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A generalized Armington trade model: Respecification

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ABSTRACT

Even if the original version of the Armington model is simple and plausible in estimation, the single CES and homotheticity assumptions are too restrictive for practical application and give biased price elasticities. This paper develops a generalized Armington model, which relaxes the single CES and homotheticity restrictions, and includes the Armington model as a special case. The Armington and generalized Armington trade models are applied to the Japanese meat import demand to demonstrate their performance. The empirical results rejected the two assumptions imposed on the Armington model. The generalized Armington model provides an alternative to the restrictive but indispensable Armington model.

INTRODUCTION

The theoretical derivation of the Armington model gives a linear and parsimonious specification of the import demand system. This approach models the demand system for differentiated goods in terms of the origin of export. This model assumes that goods imported from different countries (or regions) create different consumer utility, i.e., the elasticities of substitution among the goods imported from different places are not infinite. The differentiated good is called, in Armington's (1969) terminology, "product". Since this simple linear specification is consistent with the utility maximization and economizes the degrees of freedom in empirical application, the model has been widely used for international trade analysis (Babula, 1987; Ito et al., 1990; Figueroa and Webb, 1986; Sarris, 1983).

The Armington model contains two major assumptions induced by the constant elasticity of substitution (CES) subutility function: the single CES

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and homotheticity assumptions. The single CES assumption says that the elasticity of substitution between any two products is independent of the quantity demanded and is the same as that of any other pair. In the model, this assumption restricts responses of the import demand for each product to the price change (relative to the price index for the good) to be the same for all products. This assumption would be too restrictive if elasticities of substitution between any pair of products are not the same. Even under the single CES circumstance, effects of relative price changes on market shares are not likely to be the same.

The homotheticity assumption in the Armington model implies that size of market does not affect each exporting country's relative market share, and that expenditure elasticities are the same and unitary. This assumption, though simplifying the model specification, may also be too restrictive for empirical research. If a good is differentiated so that each product gives different utility, an increase in the buyer's budget may not be allocated in the same proportion to all products. A relatively high proportion would be allocated to the high-quality or more preferred product given other factors remaining constant.

A concern in using the Armington model is whether the restrictive assumptions should be imposed for data that do not support the assumptions. If the data do not satisfy the assumptions, the two assumptions in the model estimation would result in biased estimates. Past studies respecified the Armington model for empirically more sensible results (Duffy et al., 1990; Ito et al., 1990; Sarris, 1983).

The objective of this study is to reformulate the Armington model by relaxing the single CES and homotheticity restrictions in the model. The model is respecified within the theoretical Armington framework and has the original version as a special case. We therefore name the respecified model as 'generalized' Armington model. Both original and respecified Armington models are estimated with the Japanese meat import demand data to demonstrate the performance of these models.

This paper is organized as follows. In the next section, we develop the generalized Armington model that nests the original model but does not restrict parameters of relative prices and expenditure. Empirical application and comparison of the generalized model to the original model comes next. The final section summarizes and concludes this paper.

RESPECIFICATION OF THE ARMINGTON MODEL

The Armington approach is a two-stage procedure. At the first stage, the importing country decides how much of a specific good should be imported to maximize utility. At the second stage, the importer allocates its budget

to each product by minimizing total expenditure given the amount of the good imported. Under the assumptions of independence (separability among goods) and CES subutility, Armington specified the demand for good i imported from country j at time t, x_{ijt} as follows:

$$x_{ijt} = b_{ij}^{\sigma_i} x_{it} \left(P_{ijt}/P_{it} \right)^{-\sigma_i} \qquad j = 1, \ldots, m \tag{1}$$

where x_i is the total import demand for good i, b_{ij} is the coefficient of product j (i.e., good i imported from country j) in the CES function, σ_i is the elasticity of substitution among the products, p_{ij} is the price of good i imported from country j and p_i is the price index of good i in the buyer's market. For notational simplicity, the subscript i will be dropped from now on.

Since x_t is not observable in the Armington framework (Winters, 1984), equation (1) is expressed as a share function for estimation by dividing both side by x_t and multiplying by P_{jt}/P_t. Taking the logarithm of the expression yields the Armington model in market share form:

$$\log\left(E_{jt}/E_t\right) = \sigma \log b_j + (1 - q) \log\left(P_{jt}/P_t\right) \quad j = 1, \ldots, m \tag{2}$$

where E_{jt} is the expenditure on product j, and E_t is the total expenditure on good i. In equation (2), due to the single CES assumption, the coefficients for the relative price terms (i.e., $1 - \sigma$) are the same for all products $j = 1, \ldots, m$. In addition, due to the homotheticity, the change in importer's expenditure does not affect the market share (i.e., no expenditure term in the explanatory variables).

