CONSUMPTION EXTERNALITIES AND PREFERENCE COMPLEMENTARITIES

Rebecca Cleary
Associate
Analysis Group
Boston, MA
rcleary@analysisgroup.com

and

Andrea Carlson
Economist
Economic Research Service
U.S. Department of Agriculture
Washington, D.C.

Selected Paper prepared for presentation at the 2014 AAEA/EAAE/CAES Joint Symposium: Social Networks, Social Media and the Economics of Food, Montreal, Canada, May 29-30, 2014
CONSUMPTION EXTERNALITIES AND PREFERENCE COMPLEMENTARITIES

Rebecca Cleary∗ and Andrea Carlson†

Abstract

Social interactions can lead to a variety of phenomena including consumption externalities and complementarities. Consumption externalities arise when the choices of others have an effect on the total value of a household’s purchases; complementarities arise when the choices of others have an effect on the marginal value of a household’s purchases. This paper develops a conceptual model that separately identifies consumption externalities and complementarities and illustrates their significance in an application to a household’s choice of grocery store. We propose empirical measures of the value of consumption externalities and complementarities based on the benefit function and show how the two effects can be identified separately using household level data. Preliminary results show that purchases at traditional grocery stores have both positive externalities on other shoppers and complementarities with others’ purchases whereas purchases at superstores have negative externalities on other shoppers and show no complementarity with others’ purchases, even if those purchases are also made at a superstore.

1 Introduction

Consumers are social. As such, the choices of others can influence the benefits consumers get from their purchase decisions. Others’ purchases can cause externalities, influencing the benefits of consumers who are not involved in the purchase decision (Veblen, 2005). Others’ choices can also have complementarities, that is, others’ purchases can influence a consumer’s marginal willingness-to-pay for his own choices, meaning that the benefits received from a purchase decision are linked with other consumers’ actions (Milgrom and Roberts, 1995). Public policy has begun to focus on questions where social and psychological explanations of behavior seem essential in supplementing conventional economic explanations (Durlauf and Young, 2004). As health outcomes become progressively pertinent to public policy, separately identifying and quantifying consumer externalities and complementarities becomes increasingly relevant for welfare analysis in the food system.

For example, a consumer’s choice of grocery store is made in a social context. It may not only be a statement of income, as in conspicuous consumption (Veblen, 2005), but a statement on health, health-consciousness,
and even environmental awareness or social responsibility. For example, a purchase decision at a well-known organic grocery store versus a superstore seems to carry much information about the consumer beyond the purchase decision, and this information may influence others’ decisions.

Social psychologists have identified several reasons why the actions of others affect a person’s decisions (Asch, 1952) and knowing others’ choices can change the intrinsic value that one attaches to his own choice, as evidenced by the imaging of neural mechanisms (Escalas and Bettman, 2005). A consumer’s tendency to purchase a product can be influenced by the choices made by his associative reference group (Bearden and Rose, 1990; Childers and Rao, 1992), which can be defined by a variety of attributes including location/proximity (Glaeser and Scheinkman, 2000), imagined perceptions (Anderson, 1983), and incidental contact (Ferraro, Bettman, and Chartrand, 2009). It is also possible that information about the decisions of a reference group influences the consumer’s actual preferences about the product via the desire to avoid social sanctions, the need to comply with a perceived request, the drive to conform, appearance self-esteem, and self-identification or construction of self-concept (Escalas and Bettman, 2005; McFerran et al., 2010).

Our identification strategy does not require that we distinguish between these (or other) explanations. Instead, we develop a conceptual model that separately identifies and quantifies the values of externalities and complementarities from the consumer perspective and illustrate the model’s usefulness in an application to grocery store choice. The model is based on a standard welfare investigation of marginal willingness to pay. The main challenge to value consumption externalities and complementarities is that one needs to know more than just the value of a good to a consumer. Indeed, one needs to know how the marginal value of a good purchased by a consumer is influenced by other consumers’ purchase decisions. Obtaining this information requires a joint evaluation of willingness to pay across consumers and thus a welfare measure that is comparable across consumers.

A welfare measure with this property is Luenberger’s benefit function (Luenberger, 1995a). The benefit function provides a measure of willingness-to-pay for goods expressed in terms of a reference bundle, holding utility constant. As argued by Luenberger (1995a; 1996), the benefit function, since it is measured in terms of a reference bundle, is comparable across consumers thus allowing for the kind of inter-personal tradeoffs that measuring consumption externalities and complementarities necessitates. This provides the main motivation for using the benefit function in this analysis.

The presence of externalities and complementarities, under certain conditions, invalidate the necessary conditions for a representative consumer thus rendering traditional welfare analysis inappropriate (Luenberger, 1995a; Chambers, 2001). The ubiquitousness of these phenomena in the food system and the importance of welfare analysis in policy design warrant the development of an empirically-tractable model that can address the welfare implications of consumption externalities and complementarities in a theoretically-consistent fashion.

Therefore, we develop an analysis of the consumer value of consumption externalities and complementarities using Luenberger’s benefit function. We show that the benefit function provides a conceptual framework to develop a welfare measure of the value of consumption externalities and separately identify this from...
the value of consumption complementarities. The proposed framework involves the specification of and
estimation of a system of household-level inverse demands. The empirical tractability of the approach and
its usefulness are illustrated in an econometric application to grocery store choice.

Preliminary results suggest that, in the long run, purchases at traditional grocery stores exhibit positive
externalities and positive consumption complementarities. That is, others' purchases at traditional grocery
stores not only raise the level of utility of others consumers, they also increase other consumers' willingness-
to-pay for a good sold via a traditional grocery store. On the other hand, superstores exhibit negative
externalities, but others' purchases at superstores do not influence consumers' willingness-to-pay for a good
sold at one.

