Endogenous risk in weed control management

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Abstract

Weed control decisions are modeled in an endogenous risk framework where a producer invests in self-insurance and self-protection to reduce the severity of a realized pest infestation, or reduce the likelihood the infestation occurs. Self-insurance and self-protection are risk-reducing technologies that capture both the type and quantity of herbicides used. We supply conditions to unambiguously sign the effects of an increase in the probability of application or effectiveness failure and increased application or effectiveness uncertainty on optimal herbicide choices. If self-protection and self-insurance are stochastic substitutes, non-point source pollution policies targeted to reduce herbicide loadings can increase the use of more persistent herbicides. Policies that decrease loadings by reducing total mass may induce a substitution to herbicides more damaging or more likely to be transported to sensitive areas.

1. Introduction

Economists have argued that pesticides act as market insurance against uncertain crop damage. Uncertainty induces a risk-averse producer to use more pesticides than he would otherwise use to transfer wealth from good to bad states of nature (see, for example, Norgaard, 1976, and Feder, 1979). But Pannell (1990, 1991) has questioned this view. Using a damage function supported by agronomic evidence on weed control, Pannell demonstrated that uncertainty might decrease herbicide use for a risk-averse producer. Pannell points out that uncertainty about variables such as output price and yields may indeed lead to lower optimal levels of pesticide use. However, using a damage function supported by agronomic evidence on weed control, uncertainty about variables such as pest density and pest mortality lead to higher optimal levels of pesticide use. This suggests that other reasons beyond market insurance may exist to explain pesticide use under uncertainty—an argument supported by Deen et al. (1993) who show that only extremely risk-averse producers increase herbicide use under uncertainty.

This paper offers another perspective for pesticide use under uncertainty—endogenous risk. At the most fundamental level, endogenous risk redefines the standard view of pesticide use that generally presumes that the likelihood of weed damage is beyond the control of a producer. Endogenous risk implies that a producer can invest resources in risk-reducing technologies that influence the expected economic consequences of the hazards he or she confronts. This realization has profound impacts on formal evaluation of weed control strategies and the subsequent environmental impacts which have been traditionally driven by an assumption of exogenous risk.
Indeed, outside the field of economics, discussions of risk typically consider actions that modify events or reduce the vulnerability to loss (Kates, 1978). Note that we follow this substantial literature on environmental damages, and define ‘risk’ as the probability times the severity of damages, not as the variability of expected wealth (see, for example, Burton et al., 1979). The main purpose of the endogenous risk approach is to break the mind-set that equates the probability distribution of outcomes with the probability distribution of states. Both outcomes and the lotteries that define these outcomes depend on a producer’s actions.

Two decades ago Ehrlich and Becker (1972) defined the two basic technologies of endogenous risk—self-protection and self-insurance. Self-protection reduces the probability of an undesired state of the world, while self-insurance reduces the severity if the state occurs. Since their seminal article, researchers have explored how choice under risk is affected by the opportunity to reduce risk using self-protection and self-insurance. Examples include Hiebert (1983), Boyer and Dionne (1983), Chang and Ehrlich (1985), Shogren and Crocker (1991), among others. One general conclusion is that the endogenous risk model allows for a broader range of behavior than does the standard exogenous risk model. This broader, less constrained range of behavior exists because models with endogenous risk assume preferences over both outcomes and the lotteries that define these outcomes (Shogren, 1991). As a consequence, endogenous risk captures the precise factor of interest in weed control strategies—the technology of risk reduction.

Our approach adapts the concepts of self-protection and self-insurance to weed control decisions. From an endogenous risk perspective, weed control is viewed as a risk-reduction technology aimed at decreasing the expected damage from pest infestation. Whereas previous studies have focused primarily on herbicide application rates and have not addressed the complexity in producers’ choices of crop protection technologies, the endogenous risk approach provides a theoretical setting to capture some of this complexity. Some of the weed control factors that might be considered in the endogenous risk setting include herbicide types and quantities, application timings, cultivation methods and timings, and crop rotations. In this analysis we focus on three of these factors: herbicide application rate, herbicide timing flexibility, and herbicide persistence. In modeling herbicide application rates as investments in self-insurance which decreases the magnitude of yield loss, and modeling herbicide timing flexibility or herbicide persistence as investments in self-protection which reduces the probability of a loss, we capture both the type and quantity of herbicide used. This allows us to consider the substitution between rate and persistence and between rate and timing flexibility, and it allows us to consider how this substitution is affected by increased uncertainty regarding the efficacy of alternative weed control strategies. We supply conditions to unambiguously sign the effects of increased application or effectiveness risk on these optimal herbicide choices. Since a producer can replace a herbicide with a high application rate but low persistence for another with a low rate and high persistence, modeling these substitution possibilities is vital for a better understanding of producer decisions and more effective non-point source pollution policy.

Hirshleifer (1970) and other economists have argued it is always possible to redefine a problem such that the state of nature is independent of a producer’s actions. This position allows one, as Laffont (1980) noted, to continue working within the highly tractable framework of exogenous risk. But as Ehrlich and Becker (1972) point out, “a search for state probabilities that are independent of human action would be self-defeating” (p. 638). Consider a situation in which weed infestation threatens a producer’s crop. The probability and severity of weed damage can be altered if the producer selects an appropriate weed control strategy. An analyst might redefine the situation as independent of the producer’s actions by focusing solely on the likelihood of weed infestation, over which the producer likely has no control. But this definition is economically irrelevant if the question is the producer’s response to and damages from weed infestation.

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3 Thanks to a reviewer for emphasizing this point.
exercise some control over those events. The producer's risk is endogenous because by expending his or her valuable resources he or she can influence the probability that good states of nature occur, and reduce the severity of the bad state if it does occur.

Introduction of endogenous risk causes several key interdependencies to come to the fore. Our results suggest that although increased probability of loss can reduce herbicide application rates, this is likely to be offset by a shift towards the use of more persistent herbicides. For a given set of plausible conditions, this result can hold even if the probability of loss is uncertain. While this supports Pannell's argument, the result also suggests the use of more potent herbicides. If self-protection and self-insurance are stochastic substitutes, non-point source pollution policies targeted to reduce herbicide loadings can increase the use of more persistent herbicides. Policies that decrease loading by reducing total mass may induce a substitution to herbicides more damaging or more likely to be transported to sensitive areas.

The paper proceeds as follows. Section 2 presents a model of self-protection and self-insurance given the probability that a good state is known with certainty. Section 3 relaxes the assumption of certain probabilities and explores the restrictions under which one can unambiguously sign the comparative statics of increased uncertainty regarding weed control efficacy. We employ the WISH simulation model to test the plausibility of these alternative restrictions. Finally, we offer our conclusions in Section 4.

2. Self-insurance and self-protection with certain probabilities

In the US corn belt, weather uncertainty can lead to two catastrophic failures in weed control: application failure and effectiveness failure. Application failure occurs when weather conditions prevent the producer from applying a herbicide; for example, fields are too wet during the critical application times. Effectiveness failure occurs when weather conditions render an applied herbicide completely ineffective. This occurs if no rain falls after applica-

tion when rain is needed to carry the herbicide into the soil, or if herbicide effectiveness runs out during a "critical period for weed control". In both cases, weeds will grow unchecked causing a substantial loss in yield. Zimdahl (1980) cites studies of yield losses in corn for various weed species at different weed densities, indicating yield losses of 25–38% for high weed densities.

Following Archer and Shogren (1994), consider a risk-neutral farmer who employs herbicides as self-protection and self-insurance on a per-acre basis to reduce the probability of application and effectiveness failures, thereby increasing his expected returns from crop production. Let \( q \) and \( H \) represent self-protection and self-insurance efforts. Assume there are two mutually exclusive and jointly exhaustive states of the world—a good state implying no weed control failure and a bad state implying failure. The probability that the good state occurs, \( g(q) \), is a function of self-protection, \( q \), where \( g'(q) > 0 \). (Primes denote relevant derivatives.) The bad state occurs with probability \([1 - g(q)] \).

Self-protection, \( q \), is the herbicide type based on timing of application or herbicide persistence. The type depends on whether the producer is trying to reduce application or effectiveness failure. For application failure, self-protection is the timing flexibility of a herbicide. Timing flexibility is quantified as the length of time during which a herbicide can be applied and still be effective. Following Bouzaher et al. (1992), this time period is called the "application window". For example, atrazine combined with cyanazine is a common tank mix used to control weeds in corn. The period from 10 May to 25 May represents the critical period for a pre-emergence application of this tank mix in Iowa to achieve full control of weeds. In contrast, metolachlor and dicamba is also a pre-emergence tank mix, but the critical period is reduced to 10 May to 17 May. The application window for atrazine-cyanazine is 16 days, while the window for metolachlor and dicamba

\[ \text{(1991) for a more complete discussion.} \]

\[ \text{We assume risk neutrality to simplify the exposition and focus on technological effects rather than tastes.} \]
is 8 days. The atrazine–cyanazine tank mix represents a more flexible weed control strategy.

