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A FRAMEWORK FOR PRICING MULTIPLE-EXERCISE OPTION CONTRACTS FOR WATER

A THESIS
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JAY S. COGGINS, ADVISOR

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UNIVERSITY OF MINNESOTA

This is to certify that I have examined this copy of a doctoral thesis by

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and have found that it is complete and satisfactory in all respects,
and that any and all revision required by the final
examining committee have been made.

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GRADUATE SCHOOL

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"The hollow sea-shell...
...when held against the ear
proclaims its stormy parent; and we hear
The faint far murmur of the breaking flood...
It is the blood
In our own veins."

Eugene Lee-Hamilton, 1985

For Scott, the LOML

Abstract

Quality and quantity of water supplies are emerging as pressing concerns in many areas of the U.S. and the world, making efficient allocation of water a crucial issue for economic analysis. Water allocation mechanisms must be flexible enough to respond to changes in quality, quantity, location, and timing of both water supply and water demand. Historically the policy response to excess demand for water was to build dams, find new water sources, or construct storage facilities. More recently, markets for water have been developing in the western U.S. and elsewhere. Option contracts for water are emerging in some U.S. states as institutional and legal modifications allow water users to devise new mechanisms to increase reliability of water supply in dry years. Option contracts for water, though, are structurally distinct from financial derivatives and often entail a lengthy lifespan and the opportunity for multiple exercise. The Black-Scholes option-valuation framework is appropriate for options with simple payoffs but the complicated structure of water options requires an innovative computational algorithm. Adoption of the Black-Scholes assumption of geometric Brownian motion in the underlying asset price is also questionable in the scenario of water markets. In this thesis I present the framework and results of a finite-horizon, discrete-time, stochastic dynamic programming methodology for valuing multiple-exercise option contracts. The analysis examines a call option that can be exercised in any seven of fifteen years, but not more than once per year. I use data from short-term water markets in the Texas Lower Rio Grande to estimate parameters for two different price processes: mean reversion and geometric Brownian motion. Nonsychronous trading effects in the data are addressed through simulation and through the Scholes and Williams (1977) technique. Key findings of the analysis include: 1) non-zero contract values for both price processes, 2) higher contract values under geometric Brownian motion than under mean reversion, 3) greater sensitivity to price process parameters than to discount rate in the mean reverting case, and 4) greater sensitivity to price process volatility than to discount rate for low initial water prices under geometric Brownian motion.

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Chapter 1

Introduction

Water is, perhaps, the ultimate factor of production. Though rudimentary formulations of production functions rely on two inputs, capital and labor, without water there can be no life and thus no labor and no production. Water plays other more direct roles in an economy too, most obviously as an input for agriculture. By generating electricity water powers manufacturing and service in all sectors of the economy. In the U.S. in 1999 eight percent (307 billion kilowatt hours) of the electricity generated by the energy industry was garnered from water sources (U.S. Department of Energy, 2000; Table 5).

From a purely economic standpoint, then, water is a crucial resource and should be allocated with care. From a broader perspective, the decisions that individuals, cities, and countries make about how to use or conserve their water resources can have profound effects on regional economic growth, survival of plant and animal species, international relations, and human health and standards of living. Integral to societal activities from food production to recreation, water is distinctive among natural resources due to the

tangled web of legal and institutional systems that governs its use at the local, state, national, and international levels. With quality and quantity of water supply emerging as pressing concerns in many areas of the U.S. and the world, efficient allocation mechanisms are becoming more crucial.

In this thesis I present a model for valuing an innovative mechanism for transferring water between users: the multiple-exercise option contract. The theoretical and empirical results I present here will, I hope, inform decision-makers in water-scarce areas as they make choices about when and how to use this precious resource.

1.1 Water Scarcity and Market Instruments

Anecdotal evidence such as China's newly announced plans to combat its water shortage, pressure on water resources in the western United States, and brewing trouble in the Middle East over groundwater supplies suggests that water is increasingly scarce. One study estimates that by 2025 nearly one-third of the globe's population will live in areas with severe water scarcity (Barker *et al.*, 1999). However, dictionary definitions of scarcity overlook a subtlety that Dosi and Easter (2000) explicitly describe: scarcity arises from both demand and supply. Water scarcity, they note, comes in several forms. It can result from poor management of existing water sources, for instance, or, on the demand side, from extravagant consumption far beyond basic requirements. A number of trends in determinants of both supply and demand contribute to increasing water scarcity, perceived and real: population, water quality, in-stream uses, climate change.

Census data show that the U.S. population grew 13.2 percent between 1990 and

2000 (U.S. Census Bureau, 2001). Nineteen states exhibited growth higher than the national rate, as Table 1.1 details. At least ten of these nineteen face pressure on existing water resources: Nevada, Arizona, Colorado, Utah, Idaho, Texas, Washington, Oregon, New Mexico, California. All ten of these experienced selling, leasing, or otherwise formally transferring water among users in 1999 (Smith and Vaughn, 2000). Even more remarkably, the top four population growth states are also among the most water-challenged. Frederick and Kneese (1984) categorized parts of all four of these states as having serious water supply problems. Rising population in arid states is worrisome; the link between population growth and increasing water use is well-established (see for example Vorosmarty *et al.*, 2000).

Rank	Area	Percent Change, 1990 to 2000
1	Nevada	66.3
2	Arizona	40.0
3	Colorado	30.6
4	Utah	29.6
5	Idaho	28.5
6	Georgia	26.4
7	Florida	23.5
8	Texas	22.8
9	North Carolina	21.4
10	Washington	21.1
11	Oregon	20.4
12	New Mexico	20.1
13	Delaware	17.6
14	Tennessee	16.7
15	South Carolina	15.1
16	Virginia	14.4
17	Alaska	14.0
18	California	13.7
19	Arkansas	13.7
–	United States	13.2

Table 1.1: Percent Change in State Population, 1990 to 2000

Growing public interest in maintaining instream flows is a second factor contributing to increased water demand. Instream flows support habitat preservation, recreation, utilities, and aquatic species. Maintaining or even enhancing these flows has become an environmental issue in recent years. Allotting water for these purposes can reduce availability of water for agriculture or municipal use and further complicate the allocation decision. The federally imposed Central Valley Project Improvement Act, for instance, aimed in part to protect fish and wildlife populations. Weinberg (1997) found that the legislation's provision allocating water specifically for instream flows could negatively affect farmers and change crop mix, rotation, irrigation, and output in agriculture.

Water quality has an impact on the amount of water available and suitable for various purposes. In 2000, the EPA noted "Across all types of water bodies, states, territories, tribes, and other jurisdictions report that poor water quality affects aquatic life, fish consumption, swimming and drinking water' (U.S. Environmental Protection Agency, 2000; p. 1). Deterioration or improvement in water quality in a given watershed will help determine whether water demand and supply are in balance, or whether demand outstrips supply and scarcity ensues. Kaczmarek *et al.* (1996) note that, for many countries, the effects of climate change may exacerbate problems in managing water quality.

Finally, the predicted effects of global climate change on water resources are a reminder that though water is consumed locally, it is enmeshed in a larger hydrologic cycle. Snowmelt and rainfall are key contributors to surface and groundwater systems, central to water supply in the U.S. Increased variance of precipitation and more frequent occurrence of extreme events such as drought and flood are likely effects of rising temperatures in the

earth's atmosphere (Arnell *et al.*, 1996). Thus climate change will heighten the importance of careful water planning and management (Hoffman, 1993; Dellapenna, 1999). Climate change may affect water demand as well as supply, increasing the amount of water needed for irrigation and maintaining instream flows (Frederick, 2001; Kaczmarek *et al.*, 1996). Hurd *et al.* (1999) assessed the impact of climate change on water users in the Colorado River basin, for a wide range of temperature and precipitation changes. Seven out of ten of their scenarios showed an overall drop in welfare, stemming from damage to agricultural, municipal, hydropower, and recreational users, and decreased water quality. Gleick (1988) also suggested that climate change could worsen water shortages along the Colorado River and heighten tension between water users in the U.S. and Mexico.

Water management or allocation is a delicate balancing act between demand and supply. Growing population and rising concern for instream flows tend to increase demand while water quality problems and global climate change suppress supply or make it less reliable. In order for water to be put to its most valuable use it must be of sufficient quality and adequate quantity, at the correct location and at the proper time. Abundant water in the form of flooding in Minnesota in the spring does not ease the pinch of drought on Colorado agriculture in June. Water allocation mechanisms must be flexible enough to respond to changes in quality, quantity, location, and timing of both water supply and water demand.

1.1.1 Water Allocation and Transfer Mechanisms

Historically the policy response to excess demand for water was to apply a technical solution. Federal and state governments developed new water supplies, built dams, dug

irrigation ditches, and constructed storage facilities. Spurred by decreased federal funding for water projects, the high cost of infrastructure, and swelling environmental concern about dam construction and water diversion, the trend in the U.S. has shifted from infrastructure to institutions. Legislatures and governments are developing and re-designing agencies, regulations, and laws to facilitate water transfers and market mechanisms (Livingston, 1998). For example, the state legislature in Texas has allowed the State Water Development Board to form water banks in an effort to encourage water transfers (Thompson, 1997). Similarly, California has sought to clarify water property rights to facilitate short and long-term transfers (Archibald and Renwick, 1998).

Water transfers differ widely in their characteristics but can be loosely grouped into categories based primarily on the permanence of the trade. Permanent transfers of water right are at one end of the spectrum. In this type of transfer the seller gives up his right, in perpetuity, and no longer has a claim to the yield of that right. All other transfer mechanisms listed here are temporary and allow water to be moved from one location or user to another but do not change the underlying structure of water rights. California, Colorado, and several other western states permit short-term transfers. These transfers of water are generally limited to a one-year life span and avoid many of the legal hurdles faced by permanent transfers (MacDonnell and Rice, 1994).

Water banks were used in California in the early 1990s and were established in Idaho in 1994. In this system a central authority, typically part of state or local government, coordinates water transfers. Details of bank operation vary but usually entail water right holders depositing water in the bank for a set price. This water is then available for

purchase at a higher price so the bank can cover administrative costs. Water banking is a restricted form of a spot market or rental market for water, in which buyers can purchase a specific quantity of water for one-time use.

Water wheeling or exchange is a transfer of water in return for water, rather than for money. Lund and Israel (1995) describe several motivations for wheeling, including: to trade water of different qualities, to meet instream flow requirements, and to reduce conveyance cost or improve water distribution performance. This type of transfer is common, they note, "in agricultural regions in which different subareas have complementary demands for water over time" (p. 198).

The transfer mechanism of greatest interest here is the dry-year option or contingent transfer. This type of transfer is modeled on the more familiar and prevalent option contracts seen in commodity and stock markets. The key distinguishing features of options for water are the lifespan of the contract and the possibility of multiple exercise. Unlike standard financial derivatives, water options are often structured to allow the holder to use the option more than one time before the option expires. Option contracts for water are emerging in some states, California for instance, as institutional and legal modifications allow water users to devise innovative mechanisms to increase reliability of water supply in dry years.

Despite their differences, all types of water transfers have some common elements. First, the kinds and number of transfers that occur in a given water locality are determined in part by the institutions, laws, and regulations that govern water use. As these institutions and rules have changed in recent years the variety and frequency of transfers has grown.

Second, transfers take place only when both the seller and the buyer anticipate gaining from the transaction. Third, since water markets in the United States are not yet thriving in most locations, division of the gains from trade is negotiated separately for each transfer. This can erode the total gains as transaction costs accumulate.

All of these various transfers can fall under the general heading of "water markets." Initially water markets referred almost exclusively to permanent rights transfers. Permanent transfers of water rights face numerous obstacles: establishing/adjudicating property rights, assessing and mitigating the effects of the transfer on downstream or third-party users, and evaluating the current value of the future stream of water. Most of these factors increase the transaction costs of permanent rights sales, suggesting that such transfers are not an effective means for addressing short or medium-term water shortages.

Changes in laws and institutions combined with the high transaction costs associated with permanent transfers of water rights have provided the opportunity and incentive to utilize market principles to allocate water among its users. Gollehon (1999) notes that during 1996-1997 in the Western United States 78% of 282 recorded water transfers were permanent sales of water rights. In contrast, the overwhelming volume of water, 92.8 percent or 2.5 million acre feet[1], was exchanged in short-term, temporary transactions. Proponents of short-term water transfers argue that market mechanisms can be used even if water property rights are not adequately defined, and are especially attractive if the transaction costs of trading permanent water rights are prohibitive. (Howitt (1998) is one example.) Frederick (1998) argues that markets in many cases increase the value of water

[1]One acre-foot of water is the amount of water needed to cover one acre of land to a depth of one foot. This is 43,560 cubic feet, enough to fill a 50 meter by 25 meter swimming pool to 9.7 feet deep. See Appendix A for more volume equivalencies.

by shifting it from low-value agricultural uses to hydropower or urban consumption. For economists, markets for water hold natural appeal: ideally, as supply and demand fluctuate relative to each other, prices will provide signals to market participants and ensure efficient water allocation.

While development of short-term transfers and facilitation of permanent transfers of water rights have eased barriers to efficient water allocation, these mechanisms do not adequately accommodate medium-term planning needs of many water users. Further, as Howitt (1998) notes, short-term and spot markets appear to disproportionately burden buyers with risk while permanent transfers are riskiest for sellers. Sellers in the spot market are secure in their knowledge of the amount of water available currently available from their rights and, if failing to make a transfer in the spot market, can use the water for the purpose it was originally intended. Buyers in the spot market, in contrast, face uncertain future spot market supplies and prices. When exchanging permanent rights, the sellers must determine the present value of the stream of benefits from their water right. Once the right is transferred, the seller cannot rectify an incorrect assessment. Buyers of permanent rights bear less uncertainty: if they pay too high a price for the right, rising water prices will compensate them over time. Other market arrangements could spread risk more equally between the two parties. Uncertainty about future water supplies and prices, the need to time water availability to meet demand or production constraints, and the irreversibility of investments in water imply that a new market construct may be appropriate: option contracts for water.

The buyers and sellers of call options for water participate because they expect to

be better off with the contract than they would be without it. The seller loses a degree of flexibility in his water allocation decisions; he is obligated to provide the water when the buyer strikes. However, the seller receives a measure of immediate compensation, in the form of the option price or premium paid by the buyer when negotiations are complete. The seller also benefits in the event that the call is exercised since the buyer must pay the strike price then. The call buyer's main benefit from the contract is flexibility. Holding the call option entitles the buyer to access a source of water that would have been unavailable otherwise, increasing the buyer's ability to meet water obligations and expanding the range of water allocations choices. The water option price reflects the expected value of this addition flexibility in decision-making.

These option contracts for water are an exciting addition to the range of market-based mechanisms for transferring water. Clearly they are related to similar derivative instruments in financial markets. The point of this thesis, though, is that option contracts for water are structurally distinct from financial options and require an innovative pricing approach. Given that option contracts for water are being established between water users in some states, both participants in the agreement must perceive the potential for gains from trade. What is the value of the increased flexibility that is inherent in holding a call option for water?

1.2 A Description of Financial Options and Water Options

Options are a type of derivative, a financial instrument whose value is based on another, underlying asset. A stock option, for instance, derives its value from the price of

a specified stock. Calls and puts are the two basic types of options. A call option gives the holder the right but not the obligation to purchase the underlying asset by a certain date for a specified price. Similarly, a put option gives the holder the right but not the obligation to *sell* the underlying asset by a certain date for a price specified in the option contract. The date agreed upon in the option contract is called the expiration date or exercise date; the listed price is referred to as the strike price or exercise price. Options are further categorized depending on the structure of the exercise date. A European style option, for instance, can be exercised only on the expiration date; by contrast, an American option can be exercised any time on *or before* its expiration.

Perhaps the most basic option contract is the European call. On the option's expiration date its holder must decide whether to exercise her right to purchase the underlying asset. If she exercises, she receives a payoff equal to the difference between the current price of the asset and the price specified in the option contract. If she does not exercise the option her payoff is zero. More formally, the payoff from the option is

$$\max\left(S_T - K, 0\right), \tag{1.1}$$

where S_T is the price of the underlying asset at the expiration date, T is the expiration date, and K is the strike price.

An option contract must have both a buyer and a seller. The buyer of a call obtains the right to purchase the underlying asset but the seller of the call is *obligated* to provide the asset if the call is exercised. Thus while the holder of the call retains flexibility, the seller has none. In order to acquire this flexibility about future actions, the call buyer incurs an up-front cost, the option price. Determining the option's value to the buyer is

straightforward for a European call but can become quite tricky for other classes of option contracts. Chapter 2 presents the fundamental pricing approach for standard options, the Black-Scholes formula.

Many options have payoff structures that are more complicated than standard calls and puts; these fall under the general heading of "exotic" options. The payoff from a lookback option, for example, depends on the maximum (or minimum) price achieved by the underlying asset during the lifetime of the option. A few types of exotic options such as European-style compound options can be priced analytically using extensions of known formulas. Most, however, require numerical methods such as binomial trees or numerical quadrature. Chapter 3 describes a range of exotic options and computational methods typically used to value them.

Evidence from the western United States shows that option contracts for water can be even more exotic than many exotic options considered in finance. One contract negotiated between a municipal water authority and a large agricultural operation allows the municipality to exercise a call option on water up to seven times in a fifteen-year period. This is a highly non-standard option; there is no simple pricing formula to calculate its value. Standard European call options involve a binary decision of whether or not to exercise on the expiration date. In contrast, there are more than 6,400 ways to exercise the water district's call option seven or fewer times in fifteen years. Clearly this complicated payoff structure increases the difficulty of putting a value on the option contract.

In my research I have generalized the "seven times in fifteen years" option contract to a stylized "multiple-exercise" option. As its name implies this option can be used more

than one time during the life of the option contract, generally no more than m times out of n periods, with $n \geq m > 0$. The multiple-option contract must specify in advance the strike price for each period. To make the problem non-trivial, m must be greater than zero.

1.3 Thesis Overview

The central purpose of this thesis is to use financial derivative pricing theory as a foundation for a computational approach to valuing multiple-exercise option contracts in a natural resources setting. What is the economic value of the flexibility a municipality or farmer gains or loses by agreeing to a long-term option contract for water? As Jarrow (1999) observed, the Black-Scholes valuation framework for financial options enabled rapid growth in markets for financial derivatives.

> For both exchange-traded and [over the counter] markets, the Black-Merton-Scholes technology facilitated trading by allowing investors to understand the risks of derivatives better. In the late 1960s and early 1970s, at the time that the Black-Merton-Scholes discoveries were being formulated, call and put options on equities were considered esoteric instruments, and they traded only in limited quantities between a few brokers and investment banks...Since that time, the growth in exchange-traded and over the counter traded options on equities, indices, foreign currencies, commodities, and interest rates has been phenomenal. It is widely believed that the growth in these option markets was enabled by the Black-Merton-Scholes option pricing technology. (Jarrow, 1999; pp. 235-246)

While my aspirations are far more modest, it is my hope that this research will facilitate the use of option contracts for water in the western United States and improve water allocation in the face of excess demand.

I present my research in the seven chapters of this thesis. This first chapter emphasizes the importance of water allocation decisions and raises the primary questions for investigation. The last chapter summarizes my conclusions, outlines policy recommendations, and suggests a range of likely extensions of this research. The intervening chapters are divided into two sections. Chapters 2 through 4 describe my approach to the financial engineering problem of valuing multiple-exercise options, setting aside the specific details of water markets and trading. Chapters 5 and 6 plunge into the complexities of water allocation in the western United States and detail the results of my option pricing model using data from northeastern Colorado.

In the course of writing this thesis I have opted not to devote a single chapter to a traditional literature review. Instead, I weave citation and review of existing papers into the appropriate context in each chapter. Thus, I review the basic option pricing literature in Chapter 2, exotic options and computational methods in Chapter 3, and water options and markets and the Texas water scenario in Chapter 5. This integrated approach allows me to reference each source precisely when it is relevant to my discussion and, in the end, makes this interdisciplinary research more comprehensible.

Part I

Valuation Methodologies for Standard, Exotic, and Multiple-Exercise Options

Chapter 2

The Black-Scholes Approach to Pricing Plain Vanilla Options

The challenge of pricing an option is to determine the value of the option, in current dollars, before its maturity date. This is equivalent to putting a dollar value on the flexibility the option holder gains by entering into the contract. The derivative's value, then, is the amount the buyer pays the seller at the time they make the agreement. This chapter summarizes the ground-breaking option valuation framework advanced by Black, Scholes, and Merton in the 1970s. Chapter 3 builds on this basic valuation approach by describing more complicated option structures and computational methods for pricing them.

2.1 The Black-Scholes Framework

On December 9, 1997, Robert C. Merton and Myron S. Scholes received the Alfred Nobel Memorial Prize in Economic Sciences for their roles in developing the conceptual and mathematical frameworks of option pricing theory. The seminal articles by Black and Scholes (1973) and Merton (1973) sparked a torrent of academic and applied research on financial derivatives, which in turn generated modifications, extensions, and innovative applications of the original Black-Scholes model. Merton (1998) provides a brief review of the evolution of option-pricing theory since 1973. Though the original formulation of the Black-Scholes approach is appropriate only for European put and call options on stocks that do not pay dividends, the importance of this early work is unmistakable. As Chriss (1997) states "the basic ideas underlying it are very much the basis for all option pricing models."

In the context of financial derivatives that are traded on organized exchanges, the Black-Scholes formula supplies the value of an option as a function of five variables: strike price of the option, time until expiration, price of the underlying stock, risk-free interest rate, and volatility of the stock. For a thorough yet accessible explication of the Black-Scholes differential equation and pricing formulas see Hull (1997). Under the set of assumptions listed below, Black and Scholes revealed that the value of a financial option does not depend on the expected rate of return on the underlying stock. To derive their option pricing formula, Black and Scholes posited several assumptions (Hull, 1997). These are:

1. Price of underlying asset follows a geometric Brownian motion process.

2. Market participants are permitted to short-sell the asset, with full use of proceeds.

3. No transaction costs or taxes exist.

4. The underlying asset is perfectly divisible.

5. No dividends accrue during the life of the option.

6. Arbitrage opportunities are not available.

7. Trading of the asset is continuous.

8. The risk-free interest rate is constant, strictly positive, and compounded continuously.

While these assumptions may be justifiable to some extent in the presence of well-established, active markets, the validity of some them is doubtful in nascent or thinly traded water markets. As the list of assumptions inherent in the model suggests, the Black-Scholes method may not be suitable for wholesale adoption in this sphere of resource economics. Many of the conditions upon which the pricing formulas are predicated are absent from water markets at their current stage of development. In the succeeding paragraphs of this section I argue that applicability of the Black-Scholes framework to water markets is tenuous, suggesting the need for an improved option pricing model for water.

2.1.1 Derivation of the Black-Scholes Pricing Formulas

When the basic assumptions do hold, a riskless portfolio consists of the option and a specific amount of the underlying asset. The portfolio can be riskless because the price of the option and the price of the asset share the same source of uncertainty. In

an appropriately constructed portfolio fluctuations in the price of the underlying asset are always perfectly offset by fluctuations in the price of the option. For any short time period, then, we can calculate exactly the portfolio's value. The relative proportions of the asset and options necessary to keep the portfolio riskless change rapidly, even instantaneously. Therefore, to stay riskless, the portfolio must be rebalanced frequently. Yet in any short period of time the portfolio's expected return must equal the risk-free interest rate. In 1989 the importance of this observation was highlighted by Black himself, "This one principle gives us the option formula" (p. 4).

Using the reasoning of the previous paragraph Black and Scholes (1973) derived a differential equation that must be satisfied by the price of any derivative contingent on a non-dividend-paying asset. Equation (2.1) is the Black-Scholes differential equation:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC\left(t, S_t\right) \tag{2.1}$$

where C is the price of a call option or other derivative, r is the nominal risk-free interest rate, S is the price of the underlying asset, t is time, and σ^2 is the variance rate of the proportional changes in S.

