ABSTRACT: This paper develops an approach to decompose farmland price time series into three components: permanent fundamental component, temporary fundamental component, and nonfundamental component. This decomposition is useful for studying the importance of fundamental versus nonfundamental factors in explaining farmland price behavior and the dynamic response of farmland price to shocks to each of these components, among other issues. The approach is applied to annual Iowa farmland prices over the 1992-1994 sample period.
1. Introduction

A consensus appears to be forming that farmland price movements are not well-explained by the present value model with rational expectations. See, for example, Burt (1986), Featherstone and Baker (1987), Falk (1991,1992), and Hanson and Meyers (1995). Although the specific methods and data sets differ across these papers, each one formally or informally rejects the present value model as an explanation of farmland prices.

The reasons for the empirical failure of the present value model are not clear. Burt (1986) concludes that deviations of farmland price from its fundamental path can be explained in terms of overreaction to rent movements. Featherstone and Baker (1987), on the other hand, conclude that these deviations are largely determined by purely speculative forces, i.e., by fads. No one, however, has attempted to quantify the fad component to help resolve this basic issue.

The purpose of this paper is to suggest and apply an empirical strategy to decompose farmland price movements into a component driven by fundamental forces and a component driven by fad forces. This decomposition will provide measures that will help resolve the issue of the relative importance of these two components in explaining overall farmland price movements. In addition, we will estimate and compare the dynamic responses of farmland prices to nonfundamental shocks and two types of fundamental shocks.

The basic framework is a trivariate vector autoregression (VAR) formulated in terms of (functions of) farmland price,
farmland rent, and a time-varying discount rate. In this respect, the paper is closely related to Featherstone and Baker (1987). They applied innovation accounting and impulse response analysis to an unrestricted VAR representation of price, rent, and the discount rate, under the assumption that these series are trend stationary. In contrast, we assume that prices and rents are difference-stationary. This enables us to apply generic properties of unit root and cointegrated processes to formulate restrictions on the VAR that provide us with the means to, among other things, identify the fad component of the price series.

The remainder of the paper is organized as follows. The model is developed in Section 2, the data are described in Section 3, and the empirical results are presented in Section 4. A summary of the paper and its main conclusions are contained in Section 5.

2. Model

Let $p_t$ denote the log of the real price per acre of farmland in period $t$, let $d_t$ denote the log of the real rent per acre of farmland in period $t$, and let $r_t$ denote the real interest rate in period $t$. Assume that $p_t$ and $d_t$ are difference-stationary processes, while $r_t$ is a stationary process. Define the spread, $s_t$, according to $p_t - d_t$ (i.e., the log of the price-rent ratio) and assume that it is stationary, which implies that $p_t$ and $d_t$ are cointegrated with cointegrating vector $[1 -1]'$.

Campbell and Shiller (1988) used a log-linear approximate asset pricing framework to show that the VAR representation of the
bivariate stationary process \([\Delta d_t - r_t, s_t]\)' is characterized by a particular set of cross-equation restrictions if \(p_t\) is determined by current and expected future values of \(d\) and \(r\) according to the present value model of asset pricing. Falk (1992) used their framework to test (and reject) the time-varying discount rate version of the present value model as an explanation of Iowa farmland prices.

We begin under the premise that the present value model does not provide an adequate explanation of farmland prices and so we must work with a more general VAR that can account for the nonfundamental shocks that are not admitted into the Campbell-Shiller setup. Specifically, we consider the VAR representation of the trivariate stationary process \([\Delta d_t, \Delta d_t - r_t, s_t]\)' \(^1\).

Assume that price, rent, and the interest rate are subject to three types of orthogonal innovations: permanent fundamental innovations, temporary fundamental innovations, and nonfundamental innovations. Fundamental shocks are defined to be shocks that influence the time paths of rent and/or the interest rate. Permanent fundamental shocks, e.g., technology shocks, alter the s-step ahead forecast of future rents by a nonnegligible amount for arbitrarily large s. The effect of a temporary fundamental shock, e.g., a weather shock, on the s-step ahead forecast of \(d_t\) and \(r_t\) is arbitrarily small for sufficiently large s. The assumption that fundamental innovations can be decomposed into orthogonal permanent and temporary innovations is completely general so long as \(d_t\) is an I(1) process, as we have assumed. \(^2\) Nonfundamental shocks are
defined to be shocks that influence the time path of price but not the time path of rent or the interest rate.

