

**A Model of the Impact of  
Protein Payments on Nitrogen Application\***

by

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## **Abstract**

This paper develops a model of the impact of introducing a protein payments system on a farmer's nitrogen application. The model specifies a relationship between yield, protein, nitrogen and seasonal conditions and incorporates this relationship into a decision framework of optimal nitrogen application. The introduction of a protein payments system is shown to have a positive impact on nitrogen application if the critical protein level for extra payments is less than or equal to the farmer's existing expected protein level. However, if the reverse situation applies, the nitrogen response is ambiguous and dependant on both the critical protein level and the level of yield uncertainty. Further empirical work is required.

## Introduction

The Australian Wheat Board (AWB) has recently introduced a system of premiums and discounts for protein levels in wheat. With this system higher prices will be paid if measured protein levels exceed a specified level, while price discounts will be applied if measured protein levels are below the specified level.

The introduction of this system complicates a farmer's nitrogen application decision because nitrogen may be used by the plant to improve yield or protein level or both. Moreover, initial scientific research has shown that estimating these relationships is confounded by the role of uncertain seasonal factors in determining the actual relationships. For example "if rainfall is low the nitrogen will increase returns through high protein although yields may remain relatively low, if rainfall is high, the early nitrogen will increase yield" (Agribusiness Decision March, 1994 p.9). Consequently, the introduction of protein payments and discounts means that a farmer is now faced with a modified decision environment for nitrogen application where the nature of these modifications is uncertain.

The aim of this paper is to contribute to the understanding of how protein payments affect the farmer's nitrogen decision environment, and hence how these payments affect the level of nitrogen application.

The structure of the paper is as follows. Section 1 sets out the initial model of the farmer's nitrogen application decision. A system of protein payments is then introduced and its effect on the farmer's nitrogen decision rule is determined. It is shown that, because the system contains both incentive and disincentive effects on the level of nitrogen application, no unambiguous conclusion can be reached regarding the overall effect of the system on nitrogen application. In view of this, section 2 undertakes a numerical analysis of the model in order to highlight the sensitivity of the nitrogen application decision to the components of the protein payments system.

It is concluded that, although further empirical research is required, only where farmers are producing wheat of low expected protein levels relative to the specified level for premiums to apply, is the protein payments system likely to result in decreased nitrogen application. Moreover, for farmers in this situation, the level of yield uncertainty is an additional important factor determining the direction of nitrogen response to the introduction of the system, whereas the magnitude of protein payments is not.

## Section 1 The Model

In order to focus the analysis on the effect of protein payments on nitrogen application it is assumed that the farmer produces only one output (ie wheat) with a single variable input (ie nitrogen). The relationship between wheat yield ( $y$ ) and nitrogen application ( $N$ ) is uncertain due to seasonal factors, where this uncertainty is assumed to be multiplicative:

$$y = \theta g(N) \quad (1)$$

where:

$$\begin{aligned} g'(N) &> 0 \quad (g''(N) < 0) \\ E(\theta) &= 1 \\ \bar{y} &= \text{expected yield} = g(N). \end{aligned}$$

The relationship between nitrogen application and protein level ( $r$ ) is uncertain as well, but as this uncertainty is also due to seasonal factors, the uncertainty of protein level is assumed to be a function of the uncertainty of yield:

$$r = \gamma N/y \quad (2)$$

where:  $\gamma$  = parameter relating to soil type  
 $\bar{r}$  = expected protein level =  $\gamma N/\bar{y}$ .

Note that this specification features yield as first claimant on the available nitrogen subject to seasonal conditions with protein levels relatively high in seasons where yield is relatively low (and vice versa). In the absence of clear scientific evidence of the relationship between protein, yield and nitrogen for different seasonal conditions, this specification has been adopted on the basis of anecdotal evidence.<sup>1</sup>

The price of wheat ( $p$ ) is also assumed to be uncertain, but uncorrelated with yield. Finally, the farmer is assumed to be risk neutral as a first approximation of the effects of protein

payments on nitrogen application. On this basis, and in the absence of protein payments, expected profit is given by:

$$E(py) - cN \quad (3)$$

where:  $c$  = cost per unit of nitrogen  
 $\bar{p}$  = expected price.

