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Managing multiple international risks simultaneously with an optimal hedging model

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ABSTRACT

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A risk management model based on portfolio theory which accounts jointly for price, quantity, interest rate and exchange rate risks is developed and applied to cocoa and coffee production and exports in the Ivory Coast. Utilizing commodity and financial futures markets jointly, the results show that a government export agency can reduce risks from 27% to 89% by following a multicommodity hedging program which manages several risks simultaneously. The model and technique developed are applicable to many multiproduct firm and international risk management situations.

INTRODUCTION

The liberalization of trade among countries, the increased mobility of capital, and the introduction of flexible exchange rates have created an integrated world economy where domestic public policies have impacts and disturbances transmitted quickly from country to country. Multinational firms and government-controlled export agencies can take advantage of a greater pool of investment opportunities and larger markets. However, they also face greater economic risks resulting from increased variability in prices, exchange rates and interest rates.

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Especially vulnerable to these increased risks are less developed countries which depend heavily on their export sector for revenue and hard currency. Large fluctuations in export revenues for many countries can cause adverse effects on the economy as a whole. To guard against these effects most developing countries take steps to stabilize their export proceeds by acting at two levels. First, at the domestic level these countries isolate farmers from fluctuations in the world markets by guaranteeing them fixed prices for their products. Second, these countries often act at the international level in concert with other countries to reduce world price fluctuations. This second effort consists mainly of managing buffer stocks and/or allocating export quotas to maintain world prices within agreed upon ranges.

The stabilization efforts at the domestic level have been for the most part effective (Blandford, 1974; Denis, 1982), but at the international level they have been less successful. Factors cited as difficulties encountered by many international stabilization schemes range from the inability of the producing and consuming countries to reach agreements on export quotas and price ranges (MacBean, 1966; Ernst, 1982) to more fundamental economic ones. For example, Newbery and Stiglitz (1981) argue that the effectiveness of these schemes are limited because they do not take into account existing stabilization tools such as the futures markets [1]. In addition, these schemes usually manage only export commodity prices and fail to incorporate two major sources of export revenue fluctuations, exchange rate and interest rate variations.

Alternatively, several theories exist such as the purchasing power parity theorem and interest rate parity theorem which attempt to explain interrelationships among price, exchange rate, and interest rate differentials across countries. Differentials in one sector are to adjust to those of another sector, resulting in no revenue loss or gain to importers and exporters. However, empirical evidence has not supported these hypotheses, implying that multinational firms, international traders, and export agencies faced with price, currency and interest rate risks are justified in taking additional steps to manage them [2].

The objective of this paper is to present a risk management model based on modern risk management concepts which takes into account not only

[1] Newbery (1983) compared the risk reduction benefit of futures markets with domestic price stabilization schemes in developing countries and found for the most part that the futures market offers better insurance than price stabilization schemes unless the correlation between price and quantity is low.

[2] For empirical papers, see as examples Officer and Willet (1970), Solnik (1978), Otani and Tiwari (1981), and Isard (1987).

price and quantity risks but also interest rate and exchange rate risks. Specifically, we utilize portfolio theory to demonstrate how the futures markets can be used to manage simultaneously quantity, price, interest rate and exchange rate risks associated with the marketing of cocoa and coffee by the government export agency of the Ivory Coast. The approach developed here can be applied to many international trading situations and multiproduct firms.

After describing the marketing situation for the Ivory Coast in the next section, we then derive a general portfolio model for optimal futures market hedging within the context of the risks faced by the Ivory Coast in its exporting activities. Following, we present the estimating procedure and optimal hedging positions in the commodity and financial futures markets. Finally, we discuss the implications of this research and how the methodology developed here can be applied to numerous trading situations.

REVENUE RISKS FROM IVORY COAST EXPORTS

In the Ivory Coast, cocoa and coffee constitute by far the two major export crops ($611 million for cocoa, $434 million for coffee in 1982–4) despite the diversification policy promoted by the government since the 1960's. About 60% of the total export revenue is generated by the sale of these two commodities. Fluctuations in the cocoa and coffee export proceeds can be detrimental to the economy of the Ivory Coast. To protect farmers against world price fluctuations the government, through its marketing agency (Caisse), guarantees farmers a fixed price at the start of each season. Caisse, which does not take physical possession of the products, regulates the actions of private exporters who buy the commodities from the farmers at the guaranteed price and sells them in the world market. These private exporters are also guaranteed a fixed price which reflects the cost of transporting and handling the commodities from the farm gate to the ports. Any positive margin between the world price and the exporters' fixed price is collected by Caisse. When the margin is negative, Caisse pays the difference to the exporters. Over the years the typical situation has been for the world price to be higher than the domestic price (Delaporte, 1977). The margin collected by Caisse can be invested in the international financial market, or used to finance development projects. In the 1985 government investment budget the receipts from Caisse represented about 19% of total receipts and 30% of total domestic receipts.

