assuming that the shadow price of the marketed output is equal to its market price, the pollutant shadow price can be easily derived from (4), that is:

$$r_s = -p_m \frac{\partial D(x, h(b, y), t)/\partial b_s}{\partial D(x, h(b, y), t)/\partial y_m}. \quad (\mathbf{5})$$

Considering the production of several good outputs, (5) has to be rewritten as

$$r_s = -\partial D(x, h(b, y), t)/\partial b_s \sum_{m=1}^{M} \frac{p_m}{\partial D(x, h(b, y), t)/\partial y_m}. \quad (\mathbf{6})$$

Given the price of marketed outputs and the estimated distance function, the above formula can be used to compute shadow price of any non-marketed output.

### *2.3. Elasticities of Substitution between Good and Bad Outputs*

The Morishima output elasticity of substitution $MES_{by} = \frac{\partial \ln (r/p)}{\partial \ln (y/b)}$ allows to measure how the (inverse) pollution intensity (y/b) influences the good-bad (shadow) price ratio. Accordingly, we employ the MES to evaluate how the good-bad (shadow) price ratio changes with changes in the relative pollution intensity (ratio of bad to good outputs).

Based on the distance function estimates (Blackorby and Russell, 1989; Färe et al., 2005; Stern, 2011), the MES for a pair of good-bad output can be derived as follows:

$$MES_{b_s y_m} = y_m \left( \frac{\partial^2 D(x, h(b, y), t)/\partial b_s \partial y_m}{\partial D(x, h(b, y), t)/\partial b_s} - \frac{\partial^2 D(x, h(b, y), t)/\partial y_m \partial y_m}{\partial D(x, h(b, y), t)/\partial y_m} \right). \quad (\mathbf{7})$$

Details on the MES derivation for the IDF specification in (3) are available on request from the authors of the manuscript.

## 3. Data[7]

In our empirical application, we use data for a sample of Dutch dairy farms provided by the Farm Data Accountancy Network (FADN). The data set is an unbalanced panel that covers the period from 2001 to 2009. The sample consists of 1,866 observations on dairy farms, whose revenues from sales of milk and livestock products account for 70% or more of the farms' total revenues.

We distinguish between three categories of outputs: good output $y_1$, defined as the revenue from sales of milk and livestock products plus changes in the valuation of the livestock; good output $y_2$, which comprises revenues from the sales of crop and other agricultural products; and one bad output (b), which is the nitrogen surplus determined as the

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difference between the quantity of nitrogen applied on the farm and the quantity of nitrogen in the farm desirable output (Reinhard et al., 2000). [8]

The six input categories comprise Land (L), Labor (W), Capital (K), Livestock (A), Materials (M), and Purchased feed (F). Land is defined as the farm total agricultural land. The number of agricultural work units is used to measure the farm labor force. Capital is defined in terms of the value of the machines' and buildings' depreciation. Livestock is measured as the number of standardized livestock units. Materials represent farm variable costs except purchased feed, which, due to its specifics, was considered separately and consists of farm costs for purchasing feed and concentrated feeding stuff. All monetary values were deflated to the 2005 price level by using the price indices for each category as reported by Eurostat (2012). Summary statistics for all variables are in Appendix (Table B1). All model variables except the time variable were normalized by their respective geometric means.

On average, the nitrogen application per farm was 14,871 kg over the study period. The intensity of nitrogen use in the period covered by the study was 291.6 kg per ha of agricultural land and 130.4 kg as measured per livestock unit. The nitrogen surplus in the same period amounted to 9,108 kg per farm on average, which results in intensity values of 178.6 kg per ha of land and 79.9 kg per livestock unit. As our data shows, the nitrogen use and nitrogen surplus generation in the last decade decreased substantially compared to the earlier periods. According to Reinhard et al. (2010), the average nutrient input for a comparable FADN data set of Dutch dairy farms was much higher from 1991 to 1994, in particular it amounted 17,750 kg for a sample farm. The sample farms in Reinhard et al.'s (2000) analysis also generated considerably more nitrogen surplus than the farms in our sample, on average 14,628 kg. As Reinhard et al. do not report any statistics for land and livestock, it is not possible to draw a direct comparison regarding the intensity of the nitrogen use and nitrogen surplus generation between two samples. However, we can compare the data in terms of pollution intensity: Reinhard et al.f (2000) data indicate that on average for their data set the sample farms produced €17.3 (2005) [9] of revenue per kg of nitrogen surplus; the corresponding number is 52% higher in our sample, namely €26.4 per kg in 2005 prices.