The first order condition for maximising expected profit is given by:

$$\bar{p} \frac{\partial \bar{y}}{\partial N} = c. \quad (4)$$

Now consider the introduction of a system of protein discounts and premiums. In what follows it is assumed that this system features a critical protein level ( $r_c$ ) above which a constant per unit of output price premium is paid and below which a constant per unit of output price discount is deducted. It is also assumed that this payment system is symmetrical with respect to the expected market price:<sup>2</sup>

$$\bar{p}_2 = \bar{p} + \alpha; \quad \bar{p}_1 = \bar{p} - \alpha \quad (5)$$

and so:  $(\bar{p}_2 + \bar{p}_1)/2 = \bar{p}$ .

Given this system, the farmer will have an expected price for wheat as follows:

$$\begin{aligned} \bar{p}_2 & \text{ if } r > r_c \\ \bar{p}_1 & \text{ if } r < r_c \end{aligned} \quad (6)$$

Since:  $r = \gamma N / \theta \bar{y}$

(6) may be rearranged to give:

$$\begin{aligned} \bar{p}_2 & \text{ if } \theta < \gamma N / r_c \bar{y} \\ \bar{p}_1 & \text{ if } \theta > \gamma N / r_c \bar{y} \end{aligned} \quad (7)$$

As a consequence, with this system expected profit is given by:

$$E(\pi) = \bar{p}_2 \int_0^{\gamma N / r_c \bar{y}} \theta \bar{y} f(\theta) d\theta + \bar{p}_1 \int_{\gamma N / r_c \bar{y}}^{\infty} \theta \bar{y} f(\theta) d\theta - cN \quad (8)$$

where:  $f(\theta) =$  probability distribution function of  $\theta$ .

In this situation, the first order condition is given by:<sup>3</sup>

$$\begin{aligned} \frac{\partial \bar{y}}{\partial N} \left( \bar{p}_2 \int_0^{\gamma N / r_c \bar{y}} \theta f(\theta) d\theta + \bar{p}_1 \int_{\gamma N / r_c \bar{y}}^{\infty} \theta f(\theta) d\theta \right) \\ + (\bar{p}_2 - \bar{p}_1) f(\gamma N / r_c \bar{y}) = c. \end{aligned} \quad (9)$$

Equation (9) shows that in the presence of the system of protein payments the farmer determines optimal nitrogen application as in the absence of the system by comparing the marginal cost of nitrogen with the expected marginal revenue. However, whereas in the absence of the system expected marginal revenue depends simply on the expected market price ( $\bar{p}$ ) and the expected marginal productivity of nitrogen application ( $\partial \bar{y} / \partial N$ ), with the system in place, expected marginal revenue is more complex. The first term on the left-hand-side of (9) shows that the marginal expected price now depends on a weighted sum of the expected market price with premium and with discount ( $\bar{p}_2$  and  $\bar{p}_1$ ), where the weights depend on the critical protein level relative to the expected level ( $r_c \gtrless \bar{r}$ ). Moreover, the second term on the left-hand-side of (9) shows that expected marginal revenue with protein payments is also determined by the opportunity to increase the likelihood of receiving the protein premium that arises from applying additional nitrogen ( $f(\gamma N / r_c \bar{y})$ ).

Given this model development, the focus of the analysis in this paper is on the levels of optimal nitrogen application implied by equations (4) and (9) (ie without and with protein payments respectively). However, comparing equations (4) and (9) results only in analytical ambiguity regarding these levels.

For example consider the situation where the critical level of protein for the premium and the discount is set equal to the expected level of protein in the absence of protein payments ( $\bar{r}_0$ ):

$$r_c = \bar{r}_0.$$

In this situation the weight attached to  $\bar{p}_2$  in the first term of equation (9) is based on a set of values of  $\theta$  all less than unity, while the weight attached to  $\bar{p}_1$  is based on a set of values of  $\theta$  all in excess of unity, with the same overall probability weight applying in each case for a symmetrical yield distribution. This implies the overall value of the weight attached to  $\bar{p}_2$  is less than that attached to  $\bar{p}_1$ , and so their weighted sum is less than  $\bar{p}$ . However, the second term in equation (9) is always positive and so the overall value of the left-hand-side of this equation relative to that of equation (4) is unclear. Only in the situation where  $r_c$  is small relative to  $\bar{r}$  is it possible for the weighted sum of prices in the first term of equation (9) to exceed  $\bar{p}$ , thereby providing an unambiguous stimulus to nitrogen application. But even in this situation the shape of the yield distribution is also a relevant factor. Otherwise, the role of the two terms on the left-hand-side of equation (9) will typically be in conflict regarding their overall value relative to the left-hand-side of equation (4).

Prompted by this ambiguity, in the next section a numerical analysis is undertaken of the model developed in this section in order to investigate further the impact of protein payments on nitrogen application.