Here we have introduced the rationale to investigate social interactions in the context of consumption deci-
sions, specifically the consumer's choice of grocery store. The next section will develop a theoretical model
to separately identify externalities from complementarities after providing a brief introduction. Section three
will describe the empirical implementation of the model; section four will describe the data and how, using
well-understood estimation procedures, we can obtain a quantitative valuation of externalities and comple-
mentarities in the consumption decision. The last two sections will present the results and conclusions.

2 A Model of Consumption Externalities and Complementarities

This section develops a theoretical model to separately identify and quantify externalities from complementar-
ities in the consumption decision. Quantifying consumption externalities and complementarities necessitates
modeling trade-offs across consumers' bundles of goods. The utility function is not appropriate for this task
because utility units are not comparable across consumers. Therefore, the model is based on the benefit
function of Luenberger (1995a) and we provide a brief introduction to it here. For a primer on the benefit
function, please see Luenberger (1995b).2

2.1 A brief introduction to the benefit function

The benefit function is a willingness-to-pay measure that takes as arguments a level of utility, \( u \in U \), and a
bundle (i.e., vector) of choices, \( x \in X \), which is measured against some reference bundle, \( g \in X \), for utility
comparisons. It measures how many units of the reference bundle a consumer is willing to give up to move
from an indifference curve to a particular bundle of goods. For example, if the reference bundle is a dollar,
then the benefit function is a measure of the dollar amount a consumer is willing-to-pay to move from his
current utility to a particular bundle of goods.

Denote by \( I = \{1, ..., i, ..., m\} \) the index set of \( m \) households. Consider household \( i \in I \) consuming a vector
of choices \( x_i \in \mathbb{R}_+^n \), where \( x_{it} \) denotes an element of \( x_i \). Let \( g \in X = \mathbb{R}_+^{mn} \) with \( g \neq 0 \). Luenberger (1995a)
defines the benefit function with reference \( g \) for \( x_i \in X \) and \( u_i \in U \) as

2For a more in depth understanding of the benefit function when consumption externalities are present, which provides the
foundation of the measures presented here, please see Luenberger (1995a); Luenberger (1992) discusses the duality between the
benefit and expenditure functions that allow us to recover the unobservable benefits from the observable demands.
Intuitively, benefits measure the trade-off associated with the bundle of goods \( x_i \) and \( g \) to maintain a given utility for consumer \( i \). Imagine some bundle \( x_i \) that lies on an indifference curve above \( u_i \). Benefits, \( b_i(x_i, u_i) \), are the units of \( g \) an individual would be willing to give up to move from \( u_i \) to \( x_i \). To find \( b_i(x_i, u_i) \), subtract \( g \), in increments of \( \beta_i \), from \( x_i \) until \( \beta_i \) is so big that subtracting any more \( \beta g \) from \( x \) will result in a lower utility level than \( u_i \). It is the reference bundle, \( g \), that gives some cardinal meaning to this benefits interpretation of preferences and allows for interpersonal comparisons. For these reasons, the choice of reference bundle has important consequences. In the special case of a reference bundle with only private benefits, we can then aggregate across consumers by simply summing the \( \beta_i \)'s for consumers 1 through \( m \).

The expenditure function is a natural dual to the benefit function (Luenberger, 1992).

### 2.2 Value of Consumption Externalities

When the social nature of human behavior is taken into account, benefits for household \( i \) may not solely depend on \( i \)'s choices. As shown in 1, others’ purchases can impact a consumers’ benefits. For some reference bundle and utility level, we compactly rewrite 1 as

\[
b_i(x_i, x_{-i}, u_i) = \begin{cases} 
  \max\{\beta_i : u_i(x_1, x_2, \ldots, x_i - \beta_i g, \ldots, x_m) \geq u_i, x - \beta_i g \in X\} \\
  -\infty \text{ otherwise.}
\end{cases}
\]

(1)

where \(-i\) denotes all households other than \( i \) belonging to \( I \). There are a variety of measurable effects one household can have on another. One effect is directly influencing the benefits of a consumer not involved in the purchase decision thereby exerting externalities. Under the hypothesis of no externalities, benefits of household \( i \) will depend only on \( i \)'s choices so that 2 is equivalent to \( b_i(x_i, 0, u_i) \). When 2 is not equivalent to \( b_i(x_i, 0, u_i) \), then \( x_{-i} \) has an influence on \( b_i \). Therefore, the value of the externality of the purchases of \( x_{-i} \) on household \( i \) is given by

\[
V_i = b_i(x_i, x_{-i}, u_i) - b_i(x_i, 0, u_i).
\]

(3)

When \( V_i > 0 \), then household \( i \)'s benefits are greater when \(-i\) has positive consumption than when \(-i\) has no consumption and there are positive externalities of \(-i\) on \( i \). When \( V_i < 0 \), then household \( i \)'s benefits are less when \(-i\) has positive consumption than when \(-i\) has no consumption and there are negative externalities of \(-i\) on \( i \). On the other hand, when \( V_i = 0 \), then household \( i \) experiences the same benefits regardless of \(-i\)'s consumption and there are no externalities.  

---

3Please see Luenberger (1995a) for a complete explanation of this special case.

4Equation 3 is analogous to the incremental value described in Baggio and Chavas (2009).
This measure can be decomposed further to the specific product(s) of the particular household(s) that cause externalities. Let $x$ be the consumption of all $I$ households but $i$ and $j$ and let $x_{j/t}$ be the vector of all consumption choices of $j$ but $t$. Given $x_i$, the value of the externality of household $j$’s consumption of $t$ on household $i$ is given by

$$V_{i,t} = b_i(x_i, x_{j/t}, x, u_i) - b_i(x_i, 0, x_{j/t}, x, u_i),$$

which are the external benefits to $i$ from $j$’s consumption of $t$ at consumption level $x_i$.