The Wold representation theorem and the assumptions made regarding the stationarity of $\Delta d_t$, $\Delta p_t$, $r_t$ and $p_t - d_t$ guarantee the existence of a trivariate moving-average representation (TMAR) of $[\Delta d_t, \Delta d_t - r_t, s_t]'$:

$$z_t = [\Delta d_t, \Delta d_t - r_t, s_t]' = C(L)\varepsilon_t$$  \hspace{1cm} (1)

where $L$ is the lag operator (i.e., $L^n x_t = x_{t-n}$); $C(L) = [C_{ij}(L)]$, $C_{ij}(L) = c_{ij,0} + c_{ij,1} L + c_{ij,2} L^2 + \ldots$ for $i,j = 1,2,3$; and $\varepsilon_t = [\varepsilon_n, \varepsilon_n, \varepsilon_n]'$ is the vector of (linear) innovations in $z_t$, which implies that $\varepsilon_t$ is a zero-mean and serially uncorrelated process. For convenience, we choose to normalize the variance of each element of $\varepsilon_t$ to be equal to one, rather than restricting the coefficients $c_{ij,0}$ to be equal to one.

We identify $\varepsilon_n$ as the permanent fundamental innovation, $\varepsilon_n$ as the temporary fundamental innovation, and $\varepsilon_n$ as the nonfundamental innovation, by imposing the following additional restrictions on (1). First, we assume that the elements of $\varepsilon_t$ are contemporaneously uncorrelated, i.e.,

$$E(\varepsilon_t \varepsilon_t') = I$$  \hspace{1cm} (2.a)

where $I$ is the 3x3 identity matrix. Second, we assume that $\varepsilon_n$ does not have a permanent effect on $d_t$, i.e.,

$$C_{12}(1) = 0$$  \hspace{1cm} (2.b)

where $C_{12}(1) = c_{12,0} + c_{12,1} + c_{12,2} + \ldots$. Finally, we assume that $\varepsilon_n$
does not affect the time paths of \( d_t \) or \( r_t \) and, therefore,

\[ C_{13}(L) = C_{23}(L) = 0. \]  \hspace{1cm} (2.c)

Since \( c_{ij,k} \) measures the \( k \)-period ahead effect of a standard-deviation \( \sigma \) shock on variable \( i \), knowledge of the free parameters of the TMAR can be applied in a variety of commonly used ways to study how prices, rents, and interest rates are determined. For example, forecast error variance decompositions can be used to address our main concern, which is to measure the relative importance of fundamental vs. nonfundamental shocks in determining the time path of farmland price. Historical decompositions can also be used for this purpose and to isolate particular periods of time for which a particular type of shock seems to have been especially important in determining unusual movements in farmland prices (e.g., boom and bust periods).

Of course the TMAR cannot be estimated directly from the data since the innovations that appear in (1) are unobservable. However, assume that \( z_t \) has the following VAR(p) representation:

\[ z_t = A(L)z_{t-1} + u_t \]  \hspace{1cm} (3)

where \( A(L) = [A_{ij}(L)] \), \( A_{ij}(L) = a_{ij,0} + a_{ij,1}L + \ldots + a_{ij,p-1}L^{p-1} \) for \( i,j = 1,2,3 \) and \( u_t = [u_{1t} \ u_{2t} \ u_{3t}]' \) is the innovation vector, which is a zero-mean, serially uncorrelated process. Let \( \Sigma \) denote the variance-covariance matrix of \( u_t \), i.e., \( \Sigma = E(u_tu_t') \).

**Proposition:** The parameters in the TMAR (1), \( C(L) \), are over-identified by the VAR parameters in (3), \( A(L) \) and \( \Sigma \), when restrictions (2.a)-(2.c) are imposed.
The proof of this proposition is given in the Appendix and it provides the strategy for estimation of the TMAR (1) from estimates of the VAR parameters, provided the over-identifying restrictions are satisfied. This approach to identification follows along the path developed by Blanchard and Quah (1989) and Lee (1995a, 1995b, 1996).