## Section 2 Numerical Analysis

In order to undertake a numerical analysis of the model developed in section 1 it is necessary to specify further the relationship between expected yield and the level of nitrogen application. In what follows it is assumed that this relationship is represented by the Mitscherlich function:<sup>4</sup>

$$y = m(1 - de^{-bN}) \quad (10)$$

where:  $m$  = maximum yield  
 $d$  = axis parameter  
 $b$  = curvature parameter.



In addition, it is assumed that the multiplicative parameter governing the uncertainty of yield ( $\theta$ ) is characterised by a normal distribution. As a consequence, the value of the weights in equation (9) are given by:<sup>5</sup>

$$\int_0^{\gamma N / r_c \bar{y}} \theta f(\theta) d\theta = F(\gamma N / r_c \bar{y}) \left( 1 - \frac{\sigma_0 Z(\gamma N / r_c \bar{y})}{F(\gamma N / r_c \bar{y})} \right) \quad (11)$$

and 
$$\int_{\gamma N / r_c \bar{y}}^{\infty} \theta f(\theta) d\theta = 1 - \int_0^{\gamma N / r_c \bar{y}} \theta f(\theta) d\theta \quad (12)$$

where:  $\sigma_0$  = standard deviation of  $\theta$   
 $F(\gamma N / r_c \bar{y})$  = cumulative probability of the actual protein level exceeding the critical level ( $r_c$ )  
 $Z(\gamma N / r_c \bar{y})$  = ordinate of the standard normal distribution at the value of  $\theta$  corresponding to the critical protein level.

Note also that:

$$f(\gamma N / r_c \bar{y}) = Z(\gamma N / r_c \bar{y}) / \sigma_0. \quad (13)$$

Finally, the following set of parameter values is specified as a "Base Case":

$$\begin{aligned} m &= 6 \\ d &= 1 \\ b &= 1 \\ \bar{p} &= 20 \\ c &= 5 \\ \sigma_0 &= 0.2. \end{aligned}$$

Given these parameter values, the value of  $\gamma$  can be adjusted to give an initial expected protein level of ten associated with the initial optimal nitrogen application:

$$\begin{aligned} \gamma &= 18.093 \\ \bar{r} &= 10 \\ N^* &= 3.178 \end{aligned}$$

$$\begin{aligned}\bar{y} &= 5.75 \\ E(\pi) &= 99.110.\end{aligned}$$

Note that in the absence of a protein payments system this optimal solution is independent of the value of  $\sigma_0$ .

Next consider the introduction of a protein premium and discount system specified as follows:

$$\begin{aligned}\text{if } r < r_c \text{ then } \bar{p}_1 &= 18 \\ \text{if } r > r_c \text{ then } \bar{p}_2 &= 22.\end{aligned}$$

The results of the system's impact are demonstrated in Table 1, with rows (2) and (3) illustrating the ambiguity of the impact of the system. In particular, the results in row (2) represent a situation where the incentive effect of being able to increase the likelihood of receiving a protein premium by increasing nitrogen application (the second term on the LHS of (9)) dominates the disincentive effect of there being some probability of receiving a protein discount associated with a relatively high yield (the first term on the LHS of (9)). Note also that in this case the increase in optimal nitrogen application is associated with an increase in the level of expected profits. By contrast, the results in row (3) represent a situation where this disincentive effect dominates the incentive effect and optimal nitrogen application is reduced. Overall the results in Table 1 suggest that the higher is the critical protein level relative to the farmer's initial expected protein level the more likely it is that optimal nitrogen application will be reduced following the introduction of the protein payments system.

Next consider the sensitivity of the results in Table 1 to the specification of the size of the protein payments in the Base Case. Table 2 contains details of results comparable with those in Table 1 but calculated on the basis of a doubling of the size of the protein premium and discount (ie  $\bar{p}_1 = 16$ ,  $\bar{p}_2 = 24$ ). Overall, the results in Table 2 show that, although with increased protein payments the magnitude of the nitrogen response is larger, the pattern of responses is unchanged, featuring as in Table 1 a change of direction of nitrogen response

between the critical protein levels of 10 and 20. This observation can be explained by noting in equation (9) that an increase in the magnitude of the difference between  $\bar{p}_2$  and  $\bar{p}_1$  in a situation where  $r_c > \bar{r}$  increases the magnitude of both the incentive (term 2) and disincentive (term 1) effects on nitrogen response.