To relax the restrictions in the model, we first set up the following mathematical relations between the expenditures:

$$E_{jt} = \alpha_j E_t^{\beta_j} \qquad \alpha_j > 0 \quad \text{for all } j \tag{3}$$

where α_j and β_j are unknown parameters. As long as α_j is greater than zero, β_j can be any value to satisfy the above relations at any time t. As a special case, the total expenditure E_t is equally allocated to each product when $\alpha_j = 1/m$ and $\beta_j = 1$ for all products, where m is the number of countries from which the good is imported. This set-up gives room for different responses of demand to the changes in the total expenditure allocated.

However, given a set of parameters, these relations should hold for all time periods in empirical data. Thus, we multiply an i.i.d. disturbance term that is assumed to be log-normally distributed with mean zero and finite variance. Consequently, we have:

$$\begin{aligned} E_{jt} &= \alpha_j E_t^{\beta_j} \epsilon_{jt} \\ \log(\epsilon_{jt}) &\sim N\left(0, \sigma_\epsilon^2\right) \end{aligned} \tag{4}$$

Solving equation (4) with respect to E_t and substituting into equation (2) before taking logarithms gives:

$$E_{jt} = b_j^{\sigma}(P_{jt}/P_t)^{1-\sigma}\alpha_j^{1/\beta_j}E_{jt}^{-1/\beta_j}E_t^2\epsilon_{jt}^{1/\beta_j} \tag{5}$$

Through mathematical derivation, we come up with a modified specification of the Armington model as follows (for more details, see Appendix):

$$\log(E_{jt}/E_t) = \log a_j + (2\gamma_j - 1)\log E_t + (1-\sigma)\gamma_j \log(P_{jt}/P_t) + e_{jt} \tag{6}$$

where a_j is a suitable constant, and:

$$\gamma_j = \beta_j/(1+\beta_j) \quad \text{and} \quad e_{jt} \sim N(0, \sigma_e^2)$$

In this specification, the effects of the relative prices are not restricted to be the same for all equations. Furthermore, the effect of expenditure change is introduced directly and is allowed to differ among products in the market. Another advantage is that even under the single CES case, the responses of expenditure shares to changes in relative prices are not necessarily the same.

This respecified model in (6) is similar to that of Ito et al. (1990) except that the dependent variable in their model is the logarithm of quantity-share, $\ln(x_{jt}/x_t)$, and the budget allocated to the good, V_t, is used in the place of E_t in equation (6). However, x_t in the dependent variable is not observable; it is not a linear sum of x_{jt} for all j, but a CES function of x_{jt}. Unless parameters (i.e., b_j and σ) in the CES function are known, the dependent variable cannot be measured. Further, they defined $V_t^{\mu} = x_t p_t$. However, V_t is not observable as they recognized, and, thus, should be approximated by the actual expenditure, $x_t p_t = \Sigma_j x_{jt} p_{jt}$, for empirical estimation. This implies $\mu = 1$.

Like the Armington model, elasticities for the generalized model depend on the choice of the market price index, P_t. The readily constructed index as indicated in Armington's original work requires an estimate of the direct price elasticity of demand for the good from the first-stage procedure (e.g., Figueroa and Webb, 1986). If the Stone index, $\log(P_t) = \Sigma_j w_{jt} \log(P_{jt})$, where $w_{jt} = E_{jt}/E_t$, is used as in Alston et al. (1990), the first-stage procedure, which is independent of the second-stage procedure, need not be estimated. However, the current expenditure share, w_{jt}, in the Stone index, causes the simultaneity problem since w_{jt} is also the dependent variable. The own price elasticity, thus, is a function of other price elasticities as Green and Alston (1990) asserted in the AIDS model. One can consider the three-stage least squares to get around this simultaneity problem. However, the model becomes nonlinear and all equations are under-identified. An alternative is to replace w_{jt} with lagged (Eales and

Unnevehr, 1988) or average share to make the index exogenous, i.e., the effect of changes in individual price on the index is constant.