For effectiveness failure, self-protection is the temporal persistence of a herbicide once applied—the ‘effectiveness window’. For corn herbicides, Bouza­her et al. (1992) defined effectiveness windows for two broad categories of weeds: broadleafs and grasses. The pre-emergence atrazine–cyanazine tank mix was judged to remain effective on broadleafs for 70 days after application, and on grasses for 50 days. Metolachlor and dicamba were judged to remain effective on broadleafs for 20 days after application, and on grasses for 50 days. For simplicity, we focus on only one effectiveness window. We can think of this assumption as modeling a producer who is faced with an infestation of only one category of weeds (e.g. grasses only).

Therefore, let \( q \) denote the length of the relevant window, where \( q \) is continuous. The application window alters the probability that a farmer can apply the herbicide, while the effectiveness window influences the probability that a herbicide is effective.

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6 In reality, choosing a herbicide is a discrete choice, so the choice of window length is also discrete. To keep this model mathematically tractable, we assume that the discrete nature of the problem is not critical and the choices can be approximated by a continuous variable. This is similar to the assumption made by Beach and Carlson (1993).
once applied. For simplicity, we consider the choice of either an application window or an effectiveness window only as separate cases. In reality, this may be a joint decision. Also, in reality, producers have the option of post-emergence herbicides to control weed escapes from previous treatments. While these possibilities are simulated in the WISH model (Weather Impact Simulator for Herbicides) developed by Bouzaher et al. (1992), they add considerable complication to the theoretical analysis. We use the WISH model to provide a simulated estimate of uncertainty in weed control strategies because of weather (see Bouzaher et al., 1992, for a detailed discussion of WISH).

The unit cost, \( c(q) \), of the herbicide also depends on \( q \). Assuming that herbicides providing longer effectiveness or application windows are more expensive, \( c'(q) > 0 \).

Fig. 1 shows a plot of the sum of application and effectiveness window lengths versus the cost of each strategy applied on reduced tillage and a clay soil. Each point on the plot was generated using the WISH model. The solid line joins strategies that provide the longest combined window lengths for a given cost, indicative of the producer’s cost function. As window lengths increase, costs increase at an increasing rate. Fig. 1 also illustrates the effect of banning the use of various herbicides. The dashed line joins the most cost-effective strategies that do not contain atrazine. Banning atrazine shifts the cost curve upwards to the left; producers pay a higher price to purchase herbicides with equal window lengths. Also, herbicides with the longest window lengths are eliminated. Similarly, the dot/dash line joins strategies that do not contain triazines. A triazine ban further shifts the cost curve upwards to the left, increasing the price a producer pays for a given window length and restricting the choices to smaller window lengths.

Self-insurance, \( H \), is the application rate of the herbicide. Note there is a difference between herbicide self-insurance and the typical notion of self-insurance. In other discussions of endogenous risk, self-insurance typically decreases the magnitude of loss if a failure occurs. This brings the wealth in the two states closer together by increasing the returns in the bad state, thereby decreasing income variability. But, in the herbicide model, yield losses occur in both the good and bad state: herbicide use only decreases the magnitude of losses when applied, corresponding to the good state in our model. Herbicide use drives the states further apart by increasing returns in the good state, thereby increasing income variability. The damage in the good state caused by weed control failure, \( D(W) \), is a function of weed density, \( W \), which is influenced by the level of self-insurance, \( H \), such that \( D'(W)W'(H) < 0 \). This specification agrees with the observation of Pannell (1990), that increased herbicide use can increase income variability, and that herbicides are not risk-reducing inputs in the traditional economics sense.

The producer maximizes per-acre expected profits by selecting \( q \) and \( H \):

\[
\text{Max } E \pi = g(q) \left[ PY_0 [1 - D(W)H] \right] - c(q)H \\
+ (1 - g(q)) \left[ PY_0 [1 - D(W_0)] - \rho c(q)H \right]
\]

where \( Y_0 \) is weed-free yield and \( P \) is crop price. The parameter \( \rho \) is a binary zero–one variable. Let \( \rho = 1 \) for an effectiveness failure since the producer incurs a herbicide cost, \( c(q)H \), in both states. Let \( \rho = 0 \) for an application failure since the producer incurs no herbicide cost in the bad state. Let the damage, \( D(W) \), be specified as a hyperbolic function (Cousens, 1985; Pannell, 1990):

\[
D(W(H)) = \omega / [1 + \omega / (\Phi W(H))]
\]

with weed density, \( W(H) \), specified by:

\[
W(H) = W_0 e^{-kH}
\]

where \( W_0 \) is pretreatment weed density, \( \omega \) is asymptotic yield loss, \( \Phi \) is per unit yield loss, and \( k \) is a parameter. The first term in Eq. (1) represents the producer’s profit if there is no weed control failure; the second term represents profits with either an application or an effectiveness failure.

The first-order conditions for an interior maximum are:

\[
E \pi_q = g'(q) \left[ PY_0 [D(W_0) - D(W)] \right]
\]
\[-(1 - \rho) c(q) H] - c'(q) H \\
\times \{ \rho + (1 - \rho) g(q) \} = 0 \tag{4} \]

and

\[ E\pi_H = -g(q) P Y_0 D'(W) W'(H) \\
- \{(1 - \rho) g(q) c(q) + \rho c(q) \} = 0 \tag{5} \]

The second-order conditions require \( E\pi_{HH} < 0 \), \( E\pi_{qq} < 0 \), and \( G = E\pi_{HH} E\pi_{qq} - (E\pi_{Hq})^2 > 0 \).

The first term on the right-hand side of Eq. (4) represents the marginal expected benefit of self-protection, \( q \), in increasing the probability of successful weed control. For the application case, the benefits of reduced damage with a successful herbicide treatment are partially offset by the added cost of a weed control. For the application case, the benefits represent the marginal expected benefit of self-protection-a constant increase in the probability of a failure for all lengths of the effectiveness or application window, \( q \). In this case, the producer chooses \( H \) so the marginal benefit of \( H \) in the good state is equal to the marginal cost of \( H \) in the good state.

Following Hiebert (1983), let \( g(q) \) take the form \( g(q) = g_0 + \gamma h(q) \), with \( h(q) > 0 \) and with \( g_0 \) and \( \gamma \) as positive constants. An increase in the probability of a loss is modeled as a decrease in either \( g_0 \) or \( \gamma \). A decrease in \( g_0 \) represents an increase in the probability of failure that is independent of window length. This represents an exogenous change in the probability of failure that cannot be ameliorated with self-protection—a constant increase in the probability of a failure for all lengths of the effectiveness or application window, \( q \). In contrast, a decrease in \( \gamma \) represents an increase in the probability of failure that increases proportional to \( q \). We can think of it as a decrease in the efficacy of self-protection. An example of decreased \( \gamma \) can be seen by comparing two farmers in two different climates. Suppose for a herbicide to be potent, rain is required during the effectiveness window. If one farmer lives in a drier climate than another farmer, the first farmer will find a herbicide with a given effectiveness window less active than that of the second farmer. In this case, drier climate corresponds to a lower \( \gamma \).

For the application case \( (\rho = 0) \), the comparative static effects of a decrease in \( g_0 \) are given by \( \delta q^* / \delta g_0 < 0 \) and \( \delta H^* / \delta g_0 > 0 \) (see Appendix A), which is summarized by the following proposition.

Proposition 1: An increase in the probability of application failure \( (\rho = 0) \) due to a decrease in the exogenous probability of success, \( g_0 \), will decrease the optimal application rate of herbicide use and increase the length of the optimal application window.

For the effectiveness case \( (\rho = 1) \), \( \delta q^* / \delta g_0 < 0 \) and \( \delta H^* / \delta g_0 > 0 \) (see Appendix A). The following corollary summarizes this result.

Corollary 1: Assuming damage abatement is globally concave in \( H \), an increase in the probability of effectiveness failure \( (\rho = 1) \) due to a decrease in the exogenous probability of success, \( g_0 \), will decrease the optimal application rate of herbicide use and increase the length of the optimal effectiveness window.