About 80% of the cocoa and coffee exports from the Ivory Coast are priced in U.S. dollars or in dollar related currencies. In addition, the Ivorian currency (CFA) is pegged to the French franc (Ff) at a rate of 1Ff = 50 CFA francs. This monetary agreement allows the Ivory Coast to

participate in international trade with privileges similar to developed countries, but it leaves Ivorian policy markers with little control over monetary and exchange rate policy and less opportunity to manage risk internally. Thus, changes in the value of the dollar, or changing relationships between the economies of France and the United States, can affect the revenue received by Caisse [3]. Their investment of export receipts in international financial markets are also subject to interest rate variations.

Over the years, cocoa and coffee production in the Ivory Coast have fluctuated widely. For example, from 1975 to 1986 their coefficients of variation (annual deviations from the mean) were 36% and 26%, respectively. During the same period world cocoa and coffee prices varied around their respective means an average of 54% and 53%. Similarly, the U.S. dollar/CFA franc exchange rate and U.S. interest rate had coefficients of variation of 30% and 36%, respectively. Thus, the revenue generated by Caisse depends on the quantity of cocoa and coffee produced, the world cocoa and coffee prices, the interest rate in international financial markets, and the exchange rate between the U.S. dollar and the CFA franc. Each is subject to risk, making revenue receipts from exports highly uncertain.

HEDGING MODEL

Portfolio theory as developed by Markowitz (1952) and applied to hedging in the futures market by Johnson (1960) and Stein (1961) is the basis of the risk management model presented here. Several authors have used the Johnson–Stein framework to show empirical application of futures markets in managing price risk (Peck, 1975), price and quantity risk (Rolfo, 1980; Gemmill, 1985), exchange rate risk (Soenen, 1979) and interest rate risk (Franckle, 1980). Rolfo (1980) and Gemmill (1985) specifically applied their models based on utility maximization to determine optimal hedge ratios for cocoa exporting countries including the Ivory Coast. Anderson and Danthine (1980) expanded the framework to multiple risks and showed specifically the speculative and pure hedging components (minimum variance hedge ratios) of optimal hedges [4]. Recently, Bond et al. (1985) and Thompson and Bond (1987) examined the management of price and exchange rate risks simultaneously within the framework of the models. A model similar to the Anderson and Danthine paradigm is developed

[3] This fixed exchange rate between France and the Ivory Coast makes the validity of adjusting differentials suggested by the above mentioned international trade theories even more problematic.

[4] Rolfo (1980) and Newbery and Stiglitz (1981) also show these components.

here with the goal of applying it empirically. To our knowledge, no one has empirically utilized this type of model with more than two kinds of risks.

Using the mean-variance framework, it is assumed that the objective of Caisse is to maximize expected revenue subject to a certain level of risk where risk is measured by the variance of the revenue. The objective function to be maximized can be formalized as follows:

$$\Omega = E_t(Y_{t+1}) - \delta\, V_t(Y_{t+1}) \tag{1}$$

where, Y_{t+1} is the revenue in period $t+1$, E_t is the expectation operator, V_t is the variance operator, and δ is the risk aversion parameter ($\delta \geq 0$).

The revenue in $t+1$ is a function of the action undertaken by Caisse before harvest $(t-1)$ and at harvest (t) both in the cash and the futures markets. At harvest, Caisse through private exporters buys quantities Q_{cc} and Q_{co} of cocoa and coffee at fixed domestic prices (CFA francs), P^{d}_{cc} and P^{d}_{co}, respectively. These quantities are sold in the world markets (in U.S. dollars) at prices $P_{cc}X$ and $P_{co}X$, respectively (the dollar prices are converted back to CFA francs by exchange rate X). The proceeds are then invested at a one-period rate of interest r. The total revenue (in CFA francs) of the cash cocoa and coffee activities are designated as R_{cc} and R_{co}, respectively.

Before harvest Caisse sells quantity H_{cc} and H_{co} of cocoa and coffee forward in the futures market at price f^{t-1}_{cc} and f^{t-1}_{co}, respectively. At harvest, Caisse buys back these quantities at price f^{t}_{cc} and f^{t}_{co}, respectively. In local currency, the return from these activities in the cocoa and coffee futures markets are R^{f}_{cc} and R^{f}_{co}, respectively.

Before harvest, Caisse buys quantity C of foreign currency futures at price X^{t-1}_{f} to be sold back at time $t+1$ at price X^{t+1}_{f}. The return from this action is R^{f}_{c}.

Before harvest Caisse buys quantity I of interest rate futures contracts paying a rate r^{t-1}_{f} to be sold back at harvest (t) at rate r^{t}_{f}. The return from this investment is R^{f}_{r}.

In the risk management context the unknowns to be solved for by Caisse are the levels of the commodity hedges, H_{cc} and H_{co}; the amount of currency hedge, C; and the level of interest rate hedge, I.