The risk-free portfolio instrumental in this derivation for a call option is: short (sell) one option and long (buy) $\frac{\partial C}{\partial S}$units of the asset. However, equation (2.1) has many solutions and does not give a unique value for the price of the derivative. Boundary conditions that reflect the characteristics of the specific derivative in question will generate the unique solution. As the general expression for an option payoff shows (see equation (1.1)) a European call option is worth nothing at its expiration date if the price of the asset

is below the option's strike price; otherwise it is worth the difference between the price of the asset and the strike price. More formally, at the expiration date the value of the call is:

$$C(T, S_T) = \max(S_T - K, 0). \tag{2.2}$$

With this boundary condition the Black-Scholes differential equation yields a pricing formula for European call options:

$$C(t, S_t) = S_t N(d_1) - Ke^{-r(T-t)} N(d_2). \tag{2.3}$$

For the price of a European put option, G, the formula becomes:

$$G(t, S_t) = Ke^{-r(T-t)} N(-d_2) - S_t N(-d_1) \tag{2.4}$$

For both of these pricing formulas:

$$N(\cdot) = \text{standard normal cumulative probability distribution} \tag{2.5}$$

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \tag{2.6}$$

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \tag{2.7}$$

A key implication of equations (2.1), (2.3), and (2.4) is that the value of any derivative in an environment that satisfies the eight assumptions of the Black-Scholes framework is independent of the expected rate of return on the underlying asset. Further, the value of the option does not depend on any variable that is a function of investor risk preferences.

2.1.2 Example

To illustrate the option-pricing formulas in a more concrete manner, consider the following example. Suppose the current price of the asset is \$31 per unit, the strike price for a call option is \$30, the expiration date is six months from now, the nominal risk-free interest rate is 10% per year, and the volatility of the asset's price is 20% per year. Then $S_0 = \$31$, $K = \$30$, $r = 0.10$, $(T - t) = 0.5$, and $\sigma = 0.2$. Substituting these values into the definitions of $d_1$and d_2 (equations (2.6) and (2.7)) I find that $d_1 = 0.6561$, $d_2 = 0.5147$, and $Ke^{-r(T-t)} = 28.54$. With these parameters the Black-Scholes pricing formula for a European call option at the current date ($t = 0$), equation (2.3) becomes:

$$C(t, S_t) = 31N(0.6561) - 28.54N(0.5147). \tag{2.8}$$

For the calculated values of $d_1$and d_2, a polynomial approximation of the cumulative normal distribution (see Appendix B) yields:

$$N(d_1) = N(0.6561) = 0.7441 \text{ and } N(d_2) = N(0.5147) = 0.6966. \tag{2.9}$$

Inserting these into the pricing formula, equation (2.8), reveals that the value of the call option is $C = \$3.19$. The purchaser of the call option breaks even if the price of the asset rises by \$2.19, since they would pay a strike of \$30, an option premium of \$3.19 for the contract, and the current price of the asset is \$31.

2.1.3 When is the analytical result attainable?

The preceding example demonstrates the beauty of the Black-Scholes framework: given five inputs the formula calculates the exact value of the option. The mathematics underlying the pricing formula quickly become treacherous, however, as the complexity of the option structure increases. On the whole, American options prove more difficult than European ones; Black and Scholes failed to find a closed-form solution for an American put. Computational methods such as hill-climbing algorithms and other techniques described in Chapter 3 apply in cases where the differential equation is relatively easy to discern but difficult to solve analytically.

2.2 Early Efforts at Derivative Valuation

The Black-Scholes and Merton option valuation formulations were breakthrough concepts in a chain of intellectual exploration dating back to Bachelier's dissertation in 1900. Jarrow (1999) presents an accessible and brief narrative of the key contributors to the Black-Scholes approach; for a detailed, technical explanation see Smith (1976). Myron Scholes (1998) gave a first-hand account of the sequence of events in his Nobel Prize acceptance speech, published in the *American Economic Review.* This chronology draws from all three sources.

The development of option valuation theory before Black-Scholes and Merton is primarily a progression in understanding of the stochastic process assumed to guide changes in stock price. Bachelier (1900) assumed prices evolved according to arithmetic Brownian motion (a Wiener process.) Despite some economics and mathematics that Jarrow char-

acterizes as "slightly flawed" (p. 231), Bachelier derived an option pricing formula that is similar to the Black-Scholes result.[1] The arithmetic Brownian motion assumption presents problems because it implies that negative prices for the underlying asset have positive probabilities, allows option prices to exceed the stock price, and, by assuming zero expected change in stock price, "suggests both zero interest rates and risk neutrality" (Smith, 1976; p. 15).

To avoid these unpalatable implications, most subsequent models assumed that price exhibits geometric Brownian motion. This is equivalent to assuming the log price of the underlying asset follows a Wiener process. Sprenkle (1961, 1964), Boness (1964), Samuelson (1965), and Merton and Samuelson (1969) were all inspired by Bachelier. Sprenkle improved on Bachelier by assuming log normally distributed stock prices but failed to account for the time value of money. By discounting call prices back to the initial period, Boness addressed Sprenkle's omission. However he exposed a weakness in his own argument by explicitly assuming away differences in risk between various stocks and options. Geometric Brownian motion drove the stock price process in Samuelson's 1965 piece.

Sprenkle, Samuelson, and Merton and Samuelson proved difficult to use in practice. Each of their pricing methods depended on the investor's risk attitudes because the pricing formulas rely on the stock's risk premium. Risk premia, unfortunately, constantly fluctuate in response to changing preferences and economic conditions. Further, these models offered no guidance for hedging (reducing the risk of) the option (Jarrow, 1999).

Itô's 1951 efforts in stochastic calculus also descended in part from Bachelier's

[1]Courtault *et al.* (2000) present a brief biography of Bachelier, "one of the most mysterious figures in mathematics" (p. 342), who laid the foundation for mathematical finance and several areas of stochastic calculus.

work and played a key role in enabling Black and Scholes to derive their formula. As they struggled to create an option pricing approach in continuous time, Black and Scholes, and particularly Black, were motivated by Sharpe (1964) and Lintner's (1965) writings on their capital asset pricing model.

The early chronology of option valuation theory and developments since Black and Scholes and Merton's landmark contributions center on the nature and restrictiveness of assumptions. While the Black-Scholes assumptions may seem appropriate in financial markets a vastly different context is in question here. How suitable are the assumptions of the Black-Scholes model in the context of water markets?

2.3 Suitability of Black-Scholes Assumptions in the Context of Water Markets

Transaction costs are conspicuously absent from the Black-Scholes pricing formulas. Instead, the approach computes option values as though market prices reflect the total cost of buying or selling the underlying asset. Even in efficient and active arenas like the large stock exchanges, however, transaction costs are present in the form of brokerage fees and commissions. Leland (1985) noted that the presence of transaction costs make continuous portfolio adjustment prohibitively expensive. In 1992 two papers, Boyle and Vorst and Bensaid *et al.*, extended Leland's analysis to a discrete-time framework using a variation of Cox *et al.*'s (1979) numerical technique for valuing options.[2] All three papers concluded that relaxing the transaction cost assumption of the Black-Scholes model has a

[2]See chapter 3 of this thesis for an explanation of the Cox *et al.* approach.

distinct impact on the valuation results: rather than yielding a unique price for the option, the modified approach that includes transaction costs determines bounds on the option price.

All water exchanges are regulated or constrained in some way, usually by state or federal government. This governmental intervention generates uncertainties and imposes costs on both buyers and sellers in water markets (Colby, 1990). Such policy-induced transaction costs are only a subset of costs incurred in water transfers, beyond the purchase price of the commodity. Conveyance, storage, and treatment costs also contribute to the transaction cost load borne by water buyers (Lund and Israel, 1995). Transaction costs are important, as Archibald and Renwick (1998) observe, because potential buyers and sellers will participate in water transfers only if their rents, minus transaction costs, are positive. From a market perspective, transaction costs raise the price of water and reduce the amount of water exchanged (Easter *et al.*, 1998).

The overwhelming majority of empirical studies of transaction costs in water exchanges find them to be substantial. A 1991 purchase of water by San Francisco, for instance, included a purchase price of $45 per acre foot. Total costs including conveyance and storage ranged from $250 to $350 per acre foot (Lund and Israel, 1995). After analyzing the 1991 California State Water Bank, its transaction costs, and the incentives these costs presented to market participants, Archibald and Renwick (1998) concluded "policy-induced transaction costs are likely affecting the performance of California water markets" even beyond the water banking system (p. 21). Moving beyond California, transaction costs for nine permanent rights transfers in Colorado ranged from $0.40 to $1,552 per acre-foot, in

1982-1984 dollars (Howe *et al.*, 1990). Table 2.1 summarizes approval costs and time delays for water trades in Colorado, New Mexico, and Utah.

State	Applicant Costs ($ per af)	Time Delay (months)
California	187	29
Utah	66	5
New Mexico	54	4.3

(Source: Colby *et al.*, 1989)

Table 2.1: Approval Costs and Time Delays for Water Trades in Three States

Clearly a number of studies show that water markets tend to involve a high level of transaction costs arising from disputes over property rights, third party effects of water transfers, thinness of the water markets themselves, and the physical movement of the commodity. Though assuming that transaction costs are zero, as the Black-Scholes model does, is a strong assumption in the water markets context, transaction costs do not play a specific role in my model's specification either.

The no-arbitrage assumption fails to hold when two assets that are virtually identical are being sold at different prices. An arbitrage opportunity is defined as "an investment strategy that guarantees a positive payoff in some contingency with no possibility of a negative payoff and with no net investment (Dybvig and Ross, 1989)." A direct implication of the absence of arbitrage is the law of one price: two perfect substitutes must trade at the same price. When identical assets are available at different prices, the savvy investor will buy the asset at the low price and sell immediately at the high price, gaining certain positive payoff with no net investment. In thinly traded exchanges such as evolving water markets, arbitrage opportunities do arise, violating the Black-Scholes model's no-arbitrage condition.

Several of the assumptions built into the general Black-Scholes framework serve to ensure that option writers and buyers can indeed devise a riskless hedge, or replicating portfolio. Derivation of the pricing formulas relies heavily on the existence of the riskless portfolio and also requires the ability to rebalance the portfolio over time so that it continues to mimic exactly the payoff of the option being valued. Assumptions allowing short selling, continuous trading, and divisibility of assets, in addition to the assumptions regarding arbitrage and transaction costs, ensure availability of this dynamic riskless hedge. These assumptions do not translate well when applied to water markets. Thinness in water markets undermines the validity of these assumptions, making the possibility of devising a replicating portfolio dubious at best. Water markets are not active enough to facilitate creating and maintaining the riskless hedge or to guarantee that the value of the hedge is known with certainty.

The assumption that the underlying asset's price follows a geometric Brownian motion process may also be problematic in the context of water markets. An Itô process with μ and σ constant requires that prices adhere to the following form:

$$dS = \mu S dt + \sigma S dz \tag{2.10}$$

where μ, σ, t, and S are defined as usual and where dz is the increment of a Wiener process, also called arithmetic Brownian motion. Since dz is the continuous-time increment of a Wiener process by definition the relationship between dz and dt is given by:

$$dz = \varepsilon_t \sqrt{dt}. \tag{2.11}$$

Then, since ε_t is a normally distributed random variable with mean zero and standard deviation one,

$$E\left[(dz^2)\right] = dt. \tag{2.12}$$

Furthermore,

$$E\left[\varepsilon_t \varepsilon_s\right] = 0 \text{ for } t \neq s, \tag{2.13}$$

so the values of ε_t for any two different time intervals are independent. Clearly, the assumption of an Itô process contains a series of additional, implicit assumptions: changes in the price of the asset follow a continuous-time, non-stationary, Markov process and are normally distributed.

The price of water is a stochastic process, defined by a probability law describing how price will evolve over time. One implication of this is that we can predict the probability that water price will be in a given range at a specified future time. Is the stochastic process non-stationary and Markov? Non-stationarity requires the possibility that the expected value of water price may grow without bound and that the variance of price increase over time. To satisfy the conditions of a Markov property, only the current price of water can be relevant to predicting tomorrow's price. Given the seasonality of water use and supply, the Markov assumption is questionable. This suggests the possibility of easing the Itô process assumption by exploring different types of stochastic forces driving water price. Data from Texas suggests that a mean-reverting process, for instance, may bear further investigation.

2.4 Extensions of Black-Scholes

Modifications of the Black-Scholes option-valuation methodology present in the form of relaxing the assumptions, developing and refining computational implementation, and applying the framework to innovative scenarios. Though Chapter 3 details some of the computational extensions, I present an adumbration of the new applications and assumptions in the literature here. Smith (1976), Scholes (1998), and Jarrow (1999) provide a range of perspectives on the impact of Black-Scholes on subsequent research and practice.

Almost immediately following publication of Black and Scholes's model in the *Journal of Political Economy* (1973), Merton (1973) extended their analysis in his *Bell Journal of Economics and Management* piece. Merton provided an alternative proof of the Black-Scholes method and showed its relevance to American options on stocks that pay no dividend. He also relaxed the original Black-Scholes assumptions to allow a dividend paid continuously during the option's lifespan, according to a specified yield formula. That same year Thorp (1973) argued that Black and Scholes need not assume that proceeds from short sales be unrestricted. The Black-Scholes assumption that taxes are absent was eroded by Ingersoll's 1976 article examining the effects of tax rates that differ for capital gains and ordinary income (Smith, 1976).

Focus on theoretical modifications of the Black-Scholes framework shifted to interest rates in the 1980s when interest rate volatility rose (Jarrow, 1999). This practical experience contradicted the assumption of a constant interest rate and led to the development of arbitrage-free term-structure models by Ho and Lee (1986), Black *et al.* (1990), and Heath *et al.* (1992). These models have proven versatile in a range of scenarios such

as foreign-currency-pricing problems.

The number and variety of applications of the basic Black-Scholes formulation are staggering. For a particularly lucid but concise enumeration see Merton (1998); I rely primarily on this source. As Merton comments, the Black-Scholes model "provided a launching pad for refinements of the theory, extensions to derivative-security pricing in general, and a wide range of other applications, some completely outside the realm of finance" (p. 324).

The Black-Scholes option-pricing mechanism and its extensions facilitated an explosion in the volume of derivatives traded in financial markets. The model is the basis for pricing derivative securities on individual stocks and bonds, mutual fund portfolios, and currencies. Commodities such as agricultural products, metals, crude oil, and electricity are other assets underlying options and futures contracts. Employee stock options have become a common element in compensation packages for employees in some industries (Merton, 1998). Outside financial markets, option-pricing theory has found relevance in modeling household decisions such as leasing a car (Miller, 1995) and the real options approach to firm investments (Dixit and Pindyck (1994) and Trigeorgis (1996) among others). Table 2.2 lists a selection of other applications of Black-Scholes valuation theory cited by Merton, alongside associated references in the literature.

Computational extensions of the Black-Scholes methodology are not as varied as the applications but they hold vital importance. Without computational and empirical developments many of the scenarios in Table 2.2 would have proved difficult or impossible to assess using option valuation techniques. Few extensions of the Black-Scholes frame-

Application Context	**References**
Pricing corporate liabilities	Hawkins (1982), Brennan and Schwartz (1985)
Valuing loan guarantees, deposit insurance	Merton (1977), Kraus and Ross (1982), Doherty and Garven (1986), Cummins (1988), Cummins and Geman (1995), Harrington et al. (1995)
Valuing government guarantees of pension insurance, students loans, etc.	Sosin (1980), Baldwin et al. (1983), Cunningham and Hendershott (1984), Marcus (1987), Merton and Bodie (1992), Bodie (1996), Mody (1996), Neal (1996)
Assessing the cost of government subsidies	Mason and Merton (1985), Turvey and Amanor-Boadu (1989), Kang and Brorsen (1995)
Valuing government-granted offshore drilling rights	Paddock et al. (1988)
Investigating whether Social Security should invest in equities	Smetters (1997)
Modeling the interest rate process underlying bond prices	Black (1995)
Evaluating government education expenditures	Merton (1992)

(Source: Merton, 1998)

Table 2.2: Selected Applications of Black-Scholes Option Pricing Theory

work have analytical solutions; computational approaches are essential. For my purpose, applying a variation of Black-Scholes to the new context of water markets, some of the computational algorithms are invaluable. Chapter 3 therefore explores exotic options and the computational methods that help price them.

Chapter 3

Exotic Options and Computational Valuation Approaches

Recent formulations of option valuation in water markets naturally mimic the conceptual and mathematical framework developed by Black, Scholes, and Merton (see Howitt (1998) and Watters (1995)). As I argue above, the assumptions of Black-Scholes option pricing formulas may not be fully justifiable when valuing options in nascent or thinly traded water markets. Furthermore, the original Black-Scholes model cannot be applied to non-standard options with complex payoff structures. In this chapter I describe exotic options, explain five computational methods for valuing options, and review existing option contracts for water and related literature. Of the five numerical approaches, hill-climbing algorithms and finite difference methods are closely linked to the Black-Scholes framework and rely on derivation of differential equations that must be satisfied by the option price. These approaches do not apply to most option contracts in water markets, whose complex

structures make deriving the relevant partial differential equation impossible. Monte Carlo simulation, numerical quadrature, and binomial trees use computational techniques based more directly on the option's payoff structure and estimation of the stochastic process of the underlying asset's price.

3.1 Payoff Structures of Exotic Options

Three of the four plain vanilla options of Chapter 2, European calls and puts and American calls, are straightforward to value using the Black-Scholes pricing formulas. As option writers have utilized their creativity and responded to the needs of investors, the payoff structures of these derivatives have increased in complexity. The term "exotic option" refers generally to any option contract with a payoff structure that is more complicated than the basic call or put.[1] New option structures continue to develop at an astonishing rate; in the tables below and the following paragraphs I present a small sample of the possibilities.

One logical method for categorizing exotic options is to scrutinize and group them based on their payoff structure. Ong (1996) articulates a taxonomy of exotic options that he claims is exhaustive.[2] He distinguishes among path-dependent, nested or compounded, time-dependent, multivariate, and discontinuous payoffs. Ong also discusses myriad options that involve foreign exchange, leveraged options where payoffs are not a linear function of the underlying asset's price movements, and options that are embedded in other complex financial agreements. These are less relevant for my purposes. Path-dependent options

[1] Ong (1996) attributes this use of the word "exotic" to Mark Rubinstein.

[2] For a less detailed but more digestible categorization see Hull (1997), chapter 18. Ravindran (1996) explores a range of exotic options, their payoff functions, and sample applications.

Exotic Option Type	Description
Path-Dependent	
Asian	• Also called average-value options. • Payoff depends on the average price of the underlying asset during at least some part of the option's lifespan. • True Asian options use the arithmetic average. • Variations include: average strike, partial average, and geometric average options.
Barrier	• Payoff depends on whether the underlying asset's price reaches (or falls below) a specified level during the option's timeline. • Variations and extensions include "down-and-out," knock-in, knock-out, and multiple barrier options.
Lookback	• Can be European or American-style. • Payoff depends on the underlying asset's maximum (or minimum) price during the life of the option. • Variations and extensions include partial and modified lookback, ladder, ratchet, and shout options.

Table 3.1: Path-Dependent Exotic Options

Exotic Option Type	Description
Nested or Compounded	
"As You Like It"	• Also called chooser options. • At a specified date the holder chooses whether the option is a call or a put. Complex choosers require the holder to also declare a time to maturity when deciding whether it is a call or a put.
Compound	• May be European or American-style. • The underlying asset is also an option. Configurations include any combination of calls and puts. Each option has an associated exercise price and date.

Table 3.2: Nested or Compounded Exotic Options

Exotic Option Type	Description
Time-dependent	
"As You Like It"	• See description above.
Bermudan	• Also called quasi-American or Mid-Atlantic options. • A non-standard American option for which early exercise is restricted to specified dates during the life of the option.
Forward Start	• Also called delayed options. • An option contract that is written and paid for in the current period but begins at a future time period. Extensions and variations include caps and floors, and periodic caps and floors.

Table 3.3: Time-dependent Exotic Options

Exotic Option Type	Description
Multivariate or Correlation	
Basket	• Underlying is a collection or bundle of assets. Its payoff is calculated based on prespecified weights for each asset in the basket. • Portfolio options are a close relative of basket options but the weight associated with each asset is the number of shares of that asset in the portfolio.
Rainbow	• A broad class of options based on n assets, a rainbow with n colors. • See performance options below for one variation. • Other variations and extensions include pyramid, spread or outperformance, and exchange options.
Performance	• A variation of rainbow options. • Also known as best or worst performance options. • At exercise, the option pays a fixed cash amount or the best (or worst) performing of n specified assets.

Table 3.4: Multivariate or Correlation Exotic Options

Exotic Option Type	Description
Discontinuous Payoff	
Binary	• Also called digital, bet, or all-or-nothing options. • Payoffs are discontinuous and may have jumps. • A cash-or-nothing option for example pays nothing if the underlying asset's price is below the strike price and pays a fixed amount otherwise. • Variations and extensions include correlation digital and digital barrier options.
Barrier	• See description above.

Table 3.5: Exotic Options with Discontinuous Payoffs

have payoffs that are determined at least in part by the way the price of the underlying asset evolves during the life of the option. For a plain vanilla European put option all that matters is the price of the asset at the terminal time period, regardless of the price's path since the option began. Lookback options, in contrast, are a class of exotic options in which the holder's payoff depends on the maximum or minimum price the underlying asset's price reaches during the option's lifespan.

Nested or compounded options are built on an option, rather than a stock, currency, commodity or other underlying asset. An "As You Like It", or simple chooser, option allows the holder to decide at a pre-specified time whether the contract will be a vanilla call or a vanilla put. The call and put have the same strike price and maturity date. The payoff function for a simple chooser is

$$\max[C(K, T-t), P(K, T-t); t],$$

where K is the strike price, T is the time to maturity, and t is the time until the holder must choose (Ong, 1996; p. 27).

An "As You Like It" option's payoff is also time-dependent. While all option contracts have a time component, some structures give timing a relatively important role in determining the payoff. Plain European options can be exercised at only one time period, the maturity date, and thus do not have a special dependence on time. American options, though, *are* time-dependent; the holder must decide precisely when to strike, and timing becomes part of the decision-making process. Similarly, a chooser option's payoff has an intricate relationship to time, requiring a decision in the interim between initiation and maturity.

Appropriately titled, multivariate option contracts have two or more underlying assets. Their payoffs, then, are determined by the multiple asset prices as well as their interaction or correlation. A basket option, for instance, is built on a collection of assets and its payoff is calculated based on predetermined weights for each asset in the basket. The payoff function is

$$\max\left[\sum_{t=1}^{n}\omega_i S_i^T - K, 0\right]$$

where ω_i are the n percentage weights for the n different assets (Ong, 1996; p. 29).

Finally, some options are constructed so that they have discontinuous payoffs. Ong notes that options exhibiting this characteristic are easy to value but difficult to hedge. Jumps are one common form of discontinuity in the payoff function. Binary options pay either nothing or a pre-specified, set amount. If the payoff is cash the binary option is a "Cash or Nothing" option. However, the payoff may be in terms of the underlying asset instead of cash; this is an "Asset or Nothing" option.

3.2 Even More Exotic: Option Contracts for Water

Option contracts for water are far more complicated than the relatively simple vanilla options of Chapter 2. In some respects these water options are also more complex than the exotic options described earlier in Chapter 3. Several option contracts emerged in California in the early and middle 1990's, when drought descended or threatened almost annually. These contracts serve as evidence of the variety that exists in water options, their differences from financial options, and their complicated payoff structures.