When $1$ is continuous and differentiable, then $4$ can be written as

$$\int_0^{x_{j,t}} \frac{\partial b_i(x_i, a, x_{j/t}, x, u_i)}{\partial x_{j,t}} da, j \in I, j \neq i.$$

This is household $i$’s willingness-to-pay for $j$’s consumption of $t$. That is, if benefits of household $i$ increase because of $j$’s consumption, then $j$’s purchase of $t$ has benefits beyond those privately enjoyed by $j$; $j$’s purchase made $i$ better off, even though $i$ was not involved in $j$’s purchase decisions. Put another way, $i$ would be willing to pay up to $\frac{\partial b_i(x_i, x_{j,t}, x, u_i)}{\partial x_{j,t}}$ of $g$ to subsidize $j$’s consumption of $t$. On the other hand, if $\frac{\partial b_i(x_i, x_{j,t}, x, u_i)}{\partial x_{j,t}} = 0$, then there is no amount that $i$ is willing to pay for $j$ to consume $t$ and so there is no externality. Externalities have consequences for welfare analysis. We give these in the following result. This is summarized in the following result.

**Proposition 1.** Let $x_i, x_{j,t} \geq 0$. Assume the benefit function for household $i$ is continuous and twice differentiable in $x_{j,t}$. Household $i$’s valuation of the externality exerted by household $j$ via consumption of $t$, $V_{i,t}$, satisfies (a) $V_{i,t} = 0$ if $\frac{\partial b_i(x_i, x_{j,t}, x, u_i)}{\partial x_{j,t}} = 0$; (b) $V_{i,t} < 0$ if $\frac{\partial b_i(x_i, x_{j,t}, x, u_i)}{\partial x_{j,t}} < 0$; (c) $V_{i,t} > 0$ if $\frac{\partial b_i(x_i, x_{j,t}, x, u_i)}{\partial x_{j,t}} > 0$.

Proposition 1 shows how the sign of the value of externality is determined. From (a), a sufficient condition for no externality, $V_{i,t} = 0$, is that the total benefit of household $i$ is independent of $x_{j,t}$. From (b), a sufficient condition for negative externality, $V_{i,t} < 0$, is that the total benefit of household $i$ decreases as $x_{j,t}$ increases. This means that household $i$ would like to tax $j$’s consumption of $t$. From (c), a sufficient condition for positive externality, $V_{i,t} > 0$, is that the total benefit of household $i$ increases as $x_{j,t}$ increases. This means that household $i$ is willing to subsidize $j$’s consumption of $t$.

### 2.3 Value of Consumption Complementarities

Others’ purchases may not only influence a household’s willingness-to-pay for others’ consumption, they can also influence a household’s willingness-to-pay for its own consumption. As shown previously, when $V_{i,t} \neq 0$, household $j$ exerts some externality on household $i$. This externality may influence $i$’s willingness-to-pay for
its own consumption of, say, \( r \), which may or may not be the same good as \( t \). We can measure the joint benefit to \( i \) of its consumption of \( r \) and \( j \)'s consumption of \( t \) as

\[
V_{i,rt} = b_i(x_{ir}, x_{ir}, x_{jt}, x_j, x, u_i) - b_i(0, x_i, 0, x_j, x, u_i). \tag{5}
\]

Similarly, we can measure the benefit to \( i \) of its own consumption of \( r \) as

\[
V_{i,r} = b_i(x_{ir}, x_{ir}, x_{jt}, x_j, x, u_i) - b_i(0, x_i, x_j, x_j, x, u_i), \tag{6}
\]

which are the private benefits to \( i \) from consuming \( r \) at some positive level of \( x_{jt} \). When \( V_{i,rt} = V_{i,r} \) there is no externality from \( j \)'s consumption of \( t \). However, when \( V_{i,rt} \neq V_{i,r} \) then it becomes interesting to determine whether the external benefit to \( i \) from \( j \)'s consumption of \( t \) enhances or detracts from \( i \)'s private benefit from \( r \). Whereas 4 measures the externality, the complementarity is included in both 4 and 6. One goal of this paper is to separately identify this complementarity effect from the effect of the externality on \( i \)'s benefits. Equation 4 measures the externality of \( j \) on \( i \) at some positive consumption level of \( i \). Under complementarity, we expect \( x_{jt} \) to enhance enjoyment of \( x_{ir} \), so that 4 and 6 reflect some of these augmented benefits. In contrast, 5 measures the benefits of some positive level of both \( x_{ir} \) and \( x_{jt} \) compared to no consumption of either. Therefore, complementarity is associated with the sum of the external and private benefits being higher than the joint value.\(^5\)

\[
W_{i,rt} = V_{i,r} + V_{i,t} - V_{i,rt}. \tag{7}
\]

Substituting 6, 4, and 5 for \( V_{i,r}, V_{i,t}, \) and \( V_{i,rt} \), respectively yields

\[
W_{i,rt} = b_i(x_{ir}, x_{ir}, x_{jt}, x_j, x, u_i) - b_i(0, x_i, x_{jt}, x_j, x, u_i)
+ b_i(x_i, x_{jt}, x_j, x, u_i) - b_i(x_i, 0, x_j, x, u_i)
- (b_i(x_{ir}, x_{ir}, x_{jt}, x_j, x, u_i) - b_i(0, x_i, 0, x_j, x, u_i)), \tag{8}
\]

which is equivalently written

\[
W_{i,rt} = b_i(x_{ir}, x_{ir}, x_{jt}, x_j, x, u_i) - b_i(0, x_i, x_{jt}, x_j, x, u_i)
- b_i(x_i, 0, x_j, x, u_i) + b_i(0, x_i, 0, x_j, x, u_i). \tag{9}
\]

When \( 1 \) is continuous and twice differentiable in \( x_{ir} \) and \( x_{jt} \), this is the same as

\[
W_{i,rt} = \int_0^{x_{jt}} \int_0^{x_{ir}} \left[ \frac{\partial^2 b_i(a, x_{ir}, b, x_{jt}, x, u_i)}{\partial a \partial b} \right] \, da \, db. \tag{10}
\]