3. Data

Nominal farmland price and rent data are updated versions of the annual Iowa price and rent data used by Falk (1991, 1992), covering the sample period 1922-1994. This data set is appealing because of its length and the homogeneity of the asset being priced. The price series measures the average price per acre of whole farms sold in Iowa and the rent series measures the average cash rent per acre for the rental of whole farms in Iowa. The January producer price index is used to deflate the data, January 1967 PPI = 100. Natural logs of the deflated price and rent series measure the variables \( p_t \) and \( d_t \), respectively.

The six-month commercial paper rate is used to measure the nominal interest rate. Featherstone and Baker (1987) and Hanson and Myers (1995) also used the commercial paper rate to measure the discount rate. Falk (1992) used Treasury bill rates but was forced to throw away several observations of price and rent since T-bill rate data are only available from 1926. The real interest rate, \( r_t \), was measured as the difference between the period \( t \) nominal interest rate and the inflation rate, \( \log PPI_t - \log PPI_{t-1} \).
The theoretical model developed in the preceding section began with the assumption that \( p_t \) and \( d_t \) are difference-stationary processes, while \( r_t \) and \( p_t - d_t \) are stationary processes. Augmented Dickey-Fuller and Phillips-Perron unit root tests were applied to test these restrictions. The results are summarized here and in Table I.

The null hypothesis that \( p_t \) (or \( d_t \)) is difference-stationary cannot be rejected at the 10-percent level against the alternative of trend-stationarity or the alternative of stationarity using either test procedure. However, the null hypothesis that \( \Delta p_t \) (or \( \Delta d_t \)) is difference-stationary can be rejected at the five-percent level against the alternative of stationarity using either test procedure. The null hypothesis that \( r_t \) (or \( p_t - d_t \)) is difference-stationary is rejected against the stationary alternative at the five-percent level. Thus, unit root test results are consistent with the assumptions made about the basic time series properties of \( p_t, d_t, \) and \( r_t \).

4. Empirical Results

The data series \( p_t, d_t, \) and \( r_t \) were transformed into the series \( \Delta d_t, \Delta d_t - r_t, \) and \( s_t \) (\( = p_t - d_t \)) and then fit to a second-order VAR. The lag length of two was implied by both the Akaike (1974) and Schwarz (1978) criteria. Our main interest in this VAR is to use it to identify the TMAR of \([\Delta d_t, \Delta d_t - r_t, s_t]'\) in terms of permanent fundamental innovations, temporary fundamental innovations, and nonfundamental innovations.
It is shown in the Appendix that the TMAR restrictions imply the following over-identifying restrictions on the VAR: \( A_{13}(L) = A_{23}(L) = 0 \), i.e., the spread does not Granger-cause the bivariate \( [\Delta d_t, \Delta d_t - r_t] \) process. This set of restrictions can be viewed as another preliminary test of the compatibility of the data with the theoretical model developed in Section 2. A quasi-log-likelihood test was applied to test these restrictions with the result that they cannot be rejected at the 10-percent significance level.\(^4\)

The estimated restricted VAR and restrictions (2.a)-(2.c) were used to estimate the TMAR parameters according to the procedure described in the Appendix. The remainder of this section presents the results of several applications of the TMAR.

### 4.1 Forecast Error Variance Decompositions

The first application measures the relative importance of fundamental versus nonfundamental shocks in explaining farmland price movements over various time horizons. More precisely, we compute the proportion of the variance of the k-step-ahead forecast error in \( p_t \) attributable to each of the three types of shocks: permanent fundamental shocks \((\epsilon_{1t})\), temporary fundamental shocks \((\epsilon_{2t})\), and nonfundamental shocks \((\epsilon_{3t})\). This is accomplished in two steps. First, the time paths of \( \Delta d_t, \Delta d_t - r_t \), and \( s_t \) are simulated according to the estimated TMAR in response to representative \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \) shocks in the standard manner to decompose the k-step-ahead forecast error variances for these three variables. The results of
this exercise are presented in the top panel (Panel A) of Table II, although they are not of direct interest for our purposes. Second, for each representative shock the simulated time paths of \( \Delta d_t, \Delta d_t - r_t \), and \( s_t \) are transformed into simulated time paths of \( d_t, r_t, \) and \( p_t \), which are used to obtain the forecast error variance decompositions presented in the lower panel (Panel B) of Table II.