## 4. Results

Table 1 presents the model parameter estimates.[10] Most parameters are statistically significant at the 1% significance level and some of the key results can be summarized as follows. The distance function elasticities with respect to inputs have the expected signs and are also close to those obtained by earlier studies that used the FADN data for Dutch dairy

[8] More details on the nitrogen surplus calculation technique in the Dutch agriculture can be found in Reinhard et al. (2000).

[9] Reinhard et al. measured the farm output in guilders. We adjusted their values to the 2005 Euro equivalent by applying Euro/guilders exchange rate and the consumer price index development from 1991 to 2005.

[10] In addition to the translog formulation of the hedonic output function, Cobb-Douglas and reduced translog formulations were employed. The likelihood ratio test rejected the two latter forms in favor of the translog. Accordingly, in this section we discuss our estimation results obtained for the translog formulation.

farms (Envalomatis et al., 2011; Fernandez et al., 2002).[11] The largest value is for livestock, followed by purchased feed and materials. Taking into account the homogeneity condition, the distance function elasticity regarding materials (the input used for normalizing the distance function) is 0.122. Elasticities for land, work, and capital do not substantially differ among each other. The time trend parameters' estimates are significant at the 0.01 significance level and imply a technical change rate of approximately 1.0% per annum.

The elasticities of the distance function for all three outputs also have highly significant estimates and are all negative. The highest elasticity was estimated for dairy and livestock products. The elasticity for nitrogen surplus is also rather high, which underlines its relevance as an important byproduct of dairy farms. The average returns to scale estimate computed as the sum of negative value of the inverse of the output elasticities, is relatively high, i.e., 1.23 if it is computed considering two good outputs; yet it reduces to 1.09 if the surplus nitrogen is considered.

The monotonicity condition was fulfilled at the approximation point for all inputs and outputs. However, the condition was violated for 11.6% of the observations for land and 5.8% for livestock. For all remaining inputs, the respective (violation) proportions were below 3%. The output monotonicity condition was fulfilled in almost all cases for the two good outputs and violated for 3.3% of the observations in the case of nitrogen surplus.

**Table 1. Model parameter estimates.**

| Parameters | Estimates | | Parameters | Estimates | |
|---|---|---|---|---|---|
| *h(b,y) function* | | | *Distance function (cont.)* | | |
| b | –0.104 | *** | L^2 | –0.114 | |
| y1 | –0.800 | *** | W^2 | –0.171 | *** |
| y2 | –0.015 | *** | K^2 | 0.022 | |
| y1^2 | 0.143 | *** | A^2 | –0.860 | *** |
| y2^2 | –0.003 | *** | F^2 | –0.318 | *** |
| b^2 | –0.093 | *** | L*W | 0.305 | *** |
| y1*y2 | –0.006 | | L*K | –0.181 | ** |
| y1*b | 0.145 | *** | L*A | 0.335 | * |
| y2*b | 0.006 | | L*F | –0.322 | *** |
| *Distance function* | | | W*K | –0.047 | |
| constant | 0.002 | | W*A | 0.239 | * |
| L | 0.074 | *** | W*F | –0.025 | |
| W | 0.067 | *** | K*A | 0.302 | *** |
| K | 0.094 | *** | K*F | –0.164 | ** |
| A | 0.484 | *** | A*F | 1.02 | *** |

[11] Fernandez et al. (2002) distinguished three groups of inputs only, namely capital, work, and variable inputs. However, if we sum our elasticity estimates for livestock and capital, we come up with a similar elasticity as these authors whose capital variable includes the value of livestock. The same applies to our estimates for materials and purchased feed, which, in sum, give an elasticity value close to that obtained by Fernandez et al. (2002) for variable inputs.

| | | | | | |
|---|---|---|---|---|---|
| F | 0.159 | *** | h(b,y)^2 | –0.398 | *** |
| t | 0.034 | *** | h(b,y)*L | 0.115 | *** |
| t^2 | –0.005 | *** | h(b,y)*W | –0.161 | *** |
| h(b,y)*t | 0.008 | *** | h(b,y)*K | –0.001 | |
| L*t | 0.001 | | h(b,y)*A | 0.044 | |
| W*t | 0.014 | *** | h(b,y)*F | 0.052 | |
| K*t | –0.004 | | | 0.072 | *** |
| A*t | –0.020 | *** | | 0.09 | *** |
| F*t | 0.006 | * | *Log likelihood* | 2065.9 | |

*Note*: *, ** and *** denote significant difference from zero at the 10%, 5%, and 1% significance level, respectively.