The net income generated by Caisse is:

$$Y_{t+1} = K'R \tag{2}$$

where K is a (6×1) vector of ones and futures positions:

$$K' = [1 \quad 1 \quad H_{cc} \quad H_{co} \quad C \quad I]$$

and R is a (6×1) vector of cash and futures returns:

$$R' = \left[R_{cc} \quad R_{co} \quad R^{f}_{cc} \quad R^{f}_{co} \quad R^{f}_{c} \quad R^{f}_{r}\right]$$

with the returns defined as:

$$R_{cc} = \left[(1+r_t)P_{cct}X_{t+1} - P^{d}_{cc}\right]Q_{cc}$$

$$R_{co} = \left[(1+r_t)P_{cot}X_{t+1} - P^{d}_{co}\right]Q_{co}$$

$$R^{f}_{cc} = \left(f^{t-1}_{cc} - f^{t}_{cc}\right)X_t$$

$$R^{f}_{co} = \left(f^{t-1}_{co} - f^{t}_{co}\right)X_t$$

$$R^{f}_{c} = \left(X^{t-1}_{f} - X^{t+1}_{f}\right)$$

$$R^{f}_{r} = \left(r^{t-1}_{f} - r^{t}_{f}\right)$$

The objective function (1) becomes:

$$\Omega = K'\ E(R) - \delta[K'R\ R'K]$$

The above objective function is concave; consequently the maximum is obtained when the first derivatives with respect to the decision variables H_{cc}, H_{co}, C, and I are equal to zero. That is:

$$\frac{d(\Omega)}{dK_1} = E(R_1) - \delta\ VV'\ K = 0 \tag{3}$$

where $K_1 = [H_{cc}\quad H_{co}\quad C\quad I]$, $R'_1 = [R^{f}_{cc}\quad R^{f}_{co}\quad R^{f}_{c}\quad R^{f}_{r}]$, and VV' is the following (4×6) matrix:

$$VV' = [\mathrm{cov}(i, j)]$$

with $i = R^{f}_{cc}$, R^{f}_{co}, R^{f}_{c}, R^{f}_{r} and $j = R_{cc}$, R_{co}, R^{f}_{cc}, R^{f}_{co}, R^{f}_{c}, and R^{f}_{r}.

Premultiplying equation (3) by the following (4×4) matrix Z,

$$Z = \begin{bmatrix} \mathrm{VAR}^{-1}(R^{f}_{cc}) & 0 & 0 & 0 \\ 0 & \mathrm{VAR}^{-1}(R^{f}_{co}) & 0 & 0 \\ 0 & 0 & \mathrm{VAR}^{-1}(R^{f}_{c}) & 0 \\ 0 & 0 & 0 & \mathrm{VAR}^{-1}(R^{f}_{r}) \end{bmatrix}$$

leads to:

$$1/\delta\ Z\ E(R_1) - Z\ VV'\ K = 0 \tag{4}$$

or

$$1/\delta\ S - PK = 0$$

where $P = Z\ VV'$ is a (4×6) matrix whose elements are of the form $\mathrm{cov}(R_i, R_j)/\mathrm{var}(R_j)$, or simple regression coefficients, and $S = Z\ E(R_1)$ is a (4×1) vector of ratios of expected return to variance.

In order to isolate the four unknown elements of K (i.e., H_{cc}, H_{co}, C and I), matrices P and K are written in the following partitioned forms:

$$P = (\Sigma_2 \mid \Sigma_4)$$

and

$$K' = (1\ 1 \mid H_{cc}\ H_{co}\ C\ I)$$

where $\Sigma_2 = [a_{ij}]$ with $i = 3, 6$ and $j = 1, 2$, and $\Sigma_4 = [a_{ij}]$ with $i = 3, 6$ and $j = 3, 6$.

Define a_{ij} as the coefficients of simple regressions. Letting $i = j = 1, 6$ refer to R_{cc}, R_{co}, R^{f}_{cc}, R^{f}_{co}, R^{f}_{c} and R^{f}_{r}, respectively, a_{ij} is the coefficient of the simple regression of i on j.

Equation (4) can now be expressed as:

$$1/\delta\ S - \Sigma_2 I_2 - \Sigma_4 K_1 = 0 \tag{5}$$

where $I_2' = [1 \quad 1]$.

Since the $\text{COV}(R_i, R_i)/\text{VAR}(R_i) = 1$, the matrix Σ_4 has ones on its principal diagonal. Consequently, it can be written as:

$$\Sigma_4 = \Sigma_4^1 + I \tag{6}$$

where I is the identity matrix and Σ_4^1 is the same matrix as Σ_4 except it has zeros on its first diagonal instead of ones. Substituting (6) into (5) the solution of the hedging problem becomes:

$$K_1 = 1/\delta\ S - \Sigma_2 I_2 - \Sigma_4^1 K_1 \tag{7}$$

or

$$K_1^* = \Sigma_4^{-1}(1/\delta\ S - \Sigma_2 I_2) \tag{8}$$

Appendix 1 gives the detailed expression of the optimal hedges. As indicated in previous hedging literature (Anderson and Danthine, 1980; Rolfo, 1980), the optimal futures positions K_1^* includes both a speculative $1/\delta\ \Sigma_4^{-1}\ S$ and a hedge $-\Sigma_4^{-1}\Sigma_2 I_2$ component. For a highly risk averse decision maker (δ goes to infinity) or risky futures markets (S approaches zero) the optimal hedge coincides with the risk minimization strategy and $K_1^* = -\Sigma_4^{-1}\Sigma_2 I_2$.