\(^5\)This is the exact finding of Baggio and Chavas (2009) for complementarity between products consumed by the same household.
This is household \( i \)'s willingness-to-pay for \( r \) changes with \( j \)'s consumption of \( t \). That is, if benefits of household \( i \) via consumption of \( r \) increase because of \( j \)'s consumption of \( t \), then \( j \)'s purchase changed \( i \)'s valuation of \( r \). Put another way, \( i \) would be willing to pay up to \( \frac{\partial b_i(x_{i,r})}{\partial x_{i,r}} \) of \( g \) more for \( r \) given that \( j \) consumes \( t \). On the other hand, if \( \frac{\partial b_i(x_{i,r})}{\partial x_{i,r}} = 0 \), then there is no additional amount that \( i \) is willing to pay for \( r \) if \( j \) to consume \( t \) and so there are no complementarities. Complementarities have consequences for welfare analysis. This is summarized in the following result.

**Proposition 2.** Let \( x_{i,r}, x_{j,t} \geq 0 \). Assume the benefit function for household \( i \) is continuous and twice differentiable in \( x_{j,t} \) and \( x_{i,r} \). Household \( i \)'s valuation of the complementary of its consumption of \( r \) and \( j \)'s consumption of \( t \), \( W_{i,r,t} \), satisfies (a) \( W_{i,r,t} = 0 \) if \( \frac{\partial^2 b_i(x_{i,r}, x_{j,t}, u_i)}{\partial x_{i,r} \partial x_{j,t}} = 0 \); (b) \( W_{i,r,t} < 0 \) if \( \frac{\partial^2 b_i(x_{i,r}, x_{j,t}, u_i)}{\partial x_{i,r} \partial x_{j,t}} < 0 \); (c) \( W_{i,r,t} > 0 \) if \( \frac{\partial^2 b_i(x_{i,r}, x_{j,t}, u_i)}{\partial x_{i,r} \partial x_{j,t}} > 0 \).

Proposition 2 shows how the sign of the value of complementarity \( W_{i,r,t} \) is determined. This is how household \( i \)'s willingness-to-pay for \( r \) changes with \( j \)'s consumption of \( t \). From (a), a sufficient condition for no complementarity is that the marginal benefit of \( x_{i,r} \) is independent of \( x_{j,t} \). From (b), a sufficient condition for \( W_{i,r,t} < 0 \) is that the marginal benefit of \( x_{i,r} \) decreases with \( x_{j,t} \). In this case, the benefit \( i \) gets from \( r \) is smaller when \( j \) increases consumption of \( t \). This means that \( x_{i,r} \) and \( x_{j,t} \) behave antagonistically across households; that is, household \( i \) would prefer either to consume \( r \) or for \( j \) to consume \( t \). From (c), a sufficient condition for \( W_{i,r,t} > 0 \) is that the marginal benefit of \( x_{i,r} \) increases with \( x_{j,t} \). Then, the benefit \( i \) gets from \( r \) is larger when \( j \) increases consumption of \( t \). This means that \( x_{i,r} \) and \( x_{j,t} \) behave synergistically across households; that is, household \( i \) would prefer to consume \( r \) when \( j \) consumes \( t \).

### 3 Empirical Specification

McLaren and Wong (2009) describe several empirical specifications of the private benefit function. For the analysis in this work, we use a specification of the total social benefit function that allows for flexible quantity and utility effects, analogous to what McLaren and Wong (2009) call the B&C specification which had its debut in Baggio and Chavas (2009) which is given by

\[
b(x, U) = \alpha(x) - [U \beta(x)]/[1 - U \gamma(x)]
\]  

(11)

where \( \beta(x) > 0 \), and \( [1 - U \gamma(x)] > 0 \). From Baggio and Chavas (2009), in neoclassical consumer theory, the benefit function is nonincreasing in \( U \), and non-decreasing and concave in \( x \), which imposes corresponding constraints on \( \alpha(x) \), \( \beta(x) \), and \( \gamma(x) \). They note that these constraints imply that \( \alpha(x) - [U \beta(x)]/[1 - U \gamma(x)] \) is nonincreasing in \( U \), non-decreasing in \( x \), and that \( \frac{\partial^2 \alpha(x) - [U \beta(x)]/[1 - U \gamma(x)]}{\partial x^2} \) is a negative semi-definite matrix. When \( \alpha(x) \) is a quadratic function of \( x \) then equation 11 provides a flexible representation of quantity effects. Moreover, when \( \gamma(x) \neq 0 \) equation 11 allows the model to capture flexible utility effects. Therefore, using the B&C specification provides a flexible specification of consumer preferences via benefits. Taking the derivative of equation 11 with respect to \( x \) yields the following Luenberger price equation

\[
p^L(x, U) = \left( \frac{\partial \alpha}{\partial x} \right) - \frac{\left( \frac{\partial \beta}{\partial x} \right) U}{(1 - U \gamma(x))} - \frac{\left( \frac{\partial \gamma}{\partial x} \right) \beta(x) U^2}{(1 - U \gamma(x))^2}.
\]  

(12)
From Luenberger (1995), when \( u(x + \alpha g) \) is strictly increasing in \( \alpha \) and \( x > 0 \), the benefit function evaluated at 0 is an implicit representation of the utility function \( u(x) \). Therefore, from Baggio and Chavas (2009), it follows that, using equation 11, \( \alpha(x)/\beta(x) = U/[1 - U\gamma(x)] \) when \( U = u(x) \). Substitution this term in 12 and applying the duality discussed previously, yields the inverse Marshallian demand

\[
p^*(x) = \frac{\partial\alpha}{\partial x} - \left( \frac{\partial\beta}{\partial x} \right) \left( \frac{\alpha(x)}{\beta(x)} \right) - \left( \frac{\partial\alpha(x)^2}{\partial x} \right) \frac{1}{\beta(x)}
\]