The Morishima elasticity measures the effect of changes in the output quantities ratio on their marginal rate of substitution. According to our estimates (Figure 1), in both cases, the Morishima elasticities have positive values for most observations. This result indicates that the nitrogen surplus is a complement for both marketed outputs. However, the MES value is much higher in the case of the first input. This outcome implies that a given change in the $y_1/b$ ratio causes a much bigger change in the shadow price ratio $r/p_1$ than a change in the $y_2/b$ value evokes in the $r/p_2$ ratio.

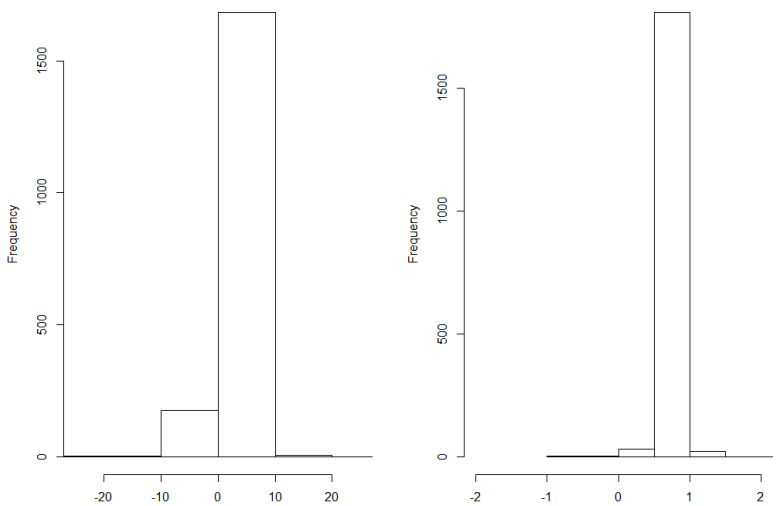


**Figure 1. *MES$_{by1}$* (left side) and *MES$_{by2}$* (right side) estimates.**

Figure 2 shows MES changes subject to the extent of specialization (in crop production in the left-hand plot and livestock production in the right-hand plot). As can be seen from the left-hand plot in Figure 2, the output elasticity between the livestock output and the nitrogen surplus seems to decrease for higher shares of crop production in the farm total agricultural output. This result suggests that a higher share of crop production allows Dutch dairy farms to decrease the marginal rate of substitution between $y_1$ and $b$, i.e. apparently a higher level of specialization on crop production lowers the amount of the revenue forgone due to reductions in nitrogen surplus generated per unit of $y_1$. The right-hand plot in Figure 2 presents the MES

estimates for the marketed output from crop production and bad output conditioning on the extent of livestock production. These estimates indicate that specialization in the livestock production does not have any clear effect on the shadow price ratio between the nitrogen surplus and the marketed crop production output.

The technical efficiency estimates indicate that Dutch dairy farms are performing quite well; the sample average technical efficiency is equal to 0.928. The distribution of the technical efficiency scores is presented in Figure 3. It shows that for most observations the estimates were above 0.9. Our technical efficiency estimates are somewhat higher than those found recently by Emvalomatis et al. (2011) for a comparable sample of Dutch dairy farms. The major difference in the estimates can be explained by differences in the output formulation. In particular, when aggregating traditional outputs in the same manner as in the above-mentioned study (distinguishing between *milk* and *other outputs* instead of between *milk plus livestock output* and *crop production output*), we obtain a similar average technical efficiency score of 0.86 (compared to 0.83 by Emvalomatis et al.). In addition, Emvalomatis et al. (2011) use the output distance function and estimate output-orientated efficiency values, we use an input distance function, which implies input orientation (Kumbhakar and Tsionas, 2008). Finally, Emvalomatis et al. used a data set for a different time period, 1995-2005.