With the replacement, price elasticities of the generalized model are:

$$e_{jk} = -\delta_{jk} + (1-\sigma)\gamma_j(\delta_{jk} - w_k) \qquad j,\ k = 1, \ldots, m \tag{7}$$

where $\delta_{jk} = 1$ if $j = k$, and 0 otherwise. The expenditure elasticity is:

$$n_j = 2\gamma_j \tag{8}$$

Conditions for Engel's aggregation, homogeneity, and symmetry are, respectively:

$$\sum_j w_j(2\gamma_j - 1) = 0 \tag{9}$$

$$(2\gamma_j - 1) = 0 \quad \text{for all } j \tag{10}$$

and

$$(2\gamma_j - 1) - (1-\sigma)\gamma_j = (2\gamma_k - 1) - (1-\sigma)\gamma_k \tag{11}$$

We cannot impose or test Engel's aggregation just with parameters since the time-depending variable w_j is included. Like negativity on the AIDS model, it can be checked locally.

In the general Armington model, imposing the homogeneity condition of equation (10) is equivalent to the homotheticity implied by the original version. This is consistent with theory, i.e., every homogeneous function is homothetic (the reverse is not true). Given the homogeneity condition, imposing the symmetry condition is equivalent to imposing the single CES in the original model. When both homogeneity and symmetry conditions hold, the generalized model reduces to the original version with homotheticity and the same price responses. Thus, the Armington model is a special case of the generalized Armington model. When empirical data do not support the homotheticity and single CES, the generalized Armington model would be suggested. A joint hypothesis test of homogeneity and symmetry would be a test for the Armington specification against the generalized Armington model.

APPLICATION OF THE GENERALIZED ARMINGTON MODEL

To determine whether the Armington model is practical for the sample data, we need to test homotheticity and the single CES (or the same price responses) assumptions. A test for the single CES can be easily conducted with the original model. However, the test for homotheticity can be conducted only with the generalized model by testing the homogeneity condition.

To determine the overall performance of the generalized Armington model, we perform a nested hypothesis test for the Armington model against the generalized Armington model. As mentioned earlier, the joint null hypothesis is composed of homogeneity and symmetry conditions, i.e., the expenditure coefficients are all zero and the price coefficients are all the same.

The import demands for the differentiated products in a country are not likely to be contemporaneously independent. The demands are commonly influenced by consumer taste, change in trade policy, and/or unexpected shock in production in the importing country, which are not included in the Armington models. Thus, Zellner's seemingly unrelated regression (SUR) technique is used to estimate the Armington models.

Data descriptions

Annual data of Japanese import demand for red meat from 1970 to 1987 were used for this analysis. Because of the assumption of independence, we used these aggregate data. Separability may not hold for disaggregate data.

The origins of products are composed of four groups: the United States (USA), European Community (EC), Oceania (Australia and New Zealand), and others. Since the United States and Australia are the major exporting countries, they are treated as independent groups. New Zealand is combined with Australia because of similar trading patterns and geographical position. No individual country in the EC is an important exporter, hence they have been combined. In the group of others, Thailand and Argentina are important exporters but are not big enough to be an independent group. Table 1 shows the average market shares of the four groups during the sample period.

Data for quantities and values (in US$) imported from various exporting countries are obtained from the Direction of Trade (United Nations). As a proxy for import prices from different countries, we used the unit value obtained by dividing the value by the quantity. These are not what con-

TABLE 1

Summary statistics for each exporter's market shares during 1970–1989

Exporter	Mean	SD	MIN	MAX
USA	0.26	0.05	0.12	0.33
EC	0.13	0.02	0.06	0.19
Oceania	0.31	0.10	0.17	0.55
Other	0.30	0.04	0.22	0.36

TABLE 2

Estimated results from the Armington and generalized Armington models for red meat import demand in Japan

	Armington model		Generalized Armington model		
	$1-\sigma$	$\bar{R}^2$	$(1-\sigma)\gamma_j$	$2\beta_j-1$	$\bar{R}^2$
USA	0.72 **	0.12	0.15	0.62 **	0.54
	(0.09)		(0.39)	(0.18)	
EC	0.72 **	0.62	1.38 **	0.04	0.59
	(0.09)		(0.19)	(0.20)	
Oceania	0.72 **	0.24	0.52	−0.62 **	0.71
	(0.09)		(0.29)	(0.13)	
Other	0.72 **	0.02	0.26	0.08	−0.01
	(0.09)		(0.18)	(0.09)	
System		0.32			0.80
Single CES [a]			$f=34.11$ **		
Homogeneity [b]			$f=12.97$ **		
Symmetry [c]			$f=9.73$ **		
Homogeneity and Symmetry [d]			$f=18.60$ **		