(13)

where \( p^*(x) \) is the price vector normalized on the reference bundle. The Baggio and Chavas (2009) analogy suggests the following for \( \alpha(x) \), \( \beta(x) \), and \( \gamma(x) \):

\[
\alpha(x) = \sum_{i=1}^{m} \sum_{r=1}^{n} \alpha_{ir}x_{ir} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{n} \sum_{t=1}^{n} \frac{1}{2} \alpha_{i,rt}x_{ir}x_{jt}
\]

(14)

where \( \alpha_{i,rt} = \alpha_{j,rt} \) as symmetry restrictions for all \( i \neq j, t \neq r \).

\[
\beta(x) = \exp \left( \sum_{i=1}^{m} \sum_{r=1}^{n} \beta_{ir}x_{ir} \right)
\]

(15)

and

\[
\gamma(x) = \sum_{i=1}^{m} \sum_{r=1}^{n} \gamma_{ir}x_{ir}.
\]

(16)

This specification is flexible. From Baggio and Chavas (2009), equation 13 identifies three components in the price equation: the first term (equation 14) is a flexible quadratic form, the second term (equation 15) means that \( \ln(\beta(x)) \) is linear in \( x \), and the third term (equation 16) has a linear specification for \( \gamma(x) \), allowing for additional flexibility by capturing quadratic utility effects. The property \( \left( \frac{\partial\beta}{\partial x} \right) = 1 \) discussed previously implies \( \left( \frac{\partial\beta}{\partial x} \right) = 1, \left( \frac{\partial\gamma}{\partial x} \right) = 0, \) and \( \left( \frac{\partial\gamma}{\partial x} \right) = 0 \) holding for all \( x \), which generates the following restrictions:

\[
\sum_{i=1}^{m} \sum_{r=1}^{n} \alpha_{ir}g_{ir} = 1
\]

(17)

\[
\sum_{i=1}^{m} \sum_{r=1}^{n} \alpha_{i,rt}g_{ir} = 1
\]

(18)

\[
\sum_{i=1}^{m} \sum_{r=1}^{n} \beta_{ir}g_{ir} = 0
\]

(19)

and
\[
\sum_{i=1}^{m} \sum_{r=1}^{n} \gamma_{ir} g_{ir} = 1. \tag{20}
\]

Therefore, the household \(i\)'s inverse Marshallian demand for \(r\) is

\[
p_{ir}^*(x) = \alpha_{ir} + \sum_{i=1}^{m} \sum_{r=1}^{n} \alpha_{i,r,t} x_{jt} - \beta_{ir} \alpha(x) - \frac{\gamma_{ir} \alpha(x)^2}{\beta(x)}. \tag{21}
\]

Reduced-form dynamics can be added to equation 21 in the form of quantity or price lags. This is discussed more in the next section.

4 Data and Estimation

The Nielsen Homescan data is a large and nationally representative panel of U.S. households measuring retail purchases. Panelists are asked to scan the bar code (UPC) on every item purchased and to identify the specific retail establishment. Nielsen also reports price and quantity purchased, deal presence and includes item characteristics such as package size, in their reported data. The data set includes household demographic data as well as a projection factor which is used to weight the data. A static sample is used which includes only households who provide minimum spending requirements for at least 10 months of a reported 12 month period. We include all households in the static sample for the years 2004-2008 in a particular Midwestern city. We consider only purchases from the refrigerated milk category.

Consumption of milk is measured in fluid ounces purchased by household-type, by week, for each store choice. There are 261 weeks and 170,239 households in the sample. We define large households as those having at least three members and small households as having at most two members; in the sample there are 88,388 large households and 81,851 small households. Given the large number of households, to make the analysis manageable, households are aggregated by week and according to household size. Households are defined by week, however the Nielsen data only reports changes in household sizes at the beginning of every calendar year. We consider four store types: 1) superstores, which includes all big-box stores except mass merchandisers and club stores; 2) health stores, convenient stores, and other small store types; 3) grocery-store chains; and 4) mass-merchandisers and club stores. Using the specifications described in the previous section, this leads to a system of eight inverse demand equations. Milk prices for each household type at each grocery-shopping outlet are calculated by dividing total dollars spent by total ounces purchased. Summary statistics are presented in table 1.

Another consideration is the choice of the reference bundle, \(g\). Here, the reference bundle is chosen as \(g = (0, 0, 0, 0, 0, 0, 0, 1)\), where the last household-store choice, that of large family milk purchases at mass merchandisers or club stores becomes the bundle referenced, or normalized on. How the normalization is achieved is thoroughly explained in Baggio and Chavas (2009). All valuations will therefore be made relative to the value of the last household-outlet choice, implying that there are no parameters to estimate for the
inverse demand of the last household-outlet choice and the last equation is dropped from the empirical analysis presented below and the system is estimated for the remaining \((mn - 1)\) equations.

In order to estimate the model, add an error term to each equation in 21. The inverse Marshallian demand for the \(i^{th}\) household-store choice is given by:

\[
p^{\star\star}_{ir}(x) = \alpha_{ir} + \sum_{i=1}^{m} \sum_{r=1}^{n} \alpha_{i,rt} x_{jt} - \beta_{ir} \alpha(x) - \frac{\gamma_{ir} \alpha(x)^2}{\beta(x)} + e_{ir}, \quad ir = 1, \ldots, (mn - 1)
\]  

(22)

where \(p^{\star\star}_{ir}\) indicates the normalized price. Capturing reduced-form price-dynamics leads to the following equation in which \(s\) indexes time.