We can use equation (8) to estimate the optimal hedges H_{cc}, H_{co}, C and I for selected values of the risk parameter δ in the interval from 10^{-5} for very low risk aversion to 10^5 for very high risk aversion.

The hedging strategy developed here will be evaluated by comparing it to the no hedge strategy. Since δ is assumed fixed in equation (1), the ratio of the expected income to the variance can be used to compare the two strategies. For large values of the risk aversion parameter, δ, the hedging

effectiveness coefficient $e = 1 - \text{VAR}\ (Y_h)/\text{VAR}(Y_u)$ can be used to compare the hedging and no hedging strategies [5]. $\text{VAR}(Y_h)$ and $\text{VAR}(Y_u)$ are the variances of the revenue associated with the hedged and the unhedged portfolios respectively.

EMPIRICAL FINDINGS

Cocoa and coffee are harvested continuously in the Ivory Coast from October up to September of the following year with the bulk of the harvest occurring from December to March. In this paper, the cocoa and coffee seasons are divided into the following four periods: October–December, January–March, April–June and July–September. (Actually, hedging decisions by Caisse are likely made more frequently, but less aggregated data are not publicly available.)

The optimal hedging strategy proposed here assumes that just before each period (in September, December, March and June), Caisse takes a position in the cocoa, coffee, currency and interest rate futures markets to be reversed at the end of the period (December, March, June and September) when cash commodities are sold in spot markets and proceeds are invested [6]. The contracts used for each quarter are December, March, June and September. However, since there are no cocoa and coffee futures contracts in June, the July futures contract is used instead for those two commodities. Also, there is no futures market for the CFA franc, and the French franc, to which the CFA franc is tied, has no active futures market. Consequently, an alternative futures currency, the British pound, is used to cross hedge the CFA franc. The return from the futures currency market R_c^f becomes $R_c^f = (\text{BP}^{t-1} - \text{BP}^{t+1})Q_{t+1}$, where BP is the British pound futures price in U.S. dollars and Q is the British pound/U.S. dollar exchange rate. This adjustment allows the optimal hedge C to be interpreted as CFA francs.

The basic data needed to calculate the commodity and financial optimal hedges are the monthly average futures prices for those months futures contracts are bought and sold and the total quantities of cocoa and coffee exported by the Ivory Coast during the quarter. The U.S. 90-day T-bill futures is used to hedge interest rate risk. Proceeds are invested in 3-month

[5] Large values of δ correspond to the situation when the speculative component is zero and the optimal hedge is the same as risk minimization.

[6] The currency position is held for two quarters. Consequently, Caisse reverses its initial currency positions taken in September, December, March and June at the end of March, June, September and December, respectively.

T-bills. Also needed are cocoa and coffee cash prices for each of the four decision months. Finally, the domestic prices for cocoa and coffee are fixed for each year. The analysis of this paper covers the period from 1976 to 1986.

Expectations about futures returns in the commodity and financial futures markets are needed to estimate the optimal hedges. Most hedgers presumably are also profit seekers (Working, 1953) who make use of all the available information in forecasting futures returns. One convenient way to take into account all the available information is to use time series techniques (Bond et al., 1985) which by design assures that the forecast residuals are white noise (i.e., expected value is zero). Consider the following expectation models:

$$R_i^{t+1} = E_t\left(R_i^{t+1}\right) + U_{t+1}$$

$$R_j^{t+1} = E_t\left(R_j^{t+1}\right) + V_{t+1}$$

where U and V are white noise residual terms (i.e., $E(U) = 0$ and $E(V) = 0$). The variance of R_i and R_j are, respectively:

$$\text{VAR}_t(R_i) = \text{VAR}_t(U)$$

$$\text{VAR}_t(R_j) = \text{VAR}_t(V)$$

The covariance between R_i and R_j is:

$$\text{COV}_t(R_i, R_j) = \text{COV}_t(U, V).$$

Thus, the parameters of equation (8) which are of the form $\text{COV}(R_i, R_j)/\text{VAR}(R_j)$ can be estimated by the coefficient of the regression U on V since $\text{COV}(R_i, R_j)/\text{VAR}(R_j) = \text{COV}(U, V)/\text{VAR}(V)$.

In this paper due to the limited number of observations, an expectation model based on a time series technique may not lead to white noise residuals. Consequently, it is assumed that the best forecast of future returns are the average of past returns. The parameters of the hedge component in equation (8) are estimated by simple regression on returns instead of forecast errors [7].

Table 1 gives the ratios of the futures returns to their respective variances for the four periods being studied. All but one ratio are less than

[7] An important feature of this model is that optimal hedges can be determined by estimating only simple regressions, eliminating any potential influence of multicollinearity. Nevertheless, hedge ratios are the direct product of levels of correlation among the prices (risks). A very high level of price correlation between two risky variables can result in placing hedges for only one of the commodities which will manage the risks for both commodities. Zero correlation among risk variables will generate hedge ratios as if each commodity were treated individually, rather than simultaneously as is done here.