3.2.1 Examples of Option Contracts for Water

Several authors have written descriptions of a wide range of option contracts for water. See, for example, Shupe *et al.* (1989), Rice and MacDonnell (1993), Pinnes (1994), Watters (1995), Howitt (1998), and Rozaklis and Weaver (1999). Rozaklis and Weaver list eleven existing interruptible water supply arrangements and studies in five U.S. states, with contract durations ranging from one to 35 years. Seven of these examples entail agriculture-to-municipality water transfers; three cases involve agriculture or agriculture and municipality water transfers to support instream flows or other environmental services. The remaining arrangement transfers water from industrial to municipal use. Less than half (four of eleven) of the contracts had been implemented by the time Rozaklis and Weaver published their report. I describe two of these cases and two others that were put into place later.

The agreement signed by the City of Grand Junction, Colorado and Redlands Water and Power Company in 1979 is a very early example of an option-like contract for

water. Pinnes (1994) provides details of the arrangement. The contract gives the City of Grand Junction the right to divert water allocated to the power company in order to meet municipal demand. The city must give two days' notice before exercising its option, must pay all fees and expenses related to the transfer, and is responsible for clearing the legal hurdles to the water exchange. In return for this form of insurance against inadequate water supply the City pays an annual option premium to Redlands Water and Power and an exercise price that varies depending on Redlands' revenue losses or increased expenses due to the reduced flow of water. As of 1994 the contract had not been terminated nor had the City exercised the option.

The Metropolitan Water District (MWD) in Southern California has been a leading instigator of a variety of option contracts for dry-year water supply. Rice and MacDonnell (1993) describe one such contract in detail. A one-year arrangement with Dudley Ridge Water District in 1992 obligated MWD to purchase part of Dudley Ridge's water supply for $125 per acre-foot, if MWD's water allocation from the State Water Project (SWP) for 1993 was below 50% of its entitlement. This contract was a put option of sorts; Dudley Ridge water users had a choice about whether to sell their water but MWD was obligated to purchase it. The SWP awarded MWD 85% of its entitlement in 1993 so the option was not triggered.

Watters (1995) uses a draft of an option contract between MWD and Areias Dairy Farms (in the Central California Irrigation District) in her application of the Black-Scholes framework to water transfers. This MWD agreement differs substantially from the Dudley Ridge example: it spans fifteen years and allows for exercise in any seven of those years.

The maximum amount transferred per year is 5,000 acre-feet; MWD must give notice of intent to exercise by March 15 of each year. This decision timing allows Areias to adjust agricultural production plans accordingly. The payment structure of the contract is moderately complex, with installments paid by MWD at various stages of negotiation and government approval. In effect, though, the payments amount to an option premium of $87.50 per acre-foot and a strike price of $87.50 per acre-foot. MWD is also obligated to pay conveyance costs and environmental fees.

The MWD continues to be active in seeking out partners for option contracts as part of their Dry-Year Transfer program. In early 2001 MWD's efforts focused on negotiating transfers with the Kern Delta Water District and the San Bernadino Valley Municipal Water District (MWD, 2002). The San Bernadino Valley agreement has a 10-year lifespan. It allows MWD to purchase 20,000 acre-feet each year and provides flexibility for MWD to purchase additional water from San Bernadino Valley's allocation each year (MWD, 2002).

3.2.2 Contract Structure and Incentives

Option contracts for water are typically lengthy legal documents. Though valuation methodologies consider only a few parameters (strike price and time to maturity, for instance) the contract itself bears the burden of specifying the rights and responsibilities of the buyer and seller. Michelsen and Young (1993), Clark and Abt (1993), and Pinnes (1994) present comparable outlines of the central terms and provisions recommended for inclusion in option contracts for water.

Obviously the document must state the identities of the buyer and seller, and the

contract duration. Because ownership of the water right itself does not change, only use of the water itself, the contract may need to clearly identify the water right(s) that will provide the transferred water. The contract should also specify the quantity of water being optioned or provide a mechanism for determining it. Timing, minimum quality, and method of water delivery should be negotiated in advance and described in the option contract.

As with financial derivatives, option contracts for water must include payment amounts and timing. Typically this will entail both an option premium for the right to hold the option and a strike fee each time the holder exercises the option. Some of the option arrangements described earlier in this chapter structure the option premium as a yearly fee rather than a one-time payment. Two peculiar features of water options are their escalation and renegotiation clauses. The escalation provision details the timing of adjustments to contract prices over the life of the option and the economic index used as a reference point. A renegotiation clause may be included in the contract document to allow either party to reopen the contract if faced with unanticipated circumstances. The agreement may define what types of events can trigger renegotiation.

Exercising a water option is a more complicated endeavor than exercising a stock option. In general, the water option holder must give advance notice of intent to exercise so that the seller can adjust water-relevant decisions as needed. An agricultural seller, for instance, may alter crop mix to avoid some variable production costs. The option contract for water may also stipulate a trigger for the exercise of the option, as in the case of the 1992 MWD-Dudley Ridge agreement that used the MWD's allocation level from the State Water Project as a trigger.

The owner of the water right, who presumably is selling the option, may decide to sell the permanent right while the option contract is in place. For that reason, many water options contain a provision that gives the option holder right of first refusal: the option buyer may purchase the permanent rights if he matches the offered price. Alternatively, the contract could allow the seller to substitute another party's water right to support the option contract. Either of these approaches would allow the option seller to maintain flexibility in long-term management of permanent rights without sacrificing the option buyer's water security.

Finally, the option agreement may include clauses outlining arbitration procedures for resolving disputes and *force majeure* provisions. *Force majeure* provisions are common legal devices that relieve (usually temporarily) either party of their contractual obligations if they are impeded by circumstances beyond their control.

The detailed or perhaps convoluted structure of water option contracts reflects both the newness of the transfer concept and the incentives of the various parties. As Pinnes (1994) notes, the main advantage option contracts have over permanent water rights transfers is that options "foster cooperation rather than ... antagonism (p. 2)." Although this thesis does not address the sociology or politics of water markets, permanent transfer of rights from agriculture to municipalities has become a point of contention in many states in the western U.S. Since option contracts leave the permanent right with the option seller while simultaneously providing water to the option buyer in times of shortage, both parties may benefit without the long-term negative third party effects that could arise from a permanent transfer.

Existing research characterizes option sellers as agricultural rights holders and buyers as municipalities or hydropower utilities, almost uniformly. Though my model makes no presupposition about the identity of market participants, this brief discussion of incentives follows the established conventions. The farmer's main incentives to engage in a water option contract are to improve the farm's future financial health, in wet and dry years alike, while minimizing limitations on flexibility in managing water rights. The option contract allows the farmer to continue normal operations in most years and to receive income from option exercise in dry years when agricultural production might have been scaled back anyway. The strike payments, in effect, give agricultural water users "a way to survive the drought" (MacDonnell and Rice, 1994; p. 50).

Option buyers, whether municipalities or utilities, share the goal of increasing water supply reliability in dry years. In the case of hydropower, increased supply of water for hydropower in dry years means higher prices charged for electricity (from greater reliability) and, indirectly, spurs more plentiful power supplies in all years (Hamilton *et al.*, 1989). Municipalities may find that option contracts ensure water supply in times of shortage at a lower expense than they would incur from buying permanent rights. Using option contracts to augment supply may also allow a city to forego developing new water projects or storage facilities (Pinnes, 1994). Halverson's 1990 empirical estimation of the economic impact of long-term interruptible water marketing contracts showed that the benefits from increased hydropower outweighed the costs of lost agricultural production.

Compared to permanent sales of water rights, option contracts may also have fewer negative effects on rural communities and other third parties. Effects on the environment,

for example, are not well understood but may be less damaging for temporary, sporadic transfers than for permanent changes in water use and location. Short-term transfers such as option contracts can also avoid severe detrimental effects on employment, tax revenue, and farm supply businesses in rural communities by enabling continued farm operation in most years.

Cox and Rubinstein (1985) offer nine reasons to invest in options. While they refer specifically to financial derivatives, two of their points are particularly relevant to the water context. First, trading an option may involve fewer transaction costs than trading the asset itself. As mentioned in Chapter 2, transaction costs can be quite onerous in permanent water transfers. This may contribute to the attractiveness of option contracts in some water markets. Second, Cox and Rubinstein note "diversification offers many advantages" (p. 45). This simple statement summarizes the incentives on both sides of an option contract for water. The buyer of a call diversifies sources of supply in the face of uncertainty about future water availability; the seller diversifies sources of revenue to address the riskiness of future income.

3.3 Pricing Approaches for Exotic Options

The fundamental problem in pricing options is computing the expected value of a random variable, usually focusing on the price of the underlying asset or assets. As Black and Scholes demonstrated, a subset of option contracts are suitable for valuation using analytic formulas: construct a partial differential equation that accurately describes the option's payoff structure, then solve it. There are two ways to discern an option valuation

partial differential equation's solution analytically: solve it mathematically using boundary conditions (as Black and Scholes did) or compute the expectation of the discounted payoff directly (see McIver (1996) for an example using binary options.) In many cases, though, it is far simpler to represent the option as a partial differential equation than it is to find the equation's solution analytically.

Researchers and practitioners have developed explicit, exact pricing formulas for some of the simpler or convenient exotic option structures. The formulas are of course reliant upon the assumptions about the interest rate, price process for the underlying, and other assumptions about the option environment that are akin to those made by Black and Scholes. A survey of Bellalah (1999), Ong(1996), and Hull (1997) yields a partial listing of options that can and others that cannot be valued with analytic formulas. The Tables 3.6 and 3.7 summarize those results.

In the face of analytically intractable partial differential equation, option valuation becomes a numerical or computational endeavor. Finite difference methods, binomial trees, and hill-climbing algorithms can approximate the differential equation's solution. For most path-dependent options, however, neither analytic nor computational solution of the partial differential equation is a workable valuation strategy. Path dependency causes a disconnect between the expected payoff and a partial differential equation. In this case and others where the payoff structure is so complex that writing down the relevant partial differential equation is difficult or impossible, exotic options can be priced without relying on such an equation. Binomial trees, Monte Carlo simulation, and numerical quadrature are available tools to value these kinds of options.

Barrier (all categories)

- Use Black-Scholes interest rate and volatility assumptions.
- Hull (1997; p. 461) reports the basic formula.

Basket

- Restrictive assumption that value of the entire basket follows a lognormal distribution, rather than each asset's price individually.
- Hull (1997; p. 469) discusses this pricing problem briefly.

Binary/Digital

- Underlying is a stock paying a continuous dividend yield rate.
- See Hull (1997; p. 463) for formulas.

Chooser (simple)

- The put and call have the same strike price and time to maturity.
- Bellalah (1999; p. 21) and Hull (1997; p. 460) specify the formula, following Rubinstein (1991).

Compound (simple)

- Both underlying options have the same exercise price and time to maturity.
- Geske (1979) valued a call on a call.
- Rubinstein (1991) generalized to other simple compounds.

Digital Barrier

- A variation of digital options, not path-dependent.
- Digital barriers include 28 different types; 44 formulas for valuing them.
- Formulas proposed in Rubinstein and Reiner (1991) and reiterated in Bellalah (1999; p. 26).

Forward Start

- Can be priced with or without a dividend.
- See Hull (1997; p. 459).

Geometric Average

- A variation of Asian options.
- Can be priced under standard Black-Scholes assumptions.
- Valuation approaches in Turnbull and Wakeman (1991) and Kemna and Vorst (1990).

Lookback

- European-style are priced analytically.
- Bellalah (1999; p. 33) and Hull (1997; p. 459) show pricing formulas.

Table 3.6: Exotic Options With Closed-Form Pricing Solutions

American put on a non-dividend paying asset

- Geske and Johnson (1984), among others, proposed analytic approximations.

Asian/arithmetic average

- Bellalah (1999) recommends Turnbull and Wakeman (1991), Conze and Viswanathan (1991), Curran (1991-1992), and Geman and Yor (1993) for insights to pricing Asian options.

Chooser (complex)

- The included options may have different exercise prices and times to maturity.
- Rubinstein (1991) presents a pricing formula, but it entails finding the solution to a non-linear equation.

Compound (complex)

- The underlying options may have differing strike prices, times to maturity, and volatilities.
- Again, Rubinstein (1991-92) suggests a pricing method that relies on solution of a non-linear equation.

Lookback (American-style)

- Numerical methods for valuing American lookbacks are summarized and referenced in Hull (1997).

Table 3.7: Exotic Options Lacking Closed-Form Pricing Solutions

3.4 Hill-climbing Algorithms

Compound options present a case where solving the differential equation generally requires a numerical approach. A compound option is an option on an option: a call on a call, a put on a put, a call on a put, or a put on a call. For example, federal agricultural price support programs that require farmers to enroll annually can be viewed as compound options: a call on a put. Deciding whether to enroll or not is a decision of whether to exercise a call; price protection acts as a put, specifying a purchase price for crops (Stinson *et al.*, 1998). Complex compound options thus have two different strike prices and two different expiration dates. Rubinstein (1991-1992) provides generalized pricing formulas for all possible vanilla combinations, where all options are European style, building on Geske's

1979 treatment of a call on a call. The pricing formulas are specific solutions to nonlinear partial differential equations but are not closed-form results from boundary conditions like I illustrated above in section 2.1.1, equations (2.3) and (2.4). Instead, Rubinstein's approach uses iterative numerical techniques to search for the solutions to the pricing formulas.

Hill-climbing algorithms like the Newton-Raphson procedure are one set of iterative computational techniques that are often used to solve nonlinear equations. The Newton-Raphson method requires that the equation to be solved have the form $g(x) = 0$, and that the researcher specify a starting point for the iterations, $x = x_0$. The procedure then improves on this initial estimate by iteratively calculating x_1, x_2, x_3, and so on according to:

$$x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)} \tag{3.1}$$

where $g'(x_i)$ denotes the first derivative of the equation being solved, evaluated at the current iteration point. The Newton-Raphson method and other iterative numerical algorithms have two main pitfalls: the iterations may not converge to a solution, and the solution obtained may depend on the initial starting point specified by the researcher.

3.5 Finite Difference Methods

Finite difference methods are popular techniques for solving the partial differential equations in financial applications. Rather than solving the equations analytically, finite difference approaches discretize the independent variables and numerically estimate the derivatives in the partial differential equations. This section mimics Hull's (1997)

demonstration of implicit finite difference methods to value an American put. Read Judd (1999; chapter 10) for in-depth explication of variations on the technique.

Finite difference calculations are based on a grid that represents the variables in the partial differential equation under consideration. Typically, time is on the x-axis and price (or log price) of the underlying asset is on the y-axis.[3] The solution algorithm prescribes boundary values for the option, then approximates the function at the remaining gridpoints. Determining the function's value at non-gridpoints requires interpolation.

The Black-Scholes framework yields a partial differential equation for the price of any vanilla option on a non-dividend paying stock:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf\left(t, S_t\right) \tag{3.2}$$

where $f\left(t, S_t\right)$ is the value of the derivative. The boundary conditions are dictated by the specific nature of this American-style put option. Specifically, there are three: the option's value is equal to the strike price when the stock price is zero, as the stock price approaches infinity as the option price moves toward zero, and the value of the put in the terminal period is equal to the maximum of zero and the strike price minus the stock price. Using Hull's (1997) notation, discretize the time and price states such that

[3]The methodology can be extended to encompass additional variables, in which case the grid is multi-dimensional.

T = time to maturity of the option

$N + 1$ = number of time slices in the grid

$\Delta t = \frac{T}{N}$ = the time increment

$S_{\max}$ = a high stock price for which the put has almost zero value

$M + 1$ = number of price slices in the grid

$\Delta S = \frac{S_{\max}}{M}$ = the price increment.

Then the resulting grid has $N+1$ time elements, $(0, \Delta t, 2\Delta t, ...T)$ and $M+1$ price elements, $(0, \Delta S, 2\Delta S, ...S_{\max})$. The grid has a total of $[(M+1) * (N+1)]$ points, each point (i, j) represents time $i\Delta t$ and price $j\Delta S$. The put option's value at point (i, j) is denoted $f_{i,j}$.

The boundary conditions are now expressed as mathematical equations:

$$\begin{aligned} f_{i,0} &= K \qquad \text{for } i = 0, 1, ..., N \\ f_{i,M} &= 0 \qquad \text{for } i = 0, 1, ..., N \\ f_{N,j} &= \max(K - j\Delta S, 0) \qquad \text{for } j = 0, 1, ..., M. \end{aligned}$$

These boundary equations assign option prices to three edges of the grid: the x-axis, all gridpoints on the vertical line at time T, and all gridpoints on the horizontal line at $S_{\max}$.

Computing the put's price at the remaining gridpoints involves calculating the value of the difference equation at each point as a means of estimating the function's derivative. Finite difference methods vary in the approach to solving these discrete translations of the calculus into derivatives of the partial differential equation. Here I employ the implicit finite difference technique.

Referring back to equation (3.2), notice the three derivatives it contains: $\frac{\partial f}{\partial t}$, $\frac{\partial f}{\partial S}$, and $\frac{\partial^2 f}{\partial S^2}$. From the structure of the discretized problem,

$$\frac{\partial f}{\partial t} = \frac{f_{i+1,j} - f_{i,j}}{\Delta t} \tag{3.3}$$

This is forward difference approximation and is appropriate since the problem restricts time from reverting to the past. Time is allowed only to march forward. Stock prices, though, may increase or decrease over time and therefore could be represented with either a forward or a backward difference approximation:

$$\begin{aligned} \frac{\partial f}{\partial S} &= \frac{f_{i,j+1} - f_{i,j}}{\Delta S} \qquad \text{(forward)} \\ \frac{\partial f}{\partial S} &= \frac{f_{i,j} - f_{i,j-1}}{\Delta S} \qquad \text{(backward).} \end{aligned}$$

To be subtle, use the average of these two to approximate the option pricing function's partial derivative with respect to stock price:

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j-1}}{\Delta S}. \tag{3.4}$$

Construct the second derivative with respect to asset price by subtracting the backward difference approximation at (i, j) from the backward difference at $(i, j+1)$, dividing by the price increment, and simplifying:

$$\frac{\partial^2 f}{\partial S^2} = \frac{f_{i,j+1} + f_{i,j-1} - 2f_{i,j}}{\Delta S^2}. \tag{3.5}$$

Substitute equations (3.3), (3.4), and (3.5) into the Black-Scholes differential equation, equation (3.2), to find:

$$f_{i+1,j} = \left(1 + \sigma^2 j^2 \Delta t + r\Delta t\right) f_{i,j} - \left(\frac{rj\Delta t + \sigma^2 j^2 \Delta t}{2}\right) f_{i,j+1} + \left(\frac{rj\Delta t + \sigma^2 j^2 \Delta t}{2}\right) f_{i,j-1}. \tag{3.6}$$

This equation is the key to calculating the option price at the remaining gridpoints. Since σ^2, r, and Δt are parameters and j is determined by the current evaluation point, the parenthetical expressions preceding $f_{i,j}$, $f_{i,j+1}$, and $f_{i,j-1}$ are simply scalar coefficients as a function of current price.

The process for computing the option prices associated with the remaining gridpoints is somewhat tedious. Begin by choosing the time period of $T - \Delta t$. This iteration of the solution algorithm finds the value of the option pricing function associated with each gridpoint having $i = N - 1$. When $i = N - 1$, equation (3.6) becomes:

$$f_{N,j} = \left(1 + \sigma^2 j^2 \Delta t + r\Delta t\right) f_{N-1,j} - \left(\frac{rj\Delta t + \sigma^2 j^2 \Delta t}{2}\right) f_{N-1,j+1} + \left(\frac{rj\Delta t + \sigma^2 j^2 \Delta t}{2}\right) f_{N-1,j-1}.$$

Note that there exists one such equation for each stock price except 0 and M, yielding $M-1$ simultaneous equations. From the boundary conditions,

$$\begin{aligned} f_{N-1,0} &= K \\ f_{N-1,M} &= 0 \\ f_{N,j} &= \max(K - j\Delta S, 0) \qquad \text{for } j = 0, 1, ..., M. \end{aligned}$$

The $M - 1$ simultaneous equations contain $M - 1$ unknowns: $f_{N-1,1}$, $f_{N-1,2}$,..., $f_{N-1,M-1}$.

The first simultaneous equation, for $f_{N,1}$, depends on parameters $f_{N-1,0}$, $f_{N-1,1}$, and $f_{N-1,2}$. Because $f_{N,1}$ and $f_{N-1,0}$ are supplied by the boundary conditions, this first simultaneous equation can be rearranged into an expression of $f_{N-1,2}$ in terms of $f_{N-1,1}$. Proceeding iteratively each successive equation can also be written as a function of $f_{N-1,1}$. The final simultaneous equation will then consist of a complicated function in one unknown, $f_{N-1,1}$, set equal to $f_{N-1,M}$. From a boundary condition $f_{N-1,M} = 0$ so the final simultaneous equation will yield a value for $f_{N-1,1}$ that enables calculation of all other $f_{N-1,j}$ option prices.

After computing each $f_{N-1,j}$, the technique determines whether to exercise the option for each gridpoint (i,j), where $j = 1,2,...,M$. To do this compare $f_{N-1,j}$ to $K - j\Delta S$, the payoff from exercising. For every j where $f_{N-1,1} > K - j\Delta S$ the put holder should strike. In this event $f_{N-1,j}$ must equal $K - j\Delta S$. At this stage in the solution, all gridpoints for time $T - \Delta t$ have been assigned option values. The same process fills in values for time $T - 2\Delta t$, then $T - 3\Delta t$ until at last the option prices for the initial period $(i = 0)$ result and all gridpoints have been processed. The price of the put option is approximated by f_{0,j^*}, where j^* is the stock price observed in the initial time period.

Finite difference approaches share many of the common drawbacks of other computational techniques. Estimates of function values are not as precise as analytic solutions and are subject to errors in approximation. Finite difference methods evaluate the pricing function many times; this can lead to "excessively large errors" (Judd, 1999; p. 37). McIver (1996) notes that most finite difference methods, including the one explained here, are not reliable if second order derivatives with respect to time are part of the differential equation.

Implicit finite difference procedures can be demanding of computer memory and processing ability. These issues are likely to increase in prominence for problems with more than one independent variable in addition to time. This set of techniques is not appropriate for option pricing problems with poorly defined partial differential equation formulations.

3.6 Binomial Tree Estimation

Binomial tree estimation does not rely on partial differential equations to value derivatives. Instead, a tree structure represents possible movements in the price of the underlying asset during the derivative's life. Using the same inputs as the Black-Scholes approach (initial asset price, strike price, expiration date, risk-free interest rate, price volatility) calculate the asset price for each node of the tree. Then determine the value of the option at each terminal node corresponding to the expiration date. Finally, work backward through the tree to assess the value of the option at each intermediate time period or node. When the algorithm reaches the initial node, it yields a single number, the option price. In short, the value of the derivative is the discounted value of the sum of the payoffs in each terminal node, weighted by the probability of achieving that node.

To illustrate a simple, two-step binomial tree consider the standard European call example from Section 2.1.2. Recall that $S = \$31$, $K = \$30$, $r = 0.10$, $(T - t) = 0.5$, and $\sigma = 0.2$. Since this is a two-step tree, divide the timeframe of six months into two periods of $\Delta t = 0.25$ years each. Using the Cox *et al.* (1979) procedure, the percent change in price for an upward movement is $u = e^{\sigma\sqrt{\Delta t}}$, the percent change in price for a downward movement is $d = e^{-\sigma\sqrt{\Delta t}}$, and the risk-neutral probability of a price increase is $p = \frac{e^{r\Delta t} - d}{u - d}$.

Note that these Cox, Ross, and Rubinstein specifications require that $u = \frac{1}{d}$. Therefore, an upward change in price followed by a downward change in price yields the same result as a downward change followed by an upward one. This has the practical effect of reducing the number of nodes in the tree.