Proposition 1 and Corollary 1 suggest that an increase in the probability of weed control failure will result in producers using lower doses of herbicides but with longer effectiveness or application windows. This may be a mixed blessing for the control of non-point source pollution from agrichemical use. Reconsider Fig. 1. As window lengths increase, a producer eliminates the strictly non-triazine activities and begins using only activities that contain atrazine. In the US corn belt, atrazine is the most detected herbicide in surface water, ground water, and precipitation (Goolsby et al., 1991; Holden and Graham, 1992; Nations and Hallberg, 1992). Holden and Graham indicate that the frequency of occurrence of herbicides in ground water is due to both the likelihood of their use and their persistence in the soil. Of the soil-applied herbicides used in corn, atrazine and simazine (a triazine herbicide) have the longest soil half-lives (Becker et al., 1989). Although increased application or effectiveness failure reduces herbicide loadings by decreasing application rates, if the herbicide is more persistent, the potential for non-point source pollution may actually
increase. The point is that a change in the probability of weed control failure has an effect on both the types and quantities of herbicides used, and the endogenous risk approach allows us to model this effect. This is important from a non-point pollution standpoint since levels of non-point pollution and resulting damages are not only dependent on herbicide quantities, but also on herbicide types. Predicting the specific effect of changes in herbicide quantities and types on levels of non-point pollution is best handled using fate and transport models. The CEEPES modeling system is an example of a practical use of this approach to link weed control decisions (via WISH) to levels of non-point source pollution (see Bouzaher et al., 1994).

Note that although Proposition 1 and Corollary 1 give the same result for an exogenous decrease in the probability of success, the result occurs for different reasons. In the case of Proposition 1 (application failure), an exogenous decrease in the probability of success has the direct effect of decreasing the expected marginal cost of self-protection, \( q \), thereby increasing the length of the optimal application window. Additionally, \( q \) and \( H \) are stochastic substitutes, inputs where an increase in the optimal level of one input has the indirect effect of decreasing the other input. If \( G \) is the Hessian matrix of the problem and \( G_{ij} \) are the minors of \( G \), Hiebert (1983) defines inputs \( i \) and \( j \) as stochastic substitutes (complements) if \( G_{ij} < 0 \) (\( > 0 \)). Since \( q \) and \( H \) are stochastic substitutes and an exogenous decrease in the probability of success has the direct effect of increasing the length of the optimal application window, this in turn decreases the optimal application rate. There is no direct effect of an exogenous decrease in the probability of a successful application on the optimal application rate. In the case of Corollary 1 (effectiveness failure), an exogenous decrease in the probability of success has the direct effect of reducing the expected marginal benefit of \( H \), thereby decreasing the optimal application rate. Since \( q \) and \( H \) are stochastic substitutes, this in turn increases the length of the optimal effectiveness window. There is no direct effect of an exogenous decrease in the probability a herbicide will be effective on the length of the optimal effectiveness window.

In addition, knowledge on how an exogenous increase in the probability of application failure affects the net likelihood of a successful herbicide application can be useful for non-point pollution policy. Holding constant parameters such as crop prices, herbicide prices, and initial weed density, the effect of an exogenous change in the probability of application failure on the net likelihood of a successful herbicide application is found by differentiating \( g(q^*) = g^0 + \gamma h(q^*) \) with respect to \( g^0 \):

\[
\frac{\partial g(q^*)}{\partial g^0} = 1 + \gamma h(q^*) \frac{\partial q^*}{\partial g^0}
\]

The two terms in Eq. (6) represent two effects—a direct effect and an indirect effect. The first term in Eq. (6), which is positive, is the direct effect. This term shows that a decrease in \( g^0 \) has the direct effect of decreasing \( g(q^*) \). The second term is the indirect effect. Proposition 1 implies that this term is negative, indicating that a decrease in \( g^0 \) increases \( q^* \) and hence has the indirect effect of increasing \( g(q^*) \). The net result depends on which term dominates. If the second term exceeds \(-1\), an exogenous increase in the probability of an application failure reduces the likelihood of success, resulting in lower doses of herbicide that are more persistent but have a smaller likelihood of application. If the second term is less than \(-1\), it is more likely the producer will successfully apply the herbicide—an exogenous increase in the probability of an application failure results in lower doses of herbicides with longer persistence having a higher likelihood of application. This possibility is somewhat counterintuitive. It says that an exogenous decrease in the probability of a successful herbicide application may cause producers to shift to a herbicide with a long enough application window that the probability of a successful application actually increases.

Now consider the comparative statics of decreased \( \gamma \) on the optimal levels of \( H \) and \( q \). For the application case \((\rho = 0)\), \( \partial q^*/\partial \gamma > 0 \) and \( \partial H^*/\partial \gamma < 0 \) (see Appendix B), which is summarized below.

Proposition 2: An increase in the probability of an application failure \((\rho = 0)\) due to a decrease in the efficacy of self-protection will increase the optimal
application rate of herbicide use and decrease the length of the optimal application window.

The result suggests that a decrease in the efficacy of self-protection results in herbicides being applied at higher doses but with shorter application windows. Again consider Fig. 1. An increase in the probability of application failure through a reduction in the efficacy of the application window causes a shift from right to left going from the region which is predominantly atrazine and other triazine herbicides with longer application windows to predominantly non-triazine herbicides with shorter application windows. Application rates would also increase relative to the recommended rates.

As illustrated by Propositions 1 and 2, decreasing \( g^0 \) and \( \gamma \) have the opposite effect on the optimal values of \( H \) and \( q \). This is due to a combination of three factors. First, the first-order condition, Eq. (5), implies that the marginal benefit of \( H \) in the good state is equal to the unit price of \( H \) in the good state. This means \( H^* \) is not directly affected by an increase in the probability of failure (\( E_q \sigma_{e^0} = E_q \sigma_{e^y} = 0 \)), so \( H^* \) depends only indirectly on \( g^0 \) or \( \gamma \) through \( q^* \). Conversely, \( q^* \) is directly affected by an increase in the probability of failure (\( E_q \sigma_{e^0} \neq 0 \) and \( E_q \sigma_{e^y} \neq 0 \), but since \( H^* \) is not directly affected there is no indirect effect of \( H^* \) on \( q^* \). The net result is that \( q^* \) depends directly on \( g^0 \) or \( \gamma \), while \( H^* \) depends only on \( q^* \) and on whether \( q \) and \( H \) are stochastic substitutes or complements.

Second, Eq. (A4) implies that \( q \) and \( H \) are stochastic substitutes, so an increase in the probability of failure will shift \( H^* \) in the opposite direction to which \( q^* \) shifts. Intuitively, an increase in \( q^* \) increases the unit price of herbicide causing the producer to apply less, decreasing \( H^* \); decreasing \( q^* \) decreases the unit price, thereby increasing \( H^* \).

Third, examining the first-order condition, Eq. (4), a decrease in \( g^0 \) or \( \gamma \) decreases the probability a herbicide will be successfully applied, thereby reducing the expected marginal cost of \( q \). However, a decrease in \( g^0 \) leaves the marginal expected benefit of \( q \) unchanged, so the producer finds it optimal to increase the expenditure on \( q^* \). Since \( q \) and \( H \) are stochastic substitutes, the producer then reduces the expenditure on \( H^* \). In contrast, a decrease in \( \gamma \) decreases marginal expected benefits of \( q \). Furthermore, the marginal expected benefit declines faster than the expected marginal cost, so the producer finds it optimal to decrease the expenditure on \( q^* \). Since \( q \) and \( H \) are stochastic substitutes, the producer then increases the expenditure on \( H^* \).

From a policy standpoint, this indicates the importance of understanding the source of increased probability of failure to determine its effect on herbicide use and the relative environmental impacts of herbicide application rates and types of herbicides used. Suppose we know that a stream flowing through an agricultural area is a critical wildlife feeding area and that the aquatic vegetation is sensitive to almost all types of herbicides. Further suppose that increased herbicide concentrations reaching this area may cause substantial damage, but a shift in the type of herbicide that reaches this area will have little effect. In this case, increased probability of failure through a reduction in the efficacy of the application window will result in herbicides being applied at higher rates, increasing herbicide concentrations in the stream and increasing the potential for environmental damage. But increased probability of failure that leaves the efficacy of the application window unchanged results in herbicides being applied at lower rates, causing herbicide concentrations and the potential for damage to decrease.

The effect of a decrease in \( \gamma \) on the probability of successful application is given by:

\[
\frac{\partial g(q^*)}{\partial \gamma} = h(q^*) + \gamma h'(q^*) \frac{\partial q^*}{\partial \gamma} \tag{7}
\]

Here both terms are positive, implying that herbicides will have a lower probability of being applied when \( \gamma \) decreases. This suggests that herbicides with shorter application windows will be successfully applied less often but that when an application occurs it will be at a higher dose. For the effectiveness case \( (\rho = 1) \), \( \partial q^*/\partial \gamma > 0 \) and \( \partial H^*/\partial \gamma > 0 \) if \( q \) and \( H \) are stochastic complements (see Appendix B). This result is summarized below.

Corollary 2: If self-protection and self-insurance are stochastic complements, an increase in the probability of effectiveness failure \( (\rho = 1) \) through a decrease in the efficacy of self-protection results in a decrease in both the optimal herbicide rate and the optimal length of the effectiveness window.