\[
p^{\star\star}_{ir,s}(x) = \alpha_{ir,s} + \alpha_{irL1} p_{ir,s-1} + \alpha_{irL2} p_{ir,s-5} + \sum_{i=1}^{m} \sum_{r=1}^{n} \alpha_{i,rt,s} x_{jt,s} - \beta_{ir,s} \alpha(x) - \frac{\gamma_{ir,s} \alpha(x)^2}{\beta(x)} + e_{ir,s}, \quad ir = 1, \ldots, (mn - 1).
\]  

(23)

It is important to consider the stochastic properties of this demand system prior to estimation. Given that \(g = (0, 0, 0, 0, 0, 0, 0, 1)\), We estimate equation 23 for \(ir = 1, \ldots, (mn - 1)\) (after dropping the last equation, as previously discussed). The estimation is not invariant to the choice of \(g\). The empirical analysis was also conducted under alternative choices for \(g\), which affects the quantitative estimates, but not the qualitative findings discussed below. The associated error terms \(e_{ir,s}\) are assumed to be serially uncorrelated, since dynamics in the system are captured through price lags of one week and five weeks. However, we do allow for correlation across households and grocery-shopping outlet choice, with \(e_s = (e_1, \ldots, e_{mn-1}, t)\) satisfying \(E[e_s] = 0\) and \(E[e_s e_s'] = \Sigma\) for all \(s\), where \(\Sigma\) denotes the contemporaneous covariance matrix. Since equation 23 is nonlinear in parameters, estimation of it requires using nonlinear methods. The system can be estimated by a nonlinear seemingly unrelated regression (NLSUR) procedure, allowing for correlation across equations. This was done using R, a freely available statistical software, using a weighted nonlinear least squares technique. This yields consistent and asymptotically efficient parameter estimates.

We conducted a series of tests on the model specification. This was done using Wald tests and the results are reported in table 2.

First, we estimate the full model presented in equation 23, which is nonlinear in parameters and allows for the capturing of dynamics through price lags. This full model also includes the parameters \(\beta_{ir}\)'s capturing the linear effect of \(U/[1 - U\gamma(x)]\), as well as the parameters \(\gamma_{ir}\)'s capturing the quadratic effects of \(U/[1 - U\gamma(x)]\).

In order to test if the flexible form of the utility is necessary, that is, if the utility effects are indeed nonlinear, we specified a null hypothesis of \(\gamma_{ir} = 0\) jointly for all equations in the system. This null hypothesis of linearity in utility is rejected with a Wald test-statistic value of 27.98 that corresponds to a \(p\)-value of 0.0002, indicating that the non-linearity of utility is strongly supported by statistical evidence and the remaining discussion will take place under the full model.

We then investigate dynamics in the context of the non-linear model, that is where \(\gamma_{ir} \neq 0, \quad ir = 1, \ldots, (mn - 1)\).

The first hypothesis considers that dynamics may exist in the intermediate and long-run and is represented by the null hypothesis that \(\alpha_{irL1,s} = 0\) and \(\alpha_{irL2,s} = 0\) jointly for all equations in the system. This null
Hypothesis is rejected with a Wald test-statistic value of 11.69 corresponding to a p-value of 0.11. The intermediate-run and long-run dynamics were then examined separately. The hypothesis that the long-run dynamics were not significant alone, represented by the null hypothesis $\alpha_{irL2,s} = 0$ jointly for all equations in the system marginally fails to be rejected with a Wald test-statistic value of 9.69 corresponding to a p-value of 0.206. Thus, there is not strong statistical evidence that the long-run dynamics are nonzero. However, the hypothesis that the intermediate-run dynamics were not significant alone, represented by the null hypothesis $\alpha_{irL1,s} = 0$, fails to be rejected with a Wald test-statistic value of 96.8 corresponding to a p-value of approximately zero. Since both intermediate and long-run dynamics are jointly somewhat supported by statistical evidence, and the intermediate-run dynamics are strongly supported by statistical evidence, results for the short-run, intermediate-run, and long-run will be examined for economic significance.

Further, we sought to determine if the model captured evidence of social interactions. Social interactions is defined by the non-zero presence of another’s consumption bundle in a household’s utility. The hypothesis of no social interactions is represented by the null hypothesis of $\alpha_{i,r,t} = 0$ for $r \neq t$, $t \in x_j$, $j \in I_i$. This null hypothesis is rejected with a Wald test-statistic value of 39.76, corresponding to a p-value of 0.000001. Thus, there is strong statistical evidence of social interactions.

On the basis of these tests, the full model is statistically supported and the remaining discussion will take place in the context of the full model represented by equation 23.

5 Results and Discussion

The estimated specification (equation 23) allows for the investigation of social interactions and dynamics. The analysis presented here focuses on three different time horizons, based on their statistical validity previously discussed: short-run effects, corresponding to the current period effects of prices, $p_{ir,s}$; intermediate-run effects, that consider both current week prices, $p_{ir,s}^*$, and 1 week lagged prices $p_{ir,s}^{**}-1$; and long-run effects, that consider price effects after many periods. In the long-run, marginal benefits are allowed to adjust to their long-run equilibrium $p_{ir}^*$, so that $p_{ir}^* = p_{ir,s}^* = p_{ir,s}^{**}-1 = p_{ir,s}^{**}-5$.

For all time horizons, in order to conduct welfare analysis using 1, the benefit function must be concave. In each of the three time horizons examined here, the concavity of the benefit function is evaluated in the context of the full model specified in equation 23. Following Baggio and Chavas (2009), the matrix of second-order partial derivatives of the benefit function is called the Luenberger matrix. Neoclassical theory implies that when the preference set is convex, the benefit function is concave. Therefore, the eigenvalues of the Luenberger matrix must all be non-positive, with at least one eigenvalue equal to zero. To investigate the statistical properties of the Luenberger matrix and other results, we conducted simulations based on the parameter estimates. We evaluated standard errors of the simulations by bootstrapping using 50 draws from the empirical distribution of the data (Davison and Hinkley, 1997).