Combining the Cox *et al.* constructs with the stated parameters of my example, then, shows that $u = 1.105$, $d = 0.905$, and $p = 0.602$. The tree diagram in Figure 3.1 presents the resulting asset price for each node. At Node B, for instance, $S_B = (S * u) = 34.260$. Similarly, at Node E, $S_E = (S * u * d) = S$.

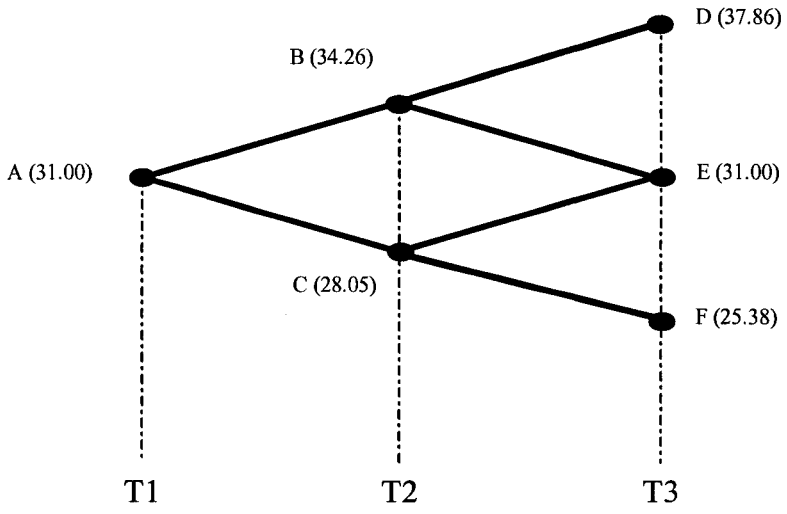

Figure 3.1: Binomial Tree of Asset Price at Each Node

Once the asset price at each node is known, calculating the value of the call at nodes D, E, and F is straightforward. The computation is based on equation (1.1), the payoff for a European call option. For Node F:

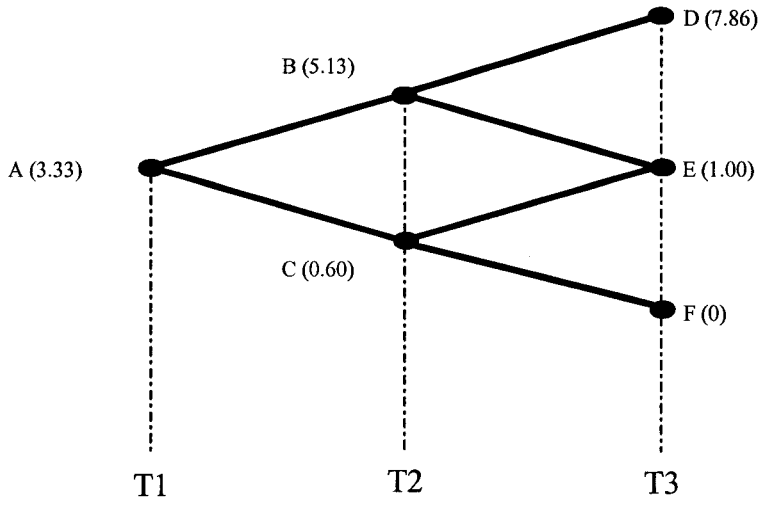

Figure 3.2: Binomial Tree of European Call Option Value at Each Node

$$C_F = \max(S_F - K, 0) = 0. \tag{3.7}$$

The value of the derivative at Node B is the discounted value of the probability-weighted sum of the call values at Nodes D and E:

$$C_B = e^{-r(\Delta t)}\left[p * C_D + (1-p) * C_E\right] = 5.132. \tag{3.8}$$

Work backward until the call value at each node is found, resulting in the tree depicted in Figure 3.2.

Thus the binomial tree estimation procedure yields a price of \$3.33 for the call option. From the analytical Black-Scholes solution in Section 2.1.2 above, the true price of this standard European call is \$3.19. The binomial tree estimate could be greatly improved by breaking the life of the derivative into a larger number of steps, calculating the option

price from a finer tree.

As this example shows, two is not a large enough number of steps to achieve a reliable estimate for the option price. Hull (1997) recommends dividing the derivative's lifespan into at least 30 steps. He also cautions that a 30-step tree represents close to one billion possible stock price paths. Binomial trees can be used for both American and European options and for some exotic options such as an American lookback option on a non-dividend paying stock (see Hull, 1997; p. 470). The binomial tree approach is versatile and applies to valuation of several classes of exotic options. The method works for American options by working backward from the terminal nodes and testing each node to see whether early exercise is optimal. Binomial trees can be modified for valuing options on assets that pay dividends according to a known yield rate or that pay dividends in specified time periods. Relaxing some of the Black-Scholes assumptions is also possible with this tree process, including time-dependent interest rates, two underlying stochastic variables, and mean reversion in the price process. Trinomial trees are an extension of binomial trees that allow up, down, and middle movements in price at each node. The trinomial tree technique is equivalent to the explicit finite difference method.

Binomial trees are more efficient than Monte Carlo simulation for valuing European options, except in the case of three or more stochastic variables. Both binomial and trinomial tree methods can accommodate path-dependent options, for instance an American lookback on a non-dividend-paying stock (see Hull, 1997; p. 470, for an illustration). Hull (1997, p. 476) presents general approaches to using modified trinomial trees for pricing barrier options and two-color Rainbow options with correlated assets.

3.7 Monte Carlo Simulation

Monte Carlo simulation is a versatile tool for approximating the path of a stochastic variable. The methodology involves repeatedly drawing a random or pseudo-random value for the underlying stochastic variable in successive trials.[4] Each trial enables calculation of the option's payoff based on the sample path of the asset price. A typical Monte Carlo simulation will include thousands of trials, resulting in a list of thousands of payoffs that are possible given the distribution and stochastic process of the asset price. Then the estimated expected value of the payoff, not yet discounted, is the arithmetic average of all the sample payoffs. This implies that the option valuation, as the sample mean of a set of random simulations, is also a random variable.

Estimating the stochastic process with acceptable accuracy can require a very large number of independent trials. This may overburden computer resources or take a long time to complete compared to other numerical techniques. As Vorst (1996) notes, "Using 100 times more simulations increases the computational costs 100 times but only increases the accuracy of the valuation result by 10 times" (p. 286). However, several modifications to the general Monte Carlo approach are designed to reduce the standard error of the estimates and improve computational efficiency.

Monte Carlo methods are flexible enough to accommodate complex payoff structures and non-standard stochastic processes. They are particularly well-suited to represent path-dependent options such as Asian options. Hull (1997) recommends the Monte Carlo approach over other numerical techniques if the problem requires estimating three or more

[4]Judd (1999) emphasizes that sampling in Monte Carlo procedures are almost never truly random. Instead, they rely on pseudorandom sequences, "deterministic sequences ... that *seem* to be random" (p. 286).

random variables. If more than one stochastic variable is entailed, each one must be sampled in each simulation trial. In general, the Monte Carlo approach cannot model American-style options. Vorst (1996) gives a brief illustration of Monte Carlo valuation of a European-style Asian option, based on an earlier paper by Kemna and Vorst (1990).

3.8 Numerical Quadrature

Numerical quadrature uses computational methods that rely directly on the option's payoff structure and the stochastic process of the underlying asset's price. One of its main strengths is that it de-emphasizes the role of differential equations in the process of valuing an option. Furthermore, this approach enables the valuation process to account for exotic option structures more easily than the Black-Scholes model and is a quicker, more robust algorithm than iterative methods for solving nonlinear equations. Subject to the constraints of computer resources, numerical quadrature can value any option whose payoff can be written as a mathematical expression. Discerning an option's payoff can be decidedly simpler than deriving and solving the differential equation that determines its value.

A variety of numerical quadrature, or numerical integration, methodologies exist. All quadrature processes are designed to approximate the value of a definite integral. Different techniques use different procedures regarding where to evaluate the integrand and how to manipulate the evaluations. In addition, some of the quicker approaches are limited by restrictions on the type or complexity of integrands that are acceptable. The general class of quadrature described here utilizes approximation and interpolation in its computa-

tions. In option valuation, quadrature is generally useful for function approximation and calculating the nodes and weights for a stochastic price process.

The numerical quadrature procedure begins with articulating the option's payoff structure and specifying a stochastic process for the price of the underlying asset. It then re-casts the problem as an integral. The quadrature technique values the option by estimating the solution to the integral. Nelken (1996) supplies a concise explanation of numerical quadrature and applies the method to complex chooser and compound options. To demonstrate how numerical quadrature can value options, I return again to the standard European call option of Section 2.1.2.

Recall again the payoff structure of the call option, represented by equation (1.1). This implies that the value of the call is:

$$C = e^{-rT} E\left[\max\left(S_T - K, 0\right)\right], \tag{3.9}$$

where E denotes the expectation operator, T is the time until the option expires (note that this is a different definition for T than I have been using), S_T is the price of the underlying asset, r is the interest rate, and K is the strike price. Assuming a lognormal random walk for the price of the underlying, and z a normally distributed random variable with mean zero and standard deviation 1, rewrite the spot price at time T as:

$$S_T = S_0 e^{\mu T + \sigma z\sqrt{T}}, \text{ where } \mu = r - \frac{\sigma^2}{2}. \tag{3.10}$$

In general, the expected value of a random variable A is given by the integral of the product of each value of A times the probability of achieving that value:

$$E(A) = \int Ap(A)\,dA. \tag{3.11}$$

Since the probability distribution $p(A)$ assigns a likelihood to every possible value of A, the integral is computed over all values of A. In practice, the range of integration is large but finite.

Combining equations (3.9), (3.10), and (3.11), re-write the option valuation problem as an integral:

$$C = e^{-rT} \int_{-\infty}^{\infty} \left[\max\left(S_0 e^{\mu T + \sigma z\sqrt{T}} - K, 0\right)\right] n(z)\,dz, \tag{3.12}$$

where $n(z)$ is the standard normal density function. To simplify equation (3.12) set the two elements within the max operator equal to each other and solve for z. This shows that the option's payoff, and therefore the value of the integrand, is zero for some values of z. Specifically, the value of the option is zero for values of z that are less than:

$$L = \frac{\ln\left(\frac{K}{S_0}\right) - \mu T}{\sigma\sqrt{T}}. \tag{3.13}$$

Thus, the integral that will be subjected to numerical quadrature is:

$$f = e^{-rT} \int_{L}^{\infty} \left(S_0 e^{\mu T + \sigma z\sqrt{T}} - K\right) n(z)\,dz. \tag{3.14}$$

Although there are several quadrature approaches, each involves choosing a finite range of integration and then evaluating the integrand at specific values within the integration range. Of the available techniques I illustrate left end-point quadrature (see Nelken, 1996). This method first breaks the integration region into a pre-specified number of slices, then

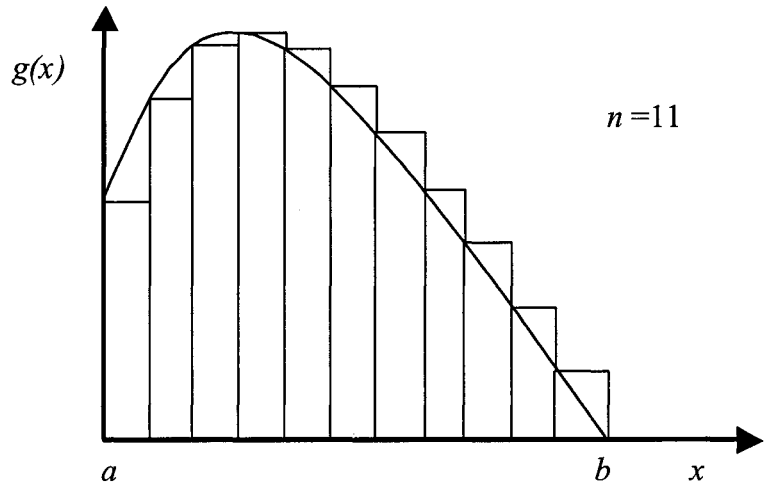

Figure 3.3: Diagram of the Quadrature Process

evaluates the integrand detailed in equation (3.14) at the left border of each slice to determine the slice's height. The sum of the slices' areas approximates the area beneath the curve. To evaluate I, the area under a function $g(x)$ between a and b,

$$I = \int_a^b g(x)\, dx, \tag{3.15}$$

first, divide the integration range into slices. For n evaluations of the function the step size is $h = \frac{(b-a)}{n}$. Then the following expression approximates I:

$$I \approx h \sum_{i=0}^{n=1} g(a + ih). \tag{3.16}$$

The quadrature process for this example is depicted in Figure 3.3.

The area encompassed by the rectangles only approximates the area under the curve. The error is evident in the space between the tops of the rectangles and the func-

tion itself. Increasing the number of slices, n, reduces this error. Notice also that the distance between the tops of the rectangles and the function being evaluated may vary as x changes. Thus the graph above alludes to a primary weakness of the left end-point quadrature method: the constant step size, h, must be specified in advance. If the method allowed, accuracy would improve by decreasing the step size where the function is changing rapidly. In this example this would mean decreasing step size as x gets larger. Some of the other quadrature techniques incorporate an adaptive control component that ensures the step size will be reduced in the regions where the integrand changes quickly (Nelken, 1996).

3.9 The Need for a Pricing Approach for Water Options

Option contracts for water, though they are few, exhibit features distinct from financial derivatives. Each agreement is negotiated individually; there is no standard format. Jercich (1997) describes the many steps in negotiating and implementing the option contracts associated with California's 1995 Water Bank Program. Strike deadlines tend to cluster in April and May to accommodate the planning needs of agricultural users. A large percentage of the contracts are long-term and allow for multiple exercises of the option. This is consistent with institutional, theoretical, and simulation-based research on water options. Hamilton *et al.* (1989), Halverson (1990), and Clark and Abt (1993) attribute both of these characteristics to the option-like transfer agreements they analyze. In a simulation of water supply options for the Fort Collins, Colorado area, Michelsen and Young (1993) construct a contract over 20 years with multiple exercise both permitted and

expected.[5]

The quantity of the underlying asset plays a prominent role in option contracts for water. While financial options usually specify the number of units of stock, for instance, that are entailed in the derivative, water options tend to allow some leeway in the volume of the commodity to be exchanged. The Dudley Ridge example included an April 1 deadline for determining the maximum volume of water for purchase. Flexible quantity provisions are reflections of the uncertainty surrounding both future demand for water and the future yield from ownership of a certain water right. Quantity can also play the role of a trigger in options contracts for water. Dudley Ridge is a good illustration of this; recall that exercise of the option was contingent on the quantity of water MWD received from the State Water Project (MacDonnell and Rice, 1994).

The distinctive attributes of option contracts for water and especially their frequent inclusion of multiple-exercise clauses provide a financial engineering challenge. Exotic options are priced using a range of numerical techniques; vanilla options rely on the Black-Scholes formula. While Ryan (2000) presents a methodology for valuing multiple options on an underlying asset but with disparate strike prices, none of these approaches is designed to address multiple exercise of a single option. This calls for computational innovation to value multiple-exercise options. Though the concern here is with water, the algorithm of the coming chapter is applicable or adaptable to any underlying asset.

[5]Read chapter 4 of this thesis for extended discussion of early water option literature.

Chapter 4

An Algorithm for Valuing Multiple-Exercise Options

Here I describe my dynamic programming method for valuing multiple-use options. I characterize the multiple-exercise option scenario as a finite-horizon, stochastic dynamic programming problem with discrete, finite state and action spaces and discrete time periods. As the chapter progresses I present the dynamic programming problem and describe the numerical techniques I use to approximate its solution. The model relies on assumptions about specific parameter values and the functional forms of the objective function and constraint equations, which I also detail here.

Option contracts for water present interesting possibilities for managing water allocation. The contracts detailed in Chapter 3 exhibit characteristics that distinguish them from both standard and exotic financial derivatives. The divergence of financial and water options prompts the central question of this research: what valuation methodology

is appropriate for long-term, multiple-exercise option contracts?

4.1 Literature on Valuing Option Contracts for Water

Research concerning the pricing of water options first appeared in the mid-1980s, coinciding with rising interest in all aspects of water markets and the early stages of institutional and legal reform to facilitate water market development in the western United States. In assessing opportunities for increasing supply of water for hydropower production in the Pacific Northwest, Whittlesey and Houston (1984) modeled "intermittent water transfers" from agriculture in periods of critically low flow. Though legal and institutional impediments made this type of transfer impossible at that time, their model predicted significant social gains from these option-like contracts for four different regions in three states: Idaho, Washington, and Oregon.

Hamilton *et al.* (1989) analyzed "interruptible water markets" to allow irrigation water to shift to hydropower production in dry years. While the authors did not label this type of contractual arrangement an option, their suggested compensation scheme for the farmer is distinctly option-like: an annual payment (akin to an option premium) plus a lump-sum payment much like a strike price "in years of water supply interruption" (p. 73). The authors provide little guidance for determining the value of the contract, noting that several variables (crop prices and production costs and technology, for instance) and a great deal of uncertainty play a role. The annual payment by the hydropower company should compensate farmers for their expected income loss due to the interruptible supply contract.

Two articles in 1993, Michelsen and Young and Clark and Abt, present detailed and highly similar methods for computing the net present value of the benefits from a water option. Clark and Abt describe a three-part water rights option agreement consisting of a water transfer plan based on engineering analysis, a financial analysis, and a contract document specifying the legal obligations of the participants. The financial analysis itself includes a water market analysis, financial calculations of the costs and benefits, and a compensation plan that outlines the terms and timing of the payment. Clark and Abt attribute their approach to assessing the worth of the option contract to Michelsen's (1988) dissertation, which is also the foundation of Michelsen and Young's analysis. Because the two valuation techniques are almost identical and draw on the same resource (Michelsen, 1988) for their formulation of the pricing problem, I omit the details of Clark and Abt, focusing instead on Michelsen and Young.

Michelsen and Young examine dry-year water supply option contracts, temporary water transfers from agricultural to non-agricultural users during drought. They use a cost-benefit approach to determining the value of the option, calculating the difference between the present value of the costs of an option and the present value of the costs of an alternative source of water supply. When the costs of the alternative exceed the costs of the option, the present value of the option benefits (PVOB) is positive. The present value of the benefits is the maximum amount a buyer should be willing to pay for the contract, that is, the maximum level for the option premium. The central equation Michelsen and Young rely on is (p. 1012):

$PVOB =$ [Cost of alternative]-[Expected exercise cost]-[Appreciation of alternative supply].

In contrast to Clark and Abt, Michelsen and Young assume that transaction costs and conveyance costs are equal for the option contract and for the alternative. Since their computation compares the costs of the option contract to the alternative, this assumption enables Michelsen and Young to disregard transaction and conveyance costs in their analysis. In contrast, Clark and Abt use a technique from Howe *et al.* (1990) to calculate transaction costs for permanent transfers and data from the City of Greeley, Colorado to approximate transaction costs for option contracts.

Even with simplifying assumptions regarding transaction and conveyance costs, Michelsen and Young's method requires input parameters that can be difficult to obtain. The expected exercise cost of the option, for instance, is modeled as the product of the exercise cost of the option (E) and the annual probability of exercising the option (P). E represents the farmer's lost benefits from giving up water during a dry year in which the option is used. This is a dollar amount negotiated between the participants but can be estimated with statistical agricultural production and input-valuation techniques. However, the data requirements for this type of analysis are quite high.

Parameter P also presents problems. Rather than making predictions about the option buyer's behavior, Michelsen and Young conducted a sensitivity analysis to examine how changes in P affected their simulated PVOB results. As expected given the underlying structure of the PVOB computation, the value of the option decreased as the number of exercises increased.

The water option contract valuation literature through the early 1990's drew most heavily from its roots in agricultural economics, hence the reliance on agricultural enter-

prise and related models. Michelsen's articulation in 1988 of a formal framework for valuing and modeling option contracts for water was a breakthrough in the field. Though Michelsen and Young represented the state of the art at the time, they and their colleagues in water option research had yet to make a connection to valuation methodologies in finance. Watters (1995) took the leap in her unpublished dissertation at the University of California-Riverside.

Watters's express purpose is to use financial models to analyze water transfer contracts in Southern California. She focuses on binomial tree estimation for discrete time analysis and the Black-Scholes formulation for continuous time analysis of the Palo Verde, Dudley Ridge, and Areias Farms contracts with the MWD. She translates the detailed contract documents into relevant option pricing parameters and uses price elasticity of demand and hydrologic data to construct a volatility estimate for water returns. While her approach is a notable advance in applying financial theories to option contracts for water, my research extends her approach.

Howitt (1998) focuses on the role of uncertainty as he builds on Watters's research.[1] He pinpoints two types of risk that hamper development of robust water markets: supply uncertainty and price uncertainty. Supply uncertainty refers to the stochastic nature of the quantity of water that will be available from a certain water right at a specific future time. Price uncertainty is the unknown future value of water, given a supply quantity. Option contracts for water, Howitt asserts, can "spread the supply and price risk between the current water user and the potential buyer and thus stimulate a fuller and more efficient market" (p. 127).

[1]Howitt cites Watters (1995) as the only available research applying financial theory to water options.

The option pricing model that Howitt presents is exactly the Black-Scholes formulation. Howitt's translation of the Black-Scholes valuation process into water market terms is articulate but some grounding in empirical evidence would strengthen his argument in favor of wholesale adoption of the Black-Scholes methodology. In addition, his analysis does not allow for one of the distinguishing characteristics of water options: multiple exercise.

My research is inspired by Michelsen and Young and extends the work presented by Watters and Howitt. The option valuation approach in this chapter begins with the Black-Scholes formulation as Howitt and Watters do. However, my contribution is distinct from Howitt's and Watters's in several respects. Though treatments before Howitt's uniformly position agricultural water users as the option writers and municipal or utility water users as the buyers, my model makes no assumption about the identity of derivative market participants. To a greater degree than my predecessors did, I explicitly identify and evaluate the Black-Scholes assumptions and those of my own model. And like Watters, I draw on empirical data to determine parameter values and use a computational algorithm to find option prices.

The fundamental innovation of my work, setting it apart from both the financial economics literature and the water option contract research, is the procedure's ability to accommodate multiple-exercise options. The multi-year, multiple-use structure that many option contracts for water exhibit, as described previously, demands such an extension of pricing methodology. Moreover, my multiple-exercise pricing program is not dependent on the context of water options in any way and can be used in other financial engineering applications.

4.2 Multiple-Exercise Option General Framework

The replicating hedge portfolio at the heart of the Black-Scholes option pricing model is also the foundation of contingent-claims analysis, an extension of Black-Scholes techniques used to value investments opportunities, or "real options." While contingent-claims analysis is not an attractive approach for water markets, Dixit and Pindyck (1994) present dynamic programming as an analytical approach for valuing "real options," or investment opportunities. Dynamic programming and contingent-claims analysis yield identical results in many scenarios but require different assumptions about discount rates and financial markets. For valuing options in water markets, the different assumptions of the two techniques are a key distinction.

Dixit and Pindyck's dynamic programming approach is simply standard theory cast in a framework of investment decision-making with uncertainty and irreversibility. The model allows for finite or infinite planning horizons, both discrete-time and continuous-time analysis, and a range of stochastic processes that may drive uncertainty. There are two primary drawbacks to the dynamic programming approach, one practical and one theoretical. On the practical side, computational requirements for solving dynamic programming systems can become onerous; this concern is mitigated by access to sufficiently powerful computers and efficient solution algorithms. More troubling is dynamic programming's treatment of the discount rate as an exogenous variable. The selected discount rate should reflect the risk inherent in the investment opportunity under scrutiny, but quantifying the risk in terms of a discount rate is an imprecise endeavor. The capital asset pricing model is a common tool for determining this discount rate; I explain and use it in Part II of this

thesis.