Intuitively, a decrease in the efficacy of self-protection has the direct effect of decreasing the marginal
benefit of self-protection, causing the producer to use herbicides with a shorter effectiveness window. Also, a decrease in the efficacy of self-protection has the direct effect of decreasing the expected marginal benefit of self-insurance, causing the producer to apply herbicides at a lower rate. That is, if there is a higher likelihood that a herbicide will be ineffective, the producer can reduce the amount of money he will lose simply by applying less of it. If self-protection and self-insurance are stochastic complements, a decrease in one has the indirect effect of decreasing the other, so both the direct effects and indirect effects move in the same direction. In this case a decrease in the efficacy of self-protection decreases the optimal levels of self-protection and self-insurance. However, if self-protection and self-insurance are stochastic substitutes, a decrease in one has the indirect effect of increasing the other. In this case, unless we can determine whether the direct effects or the indirect effects dominate, we cannot sign the effects of a decrease in the efficacy of self-protection on the optimum levels of self-protection and self-insurance.

If self-protection and self-insurance are stochastic complements, increased probability of effectiveness failure results in herbicides with shorter effectiveness windows being applied at lower rates. Using Fig. 1, we would see a shift from atrazine and other triazine herbicides with longer effectiveness windows to non-triazine herbicides with shorter effectiveness windows. We would also see herbicide application rates decrease relative to the recommended rates. Note that, although the probability of application or effectiveness failure is associated with weather uncertainty, we do not focus specifically on weather uncertainty. This is what distinguishes the endogenous risk approach from the exogenous risk approach. We focus on the probabilities that are relevant to the decision-maker (e.g. application and effectiveness risk) and not on the probabilities that are independent of human actions (e.g. weather).

If we allow the producer to be risk-averse instead of risk-neutral, our basic approach remains the same, except the producer now maximizes expected utility, where the utility function is concave in profit. Allowing for risk aversion would require us to consider the effects of self-insurance and self-protection on the variability of profit as well as expected profit. Recall that an increase in \( H \) increases the variability of profit, so we should expect herbicide application rates to be lower for a risk-averse producer than for a risk-neutral producer. Self-protection, \( q \), may increase or decrease the variability of profit depending on whether the probability of success is low or high. If the probability of success is close to zero, variability of profit is low with the weight of the distribution largely on the bad state. An increase in \( q \) will shift some weight of the distribution from the bad state to the good state initially increasing the variability of profit. However, as \( q \) continues to increase, the weight of the distribution becomes more and more centered on the good state, decreasing the variability of profit. In weed control, we generally think of producers choosing herbicides that have a high probability of success. In this case we can think of \( q \) decreasing variability of profit, so a risk-averse producer would tend to use herbicides with longer application or effectiveness windows than a risk-neutral producer.

However, the comparative statics for a risk-averse producer are more complicated. The addition of a concave utility function is an additional source of curvature. We now have to understand how an increase in the probability of failure affects the marginal productivities of self-insurance and self-protection, and marginal utilities are also affected. In general, this additional curvature makes the comparative statics ambiguous and additional restrictions will be necessary on the risk-reducing technologies, risk preferences and their interaction (see Dionne and Eeckhoudt, 1985, or Shogren and Crocker, 1991).

3. Self-insurance and self-protection with uncertain probabilities

Following the standard literature on endogenous risk, we have presumed that the producer is certain of the efficacy of self-protection. However, it is conceivable that the efficacy of self-protection is uncertain—the effect of self-protection on the probability of loss is not known with certainty (Shogren, 1991). In weed control this means that the producer is not sure how effective added herbicide flexibility or persistence will be in reducing the probability of a weed control failure. For example, a producer reads
in the extension weed control manual that atrazine should be applied during a particular time period. But his own past experience or that of his neighbor calls into question the certainty of that advice. This doubt can be reflected by the variability of the length of the application or effectiveness window or can be manifest in the variability of the efficacy of these windows. He knows the herbicide can reduce the probability of failure, but he remains unsure as to the actual capability of the window length or window efficacy. Additionally, a producer could be uncertain as to the efficacy of a weed control strategy applied to different fields due to differences in organic matter, capacity to hold moisture during a drought or hold water after a rain.

We show that if a producer doubts the self-protection a herbicide provides, it is again possible that he will increase the use of more persistent herbicides applied at lower rates. Intuitively, this says that if a producer is uncertain about how effective a herbicide is in reducing the probability of a loss he may respond to this uncertainty by choosing a herbicide that provides greater application flexibility or is more persistent.

Consider the problem of choosing the level of self-protection and self-insurance where the probabilities of each state occurring are uncertain. The profit-maximizing producer's problem is:

$$\max \mathbb{E} \pi = \int_a^b \{g(q, \epsilon)[PY_0[1 - D(W)]] - c(q) H\} \, dF(\epsilon, \alpha)$$

$$+ (1 - g(q, \epsilon))[PY_0[1 - D(W_0)]]$$

$$- \rho c(q) H\} \, dF(\epsilon, \alpha)$$

(8)

This problem is identical to Eq. (1) except for the addition of the random variable $\epsilon$, which enters the probability function $g$. Assume that a higher $\epsilon$ corresponds to a more favorable outcome, such that $g_\epsilon > 0$, and that the marginal effect of $\epsilon$ is declining, $g_{\epsilon\epsilon} < 0$.

Let $F(\epsilon, \alpha)$ represent the subjective cumulative distribution function for $\epsilon$ defined over the interval $[a, b]$ where $a$ and $b$ are constants. The parameter $\alpha$ represents the level of uncertainty. An increase in $\alpha$ increases uncertainty measured by second-order stochastic dominance:

$$\int_a^b F_a(\epsilon, \alpha) \, d\epsilon \geq 0$$

and

$$\int_a^\alpha F_a(z, \alpha) \, dz > 0$$

(9)

where the first term in Eq. (9) represents the mean effect and the second term represents the spread effect of $\alpha$ on the distribution. This representation of increased uncertainty includes a mean-preserving spread as a special case. An increase in $\alpha$ corresponds to a decrease in the expected probability of success. Formally, the expected probability of no weed control failure is:

$$E g(q, \epsilon) = \int_a^b g(q, \epsilon) \, dF(\epsilon, \alpha)$$

(10)

Differentiating Eq. (10) with respect to $\alpha$, and integrating by parts twice yields:

$$\frac{\partial E g(q, \epsilon)}{\partial \alpha} = -g_\epsilon(q, \epsilon) \int_a^b F_a(\epsilon, \alpha) \, d\epsilon$$

$$+ \int_a^b g_{\epsilon\epsilon}(q, \epsilon) \left[\int_a^\alpha F_a(z, \alpha) \, dz\right] \, d\epsilon < 0$$

(11)

Increasing $\alpha$ also corresponds to increasing the variance of $g(q, \epsilon)$, where the variance is given by:

$$\text{Var}[g(q, \epsilon)] = \int_a^b g(q, \epsilon)^2 \, dF(\epsilon, \alpha)$$

$$- \left[\int_a^b g(q, \epsilon) \, dF(\epsilon, \alpha)\right]^2$$

(12)

Differentiating Eq. (12) with respect to $\alpha$, and integrating each term by parts twice, and rearranging, gives:

$$\frac{\partial \text{Var}[g(q, \epsilon)]}{\partial \alpha}$$

$$= 2g_\epsilon(q, \epsilon)[1 - g(q, \epsilon)] \int_a^b F_a(\epsilon, \alpha) \, d\epsilon$$

$$+ \int_a^b \left[2g_\epsilon^2 - 2g_{\epsilon\epsilon}[1 - g(q, \epsilon)]\right]$$

$$\times \left[\int_a^\alpha F_a(z, \alpha) \, dz\right] \, d\epsilon$$

(13)

Intuitively, $\epsilon$ can reflect producer uncertainty about the length of the application or effectiveness window. For uncertain window length, write $g(q, \epsilon)$ as $g(q + \epsilon)$. The producer believes the window length is likely to be $q$, but it may be a few days longer or a few days shorter. An increase in $\alpha$ implies that the length of the window is more variable, while the expected window length may be
either constant or decreasing. Alternatively, \( \epsilon \) can represent uncertainty in the efficacy of the application or effectiveness window. For uncertain window efficacy, write \( g(q,\epsilon) \) as \( g(q) \). The producer believes the window length is likely to contribute a set amount to the probability of success but is not sure exactly what this set amount is. Now increasing \( \alpha \) implies an increase in the variability of efficacy, while the expected efficacy of the window may be constant or decreasing. After defining the general model and comparative statics, we will consider both of these cases to illustrate the effects of increased uncertainty on the optimal weed control decision.