[a] H_0 Price coefficients are the same for all equations of the Armington model.
[b] H_0 Expenditure coefficients are all zero for all equations of the generalized Armington model.
[c] H_0 Price coefficients are all zero and price coefficients are the same for all equations of the generalized Armington model.
[d] H_0 Expenditure coefficients are all zero and price coefficients are the same for all equations of the generalized Armington model.
** denotes significance at the 1% level.

sumers actually pay. The required assumption is the separability between domestic and import products.

Estimated results

Table 2 presents the estimated results of the Armington and generalized Armington models and test results. The Armington model is estimated with the restriction of the same price coefficients to meet the single CES assumption. The system r^2 is 0.80 for the generalized model and 0.31 for the original model, indicating that the generalized model explains more of the sample data variation than the original version.

Because of the restriction induced by the CES assumption, the estimated price coefficient for the Armington model is 0.72 for all equations. The elasticity of substitution is less than unity, i.e., 0.28. This implies from equation (2) that a relative fall (or increase) in P_j yields a decrease (or

TABLE 3

Estimated own-price and expenditure elasticities in the Armington and the Generalized Armington models

	Armington	Generalized Armington	
	Price	Price	Expenditure
USA	−0.47 *	−0.89 *	1.62 *
EC	−0.37 *	0.20 *	1.03
Oceania	−0.50 *	−0.64 *	0.38 *
Other	−0.49 *	−0.82 *	1.08 *

Armington price elasticities are calculated according to $e_{jj} = -1+(1-\sigma)-w_j(1-\sigma)$.
* indicates significance at the 5% level.

increase) in market share of X_j, which is not consistent with demand theory. Ordinarily, the elasticity of substitution is expected to exceed unity (Armington, 1969).

While the Armington model restricts the price coefficients to be the same, the estimated price coefficients of the generalized model are different for each equation: 0.15 for USA, 1.38 for EC, 0.52 for Oceania and 0.26 for others. Among them, the estimated coefficient for the EC differs significantly from zero at the 1% level.

The estimated coefficients for expenditure are significant for the USA and Oceania but not for the EC and others. The coefficient is positive for the USA while is negative for Oceania products. The negative sign on the expenditure term is mainly because the increase in total expenditure is relatively larger than the expenditure for Oceania products.

An immediate interest is whether the Armington model, which provides quite different results from the generalized model, is accepted by the sample data. The test statistics in Table 2 do not accept neither homogeneity nor symmetry condition at the 5% level. These results indicate rejection of homotheticity and single CES assumptions in the Armington model. The joint hypothesis also is rejected against the Armington model for analyzing the sample data.

Restrictions induced by inappropriate assumptions would result in biased estimates of elasticities. To demonstrate this, we calculate the price and expenditure (where applicable) elasticities from both models in Table 3. Except for the EC, the own-price elasticities from the original model are all less, in absolute terms, than those from the generalized model. This implies that the two assumptions in the Armington model leads to underestimated price elasticities. The positive own-price elasticity for the EC products may reflect the aggregation bias in the EC data.

SUMMARY AND CONCLUSIONS

Even if the original version of the Armington procedure is plausible in model estimation, the single CES and homotheticity assumptions seem to be too restrictive and give biased elasticity estimates (Winters, 1984; Alston et al., 1990). This paper developed a generalized Armington trade model, which relaxes the single CES and homotheticity restrictions assumed in the Armington model. The generalized model includes the Armington model as a special case. Given homogeneity and symmetry conditions, the generalized model reduces to the original specification.

Empirical study applied to the Japanese import demand for red meat showed that the single CES and homotheticity assumptions are not maintained. The Armington model gives biased estimates because the data do not support those assumptions. The price elasticities tend to be underestimated with the Armington model when applied to Japanese meat import demand. The expenditure elasticities from the generalized Armington model differ significantly for all products, and the United States has the largest expenditure elasticity.

The assumptions of homotheticity and the same price responses seem too restrictive for international trade analyses. For more reliable estimates for general equilibrium models or policy simulations, the generalized Armington model does provide an alternative to the restrictive but indispensable Armington model.