The decision of whether and when to exercise a multiple-exercise option is a variation on the common problem of dynamic programming: optimizing the sum of current and expected future rewards. The option holder knows that he may exercise the call option once per period, up to $\bar{R}$ times in m periods. The option contract also specifies the strike price, K. The holder knows the current price of the asset, S_t, but must estimate next period's price based on assumptions about the future price path. Exercising the option today yields a payoff equal to the difference between S_t and K while the payoff from exercising in later periods is uncertain. In contrast, foregoing use of the option today earns the option holder zero monetary reward in the current period but preserves the exercise right for possible use in the future. After the contract expires at the end of period m, however, any exercises of the option that remain unused are worth nothing.

In each period during the life of the contract, then, the option holder must decide whether or not to strike. Though he knows the immediate reward from striking in the current period, $(S_t - K)$, his future payoffs depend on the realized price of the asset in each future period and on his option use history. Since he cannot exceed the maximum number of exercises for his contract, striking today has an irreversible and limiting effect on his ability to secure higher payoffs later. Uncertainty, irreversibility, and the importance of timing characterize this derivative holder's decision, making it an excellent candidate for a dynamic programming solution algorithm.

4.2.1 Standard Dynamic Programming Formulation

Translating this multiple-exercise option-pricing problem into a standard dynamic programming formulation requires one control variable and two state variables. The control variable x_t represents the decision of whether to strike ($x_t = 1$) or hold ($x_t = 0$) in the current time period. The two states are P_t, the price of water in the current period, and R_t, the number of option exercises left in the contract. More formally,

$$\begin{aligned} P_t &\in \mathcal{S}_1 = \left[\tilde{P}, \hat{P}\right], \\ R_t &\in \mathcal{S}_2 = \{0, 1, 2, \ldots, \bar{R}\}, \\ x_t &\in \mathcal{X} = \{0, 1\}, \text{ and} \\ t &\in \{0, 1, 2, \ldots m\}. \end{aligned}$$

Here, $\tilde{P}$ and $\hat{P}$ are lower and upper limits respectively on the price, parameters of the problem. $\bar{R}$ is the maximum number of strikes during the life of the contract. Note that since the option cannot be used after the maximum number of exercises is attained, $x_t = 0$ necessarily when $R_t = 0$.

The state variables both change from period to period but their dynamics are quite distinct. The number of exercises remaining decreases whenever the option buyer strikes, evolving according to:

$$R_{t+1} = R_t - x_t. \tag{4.1}$$

The state equation that describes the price path for water is more complicated, an Ornstein-Uhlenbeck mean-reverting process:

$$dP = \eta\left(\bar{P} - P\right) dt + \sigma dP, \tag{4.2}$$

with η and $\bar{P}$ denoting the speed of reversion and mean-reverting price level.[2] These parameters are estimated from price data as described in Chapter 6.

The objective is to maximize, over the life of the contract, the present value of the sum of expected returns from the option by making the optimal decision each period regarding whether or not to exercise,

$$\max_{x\in\{0,1\}} \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t x_t \left(P_t - K\right)$$

subject to the state equations above, non-negativity of P, K, and R, and the following constraint:

$$\sum_{t=0}^{m} x_t \leq \bar{R}.$$

For convenience, the strike price may be normalized to 1.

In a given period the reward function is a function of the two state variables, asset price, and number of uses remaining:

$$f(x_t, p_t) = x_t \left(P_t - K\right). \tag{4.3}$$

For this problem the value function $V_t(P_t, R_t)$ is the value of the option at time t if the price of water is P_t and there are R_t uses of the call remaining. The value function must satisfy Bellman's equation, balancing the immediate payoff from exercising the option against the present value of expected payoffs from using the option in future periods:

$$V_t(P_t, R_t) = \max_{x\in\{0,1\}} \left\{ x_t \left(P_t - K\right) + \frac{1}{1+r} E\left[V_{t+1}\left(P_{t+1}, R_{t+1}\right)\right] \right\}. \tag{4.4}$$

[2]The conceptual model is not limited to a mean-reverting price process. Geometric brownian motion or other stochastic processes can be incorporated.

The boundary condition needed to solve the dynamic programming problem arises from the terms of the option contract since the value function equals zero with certainty for $t > m$:

$$V_t(P_t, R_t) = 0 \text{ for } t > m.$$

4.3 The Computerized Solution Algorithm

The computerized solution algorithm presented in this thesis builds directly on the standard dynamic programming model described in section 4.2. Three of the parameters my Matlab program requires are identical to those used in the Black-Scholes pricing formula: length of option life in years, option strike price, and annual interest rate. Three more inputs are necessary because I treat price as a discrete variable, and because my approach computes option prices for a range of possible initial prices: the number of discretization points for price, and the upper and lower limits on price. The program uses four parameters to characterize the mean-reverting price process for water: the speed of reversion, the mean-reverting price level, and the mean and standard deviation of the price shocks. The final parameter is dictated by the nature of the options the program values: maximum number of exercises (strikes) during the life of the option. Thus I must provide eleven parameters in all.

The valuation program generates four outputs: a vector of price nodes, a multi-dimensional array of option contract values, a multi-dimensional array of optimal policies, and a transition probability matrix for expected future prices. The first output is simply the vector of discretized price nodes. The three-dimensional array of option contract values includes one page for each possible exercise of the option. Each page of the multi-

dimensional array has a row for each time period and a column for each price state. Thus the researcher can determine, for instance, the value of having three strikes remaining when the option has four years left, for any possible starting price level for water. The array of optimal policies has the same dimensions as the array of contract values. Each element of the policy array records whether or not the option holder should strike, given the current states and time period. A value of 1 indicates that the best decision is to strike; 0 signifies holding the option for another period. Finally, the transition probability matrix has a column and a row for each price. For any current price of water the transition matrix shows the probability that each of the other price states will be realized next period.

The solution methodology entails five main steps: 1) defining the price state space, 2) creating the transition probability matrix, 3) defining the problem size, 4) finding the value of the option in the final time period, and 5) solving the model using backward recursion. The program values the multiple-exercise option for each initial water price in the specified range. The first few lines of code define and discretize the price state.

For each element of the price space, the transition matrix assigns probabilities to every possible price for next period. These probabilities are determined by the underlying stochastic price process, given in equation (4.2). I begin constructing the transition probability matrix by computing the increment between each price and every other price in the discretized price space. This is not a matter of simple subtraction, however, since I seek the value of the *stochastic* portion of the price change. The discrete-time Ornstein-Uhlenbeck process is:

$$P_{t+1} - P_t = \bar{P}(1 - e^{-\eta}) + (e^{-\eta} - 1)P_t + \epsilon_{t+1}. \tag{4.5}$$

Rearranging,

$$\epsilon_{t+1} = P_{t+1} - \bar{P} + e^{-\eta}(\bar{P} - P_t). \tag{4.6}$$

As Dixit and Pindyck note (p. 76), these increments are normally distributed with mean of zero and standard deviation that can be approximated from data. The probabilities in the transition matrix are the probabilities associated with each of the increments, under a normal distribution with mean zero and standard deviation specified as a parameter to the valuation program.

In step four of the program I define the problem size. To do so, I construct two multi-dimensional arrays with one page for each possible strike. Each page is a matrix with one row for each time period including the initial period and one column for each value of the discretized price space. Recall that the program allows for exercise in period zero. These arrays will house the option contract values and optimal exercise policies.

Computing the value of the option contract in the terminal period is straightforward. Regardless of whether one strike or multiple strikes remain in the last period, only one can be used. Thus, if $P_T > 0$, the undiscounted value of the contract in year T is

$$\max(P_T - K, 0),$$

where P_T is any price in the price space. Determining the terminal value of the option enables the dynamic programming algorithm to solve the multiple-exercise option-valuation program recursively.

The dynamic programming algorithm builds on the values established for the option contract for all states in the final period. First, the program determines the contract value for all time periods prior to the last one, for all possible price states but when only one exercise of the option remains. The equation governing this step is:

$$V_t(P_t, 1) = \max\left\{(P_t - K), \frac{1}{1+r}E\left[V_{t+1}\left(P_{t+1}, 1\right)\right]\right\}. \tag{4.7}$$

Then the program iterates from two exercises remaining through $\bar{\mathrm{R}}$ uses remaining to complete the option contract valuation. The code for this procedure parallels the Bellman equation, equation (4.4). The complete text of the Matlab program is presented in Figure 4.1.

4.4 Assumptions of the Solution Approximation

Assumptions that are designed to simplify a complex reality so that it can be simulated and modeled are familiar in economics. Of course, this multiple-exercise option-pricing model and the computer code that approximates it also rely on a host of assumptions. Some are clearly technical, some are more fundamental, and they vary in the degree of difficulty entailed in relaxing them. I group the assumptions into four categories based on their primary emphasis: 1) structures and terms of the option contract, 2) technical assumptions, 3) transition process from state to state, and 4) assumptions that mimic Black-Scholes.

The assumptions about the structure of the option contract are generally not restrictive and are easy to alter. The option can be exercised at most once in a given time

Step 1: Create the price space

```
p=linspace(a,b,n)';
insert=max(find(p<k));
p=[p(1:insert);k;p(insert+1:end)];
n=size(p,1);
```

Step 2: Create the transition probability matrix

```
increments=zeros(n,n);
for jj=1:n;
    increments(:,jj)=p-xbar+exp(-eta)*(xbar-p(jj,:));
end
incrcdf = normcdf(increments,mu,sigma);
W=zeros(n,n);
for kk=2:n;
  W(kk,:)=incrcdf(kk,:)-incrcdf(kk-1,:);
end
total=sum(W);
W(1,:)=1-total;
```

Step 3: Define the problem size.

```
V=zeros(T+1,n,Rmax);
X=zeros(T+1,n,Rmax);
nostrike=zeros(1,n);
R=(1/(1+r));
```

Step 4: Find value of option contract in last period, for all states

```
for mm=1:Rmax;
        [V(end,:,mm),X(end,:,mm)]=max([nostrike; p'-k],[],1);
end
```

Step 5: Solve the model using backward recursion.

```
for t=T:-1:1;
  [V(t,:,1),X(t,:,1)]=max([R*V(t+1,:,1)*W; p'-k],[],1);
  for nn=2:Rmax;
     [V(t,:,nn),X(t,:,nn)]=max([R*V(t+1,:,nn)*W; p'-k+R*V(t+1,:,nn-1)*W],[],1);
  end
end
X=X-1;
```

Figure 4.1: Matlab Code for Valuing a Multiple-Exercise Option

period. The total number of uses of the option must be strictly greater than zero. The number of periods in the life of the option contract is strictly greater than the total number of uses allowed. First strike can occur immediately, in period 0. Finally, the strike price is constant across time periods. The strike price assumption would not be difficult to change. This would entail re-working a few lines of code to create and use a vector of strike prices, one for each time period, rather than a single exercise price.

The technical assumptions facilitate the dynamic programming solution algorithm. Possibly the strongest assumption in this area is treating price as a discrete variable. The program divides the price state into equal intervals; the upper and lower limits on log price and the number of discretization point are parameters of the program. The actual number of discretization points is one more than the specified parameter to ensure that the strike price is included. Using a discrete price space, rather than modeling price as continuous, more accurately represents actual price movements in water markets.

My formulation of the multiple-exercise option pricing problem as a dynamic programming exercise presupposes that the price of water evolves according to a Markov process. The mean-reverting price process in equation (4.2), which I present in more detail in Chapter 6, retains the Markov property inherent in the Black-Scholes assumption of geometric brownian motion. Specifically, the probability distribution of future price depends only on the currently observed price and the action the call holder chooses:

$$\Pr(P_{t+1} = \hat{P} \mid x_t = x, P_t = P, R_t = R) = \Pr(\hat{P} \mid x, P, R).$$

The future price of water does not depend on the price history beyond the current price. The other state variable, the number of remaining uses, is deterministic; its value is known

with certainty once the current price and action are revealed.

The computer program rests on an additional requirement for the stochastic process driving the price of water: price evolves according to the simple mean-reverting process of equation (4.2). While the theoretical model I developed can accommodate other stochastic price processes, my Matlab implementation of the model uses mean reversion. In Chapter 6 I discuss the empirical justification for my choice of mean reversion, and compare model results for mean reversion and geometric brownian motion.

The remaining notable assumptions of the model mimic those of Black and Scholes: no transaction costs or taxes apply and no arbitrage opportunities exist. Because water that is traded must be transferred using available infrastructure, and due to the difficulty of coordinating potential buyers and sellers, arbitrage opportunities may arise. Transaction costs are likely present, too, as discussed in Chapter 2.

4.5 Beyond the Model: The Need for Reliable Parameters

The usefulness of this model's results depends not only on a carefully constructed theoretical and computational framework, but on the accuracy of its parameters. The specified values for the mean and speed of reversion of the price process must closely represent the true path of water price for the model to serve as a useful tool in water management and policy decision-making. Augmenting the preceding theoretical and algorithmic analysis with empirical evaluation, as illustrated in the next two chapters, will: 1) ensure high quality inputs to increase the probability of high quality output, 2) test the assumptions about the stochastic process for water price, 3) suggest a more appropriate price process,

and 4) perform comparative statics analysis to discern to which parameters the results are most sensitive.

Part II

Pricing Options for Water in the Texas Rio Grande

Chapter 5

Institutional and Economic Context of Water Allocation in Texas

Part I of this thesis describes the multiple-exercise option-pricing problem, places it in the context of existing literature in financial economics and natural resource economics, and presents a computerized algorithm for solving it. The two chapters that begin Part II build on this theoretical and computational foundation, using data from a water market in Texas to compute parameters for the pricing model. Here I provide an overview of water markets in Texas and describe the specific market setting that generated the dataset. Intertwined with this narrative is a brief review of the massive body of research on water markets. Finally, I summarize the dataset itself in preparation for the empirical analysis of Chapter 6.

5.1 General Description of Water Use in Texas

Compared to other states, Texas uses a large volume of water each year. According to estimates by the U.S. Geological Survey (USGS), Texas is second only to California in the number of gallons of groundwater and freshwater withdrawn each day (Solley *et al.*, 1998). Texas is also second to California in consumptive use of freshwater, where consumptive use is "that part of water withdrawn that is evaporated, transpired, or incorporated into products or crops" (p. 4). Of course this level of water use is in part due to the states' populations: again, Texas (18.7 million) falls behind only California (32.1 million). Taking population into account gives an alternate view of water use in Texas: the state ranks 24th highest in per capita freshwater withdrawals compared to the other 49 states, Puerto Rico, and the U.S. Virgin Islands. Offstream water use, conveyance loss, and consumptive use data for Texas and selected states appear in Table 5.1 and Table 5.2. All data are condensed from Solley *et al.* (1998; page 9).

State	Population (000s)	Per Capita Use (freshwater, gal/d)	Groundwater Withdrawals (Mgal/d)	Surface Water Withdrawals (Mgal/d)	Total Withdrawals (Mgal/d)
Arizona	4218	1620	2840	3990	6830
California	32063	1130	14700	31300	45900
Colorado	3747	3690	2270	1160	13800
Florida	14166	509	4340	13800	18200
Georgia	7201	799	1190	4630	5820
Idaho	1163	13000	2830	12300	15100
Minnesota	4610	736	714	2680	3390
Nevada	1530	1480	896	1400	2300
N. Carolina	7195	1070	535	8750	9290
Texas	18724	1300	8780	20800	29600
Utah	1951	2200	790	3670	4460
Washington	5431	1620	1760	7100	8860

Table 5.1: Total Offstream Water Use, Selected States, 1995

State	Conveyance Loss (Mgal/d)	Consumptive Use (freshwater, Mgal/d)
Arizona	1030	3830
California	1670	25500
Colorado	3770	5230
Florida	32	2780
Georgia	0	1170
Idaho	5480	4340
Minnesota	0	417
Nevada	473	1340
North Carolina	0	713
Texas	540	10500
Utah	612	2200
Washington	1090	3080

Table 5.2: Conveyance Loss and Consumptive Use, Selected States, 1995

Gollehon and Quinby (2000) use a set of Federal data sources to examine the role that irrigation plays in agricultural production in the Western United States.[1] They note that nearly 75% of freshwater withdrawals in the West are for agricultural uses. Texas has the fourth largest volume of withdrawals for irrigation of the nineteen Western states; more than half of this comes from groundwater.

The USGS estimates provide the details behind Gollehon and Quinby's general findings. Table 5.3 divides freshwater withdrawals in Texas into water use categories and specifies the proportion of freshwater for each use. Together, irrigation and thermoelectric power generation comprise more than three-quarters of freshwater withdrawals in Texas.

[1]Gollehon and Quinby (2000) consider the following nineteen states part of the West: TX, OK, KS, NE, SD, ND, AZ, NM, CO, UT, NV, WY, ID, MT, CA, OR, WA, AK, and HI.

	Withdrawals (Mgal/d)	Percentage of all freshwater withdrawals	Ranking versus all other states	Percent of withdrawal that is from surface water
Public supply	3290	13.5%	2nd highest	65.7%
Domestic	130	0.5%	7th	0.0%
Commercial	44	0.2%	13th	25.0%
Irrigation	9450	38.9%	4th	30.9%
Livestock	315	1.3%	6th	55.9%
Industrial	1300	5.4%	6th	82.3%
Mining	211	0.8%	4th	39.3%
Thermoelectric	9590	39.5%	2nd (IL)	99.4%
Total withdrawals	24300	100.1%	2nd (CA)	65.8%

Table 5.3: Freshwater Withdrawals in Texas, 1995

5.2 Development of Water Markets in Texas

In a 1969 article, Trock called for policy revision in Texas to allow trading of water rights. He argued, "a change or clarification of water law to permit purchase and sale of rights would facilitate exchange so that water would be used in higher value uses" (p. 1365). This, he said, would benefit individual users and promote land and water development in the Lower Rio Grande Valley of Texas. One impediment to water transfers that he identified, aside from the question of their legality, was poor definition of water rights. Tregarthen (1973) highlighted similar difficulties in Colorado: uncertainty in water rights, counterproductive incentives to overuse water arising from the tenet of beneficial use, and time-consuming, costly adjudication of water disputes by the courts.

Apparently Trock's analysis of obstacles to water transfers in the Lower Rio Grande was at least partly correct: water marketing became increasingly common there after water

rights were carefully defined in the Water Rights Adjudication Act in the late 1960s (Chang and Griffin, 1992). Chang and Griffin offer a concise yet informative history of water market and water rights law in Texas since 1600, during the Spanish Settlement. Focusing specifically on surface water transfers, they also analyze water use trends, water right sales, and water rentals. According to their results the Lower Rio Grande region's water market was more active than any other in Texas in 1992. Furthermore, more than 94 million cubic meters (nearly 75,000 acre-feet) of water rights were transferred from agriculture to municipalities in 152 transactions in the previous 20 years (p. 885). This accounted for 94% of the total volume in all transfers during that period. Chang and Griffin also note that the rental market was responsive to changes in climate and water availability.

A related paper by Griffin and Boadu (1992) expands the conceptual framework of water markets in Texas to include groundwater sources. This piece also highlights recommended refinements to Texas water law to accommodate concerns about return flow externalities, maintaining instream flows, and unintended secondary economic effects of water marketing. These issues were common in water markets at the time and persist even now. Howe (1998) presents an analysis of Colorado water law and markets that echoes and elaborates on many of the key points raised in Chang and Griffin, and Griffin and Boadu.

As of 1998, Griffin assesses the state of markets for water in Texas, writing "In some noteworthy situations, Texas has harnessed the positive incentives created by water markets...In other ways, however, Texas has failed to employ water rights that support efficient water allocations" (p. 51). Concerns about water quality, adequacy of instream flows, and groundwater exhaustion remain in Texas and can hamper water marketing. Sur-

face markets are rare, except in the Rio Grande Valley. Where they do exist, surface water markets have few or infrequent trades and participants. Griffin attributes this water market thinness to four factors: loose enforcement of water rights, easy availability of groundwater as a substitute for surface water, adequate water to meet existing uses, and tight control of water supply by a single regional institution. Characklis *et al.* (1999) builds on Griffin's 1998 work and examines the role of water leasing as a tool for managing water under drought conditions.

Conditions necessary for effective water markets and suitability of markets as a mechanism for allocating water are common themes in the literature. Though none of the following pieces considers Texas specifically, the insights of the authors still apply. Frohlich and Oppenheimer (1995) articulate a thought-provoking, rather philosophical critique of economic efficiency as the only consideration in allocating water. Many other authors bypass philosophical analysis in favor of an institutional, economic, or legal perspective; see, for instance, Howe *et al.* (1986a), Brajer *et al.* (1989), Gould *et al.* (1989), and Easter *et al.* (1999). Thompson (1997) focuses on the transition from a non-market system to market-based water allocation, highlighting legal, organizational, and societal obstacles to water market development.

5.3 Some Themes that Emerge from the Literature on Water Marketing in Texas

Though this thesis focuses on one specific mechanism for exchanging water, the nature and prevalence of option contracts for water must be shaped by the broader water

market. Economic and scientific publications examine a wide array of water market topics, too many and too tangential to describe exhaustively here. Vaux and Howitt (1984) develop an early model for interregional water transfers and apply it to data from California. They show large gains from trade for both buyers and sellers but caution that such water transfers might speed groundwater depletion. Using tools from game theory, Saleth *et al.* (1991) investigate the use of multilateral bargaining models to improve efficiency in thin spot markets for water. Rhodes, Miller, and MacDonnell (1992) attempt to predict possible responses by urban water suppliers to reductions in water availability due to climate change. Finally, Dinar *et al.* (1998) discuss the role of experimental economics and computerized markets in designing and evaluating policies improving water allocation. See Appendix C for additional references from the water markets literature.

Obstacles to and concerns about the development of water markets in Texas mirror the issues that arise in other geographic areas, especially in the United States. Griffin (1998) provides an overview of the status of water markets for surface and groundwater in Texas in the late 1990s. He describes institutions, organization, and economics of surface and groundwater markets but does not delve into externality issues or other concerns. An earlier article he wrote with Boadu (1992) is a better guide to the weaknesses of surface water markets in Texas. Some of the problems Griffin and Boadu pinpoint are instream flows, return flow externalities, and secondary economic effects. Other concerns raised in the literature on Texas water markets include: effects on ground water depletion and management, implications for water quality, and transboundary interactions with Mexico.

5.3.1 Instream Flow

A New York Times article in July 2002 (Yardley, 2002) illustrates the intensity and relevance of conflict over instream flows in Texas. Environmental groups fear for the health of an estuary at the mouth of the Guadalupe River and oppose a plan to pump billions of gallons of water from the mouth of the Guadalupe to San Antonio. Under the pumping plan, the amount of fresh water flowing to the mouth of the river may not be enough to sustain the estuary, which is the winter habitat for the endangered whooping crane. Many local residents and some governmental officials, however, see the pumping plan as essential to the region's economic growth. This is a classic case of the need to balance competing demands for scarce water.

Instream flow considerations like the confrontation over the Guadalupe River have become prominent in water market discussions and analyses. In Texas, Kaiser and Binion (1998) attempt to unravel the complex tangle of institutions and laws to suggest strategies for maintaining river flows at a level sufficient for wildlife habitat, navigation, and recreation. The particular challenge in Texas is reallocation of water rights from current users to the protection of instream flows since virtually all water rights were assigned before instream values were recognized as valid. Kaiser and Binion conclude that compulsory reallocation schemes would be politically infeasible and that environmental and governmental groups must pursue "consensus-building and negotiated outcomes" (p. 193). The work of Turner and Perry (1997) assessing the trade-offs between instream and agricultural water use in Oregon may offer lessons for Texas.