According to Panel B, nonfundamental shocks account for fifty percent of the year-to-year volatility in (logged real) farmland price. That is, half of the year-to-year volatility in farmland prices cannot be explained by factors that influence rents or interest rates. The relative importance of these nonfundamental forces declines monotonically over time, accounting for about 25-percent of the six-year-ahead forecast error variance and 11-percent of the 24-year-ahead forecast-error variance. Thus, although half of the year-to-year volatility in farmland prices cannot be explained by factors that influence rents or interest rates, about 90-percent of the long-run volatility in farmland prices can be explained in terms of fundamental forces.

Temporary fundamental shocks are nearly as important as nonfundamental shocks in accounting for short-run forecast uncertainty in price. Their importance also falls monotonically as the forecast horizon increases, although the decline occurs much more rapidly than it does with respect to nonfundamental shocks: temporary fundamental shocks account for only about ten percent of the six-year-ahead forecast error variance in price and only about six percent of the 24-year-ahead forecast error variance. As the
forecast horizon is extended further, this percentage would decline further since, by construction, its limit must be zero as forecast horizon goes to infinity.

Permanent fundamental shocks explain nearly 85-percent of the 24-year-ahead forecast error variance in price, playing a more important role as the forecast horizon increases. Interestingly, permanent fundamental shocks are far less important than temporary fundamental shocks in explaining annual variation in price. By the two-year horizon they are about of equal importance and subsequently permanent fundamental shocks become increasingly important.

In summary, year-to-year movements in farmland prices are determined mostly by temporary fundamental shocks and nonfundamental shocks, these two types of shocks being about equally important in this regard. In the long-run, however, farmland prices are mostly explained by permanent fundamental shocks. Thus, purely speculative forces do seem to be important in explaining short-run price volatility in the Iowa farmland market, where the short-run can be interpreted as long as about five years. But the effects of these speculative forces eventually dissipate, as one would expect.

To conclude the analysis of Table II, notice that permanent fundamental shocks appear to be much more important relative to temporary fundamental shocks in explaining rent uncertainty than in explaining price uncertainty at all horizons, but especially at shorter horizons. Permanent fundamental shocks appear to be much less important relative to temporary fundamental shocks in
explaining real interest rate uncertainty than in explaining price uncertainty, except for the one-year ahead horizon. Nonfundamental shocks do not affect the time paths of $d_t$ or $r_t$ by construction.

### 4.2 Impulse Response Functions

Next we turn to Figure 1, which graphically illustrates the impulse response functions. Panels A and B illustrate the dynamic responses of $q$ and $r_t$, respectively, to a positive one-unit permanent fundamental shock and to a positive one-unit temporary fundamental shock. Panel C illustrates the dynamic response of $p_t$ to a positive, one-unit permanent fundamental shock, a positive, one-unit temporary fundamental shock, and a positive, one-unit nonfundamental shock.

In response to a one-unit positive permanent fundamental shock (log) rent increases initially by about .05 units, then gradually increases over about the next ten years toward a new long-run value, which is about .075 units greater than the initial value. The real interest rate, which is assumed to be a stationary process, initially decreases by about .02, then gradually increases back toward its initial level, which it reaches in about six to eight years. Thus, during the first six to eight years following a positive permanent fundamental shock current and expected future discount rates and rents are increasing, which increases the fundamental value of farmland. After this interval, expected future rents remain higher but the discount rate has returned to its normal level. So the fundamental value should decline a bit after
the initial run-up, but remain at a permanently higher level. This is exactly the pattern of response of (logged) farmland price (Panel C) to this shock, indicating that the responses of farmland price to permanent fundamental shocks are consistent with the predictions of the present value model.