Finally, consider the sensitivity of the results in Table 1 to the farmer's level of yield uncertainty. Table 3 contains details of results comparable to those in Table 1 but where the standard deviation of yield has been increased ( $\sigma_0 = 0.3$  instead of 0.2). Overall the results in this Table show that the pattern of nitrogen response for various critical levels of protein is sensitive to the farmer's level of yield uncertainty, with a change of direction of response occurring at the 20 critical level of protein between Tables 1 and 3 (ie row (3)). This sensitivity reflects the impact of increased yield uncertainty on the size of the disincentive effect identified in equation (9). In particular, for a critical protein level in excess of the initial expected level, an increase in yield uncertainty increases the likelihood of receiving a protein premium, thereby reducing the size of the disincentive effect. In this case optimal nitrogen is increased despite the associated decrease in expected profits.

## Conclusion

This paper has developed a model of the impact of introducing a protein payments system on a farmer's nitrogen application. It was shown that the farmer faces two conflicting effects from the introduction of the system: an incentive effect arising from the opportunity to increase the likelihood of receiving a protein premium by increasing nitrogen application; and a disincentive effect arising from the likelihood of receiving a protein discount in seasons of relatively low protein (section 1).

As a consequence of this ambiguous analytical response, the impact of introducing a protein payments system was investigated numerically in section 2. It was shown that if the critical

protein level is sufficiently in excess of the farmer's initial expected level then the optimal nitrogen application is reduced following the introduction of the system. In addition, it was shown that the potential for a negative nitrogen response is also dependent on the farmer's level of yield uncertainty. Because higher yield uncertainty increases the likelihood of a protein premium in a situation where critical protein level is in excess of the initial expected level, the critical protein level at which this disincentive effect dominates the nitrogen response decision is increased. Finally, it was shown that because the size of the protein payments affects the incentive and disincentive effects similarly, this size does not play a significant role in determining the direction of nitrogen response, only its magnitude.

Consequently, this paper has both clarified the role of the components of a protein payments system and highlighted yield uncertainty as a farm-specific factor in influencing a farmer's nitrogen response to the introduction of such a system. However, the model has been based on an unverified structure of the relationship between yield, protein, nitrogen and seasonal conditions which needs further empirical investigation.

**References**

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## FOOTNOTES

- <sup>1</sup> Note in addition to the quote in the Introduction: "If the nitrogen supply stays the same but the yield is increased - due to a high yielding variety or a good season - the percentage protein in the grain will fall" *Agribusiness Decision* June 1994 p.8.
- <sup>2</sup> The implications of an asymmetrical payments system for nitrogen response are clear and therefore are not considered in this paper. In addition, extending the system to a sliding scale of protein payments is considered to be an unwarranted complication. Moreover, subsequent numerical results suggesting the unimportance of the magnitude of payments in determining the direction of nitrogen response support this approach.
- <sup>3</sup> Note that  $\partial(\gamma N / r_c \bar{y}) / \partial N > 0$  as long as the elasticity of expected yield with respect to nitrogen is less than unity.
- <sup>4</sup> See Paris (1992) for details of empirical support for this functional form.
- <sup>5</sup> See Fraser (1988) for details.

**Table 1**

Effect of Introducing Protein Payments on Optimal  
Nitrogen Application: Base Case

	$\bar{r}$	$N^*$	$\bar{y}$	$E(\pi)$
(1) Initial Situation	10	3.178	5.750	99.110
$r_c$				
(2) 10	12.789	4.176	5.908	106.454
(3) 20	9.901	3.141	5.741	87.683

Table 2

Effect of Introducing Protein Payments on Optimal  
Nitrogen Application with Increased Size  
of Payments<sup>a</sup>

	$\bar{r}$	$N^*$	$\bar{y}$	$E(\pi)$
(1) Initial Situation	10	3.178	5.750	99.110
$r_c$				
(2) 10	14.330	4.709	5.946	113.608
(3) 20	9.743	3.083	5.725	76.284

Note a:  $\bar{p}_1 = 16$ ;  $\bar{p}_2 = 24$ . All other Base Case parameter values apply



Table 3

Effect of Introducing Protein Payments on Optimal  
Nitrogen Application with Increased  
Yield Uncertainty<sup>a</sup>

	$\bar{r}$	$N^*$	$\bar{y}$	$E(\pi)$
(1) Initial Situation	10	3.178	5.750	99.110
$r_c$				
(2) 10	13.017	4.256	5.915	103.416
(3) 20	10.827	3.480	5.815	87.874

Note a:  $\sigma_0 = 0.3$ . All other Base Case parameter values apply