A study of community beliefs and expectations surrounding a proposed water de-

velopment project in Colorado Springs, Colorado, underscores the prominence of instream flow values in the public's perception of changes in water allocation and supply (Cortese, 1999). Instream flows and other environmental resources were "among the most important amenities in mountain communities" (p. 577). Howe and Goodman (1995) and Supalla (2000) acknowledge streamflow effects on recreation and ecosystems as factors that contribute to conflicts over water allocation and markets, then suggest processes for resolving disagreements. Howe and Goodman promote water banks as the mechanism of choice, arguing that the bank could use pricing schemes to compensate third party players for their losses. Taking a different approach, Supalla draws on game theory for insight on strategies for achieving negotiated agreements for water allocation. This is more in line with Kaiser's analysis of instream flow problems in Texas.

5.3.2 Return Flow Externalities

Return flow externalities bear some relation to instream flow issues, primarily because both concepts refer to the effects of water transfers on the amount of water available for third parties. Green and Hamilton (2000) give a simple mathematical model of how an individual water user's available supply depends on other water users, in part through return flows.[2] Thus a water transfer that changes the location or use of water can alter the amount of water supplied to users not involved in the transfer by altering the location or volume of return flow. Return flow effects, however, are highly dependent on local geology and hydrology and can be difficult to predict. Gould (1989) notes that effects on return

[2]Green and Hamilton define return flow as "excess water that is diverted for use, but is not consumed. Instead it returns to the hydrologic system and is a water supply for other users" p. 208.

flows are a major obstacle to water transfer since most states require third-party effects to be identified and measured before a transfer is approved. In the opinion of Griffin and Boadu (1992), Texas water law adequately accounts for the return flow externalities of water transfers. They note, however, that the same protections are not in place for return flow effects from changes in water use *practices* that do not involve a transfer between users.

5.3.3 Secondary Economic Effects

Secondary economic or area-of-origin effects typically become a concern when market exchange transfers water from agricultural areas to urban ones. While this may cause a net economic benefit to a region or state by reallocating water to higher-valued uses, local communities that have exported the water may experience net losses. Transferring water away from agriculture directly affects the farmer-seller, who gains income from the sale of water but must usually change crop mix or reduce planted acreage in response to reduced water availability. These changes in farming practices can cause negative, rippling effects through farm communities, indirectly hurting local agricultural supply stores, processing facilities, and businesses. Howe and Goodman (1997) divide secondary economic effects into two categories: backward and forward linkages from on-farm production. Suppliers of farm inputs such as fertilizer and equipment are backward links; processors of crops and livestock are forward links. Gollehon (1999) describes case studies of the secondary economic effects of water transfers in Arizona and Colorado. Secondary economic effects were also a concern of the focus groups in Cortese's Colorado Springs study (1999).

5.3.4 Groundwater Management and Depletion

The question of whether and how surface water transfers might affect groundwater resources entered the water markets literature early on. In 1984, for instance, Vaux and Howitt observed that one of the consequences implied by their simulations of interregional water trades was accelerated depletion of groundwater reserves. Ten years later, Vaux reiterated this concern and cited evidence of groundwater overdraft in some areas of California (Vaux *et al.*, 1994).

Non-renewable groundwater use is also a central concern of an in-depth study of the Lower Colorado River Basin (Morrison *et al.*, 1996). The study estimates that each year groundwater withdrawals in the region exceed recharge by 1.24 million acre-feet. McCarl *et al.* (1999) cite a USGS survey that indicates a similar problem of pumping volumes outstripping recharge in the Edwards Aquifer in Texas.

The interactions between surface water markets and groundwater use are part of a larger problem of how to manage freshwater resources sustainably. Though an in-depth look at this issue is well beyond the scope of this research, it is important to note that decisions about surface water allocation can affect demand for groundwater. Researchers in Texas have examined pumping limits (McCarl *et al.*, 1999), structural and institutional responses (Watkins and McKinney, 1999), and groundwater markets (Griffin and Boadu, 1992) as possible policies for groundwater management. Griffin and Boadu emphasize the need for coordinated, or "conjunctive", management of surface and groundwater resources in Texas. They suggest that the state draw on its "positive experience with surface water markets" to design effective groundwater laws and policies (p. 285).

5.3.5 Transboundary Water Issues

Water resources do not follow the dictates of state or national boundaries. The complexities and challenges of water allocation are amplified when legal, political, cultural, and economic differences come into play. Even when water markets are proposed between neighboring states in the same country, incompatible laws and institutions often pose significant barriers to water transfers. Huffman (1997) examines interstate water marketing in the U.S., focusing on legal and institutional impediments at both the state and federal levels. In contrast, Sturgess (1997) presents the case of two Australian states that have overcome a history of disagreement to successfully create an intergovernmental forum for managing the Murray River.

Taking analysis of transboundary water resource management to the nation-state level, Blatter and Ingram (2000) evaluate two models: market-based and nation-based allocation. They find both paradigms lacking and suggest "effective governance of fluid resources is increasingly and necessarily founded on the cooperative interrelationships of various institutions" (p. 439). This conclusion provides little guidance for addressing the Texas-Mexico transboundary issues identified by Armstrong (1982): accurate streamflow information, careful analysis of existing hydrologic and water quality data, basin-wide planning that considers both Mexico's and Texas' perspectives, and continued attention to water use in times of shortage. Griffin's more recent (1998) observation on transboundary water management stems from the Rio Grande Valley drought in 1996. Texas and Mexico share the benefits and responsibilities of the Rio Grande basin which forms the border between them. The water market along the border will remain workable, in Griffin's estimation,

only if governments on both sides of the river enforce and protect property rights to water.

5.4 Evidence for Consideration of Dry-Year Option Contracts Outside California

Though the decided majority of option contracts for water that have been successfully negotiated are located in the state of California, there is substantial evidence that this allocation mechanism is relevant elsewhere in the western and southern U.S. In their general analyses of the current and future configurations of water markets, MacDonnell and Rice (1993), Whittlesey and Huffaker (1995), and Howitt (1998) all advocate or predict an increased role for option contracts. Options and forward contracts seem "likely to be a growing trend given the advantage of sharing the future uncertainty" (Howitt, 1998; p. 136). Halverson (1990) advises regional power planners in the Pacific Northwest to explore the possibility of option-like water contracts for hydropower generation. Researchers at Texas A&M University have been exploring dry-year options as a means for balancing competing water demands in the Edwards Aquifer (Texas Water Resources Institute, 1996). Another investigation of water supply possibilities in the Edwards Aquifer included option contracts as a supply alternative in its simulations (Watkins *et al.*, 1999).

Option contracts are also a topic of discussion in Colorado. A report to the Northern Colorado Water Conservancy District (Pinnes, 1994) outlines possible structures, obstacles in negotiation, third-party impacts, and feasibility of option contracts in northeastern Colorado. Howe (1997) cites options and futures markets as water allocation mechanisms to explore, though he places them primarily in the context of water banks. "Interruptible

supply agreements" are one of five water supply opportunities examined for the Denver area, as described in the Metropolitan Water Supply Investigation Final Report (Rozaklis and Weaver, 1999). The report acknowledges that this allocation mechanism holds promise but that political and infrastructure considerations limit its immediate viability. Finally, an observation in a later report by Pinnes (1996) sets the stage perfectly for my research:

> "Because there has been no active market in interruptible supply contracts involving [Colorado-Big Thompson water], there is no 'market price' for this type of transaction. A computer program or model to assist in making price determinations could be helpful to both potential buyers and sellers."

5.5 The Texas Rio Grande Spot Market for Water

According to the U.S. Geological Survey, the Rio Grande River is one of our nation's longest rivers (USGS, 2000). From its headwaters in the Rocky Mountains of Colorado it flows approximately 3000 kilometers through New Mexico and along the Texas-Mexico border. The mouth of the river is in the Gulf of Mexico, west of Brownsville, Texas. Armstrong (1982) presents an excellent though dated summary of the geology, climate, distribution of surface water, presence of groundwater, and water usage of the Rio Grande basin of the U.S.-Mexico border region. He notes that, from 1931-1960, El Paso's average annual precipitation was less than 8 inches (203 mm). At the opposite end of the Texas Rio Grande, Brownsville averaged 30 inches (762 mm) of precipitation per year. More recently, Chang and Griffin (1992) cite a projected 54 percent increase in population in the 1990s in the four southernmost Texas counties along the Rio Grande (Starr, Hidalgo, Willacy, and Cameron.) These same four counties irrigated two-thirds of their agricultural lands in the mid-1980s, producing sugar cane, citrus crops, and both deep and shallow root

vegetables on irrigated acreage.

Use of surface water in Texas is carefully regulated by state government agencies. The Rio Grande Watermaster Office (RGW), an arm of the Texas Natural Resource Conservation Commission (TNRCC), monitors water use and availability, coordinates water transfers, and manages reservoirs in the Rio Grande basin. Carlos Rubinstein, the current Rio Grande Watermaster, has jurisdiction over 16 Texas counties in the Rio Grande basin, as shown in the map in Appendix D (TNRCC, 2002).[3] This area extends from Fort Quitman in Hudspeth county to the mouth of the river at the Gulf of Mexico.[4] Two reservoirs are included in this jurisdiction: the Amistad and Falcon reservoirs.

The executive director of the TNRCC has the power to appoint watermasters in Texas.[5] Each watermaster reports to the executive director and may serve in the watermaster position until removed by the executive director; there is no term limit. The foremost duty of the water master is to "divide the water of the streams or other sources of supply of his division in accordance with the adjudicated water right" (Texas Water Code, 2003). Thus the watermaster and his office are responsible for implementing the water rights regime established in the courts and in other branches of the TNRCC. The watermaster regulates water distribution, approves and implements short-term transfers, monitors water usage to ensure that users do not exceed their allocations, allocates water during shortages, and can penalize users who violate the terms of their rights.

The executive director also appoints a Watermaster Advisory Committee for each

[3]The counties are: Brewster, Cameron, Dimmit, Edwards, Hidalgo, Hudspeth, Davis, Jim Hogg, Kinney, Maverick, Presidio, Starr, Terrell, Verde, Webb, Willacy, and Zapata.

[4]Water use along the Rio Grande north of Fort Quitman is governed by the Rio Grande Compact.

[5]See Chapter 11 of the Texas Water Code for further legal details regarding watermasters. The Water Code is available at: http://www.capitol.state.tx.us/statutes/wa/wa0001100toc.html.

watermaster, consisting of nine to fifteen water right holders in the watermaster's territory. According to the TNRCC website (TNRCC, 2002), the advisory committee reviews the watermaster's annual operations budget and suggests procedures and activities that may benefit local water right holders. The advisory committee meets infrequently and members receive no compensation for participating.

Water rights in the RGW territory vary depending on location in the Rio Grande basin and are governed by the regulations detailed in Chapter 11 of the Texas Water Code. This description of water rights in the Rio Grande basin draws primarily on the TNRCC website (TNRCC, 2002). Upstream from Amistad Reservoir, water rights are based on a prior appropriation system as in the rest of the state. Water rights downstream from Amistad are supplied by both Amistad and Falcon reservoirs and are based on an account system. The TNRCC watermaster program description likens this water rights scheme to "a bank account with a constantly changing balance." The total annual allotment for all uses in the Falcon-Amistad system is roughly 225,000 acre-feet. In the Lower Rio Grande, unlike in the rest of the state, municipal and domestic water uses are prioritized over agricultural use in accordance with a 1970 water management plan approved by the Texas courts. Each year municipal account balances are re-set to the total amount authorized by the city's water rights. Agricultural users, however, receive a monthly allocation based on the volume of unallocated water within the Falcon-Amistad system and available to the U.S. under a treaty with Mexico. When the water is used, the irrigation account balance is adjusted accordingly.

Yoskowitz (1999) presents three categories of water trades along the Texas Rio

Grande: permanent exchange of water rights, term transfers of a specific quantity of water over a stated time period, and one-time spot market transfers of a quantity of water between two users. In Texas, Yoskowitz explains, term and spot transactions are lumped together in one category. Carlos Rubinstein confirms that his office makes no record-keeping distinction between spot and lease transfers and that prices and uses for these two types of transactions are the same (Rubinstein, 2003). Permanent transfer prices ranged between $800 and $1000 per acre-foot in 1998; temporary transfers brought $10 to $50 per acre-foot that year (Hines, 1998). Spot prices have exhibited sensitivity to drought conditions with prices nearly doubling during the summer of 1998 (Hines, 1998). For a discussion of water leasing procedures and restrictions in Texas, see Characklis *et al.* (1999).

The dataset of water prices for this research comes from short-term market transfers[6] in the Texas Rio Grande. Yoskowitz (1999) provides an in-depth description of the market; I summarize his observations here. The 813 active water rights in the Texas Rio Grande include both firms and individuals, and supply agricultural, municipal, industrial, and domestic uses. Water rights ownership is dominated by the agricultural sector, which holds 80% of all rights. The RGW keeps up-to-date water account balances for each right holder and can often help link potential buyers and sellers.[7] In the five-year period from 1993 to 1998, 1,504 spot transactions took place in the RGW jurisdiction. Irrigators were the dominant buyers and sellers; 1,274 of the spot market trades went to irrigation.

As noted earlier, the RGW plays several pivotal roles in facilitating and executing short-term water transfers in the Middle and Lower Rio Grande, and in the routine allo-

[6]Though these include spot and lease transactions, I will use the terms "spot markets" and "short-term markets" interchangeably.

[7]The office does not, however, charge for this service.

cation of water to users in his jurisdiction. Any time a water right holder plans to divert surface water for his own use he must alert the RGW of the time and volume of water he intends to use. The RGW and his staff verify that the water is available and that this specific user has not and will not exceed his allowed amount of water. Assuming that these two conditions are met, the RGW authorizes the withdrawal and deducts the volume of water from the user's account balance. Field staff in the RGW's office inspect diversions regularly to monitor compliance.

The RGW processes applications for temporary water rights when the amount of water is ten acre-feet or less, and when the duration of the temporary permit is one year or less. Temporary permits like this are distinct from short-term transfers since they are not transactions between two water right holders. Instead these temporary permits are allocations of surplus water, beyond the volume needed to sustain adjudicated rights. A permit of this type might be used, for instance, to provide water to a highway construction site for erosion control and landscaping (TNRCC, 1996).

Short-term transfers between water right holders, including both spot transactions and year-long leasing, also channel through the RGW office. Sellers and buyers may search for each other privately or can solicit assistance from the RGW. Because of the RGW's expertise and monitoring of water rights and usage, he and his staff can often provide valuable information about potential buyers and sellers. Anecdotal data show that spot buyers and sellers try to reduce transaction costs by finding a small set of amenable transfer partners and conducting exchanges with the same users repeatedly, rather than searching anew each time (Rubinstein, 2003). Buyer and seller freely negotiate the price per acre-foot

of the transfer; the watermaster does not control or advise in this decision. The fact that the market sets the price of water is one of the distinctive features of this dataset. The other dataset I examined closely as a candidate for this research fell victim to the same weakness in other water data I have seen: prices of transactions are determined by governmental agencies or committees rather than driven by the perceived costs and benefits of the buyer and seller.

Transfer approvals are generally quick, taking 24 hours or less after submission to the RGW. The permitting process for permanent water rights, by contrast, takes four to six months even in the simplest cases (TNRCC, 2003). The water buyer and seller in the spot market specify the date the transfer should take effect. This can be any day of the week but must precede the last Saturday of the month, which is the end of the monthly water accounting period. The watermaster's office ensures that the proposed transfer does not change the water's general use category. Irrigation water, for example, can be sold only to another irrigator. Upon approving the transfer the RGW records the effective date, identities of the two parties, quantity of water, use category, and price per acre-foot. This record-keeping process provides the necessary data for my multiple-exercise option-valuation model.

5.6 The Dataset: Spot Market Transfers in the Texas Rio Grande

The dataset for my research is from the office of the Rio Grande Watermaster and represents spot market exchanges of water from January 1, 2001 through June 30,

2002. The RGW provided a text file containing a log of all approved spot market transfers from December 22, 2000 through July 23, 2002, with information on effective date and termination date of the trade, acre-feet transferred, price per acre-foot, and type of use intended for the water. In all, 565 transactions were approved: five for mining purposes, 34 for municipal use, one for industrial processes, one unclassified, and the rest for irrigation. I discarded all observations for which the price was missing (230 trades) or recorded as zero (12 cases). I eliminated the first and last months of the time period (6 cases); the data for these two months were incomplete. I also excluded the five mining transactions because their per acre-foot prices ($250 and $100) far exceeded all other exchange prices (maximum of $52.50). My final dataset thus includes 312 trades. Trades with recorded prices, excluding those for mining, had the monthly occurrence described in Table 5.4.

My interest in this dataset was piqued because the recorded prices are the outcome of private negotiation between the transferring parties. This suggests that the prices in the dataset reflect, at least in part, the buyer and seller's perceived value of the water. Other data on water prices tend to be a product of administrative processes and thus provide less information about the market than prices set by individual buyers and sellers. Figure 5.1 assesses the number of transfers in each of eight price ranges. More than five out of every ten transactions shows a price of $25.00 to $29.99 per acre-foot. This is due primarily to the dominance of $25.00 as a transfer price. With 157 transactions, $25.00 is the mode of the dataset. Still, the data show adequate variability in water prices. At least five percent of trades are in the following price ranges: $7.50-$9.99, $30.00-$34.99, $35.00-$44.99, $45.00-$49.99.

Month	Number of Transactions	Total Acre-feet Traded	Average Price per Acre-foot
January 2001	16	9290	25.78
February 2001	2	150	27.50
March 2001	16	3042	24.48
April 2001	19	1923	24.33
May 2001	6	500	25.00
June 2001	18	3384	27.09
July 2001	8	2070	23.00
August 2001	29	8467	25.07
September 2001	10	2700	29.85
October 2001	18	4515	28.28
November 2001	28	16948	24.96
December 2001	13	3181	23.92
January 2002	23	5817	30.20
February 2002	10	2002	28.45
March 2002	19	3605	29.84
April 2002	26	7265	29.80
May 2002	35	6406	30.97
June 2002	16	5490	29.67
Total	312	77464.87	27.12

Table 5.4: Monthly Transactions and Prices, Texas Rio Grande: Jan 2001-July 2002

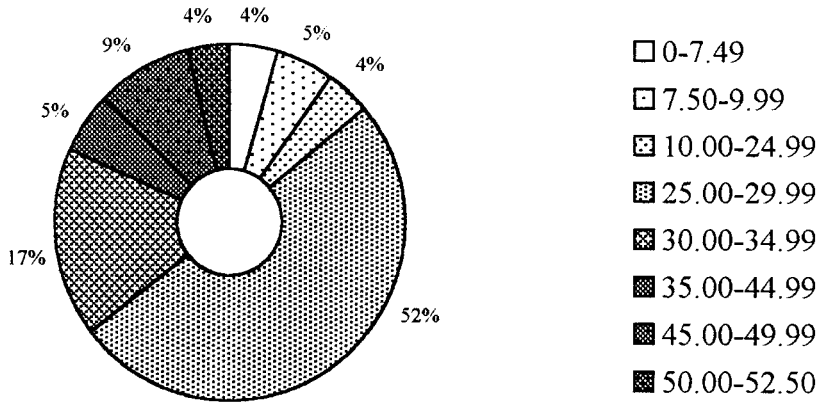

Figure 5.1: Price Ranges for Spot Transfers, As Percentage of Total Number of Transactions

Considering the price ranges as a percentage of total transaction volume changes the picture slightly, de-emphasizing the three lowest price ranges. The $25.00-$29.99 price range still dominates in Figure 5.2, constituting 47% of overall water volume. The higher price ranges figure more prominently when examining volume rather than number of transactions. This indicates that low-price transfers generally entail low volumes of water. Variability of prices is still adequate in the transaction volume perspective, an important consideration since my analysis plan requires computing volume-weighted average prices for each week in the dataset.

To facilitate later analysis of model parameters, I computed a single transaction price for each of the 78 weeks in my dataset. The weekly price is equal to the weighted average of the prices for all trades that occurred Sunday through Saturday. Each transaction's

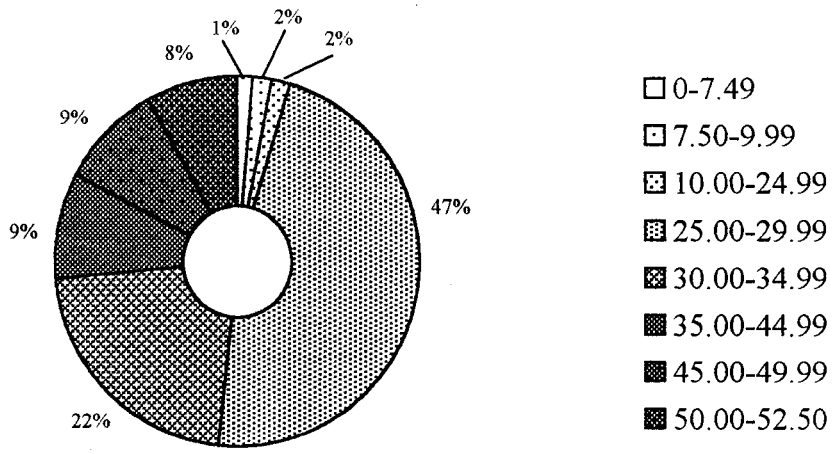

Figure 5.2: Price Ranges for Spot Transfers, As Percentage of Total Transaction Volume

price for a given week is weighted by the number of acre-feet exchanged during that trade. This resulted in 55 weeks with associated prices; 23 weeks have no price because no trade occurred in that seven-day period. A graph of the 55 weekly aggregated prices, Figure 5.3, indicates that variability in price per acre-foot is preserved though somewhat muted. Note, for example, that the weighted-average weekly price does not reach the highest price

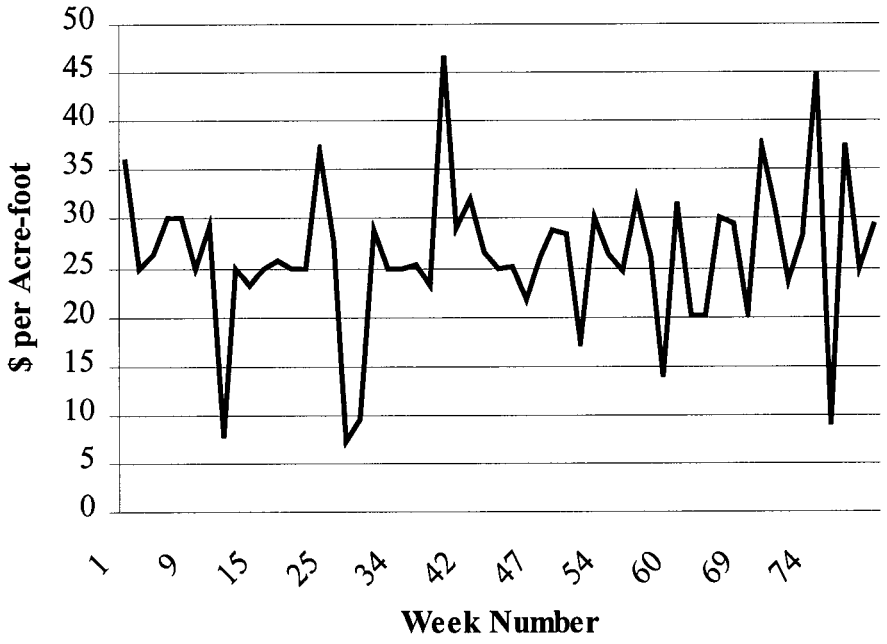

Figure 5.3: Weighted Average Weekly Price of Water

range ($50.00-$52.50). This loss of information is an inevitable result of aggregating the specifics of individual transfers into daily and then weekly indicators. The techniques I used to compute model parameters based on this price data require minimal gaps in the price series. This necessitated the conversion from individual to weekly observations.