TABLE 1

Ratios of the expected futures returns to variances for the cocoa, coffee, currency and interest rate futures markets (returns in CFA francs)

Periods	Cocoa	Coffee	Currency [a]	Interest rate
September –December	1.9×10^{-6}	-2.94×10^{-6}	−0.00036	−0.007
December –March	-16×10^{-7}	2.0×10^{-7}	−1.97	−0.28
March –June	-1.25×10^{-7}	1.53×10^{-7}	0.65	0.75
June –September	1×10^{-6}	-11×10^{-7}	−0.39	−0.004

[a] The currency hedges span two periods, not one (see text).

absolute one, and in most cases these ratios are very close to zero, suggesting that little speculative opportunity exists.

Actual estimation of the optimal hedge for values of δ varying from 10^{-5} to 10^{5} show that the optimal hedges do not vary significantly. Consequently, only the results corresponding to the risk minimization strategy ($\delta = 10^{-5}$) are reported. The hedging model can then be evaluated using the hedging efficiency criteria ($e = 1 - \mathrm{VAR}(Y_h)/\mathrm{VAR}(Y_u)$).

Table 2 reports the optimal cocoa, coffee, currency and interest rate hedge ratios and the proportion of total risk eliminated by the hedging

TABLE 2

Optimal commodity and financial hedge ratios, and hedging effectiveness (in %)

Periods	Optimal hedges				Hedging effectiveness
	Cocoa	Coffee	Currency [a]	Interest rate	
September –December	−5 [b]	190	339	46	60
December –March	17	−40	−147	−92	27
March –June	6	−314	−15	51	89
June –September	50	22	8	8	42

[a] The currency hedges span two periods, not one (see text).

[b] Positive signs indicate normal positions for Caisse, short coffee and cocoa and long currency and interest rate. Negative signs indicate opposite positions.

strategy [8]. The normal positions for Caisse of selling cocoa and coffee futures and buying currency and interest rate futures are represented by positive signs, while opposite positions have negative signs. The hedge ratios range from buying futures contracts larger than the cash position (currency in September–March), to very small futures positions (cocoa in September–December), to buying more futures contracts than the size of the cash position when selling contracts is the normal position (coffee in March–June). Specifically reading the first row, for a given unit size of cocoa and coffee, Caisse in September should buy cocoa futures contracts equivalent to 5% of the cocoa units, sell coffee futures contracts equivalent to 190% of the coffee units, buy British pound futures contracts equivalent to 339% of the value of the foreign exchange for both cocoa and coffee, and buy U.S. 90-day T-bill futures equivalent to 46% of the total sales proceeds which will be invested in T-bills. All the other rows can be interpreted likewise. However, and most importantly for this analysis, the hedging effectiveness column indicates that commodity and financial futures markets are useful risk management tools since Caisse can reduce between 27% and 89% of the risk associated with cocoa and coffee export revenue. These represent substantial reduction in risks.

Several observations can be drawn from the above results. First, across the four quarters the average risk reduction is 54%, which is noteworthy. The results also obtained in this analysis suggest that contrary to traditional hedging theory, taking futures positions greater than the cash position may be consistent with optimal hedging decisions. Sometimes these results show the futures position to be on the same side, rather than the opposite side, of the cash position. Gemmill (1985) found previously that long positions in futures were consistent with risk minimization. Also, Rolfo (1980) found that for low risk aversion long positions in the futures were optimal. However, both of those studies involved only single commodity hedges [9,10].

Second, there is considerable variability in the hedge positions from

[8] These ratios would be recommended hedge ratios for each quarter of 1987 based on regressions using data from 1976 to 1986.

[9] See also Berck (1981) and Bond et al. (1985) for additional studies with negative hedge ratios. Berck specifically demonstrated that speculative positions in futures markets can be consistent with risk aversion.

[10] In general, the results obtained here were quite different from those obtained by Rolfo (1980) and Gemmill (1985). This could be due to the method of separating the data quarterly, or the fact that this study includes the effects of multiple risks. Indeed, when testing single commodity hedges as in a more traditional approach, we found results for cocoa similar to Rolfo's results. This suggests that attempting to manage only commodity risks may not be an effective strategy.

quarter to quarter [11]. This means a government agency needs to be very flexible in establishing futures positions and be willing to alter them in the next quarter. This need for flexibility may suggest why many stabilization schemes fail where countries are locked into specific trading scenarios for a whole year.

Third, some of the results of this paper may be difficult to implement because political leaders in many developing countries believe that participation in futures markets is speculation. Therefore, it may be difficult to convince decision makers at Caisse to take futures positions at all, let alone futures positions which are greater than cash positions [12]. Consequently, upper and lower bounds corresponding to the expected cash position and zero, respectively, can be put on the different hedges. For example, when the hedge is found to be negative, set it equal to zero. Similarly, a futures position greater than the corresponding expected cash position is set to that expected cash position. Experimentation with this constrained risk minimization strategy reduced the hedging effectiveness for all four periods on average from 54% to 21%, making these results based on constrained trading positions less interesting and appealing. These results are also nonoptimal. Still, these results show more effective risk management potential than a no hedging strategy.