Positive, temporary fundamental shocks temporarily increase rent and the interest rate according to Panels A and B of Figure 1. Rent and the interest rate initially increase by about .05 and decline monotonically toward zero, dissipating in about six to seven years. Thus, over the six to seven year period current and expected future rents will be higher but current and expected discount factors will be lower. At the end of the period, current and expected rents and discount factors will be at their initial values. If the farmland market responds to these shocks according to the present value model, the impact on farmland price over the first six to seven years will be ambiguous, but there should be no effect on farmland price after this interval. According to Panel C, however, farmland price increases above its initial value immediately after the shock then decreases monotonically over the next six to seven years, falling below its initial value after the first four years, and then slowly increases back toward the initial value. The nature of the response and length of the response period indicate that farmland price overreacts to temporary fundamental shocks.

Panel C also illustrates the reaction of farmland price to a positive, one-unit nonfundamental shock. The initial and long-run
effects are about the same as the effect of a temporary fundamental shock. However, farmland price remains above its initial value over the entire adjustment process.

4.3 Historical Decomposition

The estimated VAR and TMAR are used to decompose the actual (logged) real farmland price series into three components: the permanent fundamental component, the temporary fundamental component, and the nonfundamental component. The estimated VAR and TMAR and the relationship between the $u_t$'s and $\epsilon_t$'s enable us to estimate the $\epsilon_t$ time series. The estimated $\epsilon_{1t}$'s ($\epsilon_{2t}$'s, $\epsilon_{3t}$'s) are used to simulate the time path of $p_t$ and derive the permanent fundamental component (temporary fundamental component, nonfundamental component) of price.\(^5\) This decomposition is illustrated in Figure 2.

First consider the permanent fundamental component of farmland price, illustrated in Panel A. This component of farmland price appears to be a smoothed version of the actual price series, capturing the overall long-run behavior of price, but missing many of the short-run cycles in price. Notice, however, that the upward trend in actual price from about 1950 until about 1980 and the subsequent rapid fall in price during the 1980's is largely explained by the permanent fundamental component. The sample correlation between price and the permanent fundamental component is 0.92.

The (stationary) temporary fundamental and nonfundamental
components are of relatively minor significance in explaining the overall behavior of the (nonstationary) price series, which can be seen from the differences in the vertical scales of Panels B and C relative to Panel A. However, these two components in price explain the short-run volatility that the permanent fundamental component does not capture. So, for example, the short-run fluctuations in price prior to the early 1950's and the short-run decline in farmland prices around 1972 can be explained by movements in the temporary fundamental component of price. That part of the major boom and bust of the 1970's and 1980's not explained by the permanent fundamental component can be explained by the nonfundamental component. In particular, the very rapid growth in price during the late 1970's and the steep decline following the peak several years later, seem to be accounted for by the behavior of the nonfundamental component.

4.4 Further Discussion

The empirical results presented in this section indicate that there is an important nonfundamental (or fad) component to Iowa farmland price movements. This is indicated by the forecast error variance decomposition and the historical decomposition. The forecast error variance decomposition of price implies that about one-half of the year-to-year variation in farmland price is driven by nonfundamental shocks. Even over a six-year forecast horizon approximately one-quarter of the forecast error variance in price is attributable to nonfundamental shocks. The historical
decomposition provides an estimate of the nonfundamental component of the actual farmland price series and although this component generally is very small relative to the actual price, it has occasionally played an important role in the short-run dynamics of price, particularly in the several years before and after 1980.

Fads provide one explanation of the failure of price to move according to the predictions of the present value model. Another part of the story might be that prices overreact to fundamental shocks, i.e., market participants put more weight on news about rents and interest rates than the news deserves. The impulse response analysis provided some support to the overreaction hypothesis. In particular, the response of price to a temporary fundamental shock displayed in Figure 1, Panel C appears to be consistent with overreaction for reasons discussed earlier.

Falk (1991) characterized the failure of Iowa farmland price to satisfy the statistical restrictions implied by the present value model using the time series relationship among the real price, real rent, and the ex-ante rational price (i.e., the price implied by the present value model) to help make his point. Specifically, he showed that the ex-ante rational price typically moves less than proportionally with respect to changes in rent, while actual price moves more than proportionally with respect to changes in rent. In his setup, however, there was no room for a nonfundamental component in price and the discount rate was assumed to be constant.