The Texas data on short-term transfers through the RGW are an exciting platform

for my analysis. Despite the limitations of missing observations, the requisite aggregation into weekly prices, and the brief timespan of 18 months, the dataset offers several attractive features. First, from a policy-making standpoint, it is crucial that I base my analysis on information acquired from the institutions and staff who are closest to the water transfer process. In southwest Texas, the RGW is the central figure in short-term water transfers; thus he is the best source of data. Second, this dataset is more extensive and contains more observed transactions than other available datasets. By comparison, data from northeastern Colorado include a yearly price for the past fifty years. The Colorado data may supply additional insights upon further research but the Texas data are clearly better suited to my current analysis. Third, the market generating this price data is relatively mature. The prices in this dataset are not the first, experimental prices of a brand-new transfer mechanism. The RGW has been taking note of short-term transfers and prices for ten years. Unfortunately, his office has collected data systematically only since late 2000. Finally, the prices in the Texas Rio Grande dataset represent the true value of water better than prices that are set artificially. Carlos Rubinstein observes that price is "market driven" and that prices fluctuate according to water availability (Rubinstein, 2003).

Chapter 6

Empirical Valuation of Multiple-Exercise Options for Water in Texas

This chapter describes the procedures I used to compute input parameters and presents the empirical results of my research. The preceding five chapters have paved the way for this one, first laying a foundation of theory and computational processes, then adding the contextual details of water markets in Texas. The Texas dataset serves as the raw material for calculating four parameters of my multiple-exercise option valuation program: the discount rate and three parameters that describe the stochastic water price process. Determining the values of these parameters based on spot market prices for water in the Texas Rio Grande completes the set of inputs needed to implement the Matlab computer algorithm detailed in Chapter 4. This chapter addresses four core topics: computation

of parameters, initial option valuation results, analysis of option price sensitivity to key parameters, and program results under an alternative price process.

6.1 Nonsynchronous Trading in the Dataset

Time series data on asset prices exhibit nonsynchronous trading or nontrading effects when they "are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possibly irregular, lengths" (Campbell *et al.*, 1997; p. 84). Clearly, the spot market prices in my dataset are sampled at irregular intervals: some weeks and many days are void of transactions, and the time of day each transaction price was recorded is unknowable. Ignoring the nonsynchronous nature of time series data can result in biased means, variances, and other statistical measurements, and can invalidate the results of the capital asset pricing model (CAPM). Since my data undergo tests for random walk properties and are used in a CAPM setting to determine the proper discount rate, I must address nontrading effects in my analysis.

Reviewing the related literature, including Campbell *et al.* (1997), Dimson (1979), Fowler and Rorke (1983), Lo and MacKinlay (1990), and Scholes and Williams (1977), yielded an approach for accounting for the nonsynchronous effects on CAPM results but was less helpful for modifying the price process analysis procedure. I compensated for nonsynchronous trading by using different techniques in the CAPM and price process analysis. To analyze the price process for water, I first simulated values for the weeks that lack observed prices. I describe this simulation process below as I lay out the steps in my stochastic process analysis. For the purpose of CAPM, however, I used the 55 pure observations in

the Texas dataset but compensated for the nontrading effects by using a technique devised by Scholes and Williams. Again, this approach is detailed later in this chapter.

6.2 Analysis of Stochastic Water Price Process

Financial derivative pricing models based on the Black-Scholes framework typically make restrictive assumptions about how the price of the underlying asset evolves over time. My multiple-exercise option valuation methodology also requires information about the stochastic price process. However, rather than assume from the start that water prices follow a random walk, I conducted a series of tests to see whether or not water prices from Texas show this type of unpredictability. I then built the appropriate price process into my computerized implementation of the valuation algorithm. Assessing the characteristics of the stochastic process driving the price of the option contract's underlying asset supplies necessary parameters to the valuation algorithm. If the process is mean reverting, I must determine the mean-reverting price level, the speed of reversion, and the standard deviation of the stochastic term of the price process. If the stochastic process is a random walk, I must compute the volatility and trend parameters.

6.2.1 Variance Ratio Tests: Campbell, Lo, and MacKinlay Method

Variance ratio tests are a prominent empirical approach for detecting predictability over time in a dataset. Researchers have applied variance ratio tests to assess whether stock prices follow a random walk or exhibit mean reversion; Lo and MacKinlay (1999) use financial data from the U.S. while Grieb and Reyes (1999) focus on Mexico and Brazil.

Faust (1992) examines variance ratio tests and concludes that they are generally the best method for detecting mean reversion.

Campbell *et al.* (1997) outline a versatile variance ratio procedure to test price data for random walk characteristics. Recall that the typical assumption under the Black-Scholes option valuation framework is that asset prices follow geometric brownian motion. More specifically for the purpose of the variance ratio test, the stock returns are mutually independent. In the language of Campbell *et al.*, this is a random walk type 1 (RW1). Under RW1 returns are independently and identically distributed, and thus the variance of the price increments is constant over time. The important implication of this for variance ratio tests is that "the variance of random walk increments must be a linear function of the time interval" (p. 48). More precisely, as Campbell *et al.* explain, under RW1 the variance of returns in two consecutive periods must be twice the variance of the return in one period.

Capitalizing on this characteristic of RW1, Campbell *et al.* devise a statistical test for RW1 based on variance ratios (see Campbell *et al.*, 1997; pp. 48-55). Rather than limiting the analysis to two-period returns, I followed the more general treatment that allows for multi-period returns. I also incorporated the two refinements Campbell *et al.* recommended: using *overlapping* multi-period returns and correcting the variance estimators' bias prior to computing their ratio. Thus the variance ratio, VR(q), has as its numerator the variance of returns computed for overlapping timespans of q periods each. The denominator of the variance ratio is the variance of one-period returns in the dataset. Both of these variance estimators are unbiased after Campbell *et al.*'s correction procedure.

The unbiased variance estimator for one-period returns in the sample is given by

$$\bar{\sigma}_a^2 = \frac{1}{nq-1}\sum_{k=1}^{nq}(p_k - p_{k-1} - \hat{\mu})^2.$$

In this equation, nq is one less than the number of observations in the sample, and q is the number of periods in each overlapping return, a positive integer greater than 1. The log price in period k is denoted p_k, and $\hat{\mu}$ is the usual estimator of the sample mean:

$$\hat{\mu} \equiv \frac{1}{nq}\sum_{k=1}^{nq}(p_k - p_{k-1}) = \frac{1}{nq}(p_{nq} - p_0).$$

The variance estimator for the q-period overlapping returns, again adjusted by Campbell *et al.* for bias, is

$$\bar{\sigma}_c^2 = \frac{1}{nq-1}\sum_{k=q}^{nq}(p_k - p_{k-q} - q\hat{\mu})^2,$$

where

$$m \equiv q(nq - q + 1)\left(1 - \frac{q}{nq}\right).$$

The term m is the bias adjustment. Campbell *et al.* note that, even though the individual variance estimates $\bar{\sigma}_a^2$ and $\bar{\sigma}_c^2$ are unbiased, the variance ratio itself remains biased.

Using the equations for $\bar{\sigma}_a^2$, $\bar{\sigma}_c^2$, $\hat{\mu}$, and m, the variance ratio is calculated as

$$VR(q) \equiv \frac{\bar{\sigma}_c^2(q)}{\bar{\sigma}_a^2}.$$

The ratio is distributed asymptotically as

$$\sqrt{nq}\,(VR(q) - 1) \sim N\left(0, \frac{2\,(2q-1)\,(q-1)}{3q}\right).$$

I then use the standard test statistic (from Campbell *et al.*; p. 53) to test the null hypothesis

of random walk:

$$\frac{\sqrt{nq}\,(VR(q)-1)}{\sqrt{\frac{2(2q-1)(q-1)}{3q}}}. \tag{6.1}$$

When analyzing my data, then, I computed the value of the test statistic in equation (6.1). I rejected the null hypothesis of RW1 with 95% confidence when the statistic's value was outside the interval $[-1.96, 1.96]$.

6.2.2 Matlab Implementation of Variance Ratio Test

I used Matlab to implement Campbell *et al.*'s multi-period overlapping returns variance ratio test described above. The Matlab code is in Appendix E and follows Campbell *et al.*'s formulation precisely. I subsequently used this program to test the RW1 hypothesis with the Texas spot price data. To make the data usable, however, I had to address the nonsynchronous trading issue. Relevant literature on nonsynchronous trading (cited above) focuses primarily on estimating β, the CAPM coefficient, and provided little guidance on modifications for the variance ratio tests. Though Campbell *et al.* (1997) discuss variance ratios at length and nonsynchronous trading effects in detail, even they did not link the two topics. Nevertheless, non-trading effects "increase the variance of observed individual security returns that have non-zero means" (Campbell *et al.*, 1997; p. 129). Clearly this variance effect could weaken the variance ratio test results and muddy their interpretation.

My main concern about the dataset was that the missing observations in the series of weekly prices would cause misleading or inaccurate empirical results from the variance ratio test. Since I was unable to find a precedent in the literature I developed a new

technique to patch the gaps in weekly prices. I began by using linear interpolation to generate a base price value for each of the 23 weeks without trades. I computed the non-stochastic linear interpolation between the observed prices preceding and succeeding a week with no transactions. Suppose, for instance, that Week 3 was void of water trades, the price in Week 2 was $25 per acre-foot, and the price in Week 4 was $26.29 per acre-foot. The deterministic linear interpolation yields a Week 3 price of: $\frac{25+26.29}{2} = 25.64$. I filled two-week gaps by distributing the observed price change (from the week before the gap to the week after the gap) equally across the missing weeks. Upon completing the linear interpolation phase I had one full set of weekly price observations, the base dataset: 23 interpolated prices and 55 observed prices.

The weeks with interpolated data in the base dataset were qualitatively distinct from the weeks with observed data: the interpolated prices were merely an approximation of possible transfer prices rather than the outcome of actual transactions. To address this concern I decided not to accept the interpolated prices as known values. Instead, I used Matlab to generate a series of 23 random shocks from the standard normal distribution, one for each week with interpolated data. To generate a data series with simulated observations, I added the product of each random shock and the standard deviation of the observed data to the corresponding week's linear interpolated value from the base dataset.

To summarize, for a one-week gap in the original dataset, the simulated weekly price was:

$$P_i = \frac{P_{i-1} + P_{i+1}}{2} + \lambda\epsilon.$$

In this equation, i denotes the index of a week without observed trades, λ is the standard

deviation of the prices in the 55 weeks with observed prices, and ϵ is the disturbance term drawn from the standard normal distribution.

Recognizing that this dataset including simulated observations was no longer a static object, I conducted the variance ratio analysis by using this Monte Carlo method to generate thousands of shock sets at a time. Each iteration recorded the percentage of simulated datasets that could not reject the RW1 hypothesis with overlapping 2-period returns. Roughly 87% of the simulated datasets, in nine batches of 10,000 datasets each, rejected the null hypothesis: 87.2%, 87.0%, 87.5%, 87.0%, 87.5%, 86.9%, 87.3%, 87.6%, and 87.4% respectively. This overwhelming rejection of RW1 strongly implied that the standard assumption of geometric brownian motion in the underlying asset price does not fit weekly spot market water data from Texas. This result is consistent with Lo and MacKinlay's (1999) observation that weekly stock returns do not fit the random walk model.

6.2.3 An Alternative Process: Mean Reversion

Having ruled out RW1 as the stochastic process for weekly spot water prices, I turned to mean reversion. In doing so, the timespan of my dataset (18 months) became something to consider; as Dixit and Pindyck (1994) note, "it usually requires many years of data to determine with any degree of confidence whether a variable is indeed mean reverting" (p. 77). Dixit and Pindyck's assertion is made in the context of financial data, some of which is available in extremely long time series. Information on spot market prices for water is not plentiful and the organizations that track such prices have generally done so for relatively short timespans. Thus, though the ideal research plan would proceed only with two or three decades of water prices, the reality of applied natural resource

economics makes this impossible. Since the Texas data convincingly rejected the random walk hypothesis through the variance ratio tests, I proceeded to explore mean reversion despite the limitations of my dataset. I also, however, chose to more briefly examine my model's results under a random walk price process, for comparison to the mean reverting case.

In modeling the mean reverting process for water price, I use the Ornstein-Uhlenbeck process employed by Schwartz (1997) and Dixit and Pindyck (1994).[1] In this simple process the long-run price of water tends toward some specific level, rather than fluctuating as unpredictably as a random walk. The process does not have independent increments since the expected change in price depends on whether the current price is greater than or less than the mean-reverting price, $\bar{P}$. The Markov property is preserved by this process; tomorrow's price depends on today's price and a parameter, $\bar{P}$. If the speed of reversion is denoted η, the continuous-time Ornstein-Uhlenbeck process is:

$$dP = \eta\left(\bar{P} - P\right)dt + \sigma dP. \tag{6.2}$$

The discrete version of this process is equation (4.5), reprinted here for convenience:

$$P_{t+1} - P_t = \bar{P}(1 - e^{-\eta}) + (e^{-\eta} - 1)P_t + \epsilon_{t+1}. \tag{6.3}$$

Dixit and Pindyck, and Schwartz, provide equations for expected value and variance

[1] I follow Ddixit and Pindyck's representation since the notation is simpler than Schwartz's.

(Dixit and Pindyck, 1994; pp. 74-75):

$$\begin{aligned} E[P_t] &= \bar{P} + (P_0 - \bar{P})\, e^{-\eta t} \\ Var\left[P_t - \bar{P}\right] &= \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right). \end{aligned}$$

Here, the current price of water is P_0. The discrete form of the process can be estimated by OLS regression (pp. 76-77):

$$P_t - P_{t-1} = \alpha + \beta P_{t-1} + \varepsilon_t. \tag{6.4}$$

Regression coefficients, then, determine the mean-reverting process parameters:

$$\begin{aligned} \bar{P} &= \frac{-\hat{\alpha}}{\hat{\beta}} \\ \text{and } \hat{\eta} &= -\ln\left(1 + \hat{\beta}\right). \end{aligned} \tag{6.5}$$

The regression also enables me to estimate the standard deviation of the stochastic component of the price process (ϵ), which is equal to the standard error of the regression.

I used one of the datasets I generated for the variance ratio tests in the OLS regression to estimate the mean-reverting price and speed of reversion, . A simple Matlab random number generator supplied a four-digit decimal number between 0 and 1; I chose the simulated dataset in the column with that index. The chosen dataset rejected the RW1 hypothesis and therefore was not an unusual outlier.

Parameter estimation revealed reversion speed of 1.94 and long-run mean-reverting level of 25.97. The t-statistic on $\hat{\beta}$ is -7.555, clearly outside the critical interval of [-1.994, 1.994] for a sample of 70 observations.

Dixit and Pindyck (1994) and Greene (1993) suggest the importance of conducting unit root tests to help ascertain whether the data series exhibits some variety of random walk. The seminal paper in this regard is Dickey and Fuller (1981); Greene's discussion of unit root tests relies heavily on their results. With the formulation I have used for the mean-reverting stochastic price process, the appropriate Dickey-Fuller test for a unit root is implemented by testing the hypothesis that $\hat{\beta}$ equals zero (Greene, 1993; p. 564). However, the standard t test is not useful in this context and will tend to erroneously reject the null hypothesis of a unit root. Dickey and Fuller's proposed solution is to conduct a standard t-test but refer to a revised table of critical values. The critical value for a Dickey-Fuller test of the null hypothesis that $\hat{\beta}$ equals zero, with approximately 50 observations and at the 95% level, is -3.22 (Greene, 1993; p. 565). The t-statistic on $\hat{\beta}$ is -7.555. So, the hypothesis of a unit root is rejected. This is further evidence against the random walk assumption for spot market water prices in Texas.

6.3 The Capital Asset Pricing Model

The capital asset pricing model (CAPM) is a general purpose model for establishing the market price of risk, the first equilibrium model of asset pricing under uncertainty (Brennan, 1989). It is used extensively to price risk associated with assets that are not traded, or whose returns cannot be replicated directly as in Black-Scholes. At its heart CAPM quantifies the relationship between risk and return, concluding that the return required by investors on a specific asset is equal to the return on a risk-free asset plus an adjustment for risk, the risk premium. For a concise description of the model, see Copeland

and Weston (1989).

In order to calculate the risk premium for an asset or portfolio, CAPM measures how sensitive the asset's return is to market movements. This sensitivity is measured by β, the slope of the best-fit line obtained in a regression of excess return on the asset over the risk-free rate against the excess return on the market over the risk-free rate. More formally, in its simplest presentation, the pivotal equation of CAPM is:

$$\mu_j - r = \beta_j (\mu_m - r),$$

where μ_m is the expected return on the market portfolio, r is the risk-free rate of interest, and μ_j is the expected rate of return on the asset in question. In fact, β_j is the covariance of the asset return with the return on the market portfolio, normalized by the variance of the market returns. Assets with a β greater than one tend to amplify market movements; those with a β below one move in the same direction as the market but with smaller magnitude.

I model my application of CAPM in the water markets environment after the technique Scholes and Williams (1977) describe. Their procedure yields a consistent estimator of β and corrects for distortions introduced by nonsynchronous trading. Ordinary least squares (OLS) estimates of β are biased downward for very infrequently traded assets; estimates of α are biased upwards. Dimson (1979) presents an alternative method for finding unbiased and consistent estimates of β in the presence of nontrading. Fowler and Rorke (1983), however, show that Dimson's original formulation is flawed and that the corrected Dimson approach provides results identical to Scholes and Williams.

Scholes and Williams base their methodology on a simple observed market model:

$$r_{nt} = \alpha + \beta r_{mt} + \varepsilon_t,$$

where r_{nt} and r_{mt} denote the measured returns on a risky asset and the market portfolio M. They show that OLS estimation of the observed-market equation gives biased and inconsistent estimations of α and β when observed returns differ from true returns. Nontrading effects in the Texas water data guarantee that simple OLS will be insufficient for CAPM execution in my research. Instead of a single, routine OLS estimation, Scholes and Williams recommend using three distinct OLS regressions to construct β when data exhibit nonsynchronous trading. They derive the following estimator (p. 317):

$$\hat{\beta}_n \equiv \frac{b_n^- + b_n + b_n^+}{1 + 2\hat{p}_m}.$$

The variables b_n^-, b_n, and b_n^+ are the OLS estimators associated with regressing observed returns to the asset against observed returns to the market portfolio where the market returns are lagged one period, for the current period, or leading one period, respectively. The value denominator of the formula for the consistent estimator $\hat{\beta}_n$ is determined by $\hat{p}_m$, the estimated autocorrelation coefficient for the market portfolio.

Nontrading in the dataset affects the computation of returns. Any period without an observed price precludes computation of returns both in that period and in the subsequent period. The dataset used for OLS regressions in the Scholes and Williams method includes all periods for which returns can be calculated and ignores all others. For the Texas dataset, this leaves 37 weeks of data.

My implementation of the three-part Scholes and Williams methodology used returns based on weekly water prices in Texas and weekly returns to the S&P 500. I computed weekly S&P 500 returns from daily closing prices, as recorded at Economagic, an on-line dataset source (Economagic, 2002). The null hypothesis for this CAPM exercise is that

β=0. If the empirical results do not reject the null hypothesis, movements in water returns are indistinguishable from movements in market returns. Then the appropriate discount rate for my valuation model is equal to the rate of return on a risk-free asset.

Table 6.1 presents the results from my analysis of the Texas water data using the Scholes and Williams procedure. None of the individual estimates of beta is statistically

	Symbol	**Coefficient**	**t-statistic**
One -period lag	b_n^-	-4.871	-1.1718
Current period	b_n	-1.007	-0.2400
One-period lead	b_n^+	-1.055	-0.2467

Table 6.1: Scholes and Williams Estimated Betas for Texas Water Data

different from zero; I cannot reject the null hypothesis. The discount rate parameter for the multiple-exercise option valuation program is therefore the riskless rate. Since the 20-year treasury bond rate from 2002 is 0.0543 (Federal Reserve, 2003), I take this as the nominal risk-free rate for my analysis.

6.4 Valuation Results for Texas: Base Case

Using the Texas spot price dataset to compute the price process parameters for mean reversion and to determine the appropriate discount rate completes the set of parameter inputs for the multiple-exercise option valuation program. The base case parameters are listed in Table 6.2.

Parameters a and b are taken from the Texas data; T and $Rmax$ come from the MWD-Areias Farms contract. As I noted in Chapter 4, the mean of the price shocks is zero. I computed the distribution's standard deviation as described earlier, with a result of

Parameter	Description	Value
xbar	Mean-reverting price	25.969
eta	Speed of reversion	1.941
r	Annual interest rate	0.0543
mu	Mean of price shock distribution	0
sigma	Standard deviation of shocks	8.5
T	Lifespan of option (years)	15
Rmax	Maximum number of strikes	7
k	Strike price	25
n	Discretization points for price	500
a	Minimum starting price	7
b	Maximum starting price	115

Table 6.2: Parameter Values for Base Case Analysis of Texas Data Under Mean Reversion

8.485. Based on these parameters and a mean-reverting price process, the option valuation program reports the contract values in Table 6.3.

Starting Price for Water	Value in Initial Period
\$6.94 per acre-foot	\$35.54
\$25	\$36.69
\$49.89	\$61.39
\$99.92	\$116.90

Table 6.3: Option Contract Values for Base Case Texas Data, Under Mean Reversion

As Table 6.3 shows, the value of a 7/15 option contract at its initiation depends on the price of water at that time. The mean-reverting price process causes price level to fall toward \$26.00 when the starting price is high, and causes the price level to rise toward \$26.00 when the starting water price is low. For a low initial price, roughly \$7.00 per acre-foot for instance, the buyer expects water prices to rise toward \$26.00 then level off. He will not exercise the option until mean reversion has pulled water price above \$25.00. The value of the 7/15 option contract for a starting price of \$6.94 is \$35.54.

When the starting price of water is above the mean-reverting level, the option buyer

expects the mean-reverting process to force prices to fall and to eventually hover around the mean-reverting price. In this case the buyer will strike quickly to take advantage of temporarily elevated prices and large payoff to exercising the option. When the initial price of water is near $100.00, for example, the 7/15 option contract's value is $116.90.

Using the base case parameters of Table 6.2, I compared the value of the 7/15 option with the value of option contracts with fewer than seven strikes in fifteen years. In Figure 6.1 the seven lines correspond to the value of the option contract, as assessed in the initial time period, when the option has one to seven strikes. The lowest line on the graph represents one strike available in fifteen years; the highest line shows contract values for seven uses. As expected, additional strikes add value to the option contract; notice that contract value rises as the number of exercises increases for any given initial price for water. The marginal value of additional strikes decreases rapidly then becomes almost constant. Again this is due to the mean-reverting process: in time periods after the price approaches the mean-reverting level, the undiscounted expected payoff from exercising the option is small.

The Texas data described in Chapter 5 reveal that most trades with a transfer price different from $25.00 per acre-foot have prices higher than the mean-reverting level of $26.00. This suggests that options written in the Texas Rio Grande water market would most often entail initial water prices above the mean-reverting price. Thus the option contract values would likely be similar to the base case results for a starting water price of about $50.00 per acre-foot, about $61.39 for a 7/15 option.