Fourth, data limitations affect the results. We were constrained to quarterly data, but certainly Caisse makes decisions more frequently. Presumably, Caisse has additional data for analysis that would conform more closely with their decision horizons [13].

Since the empirical results are based on simple regressions, the general nature of the results are expected to be valid over alternative time-frames of these data because the general level of price correlations is not expected to change dramatically. However, the results in Table 2 are very data specific and are shown here primarily for illustrative purposes. As just mentioned, we were limited to quarterly data, and daily users of these markets will have access to more detailed data. Different data will generate different coefficients. Most important about this paper are the develop-

[11] An explanation for this variability may be large-basis risks (Peck, 1975).

[12] The instability in futures positions can represent a valid reason that some governments or agencies do not participate in futures markets.

[13] An alternative risk management model based on safety first criteria was also constructed and estimated. Instead of maximizing expected utility, this approach assumes that one seeks to maximize income provided that a certain level of return (i.e., above some disaster level) is obtained with a particular confidence level. The results from this alternative approach were in correspondence with the portfolio shown here, adding credibility to the demonstrated results.

ment of a methodology for hedging multiple risks simultaneously, and the empirical demonstration that multicommodity hedging strategies can reduce risks considerably compared to either no hedging or hedging each risk independently. For most effective risk management, firms and countries must assess all their commodity risks simultaneously.

CONCLUSION

This paper demonstrates how a marketing agency faced with multiple international risks (quantity, price, exchange rate and interest rate) may use commodity and futures markets jointly as management tools. In particular, this paper develops a multicommodity hedging model based on portfolio theory and applies it to Ivory Coast cocoa and coffee exports.

When the average of past returns is used as an expectation model, the cocoa, coffee, British pound and 3-month U.S. T-bill futures markets offer little speculative opportunity. However, when the objective of the decision maker is to minimize risk, the cocoa, coffee, exchange rate and interest rate futures markets provide substantial risk reduction opportunity. In particular, the government export agency of the Ivory Coast may reduce from 27% to 89% of the risk it faces in marketing cocoa and coffee. The futures positions have considerable range, and on occasion risk minimization would require futures positions greater than the expected cash positions, and on the same side as the cash positions. A policy utilizing such trading strategies may be politically difficult to implement in many developing countries. However, even setting hedge ratios between 0 and 1 still leads to better risk management results than the no hedging strategy. Also, these results are quite different from those obtained when managing each risk independently, presumably due to the interactions among the various risks. Multicommodity strategies are demonstratably more efficient risk management scenarios than single commodity strategies.

This study demonstrates the potential for substantial risk reduction when managing several risks simultaneously. This technique has applicability to many situations facing numerous governments and private international trading firms.

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APPENDIX 1

Derivation of the optimal hedges

The objective function to the maximized is:

$$E(Y) - 1/2\ \delta\ \mathrm{VAR}(Y) \tag{A1}$$

where

$$Y = R_{cc} + R_{co} + H_{cc}R^{f}_{cc} + H_{co}R^{f}_{co} + CR^{f}_{c} + IR^{f}_{r}$$

and δ is the risk parameter coefficient, $E(\cdot)$ is the expectation operator, and $\mathrm{VAR}(\cdot)$ is the variance operator. The detailed expression of the variance of Y is:

$$\begin{aligned}\mathrm{VAR}(Y) = {} & \mathrm{VAR}(R_{cc}) + \mathrm{VAR}(R_{co}) + H^2_{cc}\,\mathrm{VAR}(R^{f}_{cc}) + H^2_{co}\,\mathrm{VAR}(R^{f}_{co}) \\ & + C^2\,\mathrm{VAR}(R^{f}_{c}) + I^2\,\mathrm{VAR}(R^{f}_{r}) + 2(\mathrm{COV}(R_{cc}, R_{co}) \\ & + H_{cc}\,\mathrm{COV}(R_{cc}, R^{f}_{cc}) + H_{co}\,\mathrm{COV}(R_{cc}, R^{f}_{co}) + C\,\mathrm{COV}(R_{cc}, R^{f}_{c}) \\ & + I\,\mathrm{COV}(R_{cc}, R^{f}_{r}) + H_{cc}\,\mathrm{COV}(R_{co}, R^{f}_{cc}) + H_{co}\,\mathrm{COV}(R_{co}, R^{f}_{co}) \\ & + C\,\mathrm{COV}(R_{co}, R^{f}_{c}) + I\,\mathrm{COV}(R_{co}, R^{f}_{r}) + H_{cc}H_{co}\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{co}) \\ & + H_{cc}C\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{c}) + H_{cc}I\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{r}) + H_{co}C\,\mathrm{COV}(R^{f}_{co}, R^{f}_{c}) \\ & + H_{co}I\,\mathrm{COV}(R^{f}_{co}, R^{f}_{r}) + CI\,\mathrm{COV}(R^{f}_{c}, R^{f}_{r}))\end{aligned}$$