The long-run Luenberger matrix and its eigenvalues are presented in table 3. Again, all the nonzero eigenvalues are negative and statistically different from zero, providing statistical evidence that the long-run Luenberger matrix is negative semi-definite, meaning that, likewise, the long-run benefit function is concave. As discussed previously, the concavity of the benefit function is implied by the neoclassical assumption of
a quasi-concave utility function. The tests for dynamics previously discussed provides statistical evidence of dynamics, however, the difference in parameter values is not economically meaningful.\textsuperscript{6} Therefore, the remainder of the discussion will take place in the long-run context.

5.1 Household value of externalities in store choice

Externalities are given by the first order derivative of the benefit function. Here we find that large and small households provide positive external benefits in almost all cases. However, small households exert a social cost of $0.39/gallon when they buy their milk at superstores. There are many possible explanations for this finding. Large households benefit more from superstores than small households and thus small households congest superstores without deriving benefits to offset this cost. It may be that small households are small basket shoppers and the most benefit from superstores is to large basket consumers. On the other hand, there are social benefits when small or large households shop at grocery stores.

5.2 Household value of complementarities of store choices

As discussed previously, Table 3 gives the signs of complementarity, revealing if different households’ purchases act in an antagonistic or complementary fashion. The signs of complementarity for the present application. Purchases from superstores by both household types act antagonistically and lead to a decrease of social benefits, whereas purchases from grocery stores by both household types act complementary and lead to an increase of social benefits. The consumer value of complementarity is given in Table 5.

5.3 Empirical evidence of potential explanations of social interactions

As discussed previously, this paper does not attempt to model the source of social interactions. Indeed, in the current framework, this would require a theory of preference formation (Hausman, 2012). However, the model presented here does lend itself to provide evidence for several testable hypotheses.

In order to determine if superstore avoidance leads to consumption externalities, it is important to look across groups that are not experiencing reference groups or dissociative groups effects. Reference groups or aspiration groups are groups that enjoy positive consumption externalities when all members act similarly. Therefore, if groups are reference groups or aspiration groups the marginal benefit across groups will be positive when group members act similarly. A test for reference groups, therefore, would be if the Luenberger matrix elements representing group members acting similarly are all positive and the Luenberger matrix elements representing group members acting differently are all negative. This is not supported by statistical evidence in the short-run, intermediate-run or long-run. Therefore, statistical evidence supports the claim that small household and large households are neither reference groups or aspiration groups. However, another type of inter-household effect that is possible is dissociative groups. Dissociative groups occur when agents suffer negative externalities when members of different groups act similarly and positive or no

\textsuperscript{6} Parameter results for all time horizons are available upon request.
externalities when members of different groups act differently. A test for dissociative groups, therefore, would be if the Luenberger matrix elements representing agents acting differently are all positive and agents acting similarly are all negative. This is not supported by statistical evidence in the short-run, intermediate-run, or long-run. Therefore, statistical evidence supports the claim that small households and large households are not dissociative groups.

6 Concluding Remarks

This analysis has addressed the question of how households value others’ purchase decisions and how their willingness-to-pay for their own choices varies with others’ purchases. The main challenge to value consumption externalities and complementarities is that one needs to know more than just the value of a good to a consumer. Indeed, to evaluate consumption externalities and complementarities, one needs to know how the marginal value of a good purchased by a consumer is influenced by other consumers’ purchase decisions. Obtaining this information requires a joint evaluation of willingness to pay across consumers and thus a welfare measure that is comparable across consumers. This was done by specifying and estimating household benefits. The analysis is illustrated by an application to grocery-outlet shopping choice using panel data for two household types by week for five years.

The proposed approach is based on the benefit function developed by Luenberger (1995). The benefit function provides a measure of willingness to pay for goods expressed in number of units of a reference bundle, holding utility constant. As argued by Luenberger (1995; 1996), the benefit function, since it is measured in terms of a reference bundle, is comparable across consumers thus allowing for the kind of inter-personal tradeoffs that measuring consumption externalities necessitates. The benefit function provides a conceptual framework to develop a welfare measure of the value of preference externality. The framework proposed herein involves the specification of and estimation of a system of household-level inverse demands. The empirical tractability of the approach and its usefulness are illustrated in an econometric application to the store choice.
References


Asch, S.E. 1952. Group forces in the modification and distortion of judgments.. Prentice-Hall, Inc.


<table>
<thead>
<tr>
<th>Household Choice</th>
<th>Consumption (in 1,000 ounces)</th>
<th>Prices (dollars/ounce)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>Small Superstores</td>
<td>0.5860</td>
<td>0.4581</td>
</tr>
<tr>
<td>Small Convenience</td>
<td>2.0131</td>
<td>0.5494</td>
</tr>
<tr>
<td>Small Grocery</td>
<td>26.3253</td>
<td>6.7616</td>
</tr>
<tr>
<td>Small Club</td>
<td>2.8650</td>
<td>0.6653</td>
</tr>
<tr>
<td>Large Superstores</td>
<td>1.3174</td>
<td>1.1089</td>
</tr>
<tr>
<td>Large Convenience</td>
<td>1.6759</td>
<td>0.5968</td>
</tr>
<tr>
<td>Large Grocery</td>
<td>27.5346</td>
<td>9.4725</td>
</tr>
<tr>
<td>Large Club</td>
<td>4.8453</td>
<td>1.0308</td>
</tr>
</tbody>
</table>
Table 2: Model Diagnostics