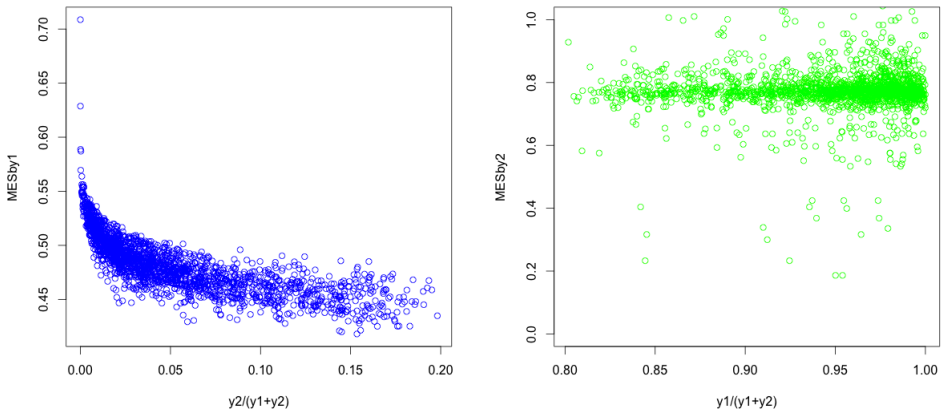


**Figure 2. $MES_{by1}$ and $MES_{by2}$ estimates subject to the extent of specialization.**

*Note*: The left-hand plot depicts the elasticity of substitution of the bad output with respect to the good output $y_1$ conditioning on the good output $y_2$; while the right-hand plot shows it with respect to the good output $y_2$ conditioned on $y_1$; MES values were computed by using the model parameter estimates and setting all relevant variable values to their means except the values of the good output used for conditioning.

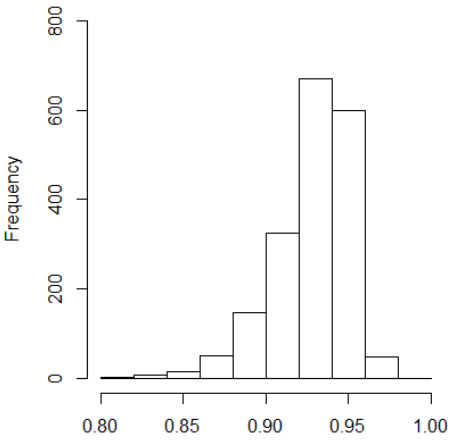


**Figure 2. Technical efficiency estimates.**

Shadow price estimates are shown in Figure 4. For almost all observations (96.6%), the shadow price estimates have the right sign and vary between €0.6 and €35.8 (2005) per 1 kg of nitrogen surplus. The average shadow price value (considering positive estimates only) is €12.4 (2005), which is substantially higher than the €1.96 (2005 prices)[12] estimated by Reinhard et al. (1999) for the 1991-1994 period. Our estimates might differ from Reinhard et al.'s (1999) due to differences in the modeling approach. Reinhard et al. (1999) did their estimates by applying the production function approach and regarded nitrogen surplus as an input. Moreover, our estimates of the nitrogen surplus shadow price show a highly significant linear trend suggesting a growth rate of 0.21 per annum from 2001 to 2009. This finding suggests lower substitution possibilities between conventional outputs of Dutch dairy farms and nitrogen surplus, i.e., obviously it becomes more costly to Dutch dairy farms to cut their nitrogen surpluses. This study result is also consistent with the finding by Hailu and Veeman (2000), who found that shadow prices grew over time.

A look at the distribution of shadow price estimates shows that, for most sample farms, the price estimates vary from €5 to €15 (2005) per kilogram of nitrogen surplus. However, for a fraction of the farms, the shadow prices are above €15. To analyze this substantial variation in the shadow price estimates across sample farms, we used an econometric model. A Tobit regression model with 11 regressors was applied[13]. The estimation results of this model are presented in Table 2.

[12] To be able to compare our shadow price estimates with those derived by Reinhard et al (1999), we adjusted the average shadow price estimate of 3.14 guilders of 1991 per kilogram estimated by these authors to the 2005 price level by using the guilders/Euro equivalent and the consumer price index development from 1991 to 2005.

[13] The estimation results were quite robust to the choice of the model. The OLS estimates are almost identical with the Tobit model estimates. The corresponding R-squared value was 11.5, which indicates that the considered factors explain a relatively small portion of the variability in the estimated shadow prices.