The concavity of the objective function assures that the maximum is obtained at the point where the first derivatives of (A1) with respect to the decision variables H_{cc}, H_{co}, C and I are equal to zero. These first derivatives are, respectively:

$$\begin{aligned}\frac{\mathrm{d}\,E(U(Y))}{\mathrm{d}H_{cc}} = {} & E(R^{f}_{cc}) - \delta[H_{cc}\,\mathrm{VAR}(R^{f}_{cc}) + \mathrm{COV}(R_{cc}, R^{f}_{cc}) \\ & + \mathrm{COV}(R_{co}, R^{f}_{cc}) + H_{co}\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{co}) + C\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{c}) \\ & + I\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{r})] = 0 \qquad \text{(A2)}\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\,E(U(Y))}{\mathrm{d}H_{co}} = {} & E(R^{f}_{co}) - \delta[H_{co}\,\mathrm{VAR}(R^{f}_{co}) + \mathrm{COV}(R_{cc}, R^{f}_{co}) \\ & + \mathrm{COV}(R_{co}, R^{f}_{co}) + H_{cc}\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{co}) + C\,\mathrm{COV}(R^{f}_{c}, R^{f}_{co}) \\ & + I\,\mathrm{COV}(R^{f}_{r}, R^{f}_{co})] = 0 \qquad \text{(A3)}\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\,E(U(Y))}{\mathrm{dC}} = {} & E(R^{f}_{c}) - \delta[C\,\mathrm{VAR}(R^{f}_{c}) + \mathrm{COV}(R_{cc}, R^{f}_{c}) \\ & + \mathrm{COV}(R_{co}, R^{f}_{c}) + H_{cc}\,\mathrm{COV}(R^{f}_{cc}, R^{f}_{c}) + H_{co}\,\mathrm{COV}(R^{f}_{co}, R^{f}_{c}) \\ & + I\,\mathrm{COV}(R^{f}_{r}, R^{f}_{c})] = 0 \qquad \text{(A4)}\end{aligned}$$

$$\frac{\mathrm{d}\ E(U(Y))}{\mathrm{d}I} = E(R_{\mathrm{r}}^{\mathrm{f}}) - \delta\big[I\ \mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}}) + \mathrm{COV}(R_{\mathrm{cc}}, R_{\mathrm{r}}^{\mathrm{f}}) + \mathrm{COV}(R_{\mathrm{co}}, R_{\mathrm{r}}^{\mathrm{f}}) + H_{\mathrm{cc}}\ \mathrm{COV}(R_{\mathrm{cc}}^{\mathrm{f}}, R_{\mathrm{r}}^{\mathrm{f}}) + H_{\mathrm{co}}\ \mathrm{COV}(R_{\mathrm{co}}^{\mathrm{f}}, R_{\mathrm{r}}^{\mathrm{f}}) + C\ \mathrm{COV}(R_{\mathrm{c}}^{\mathrm{f}}, R_{\mathrm{r}}^{\mathrm{f}})\big] = 0 \tag{A5}$$

Dividing equations (A2), (A3), (A4) and (A5) through by $\mathrm{VAR}(R_{\mathrm{cc}}^{\mathrm{f}})$, $\mathrm{VAR}(R_{\mathrm{co}}^{\mathrm{f}})$, $\mathrm{VAR}(R_{\mathrm{c}}^{\mathrm{f}})$ and $\mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}})$, respectively, and solving for H_{cc}, H_{co}, C and I give the following equations:

$$H_{\mathrm{cc}} = \frac{E(R_{\mathrm{cc}}^{\mathrm{f}})}{2\delta\ \mathrm{VAR}(R_{\mathrm{cc}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{cc}}^{\mathrm{f}}, R_{\mathrm{cc}})}{\mathrm{VAR}(R_{\mathrm{cc}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{cc}}^{\mathrm{f}}, R_{\mathrm{co}})}{\mathrm{VAR}(R_{\mathrm{cc}}^{\mathrm{f}})} - H_{\mathrm{co}}\frac{\mathrm{COV}(R_{\mathrm{cc}}^{\mathrm{f}}, R_{\mathrm{co}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{cc}}^{\mathrm{f}})} - C\frac{\mathrm{COV}(R_{\mathrm{cc}}^{\mathrm{f}}, R_{\mathrm{c}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{cc}}^{\mathrm{f}})} - I\frac{\mathrm{COV}(R_{\mathrm{cc}}^{\mathrm{f}}, R_{\mathrm{r}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{cc}}^{\mathrm{f}})}$$

$$H_{\mathrm{co}} = \frac{E(R_{\mathrm{co}}^{\mathrm{f}})}{2\delta\ \mathrm{VAR}(R_{\mathrm{co}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{co}}^{\mathrm{f}}, R_{\mathrm{cc}})}{\mathrm{VAR}(R_{\mathrm{co}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{co}}^{\mathrm{f}}, R_{\mathrm{co}})}{\mathrm{VAR}(R_{\mathrm{co}}^{\mathrm{f}})} - H_{\mathrm{cc}}\frac{\mathrm{COV}(R_{\mathrm{co}}^{\mathrm{f}}, R_{\mathrm{cc}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{co}}^{\mathrm{f}})} - C\frac{\mathrm{COV}(R_{\mathrm{co}}^{\mathrm{f}}, R_{\mathrm{c}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{co}}^{\mathrm{f}})} - I\frac{\mathrm{COV}(R_{\mathrm{co}}^{\mathrm{f}}, R_{\mathrm{r}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{co}}^{\mathrm{f}})}$$