In Figure 3 we illustrate the time series relationship among
the real price, real rent, and the fundamental component of price, where the fundamental component is the sum of the permanent and temporary fundamental components described in Figure 2. The fundamental component and the actual price series tend to fall on the same side of weighted rent, indicating that they both tend to move more than proportionally with respect to rent movements. This is in contrast to the behavior of Falk's ex-ante rational price, which moves less than proportionally with respect to rent movements. On this basis it appears that the fundamental component of price is not equivalent to the fundamental value of land implied by the present value theory: price appears to overreact to fundamental shocks. However, in Figure 3, the fundamental component tends to fall between actual price and weighted rent, indicating in yet another way that there is a fad component in farmland price.

5. Summary and Conclusions

The main purpose of this study was to propose and apply a procedure to decompose farmland price movements into movements attributable to fundamental factors (i.e., factors that influence the time paths of rents and interest rates) and movements attributable to nonfundamental factors. We assume that the real interest rate is a stationary process and that the bivariate log real price and log real rent process is a cointegrated process. Then we can formulate a trivariate moving average representation (TMAR) of the growth rate of real rent, the growth rate of real rent minus the real interest rate, and the log of the real price-
rent ratio. The innovations in this TMAR can be interpreted as permanent fundamental shocks, temporary fundamental shocks, and nonfundamental shocks. Knowledge of the parameters of the TMAR can be used in a variety of ways (e.g., impulse response analysis, forecast error variance decompositions, and historical decompositions) to study the influence of fundamental shocks and nonfundamental shocks on the time path of farmland prices. We prove that the parameters of the TMAR are overidentified by the parameters of a finite-order trivariate vector autoregression and so can easily be estimated from price, rent, and interest rate data.

The procedure is applied to study Iowa annual farmland prices and rents over the 1922-1994 sample period, using the six-month commercial paper rate (adjusted for inflation) to measure the real interest rate. Unit root tests indicate that the behavior of the data is consistent with the time series restrictions that the model imposes on price, rent, and the interest rate. Further, the overidentifying restrictions the model imposes on the VAR are not rejected. Therefore, we estimate a restricted VAR and apply it to identify the TMAR of interest to us. Based upon the estimated TMAR, our two main conclusions about the behavior of Iowa farmland prices are as follows.

First, nonfundamental shocks appear to play an important role in explaining the short-run behavior of farmland prices. In particular, short-run movements in farmland prices are mostly determined by temporary fundamental shocks and nonfundamental
shocks, with these two types of shocks being of roughly equal importance in this regard. In the long-run, however, farmland prices are mostly explained by permanent fundamental shocks. Second, the dynamic responses of rent, the interest rate, and price to permanent fundamental shocks seem to be consistent with the predictions of the present value model of asset pricing. However, their dynamic responses to temporary fundamental shocks suggest that farmland prices overreact to temporary fundamental shocks.

Thus, we conclude that deviations of farmland price from the predictions of the present value model are important in the short-run but not in the long-run. The short-run deviations appear to be a combination of overreactions to temporary fundamental shocks and reactions to nonfundamental factors.
NOTES

1. The VAR representation of the trivariate process \([\Delta d_t \Delta r_t s_t]\)' can be formally derived from the asset pricing model applied by Campbell (1991) and Campbell and Ammer (1993), although it exists more generally. The Campbell-Ammer model extends the log-linear approximate asset pricing framework developed by Campbell and Shiller (1988) by allowing for excess returns (due to overreaction to fundamentals or reactions to nonfundamentals). Note that we cannot work directly with the \([\Delta p_t \Delta d_t r_t]\)' process because the assumption that \(p\) and \(d\) are cointegrated means that this trivariate process does not have a finite-order VAR representation.

2. See, for example, Quah (1992).

3. The price and rent data are actually available since 1921. However, we followed Falk (1991,1992) in pushing each price data point up a year since the published prices (at least since 1950) are end-of-the-year prices. Thus, land purchased at the beginning of year \(t\) at a price per acre of \(p_t\) is assumed to generate per acre rent \(d_t\) during year \(t\). Further discussion of these data and their sources can be found in Falk's papers. The data we use here are available upon request.