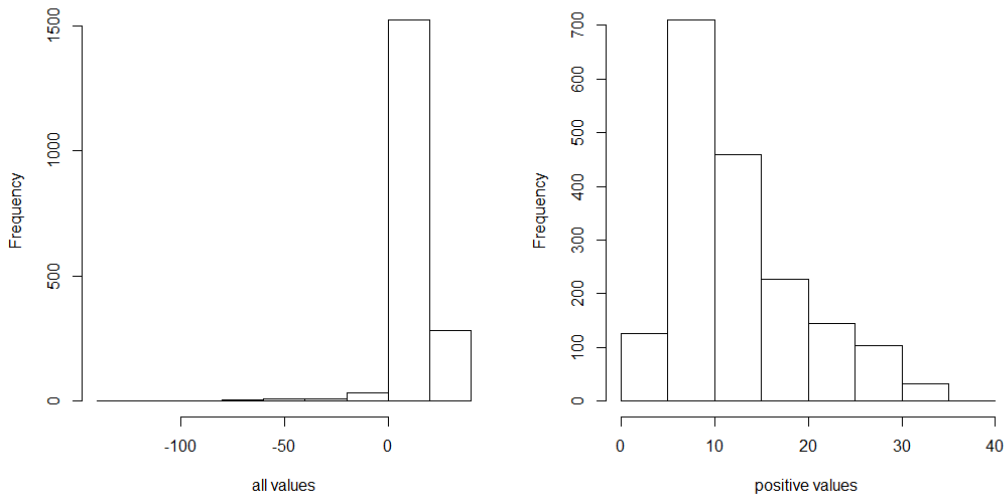


**Figure 4. N surplus shadow price estimates.**

The coefficient estimates show that lower values of shadow prices were obtained for farms with a lower livestock density and a higher magnitude of off-farm manure displacement when all other factors are kept constant. Though the variables 'input contracting' and 'off-farm employment' seem to have tendency to reduce the polluter shadow price, their parameter estimates are not statistically significant. However, the magnitude of the coefficient estimates for all four variables negatively influencing shadow prices is rather low. This finding suggests a relatively limited scope for reduction of nitrogen pollution. So, an increase of grazing land per LSU by 100 percent would decrease the nitrogen surplus shadow price by merely €2.78 (2005) ceteris paribus. Off-farm manure displacement seems not to be effective at all, since each Euro spent on off-farm manure displacement leads to a reduction in the pollutant shadow price smaller than 1 Euro cent, keeping the effect of all other factors constant.

**Table 2. Determinants of shadow prices: Tobit model estimates.**

| Variable | Coefficient estimate | t-value |
|---|---|---|
| const. | 6.03 | 3.04 |
| t | 0.2 | 3.16 |
| oldest shareholder age | 0.03 | 1.92 |
| off-farm employment | -0.01 | -0.38 |
| own land share | 2.32 | 4.54 |
| I/K ratio | 0.33 | 2.94 |
| off farm manure displacement | -0.0001 | -1.81 |
| grazing land per LSU | -2.79 | -1.73 |
| input contracting | -0.16 | -0.12 |
| on-farm processing | 0.55 | 0.58 |
| LSU number | 0.57 | 1.66 |
| total subsidies | 0.0001 | 4.4 |
| *Log likelihood* | 5967.65 | |

The study farms with lower shares of own land and younger management seem to have a lower shadow price of nitrogen surplus.

Additionally, the shadow price tends to be higher for farms with higher investment rates. This result indicates that further cuts in nitrogen surplus due to switching to new (potentially more environmentally friendly) technological solutions are possible only at higher costs. This finding is supported as well by a significantly positive time trend found in shadow price development. Finally, government subsidies have a barely noticeable but significantly positive effect on farm costs to lower pollution.

## 5. Conclusions

The paper presents an approach for modeling a joint production of desirable and undesirable outputs. In contrast to distance functions, directly incorporating both inputs and outputs, our approach distinguishes between two components in the technology representation. A hedonic aggregate output function is used to describe the technological relationship between different categories of outputs. The aggregate output index from this function is used as an output measure. In the second component of the modeling, an input distance function links multiple inputs with the index output. Such representation of the production technology allows the derivation of shadow prices for bad outputs considering their technological relationships with good outputs as it is inferred from empirical data.

The application of the proposed approach to the nitrogen surplus generation problem in the Dutch dairy sector exemplifies complexity of technological relationships between single categories of outputs: in the considered empirical case it refers to substitutability between milk and livestock products, crop production outputs, and nitrogen surplus. On the one hand, traditional outputs compete among each other in view of a scarce input endowment; on the other hand, single good outputs might exhibit different relations with the polluting output.

We have found relatively high shadow prices for nitrogen surplus. For most farms, these prices are above €5.0 (2005). Our estimates show a statistically significant increase in shadow prices during the study period. These findings suggest that Dutch dairy farms have only limited opportunities for reducing their nitrogen surplus without substantial cuts in traditional outputs. In addition, we have found that Dutch dairy farm investments lead to an increase in nitrogen shadow prices. This result implies that further cuts in nitrogen surplus are possible only at relatively high costs given the current stage of technological development. Besides, we have found a positive effect of governmental subsidies on the nitrogen surplus shadow prices. This finding is in line with empirical evidence: as farm subsidization is conditioned on farm compliance with certain environmental regulations, farms become more strongly involved in pollution abatement activities.

Based on the available data we were able to reveal several important factors explaining differences in farms shadow prices, however a large part of the variation remains unexplained. Considering the importance of the contamination of the groundwater aquifers with nitrogen, this aspect requires further investigations.

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