The first-order conditions for Eq. (8) are:

\[
E\tilde{\pi}_q = \int_a^b \left\{ g_q(q,\epsilon) PY_0 \left[ D(W_0) - D(W) \right] \right. \\
- (1 - \rho) H \left\{ g_q(q,\epsilon) c(q) + g(q,\epsilon) c'(q) \right\} \\
- \rho c'(q) H dF(\epsilon,\alpha) = 0
\]

and

\[
E\tilde{\pi}_H = \int_a^b - g(q,\epsilon) PY_0 D'(W) W'(H) \\
- (1 - \rho) g(q,\epsilon) c(q) \\
- \rho c(q) dF(\epsilon,\alpha) = 0
\]

The second-order conditions require \( E\tilde{\pi}_{HH} < 0 \), \( E\tilde{\pi}_{qq} < 0 \) and \( EG = E\tilde{\pi}_{HH} E\tilde{\pi}_{qq} - (E\tilde{\pi}_{qq})^2 > 0 \).

The comparative statics of an increase in uncertainty for the application case (\( \rho = 0 \)) are given by \( \partial g_q^*/\partial \alpha > 0 \) and \( \partial H^*/\partial \alpha < 0 \) (see Appendix C), which is summarized by the following proposition.

Proposition 3: Assuming \( g_{qe} \leq 0 \) and \( g_{qee} > 0 \), an increase in application uncertainty (\( \rho = 0 \)) decreases the optimal herbicide application rate and increases the length of the optimal application window.

If \( g_{qe} \leq 0 \) and \( g_{qee} > 0 \), uncertainty causes producers to use herbicides with a higher probability of being applied than those they would use in the absence of uncertainty. It also causes producers to apply more expensive herbicides than they would use in the absence of uncertainty.

Similarly, for the effectiveness case (\( \rho = 1 \)), if damage abatement is globally concave in \( H \), then \( \partial g^*/\partial \alpha > 0 \) and \( \partial H^*/\partial \alpha < 0 \). This result is summarized below.

Corollary 3: Assuming \( g_{qe} \leq 0 \) and \( g_{qee} > 0 \) and that damage abatement is globally concave in self-insurance, \( H \), an increase in effectiveness uncertainty (\( \rho = 1 \)) decreases the optimal herbicide application rate and increases the length of the optimal application window.

If \( g_{qe} \leq 0 \), \( g_{qee} > 0 \) and damage abatement is globally concave in \( H \), uncertainty causes producers to use herbicides that have a higher probability of being effective than those they would use in the absence of uncertainty. In this case, although uncertainty tends to decrease the optimal herbicide application rate, it leads to more flexible and persistent herbicides. Note that Proposition 3 and Corollary 3 are similar to Proposition 1 and Corollary 1. This is not too surprising since an increase in application or effectiveness uncertainty reduces the expected probability of success, which is the effect we considered in the certain efficacy case. Similar to the problem of self-insurance and self-protection with certain efficacy, an increase in application uncertainty has no direct effect on herbicide application rates since this decision occurs as the herbicide is being applied. Also similar is that an increase in effectiveness uncertainty has the direct effect of increasing the probability a herbicide application will be wasted thereby decreasing the amount of herbicide applied. Both the certain and uncertain efficacy cases also require restrictions on the curvature of the probability functions to achieve this result. While we assumed a specific functional form for the certainty case, we consider a more general probability function for the uncertainty case. The conditions \( g_{qe} \leq 0 \) and \( g_{qee} > 0 \) imply that \( g_q \) is decreasing or constant and is convex in \( \epsilon \). Intuitively, this means self-protection is less effective in shifting the probability of success as \( \epsilon \) increases, but this decrease in effectiveness diminishes as \( \epsilon \) increases. This has different implications for the shape of \( g(q,\epsilon) \) depending on how \( \epsilon \) enters the model. We now consider the implications of both uncertain window length and efficacy.

3.1. Uncertain window length

Consider the case of uncertain window length represented by \( g(q,\epsilon) = g(q + \epsilon) \). In this case,
g_{qe}(q + \epsilon) = g"(q + \epsilon) and g_{qee}(q + \epsilon) = g""(q + \epsilon). If we think of \(g(q,\epsilon)\) as a cumulative distribution function, the condition \(g_{qe} \leq 0\) implies that the probability distribution function, \(g'(q + \epsilon)\), is non-increasing which is guaranteed by our assumption that \(g_{ee} < 0\). The condition \(g_{qee} > 0\) implies the probability distribution function is a convex function. Using a parallel argument to the idea of non-increasing Arrow–Pratt absolute risk aversion, Shogren (1991) shows that \(g''(q + \epsilon) > 0\) if we assume non-increasing aversion to uncertain protection efficiency. That is, the marginal efficacy of the probability function is convex if a producer’s willingness to pay a protection premium decreases monotonically with the level of self-protection. A protection premium is defined as the amount an individual is willing to pay to remove the uncertainty about the efficacy of self-protection (see Shogren, 1991, Proposition 2).

For \(g(q,\epsilon) = g(q + \epsilon)\), distributions that meet both the conditions \(g_{qe} \leq 0\) and \(g_{qee} > 0\) include the exponential and Pareto distributions. Intuitively, the exponential distribution makes sense for the application uncertainty model since the probability of successful application can be described as a waiting time problem. For the application problem, success occurs when weather conditions permit an application. Let \(\theta\) represent the probability of a successful application occurring during an interval \(t + \Delta t\). Following Freund and Walpole (1980), if the probability of more than one success during that interval is negligible and the probability of success during such an interval does not depend on what happened before \(t\), then the probability of zero successes during an interval of length \(q + \epsilon\) is given by \(e^{-\theta(q+\epsilon)}\). The probability of at least one success during interval \(q + \epsilon\) is then the exponential cumulative distribution function \(1 - e^{-\theta(q+\epsilon)}\).

We use the WISH model to explore the relationship between the length of the primary application window and the probability that a herbicide is successfully applied. Let \(\epsilon = 0\) represent the weed control strategies as defined by Bouzaher et al. (1992). We modify each strategy by adding or subtracting days from the window lengths defined in WISH and rerunning the simulation. For example, \(\epsilon = 1\) represents adding 1 day to the application window, and \(\epsilon = -1\) represents subtracting 1 day from the application window. With the new values generated with

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
& & & & \multicolumn{2}{c|}{\text{Soil type: sand (}n = 35\text{)}} & & \\
\hline
& & & & \text{Linear} & \text{Exponential} & \text{Linear} & \text{Exponential} \\
\hline
\(\beta_0\) & 0.1128 & 0.0753 & 0.0680 & 0.0442 \\
& (0.0194) & (0.0244) & (0.0133) & (0.0164) \\
\(\beta_1\) & 0.0226 & -0.0470 & 0.0196 & -0.0338 \\
& (0.0010) & (0.0014) & (0.0007) & (0.0009) \\
\(\beta_2\) & 0.0073 & -0.0147 & 0.0067 & -0.0116 \\
& (0.0009) & (0.0015) & (0.0006) & (0.0009) \\
\(R^2\) & 0.9465 & NA & 0.9656 & NA \\
MSE & 0.00095 & 0.00048 & 0.00045 & 0.00029 \\
\hline
\end{tabular}
\caption{Uncertain window length: estimated probability function coefficients}
\end{table}
a clay soil. The coefficient $\beta_0$ represents the probability of successful application that is not affected by window length. Regardless of window length, there is at least an 11.28% probability of successful application on sandy soils and at least a 6.80% probability of successful application on clay soils. For the exponential model, $-\beta_1(e^{\beta_0+\beta_1(q+\epsilon)} - 1)$ represents the marginal effect of the early preplant application window length on the probability of a successful application and $-(\beta_1 + \beta_2)(e^{\beta_0+\beta_1(q+\epsilon)} - 1)$ represents the marginal effect of pre-emergence application window length on the probability of a successful application. Unlike the linear model, these marginal effects are not constant.

The estimated coefficients for both the linear and non-linear models all have the appropriate sign. Also, as expected for a probability function, neither of the estimated functions has a range outside the interval [0,1] over the domain of data from which they were estimated. Although there is little evidence to select the exponential over the linear model, there is no evidence to reject it either. For the exponential model providing the best fit, then Proposition 3 holds unambiguously. But if the linear model provides a better fit, then from Eq. (C6) we can see that most of the terms disappear due to $g_{qe} = g_{qee} = g_{e} = 0$. The only remaining effect is the mean effect, which implies:

$$E\tilde{\pi}_{qa} = c'(q) H_{g_e} \left( \int_a^b F_a d\epsilon \right) \geq 0 \tag{18}$$

Therefore, this implies that, for all but a mean-preserving spread, the implications of Proposition 3 hold unambiguously. This suggests that the main effect of increased uncertainty will occur from a change in the mean of the distribution rather than from the spread of the distribution for uncertain window length.