4. Under the null hypothesis, the statistic \(T(\ln|V_r|-\ln|V_u|)\) is
asymptotically distributed as a $\chi^2(4)$, where $T$ is the effective
sample size for estimation of the VAR, $\ln|V_\varepsilon|$ is the natural log of
the determinant of the sample second moment matrix of the residual
vector from the restricted VAR, and $\ln|V_u|$ is the natural log of the
determinant of the sample second moment matrix of the residual
vector from the unrestricted VAR. The realized value of the
statistic was 6.19 implying a p-value of .19.

5. Since $u_\epsilon$ and $C_0\varepsilon_\iota$ are both the innovation vector in $z_\iota$, $C_0\varepsilon_\iota = u_\epsilon$.
Thus, given the VAR estimates of $u_\iota$ and the estimated $C_0$, estimates
of $\varepsilon_\iota$ can be obtained according to $\varepsilon_\iota = C_0^{-1}u_\iota$. Construct a new $u_\iota$
sequence according to $u_\iota = C_0[\varepsilon_\iota\ 0\ 0]'$. Use the estimated VAR to
simulate the behavior of $\Delta d_\iota$, $\Delta d_\iota - r_\iota$, and $s_\iota$ from the initial
conditions and this innovation sequence. This yields the permanent
fundamental components of $d_\iota$, $r_\iota$, and $p_\iota$. The temporary fundamental
component and the nonfundamental component are constructed analogously.

6. See Figure 3 in Falk (1991).

7. Rent is weighted by the constant 14.92, which is the reciprocal
of the sample mean real rate of return in this market.
APPENDIX

In this Appendix, we prove that the moving average representation of $z_t$ satisfying restrictions (2.a)-(2.c) is over-identified by its vector autoregressive representation. We also characterize the over-identifying restrictions.

Let $C_0$ denote the coefficient matrix associated with the contemporaneous innovation term $\epsilon_t$ in (1), the MA representation of $z_t$. Comparing (1) and (3), the VAR representation of $z_t$, note that $C_0\epsilon_t$ and $u_t$ are both defined to be the innovation in $z_t$ and so

$$C_0\epsilon_t = u_t. \quad \text{(A.1)}$$

Further, (1) and (3) imply

$$C(L)\epsilon_t = [I - A(L)L]^{-1}u_t, \quad \text{(A.2)}$$

which, in light of (A.1), requires that

$$C(L) = [I - A(L)L]^{-1}C_0. \quad \text{(A.3)}$$

From (A.3) it is clear that given $A(L)$, $C(L)$ can be determined once $C_0$ is determined. To determine $C_0$'s nine elements, first note from (2.c) that

$$c_{13,0} = 0 \text{ and } c_{23,0} = 0. \quad \text{(A.4)}$$

Second, from (A.1) and the normalization restrictions (2.a), we obtain the condition

$$C_0C_0' = \Sigma_u \quad \text{(A.5)}$$

where $\Sigma_u$ is the contemporaneous covariance matrix of $u_t$, imposing six additional restrictions on $C_0$. Third, setting $L = 1$ in (A.3)
and using the long-run restriction (2.b),

\[ ([I - A(1)]^{-1}C_0)_{12} = 0. \]

Thus, (A.3), (A.4), and (A.5) impose nine restrictions on \( C_0 \) that identify that matrix given the VAR parameters \( A(L) \) and \( \Sigma_u \).

Restrictions (2.c) impose additional conditions on \( C(L) \) beyond those in (A.4). These are overidentifying restrictions which imply that in the VAR representation of \( z_t \)

\[ A_{13}(L) = 0 \quad \text{and} \quad A_{23}(L) = 0, \quad (A.6) \]

that is \( z_n \) does not Granger-cause the \( [z_{1t}, z_{2t}]' \) process.
REFERENCES


Tables and Figures:

Table 1 - Unit Root Test Results
Table 2 - Forecast Error Variance Decompositions
Figure 1 - Impulse Response Graphs
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Figure 3 - Price, Weighted Rent, Fundamental Component