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Null</th>
<th>Restrictions</th>
<th>Wald</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No non-linearities in utility</td>
<td>( \gamma = 0 )</td>
<td>7</td>
<td>27.98</td>
<td>0.0002</td>
</tr>
<tr>
<td>No dynamics</td>
<td>( \alpha_{L1} = \alpha_{L2} = 0 )</td>
<td>14</td>
<td>11.69</td>
<td>0.11</td>
</tr>
<tr>
<td>No intermediate-run dynamics</td>
<td>( \alpha_{L1} = 0 )</td>
<td>7</td>
<td>96.8</td>
<td>0</td>
</tr>
<tr>
<td>No long-run dynamics</td>
<td>( \alpha_{L2} = 0 )</td>
<td>7</td>
<td>9.69</td>
<td>0.206</td>
</tr>
<tr>
<td>No social externalities</td>
<td>( \alpha_{ij,ik} = 0 ) for all ( i \neq l )</td>
<td>11</td>
<td>39.76</td>
<td>0.000001</td>
</tr>
</tbody>
</table>
### Table 3: Long-run Luenberger Matrix and Corresponding Eigenvalues

<table>
<thead>
<tr>
<th>Household i</th>
<th>Outlet j</th>
<th>Small</th>
<th>Small</th>
<th>Small</th>
<th>Small</th>
<th>Large</th>
<th>Large</th>
<th>Large</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>Superstores</td>
<td>-0.20018</td>
<td>0.00213</td>
<td>-0.00394</td>
<td>-0.00036</td>
<td>-0.00598</td>
<td>0.05997</td>
<td>-0.00685</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Convenience</td>
<td>(0.00972)</td>
<td>(0.00192)</td>
<td>(0.00114)</td>
<td>(0.00097)</td>
<td>(0.00030)</td>
<td>(0.01094)</td>
<td>(0.00112)</td>
<td>(0)</td>
</tr>
<tr>
<td>Small</td>
<td>Grocery</td>
<td>0.00213</td>
<td>-0.03974</td>
<td>0.00005</td>
<td>-0.00021</td>
<td>0.00224</td>
<td>-0.00632</td>
<td>0.00098</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00192)</td>
<td>(0.00038)</td>
<td>(0.00023)</td>
<td>(0.00019)</td>
<td>(0.00006)</td>
<td>(0.00216)</td>
<td>(0.00022)</td>
<td>(0)</td>
</tr>
<tr>
<td>Small</td>
<td>Club</td>
<td>-0.00394</td>
<td>0.00005</td>
<td>-0.00045</td>
<td>-0.00039</td>
<td>0.00032</td>
<td>0.00138</td>
<td>0.00030</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00114)</td>
<td>(0.00023)</td>
<td>(0.00013)</td>
<td>(0.00011)</td>
<td>(0.00004)</td>
<td>(0.00128)</td>
<td>(0.00013)</td>
<td>(0)</td>
</tr>
<tr>
<td>Small</td>
<td>Club</td>
<td>-0.00036</td>
<td>-0.00021</td>
<td>-0.00039</td>
<td>-0.01292</td>
<td>-0.00015</td>
<td>-0.00148</td>
<td>0.00023</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00097)</td>
<td>(0.00019)</td>
<td>(0.00011)</td>
<td>(0.00010)</td>
<td>(0.00003)</td>
<td>(0.00108)</td>
<td>(0.00011)</td>
<td>(0)</td>
</tr>
<tr>
<td>Large</td>
<td>Superstores</td>
<td>-0.00398</td>
<td>0.00224</td>
<td>0.00032</td>
<td>-0.00015</td>
<td>-0.01791</td>
<td>-0.00280</td>
<td>0.00057</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Convenience</td>
<td>(0.00030)</td>
<td>(0.00006)</td>
<td>(0.00004)</td>
<td>(0.00003)</td>
<td>(0.00002)</td>
<td>(0.00003)</td>
<td>(0.00004)</td>
<td>(0)</td>
</tr>
<tr>
<td>Large</td>
<td>Grocery</td>
<td>0.00685</td>
<td>-0.00932</td>
<td>0.00138</td>
<td>-0.00148</td>
<td>-0.00280</td>
<td>-0.16053</td>
<td>0.00418</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.01094)</td>
<td>(0.00216)</td>
<td>(0.00128)</td>
<td>(0.00108)</td>
<td>(0.00034)</td>
<td>(0.01223)</td>
<td>(0.00127)</td>
<td>(0)</td>
</tr>
<tr>
<td>Large</td>
<td>Club</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

#### Inference on Eigenvalues

<table>
<thead>
<tr>
<th>Estimated</th>
<th>Bootstrapped</th>
<th>Standard error</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0011</td>
<td>0.2229</td>
</tr>
<tr>
<td>-0.0001</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.5758</td>
</tr>
<tr>
<td>-0.0007</td>
<td>-0.0007</td>
<td>-0.0007</td>
<td>0.3165</td>
</tr>
<tr>
<td>-0.0129</td>
<td>-0.0129</td>
<td>0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>-0.0173</td>
<td>-0.0172</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>-0.0394</td>
<td>-0.0393</td>
<td>0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>-0.1184</td>
<td>-0.1179</td>
<td>0.0006</td>
<td>0.0000</td>
</tr>
<tr>
<td>-0.2555</td>
<td>-0.2445</td>
<td>0.0168</td>
<td>0.0000</td>
</tr>
<tr>
<td>Table 4: Long-Run Value of Consumption and Externality (Dollars/Gallon)*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benefits from Small Households</td>
<td>Benefits from Large Households</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Superstores</td>
<td>-0.385</td>
<td>0.303</td>
<td></td>
</tr>
<tr>
<td>Convenience</td>
<td>0.695</td>
<td>1.085</td>
<td></td>
</tr>
<tr>
<td>Grocery</td>
<td>4.622</td>
<td>2.963</td>
<td></td>
</tr>
<tr>
<td>Club</td>
<td>0.546</td>
<td>4.845</td>
<td></td>
</tr>
</tbody>
</table>

*These values were calculated using the total benefit function and reflect total, not individual, benefits.

<table>
<thead>
<tr>
<th>Table 5: Value of Preference Complementarity Evaluated (Dollar/Gallon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household type</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Superstores</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Convenience</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Grocery</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Club</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>