If a producer is uncertain about the window length associated with different herbicides, increased uncertainty is likely to lead to reduced herbicide application rates and to use of herbicides with longer application or effectiveness windows. From an environmental standpoint, when herbicides are successfully applied we have lower loadings of more persistent herbicides. Also, since herbicides with longer application windows have a higher probability of successful application, uncertainty in window length tends to increase the likelihood that herbicides will be applied.

### 3.2. Uncertain window efficacy

Now consider uncertain window efficacy represented by $g(q,e)| = g(q,e)$. In this case, $g_{qe} = g''(q,e)q = g'(q,e)q$ and $g_{qee} = g'''(q,e)q^2 + 2g''(q,e)q$. Now $g_{qe} \leq 0$ if and only if $-g''(q,e)q/g(q,e) \geq 1$. The term $\eta(q,e) = -g''(q,e)q/g(q,e)$ is the elasticity of the marginal probability of success. Similar to Shogren (1991), we might think of $\eta$ as a measure of relative aversion to uncertain protection efficiency. For a simple self-protection problem, $\eta$ indicates the size of a multiplicative protection premium.

A sufficient condition for $g_{qe} \leq 0$ is for the probability density function, $g'(q,e)$, to always be elastic. Lichtenberg and Zilberman (1986) suggest that this is a reasonable assumption for a probability density function for damage control problems. For common distributions including many gamma, exponential, and Pareto distributions, $g'(q,e)$ is elastic if $q$ is bounded sufficiently far from zero. Arrow (1984) shows that relative risk aversion tends to a limit below one as wealth approaches zero. Similarly, $\eta$ approaches a limit below one as $q$ approaches zero, implying that we need to restrict $q$ away from zero if we want to maintain $\eta > 1$.

This bound, $\eta > 1$, may be very restrictive for some distributions and may impose little restriction for others. For example, consider the Pareto distribution $g(q,e) = 1 - (k/(q,e))^{\theta}$ for $q \geq k > 0$ and $\theta > 0$. The elasticity of the marginal probability of success is $\eta = \theta + 1 > 1$. Requiring $\eta > 1$ places no additional restrictions on $q$. Alternatively, consider the exponential distribution $g(q,e) = 1 - e^{-\theta q_e}$ for $q > 0$ and $\theta > 0$. The elasticity of the marginal probability of success is $\eta(q,e) = \theta q$. Requiring $\eta > 1$ implies that $q_e > 1/\theta$, which in turn implies that $g(q^*e) \geq 1 - e^{-1} \approx 0.632$ for all $e$ in $[a,b]$. Now we can guarantee that $g_{qe} \leq 0$ only if the producer chooses an optimal window length certain to result in at least a 63% probability of success.
Consider now the restriction $g_{q\epsilon e} > 0$. Assuming $g''(q\epsilon) < 0$ and $q* \neq 0$, then $g_{q\epsilon e} > 0$ if and only if $-g'''(q\epsilon)q\epsilon/g''(q\epsilon) > 2$. Again this holds for many distributions if $q\epsilon$ is bounded away from zero. This imposes no further restrictions on a Pareto distribution, but for an exponential distribution it implies that $q\epsilon > 2/\theta$. In this case, $g(q\epsilon) > 1 - e^{-2} \approx 0.865$, so we can guarantee that $g_{q\epsilon e} > 0$ only if the producer chooses an optimal window length that he is certain will result in at least an 87% probability of success. In general:

$$-g'''(q\epsilon)q\epsilon/g''(q\epsilon) = 1 + \eta - \eta'q\epsilon/\eta$$

(19)

then $-g'''(q\epsilon)q\epsilon/g''(q\epsilon) > 2$ if and only if $[1 + \eta - (\eta'q\epsilon)/\eta] > 2$.

Fig. 2 illustrates the regions in which we can unambiguously sign Proposition 3 and Corollary 3 for uncertain window efficacy. The elasticity of self-protection is represented on the horizontal axis, while the relative marginal elasticity ($\eta'q\epsilon$) is represented on the vertical. Regions I and III represent the cases that violate either of the two restrictions $\eta > 1$ or $[1 + \eta - (\eta'q\epsilon)/\eta] > 2$. Region II represents the cases that satisfy both restrictions. If $\eta'q\epsilon$ lies entirely in Region II for all $\epsilon$ in $[a,b]$, our results hold. Assuming $\eta > 1$, a sufficient condition for $-g'''(q\epsilon)q\epsilon/g''(q\epsilon) > 2$ is $\eta' \leq 0$, implying $\eta'q\epsilon$ at the optimum lies entirely in Region IIb where the probability is elastic, and the elasticity is decreasing. This is the most restrictive set of sufficient conditions. We can weaken these restrictions by allowing the elasticity to increase, but at a slow enough rate to stay in Region IIa. In addition, if we consider a mean-preserving spread in $\epsilon$ such that:

$$\int_a^b F_{\theta}(\epsilon, \alpha) d\epsilon = 0$$

(20)

then the mean effect in Eq. (C6) disappears, and we no longer require $\eta > 1$. Therefore, our results hold if $\eta'q\epsilon$ lies entirely in Regions II and III for all $\epsilon$ in $[a,b]$. Eliminating the mean effect removes the restriction that the elasticity must exceed unity, thereby increasing the likelihood that Proposition 3 and Corollary 3 hold. In general, sufficient conditions to sign the effects of increased uncertainty on $q*$ and $H*$ are that the marginal productivity of success is elastic and the elasticity is non-increasing in $q\epsilon$ over the interval $[q^*a, q^*b]$.

Again using the weed control strategies from the WISH model, we model uncertain window efficacy as uncertainty in the amount of crop land that can be treated in a day. Let $\epsilon$ represent the percentage of land that can be treated in 1 day. For example, if a producer requires 4 days to treat all of his corn acres, then $\epsilon = 0.25$. With the values generated by WISH, we estimated coefficients for a linear model, an exponential model, and a Pareto model. Observations were generated for $\epsilon = 0.1665$, $\epsilon = 0.2$, $\epsilon = 0.25$, $\epsilon = 0.333$, and $\epsilon = 0.5$, covering producers requiring from 2 to 6 days to apply herbicides to all of their corn acres. In this specification the product $q\epsilon$ represents the maximum treatment percent, i.e. is the percentage of acreage that could be treated if there were no weather delays.

We use dummy variables to represent alternative timings in both the linear and exponential models, with the linear model given by:

$$g(q\epsilon) = \beta_0 + \beta_1 q\epsilon + \beta_2 d_1 q\epsilon + \beta_3 d_2 q\epsilon$$

(21)
Table 2
Uncertain window efficacy: estimated probability function coefficients

<table>
<thead>
<tr>
<th>Soil type: sand (n = 40)</th>
<th>Soil type: clay (n = 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>0.2652</td>
</tr>
<tr>
<td>(0.0255)</td>
<td>(0.0316)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.0550</td>
</tr>
<tr>
<td>(0.0045)</td>
<td>(0.0078)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.0137</td>
</tr>
<tr>
<td>(0.0068)</td>
<td>(0.0096)</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.0248</td>
</tr>
<tr>
<td>(0.0046)</td>
<td>(0.0086)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.8601</td>
</tr>
<tr>
<td>MSE</td>
<td>0.00520</td>
</tr>
</tbody>
</table>

Standard errors are given in parentheses.

Table 3
Uncertain window efficacy: estimated Pareto distribution coefficients

<table>
<thead>
<tr>
<th>Soil type by application timing</th>
<th>Sand</th>
<th>Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early preplant</td>
<td>Pre-emergence</td>
<td>Post-emergence</td>
</tr>
<tr>
<td>(n = 15)</td>
<td>(n = 10)</td>
<td>(n = 15)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>1.2286</td>
<td>0.9115</td>
</tr>
<tr>
<td>(0.0603)</td>
<td>(0.0603)</td>
<td>(0.0603)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>2.1334</td>
<td>2.0928</td>
</tr>
<tr>
<td>(0.1941)</td>
<td>(0.1469)</td>
<td>(0.1469)</td>
</tr>
<tr>
<td>MSE</td>
<td>0.00129</td>
<td>0.00063</td>
</tr>
</tbody>
</table>

Standard errors are given in parentheses.
probability of successful application. With the exponential model \(- ( \beta_1 + \beta_2 d_1 + \beta_3 d_2)(q_{50} + \beta_1 q_{50} + \beta_2 d_1 q_{50} + \beta_3 d_2 q_{50})\) represents the marginal effect of maximum treatment percentage on the probability of a successful application. Unlike the linear model, this effect is not constant, but it is positive as expected for all herbicide timings and soil types. The estimated coefficients for all of the models have the appropriate linear model, this effect is not constant, but it is the probability of a successful application. Unlike the linear model, this effect is not constant, but it is positive as expected for all herbicide timings. With the exponential model \(\beta_0 K \beta_0 (\beta_1 + q_{50})^{-1} - \beta_0\) represents the marginal effect of maximum treatment percentage on the probability of a successful application. Again unlike the linear model this effect is not constant, but it is positive as expected for all herbicide timings and soil types. The estimated coefficients for all of the models have the appropriate sign. As expected for a probability function, none of the estimated functions has a range outside the interval [0,1] over the domain of data from which they were estimated. Although the \(R^2\) values for the linear model are large, residual plots indicate that the linear model does not fit the data as well as the exponential model. Similarly, residual plots indicate that the Pareto models do not give as good a fit as the exponential model. However, given the small number of observations used to estimate the Pareto models it is difficult to make a conclusive judgment.