$$C = \frac{E(R_{\mathrm{c}}^{\mathrm{f}})}{2\delta\ \mathrm{VAR}(R_{\mathrm{c}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{c}}^{\mathrm{f}}, R_{\mathrm{cc}})}{\mathrm{VAR}(R_{\mathrm{c}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{c}}^{\mathrm{f}}, R_{\mathrm{co}})}{\mathrm{VAR}(R_{\mathrm{c}}^{\mathrm{f}})} - H_{\mathrm{cc}}\frac{\mathrm{COV}(R_{\mathrm{c}}^{\mathrm{f}}, R_{\mathrm{cc}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{c}}^{\mathrm{f}})} - H_{\mathrm{co}}\frac{\mathrm{COV}(R_{\mathrm{c}}^{\mathrm{f}}, R_{\mathrm{co}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{c}}^{\mathrm{f}})} - I\frac{\mathrm{COV}(R_{\mathrm{c}}^{\mathrm{f}}, R_{\mathrm{r}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{c}}^{\mathrm{f}})}$$

$$I = \frac{E(R_{\mathrm{r}}^{\mathrm{f}})}{2\delta\ \mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{r}}^{\mathrm{f}}, R_{\mathrm{cc}})}{\mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}})} - \frac{\mathrm{COV}(R_{\mathrm{r}}^{\mathrm{f}}, R_{\mathrm{co}})}{\mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}})} - H_{\mathrm{cc}}\frac{\mathrm{COV}(R_{\mathrm{r}}^{\mathrm{f}}, R_{\mathrm{cc}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}})} - H_{\mathrm{co}}\frac{\mathrm{COV}(R_{\mathrm{r}}^{\mathrm{f}}, R_{\mathrm{co}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}})} - C\frac{\mathrm{COV}(R_{\mathrm{r}}^{\mathrm{f}}, R_{\mathrm{c}}^{\mathrm{f}})}{\mathrm{VAR}(R_{\mathrm{r}}^{\mathrm{f}})}$$

or

$$H_{\mathrm{cc}} = 1/\delta\ S_1 - a_{31} - a_{32} - a_{34}H_{\mathrm{co}} - a_{35}C - a_{36}I$$

$$H_{\mathrm{co}} = 1/\delta\ S_2 - a_{41} - a_{42} - a_{43}H_{\mathrm{cc}} - a_{45}C - a_{46}I$$

$$C = 1/\delta\ S_3 - a_{51} - a_{52} - a_{53}H_{\mathrm{cc}} - a_{54}H_{\mathrm{co}} - a_{56}I$$

$$I = 1/\delta\ S_4 - a_{61} - a_{62} - a_{63}H_{\mathrm{cc}} - a_{64}H_{\mathrm{co}} - a_{65}C$$

where

$S_1 = E(R^{\mathrm{f}}_{\mathrm{cc}})/\mathrm{VAR}(R^{\mathrm{f}}_{\mathrm{cc}})$

$S_2 = E(R^{\mathrm{f}}_{\mathrm{co}})/\mathrm{VAR}(R^{\mathrm{f}}_{\mathrm{co}})$

$S_3 = E(R^{\mathrm{f}}_{\mathrm{c}})/\mathrm{VAR}(R^{\mathrm{f}}_{\mathrm{c}})$

$S_4 = E(R^{\mathrm{f}}_{\mathrm{r}})/\mathrm{VAR}(R^{\mathrm{f}}_{\mathrm{r}})$

and

$1/\delta\ S_1$, a_{31}, a_{32}, a_{34}, a_{35} and a_{36} correspond in order with the elements in (A2)

$1/\delta\ S_2$, a_{41}, a_{42}, a_{43}, a_{45} and a_{46} correspond in order with the elements in (A3)

$1/\delta\ S_3$, a_{51}, a_{52}, a_{53}, a_{54} and a_{56} correspond in order with the elements in (A4)

$1/\delta\ S_4$, a_{61}, a_{62}, a_{63}, a_{64} and a_{65} correspond in order with the elements in (A5)

In matrix form the above equations can be written as:

$$K_1 = 1/\delta\ S - \Sigma_2 I_2 + \Sigma_4^1 K_1$$

where

$$K_1' = [H_{\mathrm{cc}}\ H_{\mathrm{co}}\ C\ I]$$

$$S' = [S_1\ S_2\ S_3\ S_4]$$

$$\Sigma_2 = \begin{bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \\ a_{51} & a_{52} \\ a_{61} & a_{62} \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and

$$\Sigma_4^1 = \begin{bmatrix} 0 & a_{34} & a_{35} & a_{36} \\ a_{43} & 0 & a_{45} & a_{46} \\ a_{53} & a_{54} & 0 & a_{56} \\ a_{63} & a_{64} & a_{65} & 0 \end{bmatrix}$$