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APPENDIX

Derivation of generalized Armington model

First rewrite equation (1) as follows:

$$E_{jt} = b_j^{\sigma}\left(P_{jt}/P_t\right)^{1-\sigma} E_t \tag{A1}$$

Solving equation (4) with respect to E_t and substituting it into equation (A1) multiplied by (E_t/E_t) yields:

$$E_{jt} = b_j^{\sigma}\left(P_{jt}/P_t\right)^{1-\sigma} E_t^2 \alpha_j^{1/\beta_j} E_{jt}^{-1/\beta_j} \epsilon_{jt}^{1/\beta_j} \tag{A2}$$

This expression is the same as equation (5). Rearrange equation (A2) as:

$$E_{jt}^{1+1/\beta_j} = \alpha_j^{1/\beta_j} b_j^{\sigma}\left(p_{jt}/P_t\right)^{1-\sigma} E_t^2 \epsilon_{jt}^{1/\beta_j} \tag{A3}$$

Let $\gamma_j = \beta_j/(1+\beta_j)$. Then equation (A3) is rewritten as:

$$E_{jt} = a_j\left(P_{jt}/P_t\right)^{(1-\sigma)\gamma_j} E_j^{2\gamma_j} \epsilon_{jt}^{1/(1+\beta_j)} \tag{A4}$$

where

$$a_j = \alpha_j^{1/(1+\beta_j)} b_j^{\sigma\gamma_j}$$

To express equation (A4) as a share function, divide both sides by E_t:

$$E_{jt}/E_t = a_j\left(P_{jt}/P_t\right)^{(1-\sigma)\gamma_j} E_t^{2\gamma_j - 1} \epsilon_{jt}^{1/(1+\beta_j)} \tag{A5}$$

Taking logarithms on both sides of equation (A5) gives:

$$\log\left(E_{jt}/E_t\right) = \log\ a_j + (1-\sigma)\gamma_j\ \log\left(P_{jt}/P_t\right) + \left(2\gamma_j - 1\right)\log\ E_t + e_{jt} \tag{A6}$$

where

$$e_{jt} = 1/\left(1+\beta_j\right)\log\ \epsilon_{jt}$$

since $\log\ \epsilon_{jt} \sim N(0,\ \sigma_\epsilon^2)$, $e_{jt} \sim N(0,\ \sigma_\epsilon^2/(1+\beta_j)^2)$.

REFERENCES

Alston, J., Carter, C., Green, R. and Pick, D., 1990. Whither Armington trade models? Am. J. Agric. Econ., 72: 455–467.

Armington, P., 1969. A theory of demand for products distinguished by place of production. IMF Staff Paper, 16: 159–176.

Babula, R., 1987. An Armington model of U.S. cotton exports. J. Agric. Econ. Res., 39: 12–22.

Deaton, A. and Muellbauer, J., 1980. An almost ideal demand system. Am. Econ. Rev., 70: 312–326.

Duffy, P., Wolhgenant, M. and Richardson, J., 1990. The elasticity of export demand for U.S. cotton. Am. J. Agric. Econ., 72: 468–474.

Eales, J. and Unnevehr, L., 1988. Demand for beef and chicken products: Separability and structural change. Am. J. Agric. Econ., 70: 521–532.

Figueroa, E. and Webb, A., 1986. An analysis of the U.S. grain embargo using a quarterly Armington-type model. In: Embargoes, Surplus Disposal, and U.S. Agriculture. Agric. Econ. Rep. 564, USDA Economic Research Service, Washington, DC.

Green, R. and Alston, J., 1990. Elasticities in AIDS models. Am. J. Agric. Econ., 72: 442–445.

Hayes, D., Wahl, T. and Williams, G., 1990. Testing restrictions on a model of Japanese meat demand. Am. J. Agric. Econ., 72: 556–566.

Ito, S., Chen, D.T. and Peterson, W., 1990. Modeling international trade flows and market shares for agricultural commodities: a modified Armington procedure for rice. Agric. Econ., 4: 315–333.

Sarris, A., 1983. European Community enlargement and world trade in fruits and vegetables. Am. J. Agric. Econ., 65: 235–246.

United Nations Direction of Trade, various issues, New York.

Winters, L., 1984. Separability and the specification of foreign trade functions. J. Int. Econ., 17: 239–263.