Recall that if the Pareto model holds, we can unambiguously conclude that Proposition 3 holds, i.e. \(\frac{\partial q^*}{\partial q} > 0\) and \(\frac{\partial H^*}{\partial q} < 0\) for all application timings and soil types. If the exponential model holds, however, we need to satisfy two conditions to make these predictions, i.e. \(\eta > 1\) and 

\[
-a''(q_{50}) q_{50} / g''(q_{50}) > 2.
\]

For the exponential model, \(\eta = - (\beta_1 + \beta_2 d_1 + \beta_3 d_2) q_{50}\) and 

\[
-a''(q_{50}) q_{50} / g''(q_{50}) = - (\beta_1 + \beta_2 d_1 + \beta_3 d_2) q_{50}.
\]

Given our estimates, the condition \(\eta > 1\) requires \(q^* > 33.2\) for early preplant applications on sand, \(q^* > 25.5\) for pre-emergence applications on sand, and \(q^* > 21.8\) for post-emergence applications on sand. On clay, the condition requires \(q^* > 47.0\), \(q^* > 35.6\), and \(q^* > 31.1\) for the respective timings. Since \(8 \leq q \leq 25\) for the strategies defined in WISH, the condition \(\eta > 1\) can only be satisfied for post-emergence applications on sand.

The second condition \(- g''(q_{50}) q_{50} / g''(q_{50}) > 2\) requires \(q^* > 66.4\), \(q^* > 50.9\), and \(q^* > 43.5\) for early preplant, pre-emergence, and post-emergence applications on sand, and \(q^* > 93.9\), \(q^* > 71.2\), and \(q^* > 62.2\) for the respective timings on clay. This condition is not satisfied for any timing or soil type. Therefore, even if the exponential model holds, it is possible that the opposite result predicted by Proposition 3 occurs, i.e. \(\frac{\partial q^*}{\partial q} < 0\) and \(\frac{\partial H^*}{\partial q} > 0\).

These results lead to two observations. First, as do those of Lichtenberg and Zilberman (1986), our findings reinforce the importance of functional form. At present, more field data are needed to determine the most appropriate functional form for uncertain window efficacy. Second, our findings support the importance of two theoretical constraints paralleling the observations of Arrow (1984) on relative risk aversion. We can show that \(\eta\) approaches a limit below unity as \(q_{50}\) approaches zero and that \(\eta\) approaches a limit above unity as \(q_{50}\) approaches infinity. This indicates that the conditions \(\eta > 1\) and \(\eta' < 0\) are violated for some \(q_{50}\) in the interval \(0 < q_{50} < \infty\). Our exponential model shows that these theoretical constraints are binding over the relatively small interval \(aq^* \leq q_{50} \leq bq^*\), where \(a = 0.1665\) and \(b = 0.5\).

In sum, for certain probability functions, we can conclude that uncertainty about self-protection will lead to herbicides with longer application or effectiveness windows being used at lower application rates. Again this implies lower loadings of more persistent herbicides when an application occurs, but it also implies a greater probability of successful application. However, these results are less convincing in the uncertain window efficacy specification than in the uncertain window length specification.

Again, if we drop the assumption of risk neutrality, and allow for risk-averse producers, the results are still complicated by the necessity to consider effects of self-protection and self-insurance on the variability of profit as well as expected profit. The net result depends on the relative curvature in the utility function compared to curvature in the profit function. As before, the direction of the comparative statics for a risk-averse producer are more difficult to determine due to the interaction of tastes and technology (Babcock and Shogren, 1995).

4. Conclusion

The concept of endogenous risk allows us to escape the simplified assumption that the likelihood
of weed damage is beyond the control of a producer. As a result this approach provides a theoretical framework to capture some of the complexity involved in weed control decisions, and particularly to model decisions involving herbicide types as well as herbicide quantities.

Endogenous risk in weed control management reveals two key points—a clear trade-off between the herbicide application rate and flexibility or persistence, and the idea that the source of a probability change affects this trade-off. First, a policy-maker concerned with non-point source pollution must understand the trade-off between the type and quantity of herbicides used. Policies that reduce loadings because they reduce total mass may induce substitution to chemicals more damaging or more likely to be transported to sensitive areas (e.g. low-dose sulfonylureas such as nicosulfuron and primisulfuron). This point is supported by Wagenet and Hutson (1991) who indicate that the fate and transport of pesticides in the soil depend on specific characteristics such as persistence and total mass. Our endogenous risk framework captures the mechanisms behind this trade-off.

Second, different sources for changes in probability have different implications for weed control. An increase in the exogenous probability of loss—probability that cannot be ameliorated by self-protection—will decrease application rates and increase persistence. Similarly, uncertain window length will decrease rates and increase persistence. In contrast, a decrease in the efficacy of self-protection will decrease persistence and may increase application rates. Finally, for uncertain window efficacy, definitive predictions are difficult to guarantee. Based on the elasticity of the probability function, we provide a set of conditions to unambiguously sign the effects of uncertain window efficacy, but our empirical results reveal a need for additional field data to determine conclusively whether these conditions are satisfied.

There are several extensions to our model that may prove useful. The first is determining the relative importance of risk-reducing technologies and risk preferences on weed control strategies. By focusing on risk-neutral producers we were better able to see the technological effects of herbicide use on weed control decisions. There needs to be further empirical work to determine the importance of these technological considerations relative to taste considerations embodied in risk aversion (see Babcock and Shogren, 1995). Second, our model did not consider the market effects of policy decisions. This may be realistic for policies targeted to limited areas where shifts in herbicide use might not be expected to have an effect on herbicide and crop prices. However, large-scale policies such as nationwide bans of key herbicides such as atrazine may have significant effects on crop prices and the prices of other herbicides. Third, our model considered a single herbicide treatment. In reality, producers generally have the option of treating herbicide escapes with additional herbicides or cultivation. The availability of treatments for weed escapes may have a significant effect on both the quantities and types of herbicides used. This remains to be determined.

We conclude by stressing our main point. In general, self-protection and self-insurance in weed control can be viewed as stochastic substitutes. Therefore, non-point source pollution policies aimed at restricting one will likely increase the other. Furthermore, we know that increases in the probability of application failure will lead to a trade-off between herbicide rates and persistence. The net environmental effect will depend on the relative importance of herbicide loadings and persistence, a remaining question to be addressed by integrated environmental economic models (e.g. CEEPES) that include fate-and-transport systems.

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Appendix A

The comparative static effects of a decrease in $g^0$ representing increased probability of application or
effectiveness failure are:

\[
\frac{\partial q^*}{\partial g^0} = \frac{\left[ E\pi_{q^0} E\pi_{HH} + E\pi_{Hq^0} E\pi_{Hq} \right]}{G} \tag{A1}
\]

and

\[
\frac{\partial H^*}{\partial g^0} = \frac{\left[ E\pi_{Hq^0} E\pi_{qq} + E\pi_{qg^0} E\pi_{qH} \right]}{G} \tag{A2}
\]

The first two terms on the right-hand side of Eq. (A1) represent the direct effects of a change in the probability of failure on \( q^* \), while the second two terms are the indirect effect on \( q^* \) through a change in \( H^* \). Similarly, the first two terms and the second two terms in Eq. (A2) are the direct and indirect effects of a change in the probability of failure on \( H^* \). The sign of \( E'ITqH \) defines whether the inputs are stochastic substitutes or stochastic complements. If \( G \) is the Hessian matrix of the problem and \( G_{ij} \) are the minors of \( G \), Hiebert (1983) defines inputs \( i \) and \( j \) as stochastic substitutes (complements) if \( G_{ij} < 0 \) (\( > 0 \)). In our model, stochastic substitutes are inputs where an increase in the optimal level of one input has the indirect effect of decreasing the optimal level of the other input. Stochastic complements are inputs where an increase in one input indirectly increases the other input.

To determine the effect of a change in the probability of failure on the optimal window length and application rate, we need to sign all of the terms in Eqs. (A1) and (A2). Using the first-order condition, Eq. (5), \( E\pi_{HY} = 0 \), we know:

\[
E\pi_{Hg^0} = -PY_0 D'(W)W'(H) - (1 - \rho)c(q) \tag{A3}
\]

Additionally, using the first-order condition, Eq. (4), \( E\pi_q = 0 \), we can write:

\[
E\pi_{Hq^0} = -g'(q)PY_0 D'(W)W'(H) - (1 - \pi) \left[ g'(q)c(q) + g(q)c'(q) \right] - \rho c'(q) \\
= -(1 - \rho)g(q)c'(q) - \rho g'(q)PY_0 \left[ \frac{D(W_0) - D(W)}{H} + D'(W)W'(H) \right] \tag{A4}
\]

Using the first-order condition, Eq. (4), \( E\pi_q = 0 \), and given \( c'(q) > 0 \), we also know:

\[
E\pi_{qg^0} = -(1 - \rho)c'(q)H \tag{A5}
\]

Thus, for the application case \((\rho = 0)\), \( E\pi_{Hg^0} = 0 \), \( E\pi_{qH} < 0 \), and \( E\pi_{qg^0} < 0 \). Assuming second-order conditions hold, \( \partial q^*/\partial g^0 < 0 \) and \( \partial H^*/\partial g^0 > 0 \).

For the effectiveness case \((\rho = 1)\), \( E\pi_{Hg^0} > 0 \) and \( E\pi_{qg^0} = 0 \), but the sign of \( E\pi_{qH} \) is ambiguous. If herbicide use is considered damage abatement, the term \([D(W_0) - D(W)]/H\) represents average damage abatement, and the term \(-D'(W)W'(H)\) represents marginal damage abatement. In general, although we cannot determine which term is larger, if we assume abatement is always concave in \( H \), then average abatement exceeds marginal abatement, implying \( E\pi_{qH} < 0 \) and \( \partial q^*/\partial g^0 < 0 \) and \( \partial H^*/\partial g^0 > 0 \).

Appendix B

A decrease in \( \gamma \) increases the probability of failure by reducing the efficacy of self-protection such that:

\[
\frac{\partial q^*}{\partial \gamma} = \left[ -E\pi_{qH} E\pi_{HH} + E\pi_{Hq} E\pi_{Hq} \right] / G \tag{B1}
\]

and

\[
\frac{\partial H^*}{\partial \gamma} = \left[ -E\pi_{Hq} E\pi_{qq} + E\pi_{qH} E\pi_{Hq} \right] / G \tag{B2}
\]

Differentiating \( E\pi_{HY} \) and \( E\pi_{q} \) with respect to \( \gamma \) yields:

\[
E\pi_{HY} = -h(q)PY_0 D'(W)W'(H) - (1 - \rho)h(q)c(q) \tag{B3}
\]

and using \( E\pi_q = 0 \), yields:

\[
E\pi_{qH} = h'(q)PY_0 [D(W_0) - D(W)] - (1 - \rho)H \left[ h(q)c(q) + h(q)c'(q) \right] \\
= \left[ (1 - \rho)g^0c'(q)H \right] / \gamma \\
+ \rho [h'(q)PY_0 [D(W_0) - D(W)]] > 0 \tag{B4}
\]

For the application case \((\rho = 0)\), given \( E\pi_H = 0 \), then \( E\pi_{HY} = 0 \). Assuming second-order conditions hold and noting that \( E\pi_{qH} > 0 \) from Eq. (B4) and \( E\pi_{qH} < 0 \) from Eq. (A4), then \( \partial q^*/\partial \gamma > 0 \) and \( \partial H^*/\partial \gamma < 0 \).
For the effectiveness case ($\rho = 1$), Eqs. (B3) and (B4) imply $E \pi_H > 0$ and $E \pi_q > 0$, while the sign of $E \pi_{qH}$ is ambiguous. If we assume damage abatement is globally concave in $H$, then $E \pi_{qH} < 0$, and the signs of $\partial q^* / \partial \gamma$ and $\partial H^* / \partial \gamma$ remain ambiguous. But if $q$ and $H$ are stochastic complements, which implies that $E \pi_{qH} > 0$, then $\partial q^* / \partial \gamma > 0$ and $\partial H^* / \partial \gamma > 0$. Note that, for application failure, a decrease in the efficacy of self-protection has a direct effect on $q^*$ but no direct effect on $H^*$. Alternatively, for effectiveness failure, a decrease in the efficacy of self-protection has a direct effect on both $q^*$ and $H^*$.

Appendix C

The comparative statics of an increase in uncertainty on $q^*$ and $H^*$ are given by:

$$\frac{\partial q^*}{\partial \alpha} = \left[-E \tilde{\pi}_{qa} E \tilde{\pi}_{HH} + E \tilde{\pi}_{Ha} E \tilde{\pi}_{Hq}\right] / \text{EG} \quad (C1)$$

and

$$\frac{\partial H^*}{\partial \alpha} = \left[-E \tilde{\pi}_{Ha} E \tilde{\pi}_{eq} + E \tilde{\pi}_{qa} E \tilde{\pi}_{qH}\right] / \text{EG} \quad (C2)$$

Assuming the second-order conditions hold, we need to sign $E \tilde{\pi}_{Ha}$, $E \tilde{\pi}_{qH}$, and $E \tilde{\pi}_{qa}$ to sign Eqs. (C1) and (C2). Consider each in turn. Differentiating $E \tilde{\pi}_H$ with respect to $\alpha$, integrating by parts twice, using Eq. (14), and assuming $E g(q,\varepsilon) \neq 0$, yields:

$$E \tilde{\pi}_{Ha} = \frac{\rho c(q)}{E g(q,\varepsilon)} \left\{g_e(q,\varepsilon) \int_a^b F_c(\varepsilon,\alpha) \right\} - \int_a^b g_{ee}(q,\varepsilon) \left\{\int_a^e F_c(z,\alpha) dz\right\} d\varepsilon < 0 \quad (C3)$$

For application uncertainty ($\rho = 0$), $E \tilde{\pi}_{Ha} = 0$. For effectiveness uncertainty ($\rho = 1$), $E \tilde{\pi}_{Ha} < 0$.

The cross-effect term, $E \tilde{\pi}_{Hq}$, for application uncertainty ($\rho = 0$), is given by:

$$E \tilde{\pi}_{Hq} = -\int_a^b c'(q) g(q,\varepsilon) dF(\varepsilon,\alpha) < 0 \quad (C4)$$

This result, combined with the observation that $E \tilde{\pi}_{Ha} = 0$ for application uncertainty, implies that a change in uncertainty regarding the probability of a successful application will cause $q^*$ and $H^*$ to move in opposite directions. Producers who face greater application uncertainty will either choose herbicides with longer application windows and apply them at lower rates, or choose herbicides with shorter application windows and apply them at higher rates.

For effectiveness uncertainty ($\rho = 1$), using Eq. (14) we can write the cross-effect term as:

$$E \tilde{\pi}_{Hq} = P Y_0 \left[\frac{D(W_o) - D(W)}{H} + D'(W) W'(H)\right] \times \int_a^b g_q(q,\varepsilon) dF(\varepsilon,\alpha) \quad (C5)$$

The second-order condition $E \tilde{\pi}_{HH} < 0$ requires damage abatement to be locally concave in $H$. If we assume damage abatement is globally concave in $H$, average abatement exceeds marginal abatement, implying $E \tilde{\pi}_{qH} < 0$. Recognizing that $E \tilde{\pi}_{qH} < 0$ is again useful for policy analysis. The negative cross-term effect indicates that producers see herbicide application rates and herbicide persistence as stochastic substitutes. Therefore, a policy designed to restrict herbicide application rates should result in the use of more persistent herbicides. A policy designed to restrict persistence should be expected to induce higher application rates.

Differentiating $E \tilde{\pi}_q$ with respect to $\alpha$, integrating by parts twice, using Eq. (13), and assuming $E g_q(q,\varepsilon) \neq 0$ yields:

$$E \tilde{\pi}_{qa} = c'(q) H (1 - \rho) E g(\cdot) + \rho \int_a^b g_q(q,\varepsilon) d\varepsilon \leq 0 \quad (C6)$$
In general, the sign of \( E \tilde{q}_a \) is ambiguous. But if \( g_q \leq 0 \) and \( g_{qe} > 0 \), then \( E \tilde{q}_a > 0 \) for both cases of application and effectiveness uncertainty.

Given Eqs. (C3), (C4) and (C6), if \( g_q \leq 0 \), \( g_{qe} > 0 \) and \( \rho = 0 \), then \( \partial q^* / \partial q > 0 \) and \( \partial H^* / \partial q < 0 \).

Similarly, given Eqs. (C3), (C5) and (C6), if \( g_q \leq 0 \), \( g_{qe} > 0 \), \( \rho = 1 \), and damage abatement is globally concave in \( H \), then \( \partial q^* / \partial q > 0 \) and \( \partial H^* / \partial q < 0 \).

References