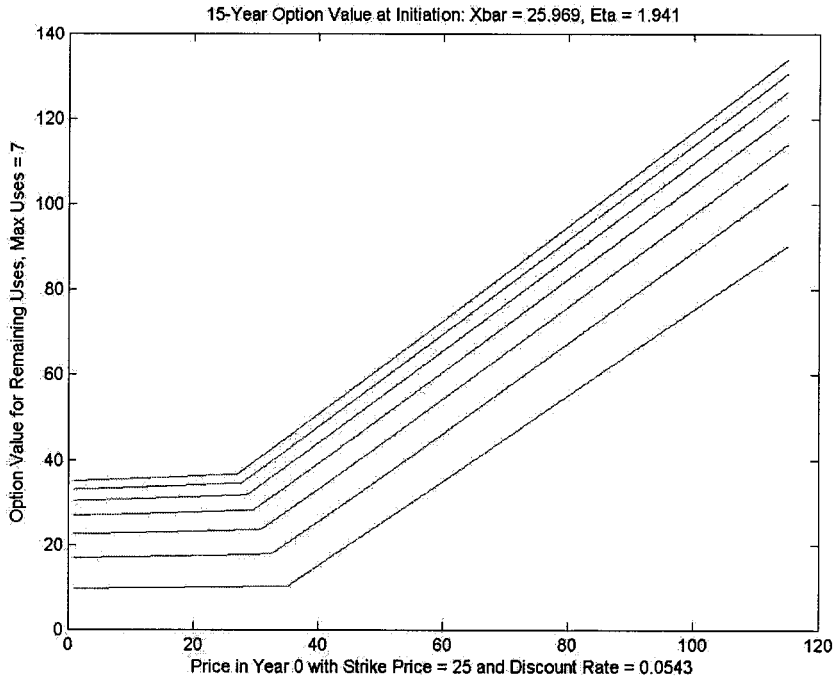

Figure 6.1: Baseline Option Contract Values Under Mean Reversion

6.5 Sensitivity Analysis of Mean-Reverting Case

Previous research on option prices for water have shown model results to be sensitive to changes in parameters. Michelsen and Young (1993), for instance, found the value of option contracts for water to be very responsive to the researcher's choice of discount rate. Since several of the parameters of the base case here were estimated from data, prudence compels me to examine the model's sensitivity to variations in key parameters. Specifically, I examine the mean-reverting price level ($\bar{P}$ in equation (6.3)), speed of reversion (η in equation (6.3)), the standard deviation of the price shocks, and the discount rate.

To assess the sensitivity of my results to these parameters, I ran the computer algorithm repeatedly, varying one parameter at a time. Initially, I set all parameters to their base case levels (see Table 6.2). Then, first for $\bar{P}$ and then for the other parameters, I varied the parameter in question proportionally by multiplying its base case value by one of a set of values: 0.25, 0.5, 0.75, 1, 1.25, 1.50, 1.75, 2. I then obtained model results in two ways: graphically and numerically. The graphical presentation shows the initial value of the option contract for starting prices ranging from \$1.00 per acre-foot to \$115.00 per acre-foot, for eight different values of the parameter under analysis ($\bar{P}$, η, σ_ϵ, or discount rate). The numerical results are in a table with one row for each value of the parameter, columns for different starting prices of water, and table cells of option contract values in the initial period. Rather than listing contract values for each possible starting price, I do so for the price states closest to the prices that are most common in my dataset (\$25 and \$50 per acre-foot) and the water price bounds (\$7 and \$100 per acre-foot).

I first explored the effect of raising the mean-reverting price level, $\bar{P}$, which I

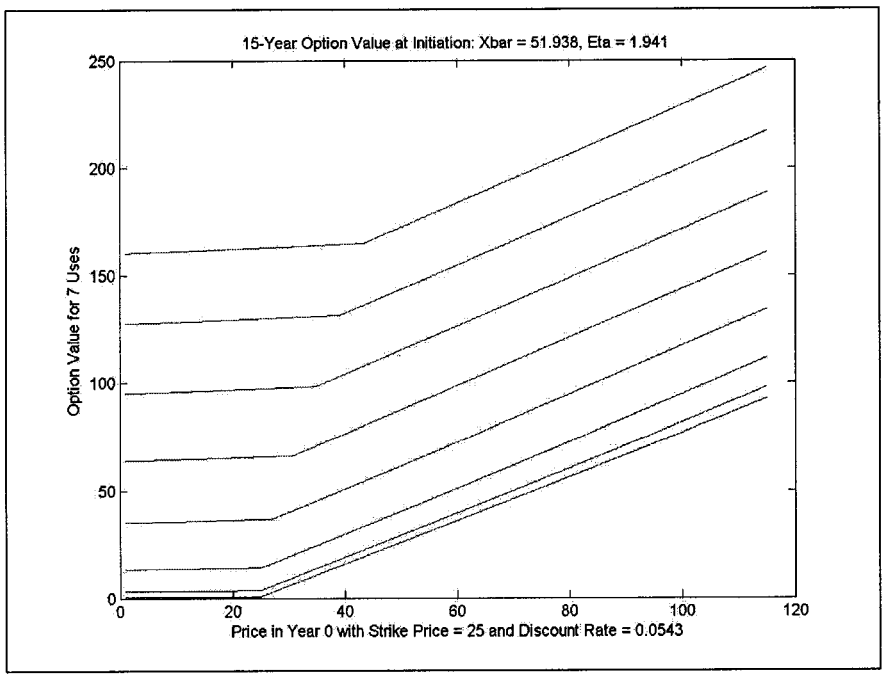

Figure 6.2: Value of Seven-Exercise Option Contract as Xbar Varies

labeled *xbar* in my program. Reasoning suggests that the value of the option contract will increase as *xbar* rises, if all other parameters are held constant. Figure 6.2 and Table 6.4 display the graphical and numerical results of the *xbar* sensitivity analysis. Both exhibits show that, as expected, the multiple-exercise option contract gains value as the mean-reverting level of the price process rises. This result applies regardless of the initial price of water. The contract value for a starting price of \$6.94 per acre-foot varies from \$0.52 when *xbar* is \$6.49 to \$160.78 when *xbar* is \$51.94. Similarly, the contract value ranges from \$76.83 to \$228.95 when the initial price of water is \$99.92.

The relationship between contract value and mean-reverting price is easy to understand if examined from the option holder's perspective. Recall that the strike price is

	Starting Price for Water			
Xbar Value	$6.94	$25	$49.89	$99.92
6.49	0.52	0.58	25.66	76.83
12.98	3.23	3.48	28.97	81.61
19.48	13.39	14.08	40.13	94.56
25.97	35.54	36.69	62.39	116.90
32.46	64.52	65.92	87.31	143.33
38.95	95.69	97.29	114.82	171.18
45.45	127.88	129.65	146.13	199.79
51.94	160.78	162.70	172.06	228.95

Table 6.4: Sensitivity Analysis of Xbar Parameter with Texas Data

$25.00 and that the holder will never exercise the option if the current price of water is below the strike price. Thus, the contract can have positive value only to the extent that the option's owner expects the price of water to exceed the strike price during the option's lifetime. If the price of water in period 0 is low ($6.94, for example) and the mean-reverting level of the price process is also low (perhaps $12.98) then it is unlikely that the price of water will be higher than $25.00 very often in the next fifteen years. Thus the option contract is worth very little, $3.23 in this case. If, by contrast, the mean-reverting price for water is high ($51.94) relative to the exercise price, the option contract has value ($160.78) even if the water price in period 0 is low ($6.94). In this scenario, the option holder expects price to rise toward the mean-reverting level over time and so waits for a few periods before exercising the option.

Varying the speed of reversion from 0.4853 to 3.882 yields the option contract values graphed in Figure 6.3. Note that the highest curve corresponds to the slowest reversion speed (0.4853) while the lowest curve is for the fastest reversion. The results of the η sensitivity analysis confirm the intuitive conclusion that a multi-year multiple-exercise

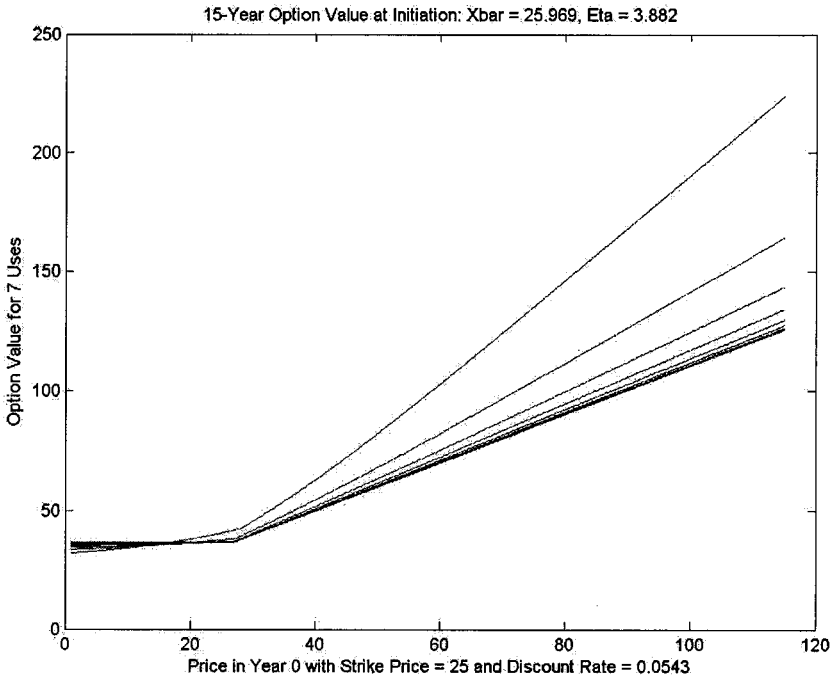

Figure 6.3: Value of Seven-Exercise Option Contract as η Varies

option contract in a mean-reverting price regime is most valuable when the initial price of water is high and option holders expect the asset price to move slowly toward the mean. This implies a longer period of time with high water prices, if the starting price is above the mean-reverting price, and thus more opportunity to capitalize on the flexibility inherent in the option contract.

Numerically, the η sensitivity analysis yields the contract values in Table 6.5.

	Starting Price for Water			
Eta Value	\$6.94	\$25	\$49.89	\$99.92
0.4853	33.51	40.50	81.92	189.97
0.9705	34.36	37.69	67.92	141.11
1.4558	34.99	36.90	63.28	124.36
1.9410	35.54	36.69	61.39	116.90
2.4263	35.96	36.66	60.54	113.36
2.9115	36.24	36.68	60.12	111.64
3.3967	36.44	36.70	59.90	110.76
3.8820	36.56	36.72	59.78	110.29

Table 6.5: Sensitivity Analysis of Eta Parameter with Texas Data

This reinforces the conclusions of both the base case analysis and the graphical η-sensitivity analysis. For the starting price well below the mean-reverting price, \$6.94, rising η yields higher contract values. High η means that the price of water reaches the mean-reverting price level quickly; compared to lower η, the option buyer does not have to wait as long for positive payoffs to exercising the option. This implies that the payoffs will not be subject to as many periods of discounting with high η values rather than low ones.

The model results demonstrate the opposite effect when the starting price of water is above the mean-reverting level: option contract values decline as η increases. Faster reversion to the mean leaves fewer early time periods with price well above the mean-

reverting level. Therefore payoffs to exercising the option fall quickly over time. Since spot transactions in the Texas dataset cluster around $25.00 and $50.00 per acre-foot, speed to reversion would have two different effects, depending on the current price of water. Contract values for the lower of these two prices are small (ranging from $36.72 to $40.50) that the costs of negotiating and implementing the option might well overwhelm the benefits the buyer could expect from added flexibility in water supply. Thus the more practically significant implication of the η sensitivity analysis is for high starting water prices: speedy reversion to the mean lowers the value of the option contract.

The third parameter I scrutinized with my sensitivity analysis program was sigma (σ_ϵ), the standard deviation of the stochastic increments of the price process. In Chapter 4 I explained that σ_ϵ played an important role in determining the probability of each price in the price space occurring in period $t+1$. As σ_ϵ gets larger the probabilities associated with price states at the extremes of the distribution increase. Rearranging equation (6.3), the discrete price process, notice that the change in price from one period to the next is:

$$P_{t+1} - P_t = (\bar{P} - P_t)(1 - e^{-\eta}) + \epsilon_{t+1}. \tag{6.6}$$

Given my base case parameters where $\eta = 1.941$, this becomes:

$$P_{t+1} - P_t = 0.856(\bar{P} - P_t) + \epsilon_{t+1}.$$

Now consider the special case where today's price is equal to the mean-reverting price level. Then the expected change in price next period depends solely on the distribution of the stochastic component, the price shocks (ϵ_{t+1}). As σ_ϵ rises, the probabilities of large positive

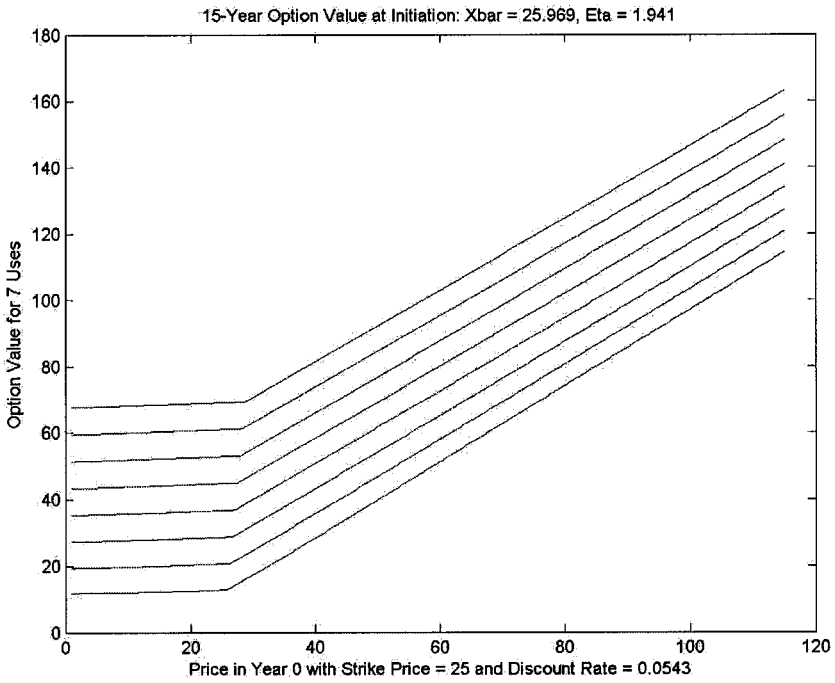

Figure 6.4: Value of Seven-Exercise Option Contract as σ_ϵ Varies

or negative price changes both increase. The value of the option contract will grow when σ_ϵ does because higher probability of high prices adds to the expected value of the payoff next period. However, amplified chances of low future prices do not offset these expected gains because future prices lower than the exercise price simply result in zero payoffs, rather than negative ones.

Theory suggests, then, that option contract values will increase with σ_ϵ. The results of my analysis, shown in Figure 6.4 and Table 6.6, exhibit the anticipated relationship between σ_ϵ and contract value. Regardless of whether the initial price of water is high or low compared to the exercise price, contract values grow with σ_ϵ.

The sensitivity analysis for the discount rate parameter provides a distinct contrast

Sigma Value	Starting Price for Water $6.94	$25	$49.89	$99.92
2.125	11.89	12.79	39.52	96.98
4.250	19.63	20.69	46.52	103.32
6.375	27.56	28.67	53.89	109.97
8.500	35.54	36.69	61.39	116.90
10.625	43.57	44.83	68.96	124.06
12.750	51.63	52.80	76.57	131.38
14.875	59.75	60.93	84.25	138.83
17.000	67.92	69.10	91.97	146.37

Table 6.6: Sensitivity Analysis of Sigma Parameter with Texas Data

to the η and σ_ϵ results. While the value of η used in the computational process caused the value of the option contract to vary by as much as $189, the largest impact that changing the discount rate had is about $22. Graphically, the option contract value curves for the full range of water prices and for differing discount rates in Figure 6.5 are close together and seemingly parallel. The tabular results underscore the conclusions of visual presentation, as shown in Table 6.5.

r Value	Starting Price for Water $6.94	$25	$49.89	$99.92
0.0136	47.38	48.47	72.12	127.53
0.0272	42.85	43.97	68.02	123.47
0.0407	38.94	40.07	64.47	119.96
0.0543	35.54	36.69	62.39	116.90
0.0679	32.58	33.73	58.70	114.22
0.0814	29.98	31.14	56.34	111.85
0.0950	27.69	28.85	54.26	109.76
0.1086	25.67	26.84	52.41	107.89

Table 6.7: Sensitivity Analysis of Discount Rate Parameter with Texas Data

In contrast to the η parameter, which had different effects on high and low initial water prices, increasing the discount rate reduces option contract values across the board.

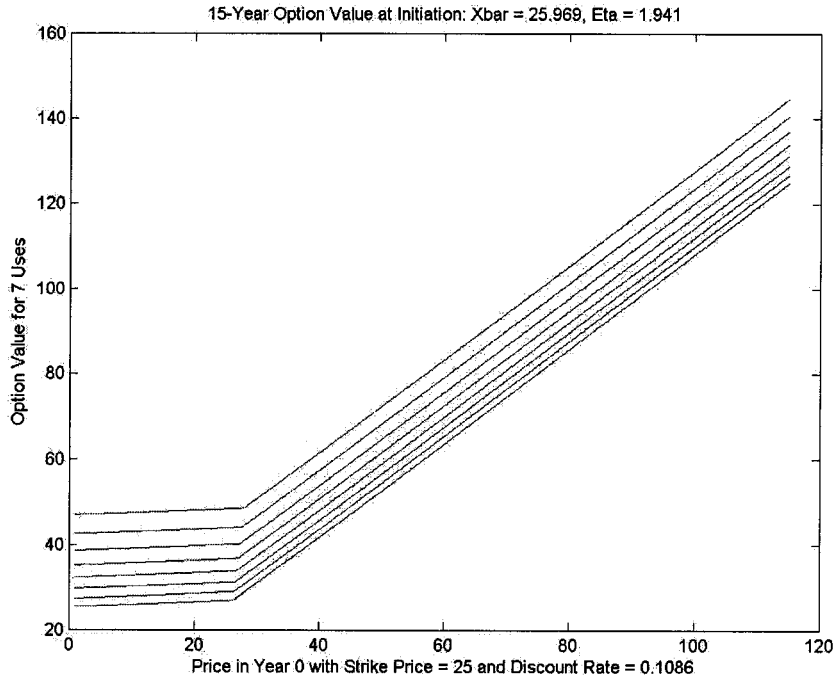

Figure 6.5: Value of Seven-Exercise Option Contract as Discount Rate Varies

Regardless of whether the initial price of water is above or below the mean-reverting level, contract values fall as the discount rate rises. The strongest effect in percentage terms is for the low starting water prices. This is explained by the optimal exercise patterns for the option contracts. Recall that, as discussed during presentation of the base case, buyers will tend to delay exercise if beginning from a low water price; they wait for the water price to rise above the strike price. This delay in exercise translates into payoffs that must be discounted several periods to compute the contract's value in the initial period. Buyers who face high water prices at the outset exercise their options quickly in response to decreasing, mean-reverting prices; their payoffs are discounted fewer periods.

The value of a 7/15 option contract with an initial water price of $6.94 per acre-foot decreases from $47.38 to $25.67 (44%) when the discount rate moves from 0.0136 to 0.1086. For starting prices of $49.89 and $99.92 per acre-foot, contract values drop 27% and 15% respectively. Determining the proper discount rate may not merit strenuous effort when water prices are well above the mean-reverting level. Though the effect on the computed value of the multiple-exercise option contract is smaller than other parameters presented earlier, varying the discount rate from 0.0136 to 0.1086 still produces the expected result: higher discount rates imply lower contract values, *ceteris paribus.*

6.6 Comparison to Brownian Motion Results

Earlier in this chapter, I described my efforts to characterize the stochastic price process for water and the steps that led to a diagnosis of mean reversion for the Texas data. How important to the empirical results of this model is the assertion of mean reversion?

Since most financial models and the Black-Scholes framework assume that prices evolve according to a random walk, I tested my model using the Texas data but based on an alternative price process: geometric brownian motion.

The Matlab program I used for the geometric brownian motion version of the analysis differs from the mean reversion program of Chapter 4. Both programs rely on dynamic programming as the solution algorithm, use a discretized price space, and make identical assumptions about option structure. The geometric brownian motion program, however, is more complicated than the mean reversion code. The geometric brownian motion program uses the basis approach to dynamic programming Miranda and Fackler (2002) articulated. This version of the option-valuation program requires the same parameter inputs as the mean reversion program, with two critical exceptions. Rather than describing the price process by specifying the mean-reverting price level, the speed to reversion, and the mean and standard deviation of the price shocks, I must determine the process's expected rate of return and volatility.

Since expected rate of return, μ, and volatility, σ, are standard parameters in financial economics, I used Hull's procedures to estimate price volatility and rate of return for the Texas time series data (Hull, 1997; p. 233). Let P_t denote the price of water in time period t. Then the continuously compounded weekly return, u_t, is

$$\ln\left(\frac{P_t}{P_{t-1}}\right).$$

The estimate of expected rate of return, $\hat{\mu}$, is then the arithmetic average of the realized returns in a given time period. I computed the price-process parameter for a one-year time

frame, so

$$\hat{\mu}_T = \left(\sum_{t=1}^{T} u_t \right) \frac{1}{T},$$

and $T = 52$ since there are 52 weeks in a year. Hull's estimate of volatility from historical data builds on the series of continuously compounded returns, u_t. Specifically, for a one-year horizon,

$$\sigma_T = \left(\frac{1}{T-1} \sum_{t=1}^{T} (u_t - \hat{\mu})^2 \right)^{1/2}.$$

Again, $T = 52$ since I used 52 weekly observations in the computation. Using these formulas I found that the Texas spot market data exhibit annual expected rate of return of -0.01419 and annual volatility equal to 0.4418, on average.[2]

The base case I computed using geometric brownian motion and the multiple-exercise option valuation program began with the parameters in Table 6.8.

Parameter	Description	Value
sigma	Annual volatility	0.4418
mu	Annual expected rate of return	-0.01419
r	Annual interest rate	0.0543
T	Lifespan of option	15
Rmax	Maximum number of strikes	7
k	Strike price	25
n	Discretization points for price	501
a	Minimum starting price	7
b	Maximum starting price	100

Table 6.8: Parameter Values for Base Case Analysis of Texas Data Under Geometric Brownian Motion

Note that the geometric brownian motion base case parameters are identical to the mean reversion base case parameters in Table 6.2, except for the price process parameters. Varying the smallest possible number of inputs facilitates comparisons between the results

[2]I computed volatility for 27 sets of 52 consecutive weeks in my sample. These volatilities ranged from 0.5125 to 0.3571. I chose to use the average result.

of the valuation algorithm under the two price processes.

Table 6.9 displays the value of a 7/15 multiple-exercise option contract beginning with the base case parameters and using geometric brownian motion to drive the price of water over time. The table contains the program's results for selected water prices in the initial period. Figure 6.6 presents the value in period zero of option contracts with from one

Starting Price for Water	**Value in Initial Period**
\$6.94 per acre-foot	\$24.94
\$25	\$111.40
\$50.14	\$221.42
\$100	\$425.51

Table 6.9: Option Contract Values for Base Case Texas Data, Under Mean Reversion

to seven strikes over fifteen years, for each possible starting price for water. The highest curve in the graph applies to a 7/15 option; the lowest curve values a 1/15 option.

Comparing the base case results for the two price processes reveals some notable differences. For the selected water prices in Tables 6.3 and 6.9, option contracts are worth more under geometric brownian motion than under mean reversion. The exception to this is the lowest starting water price, roughly \$7.00 per acre-foot. In general, geometric brownian motion allows for the possibility of higher water prices over time than mean reversion does. This translates into higher expected payoffs to exercising the option since the price of water is unlikely to hover around \$26.00 per acre-foot as it does under the mean-reversion base case. For very low initial water prices, expected future price rises quickly under mean reversion. Under geometric brownian motion, however, expected future prices from the lowest starting price (\$7.04 per acre-foot) do not often reach high enough levels to make striking worthwhile.

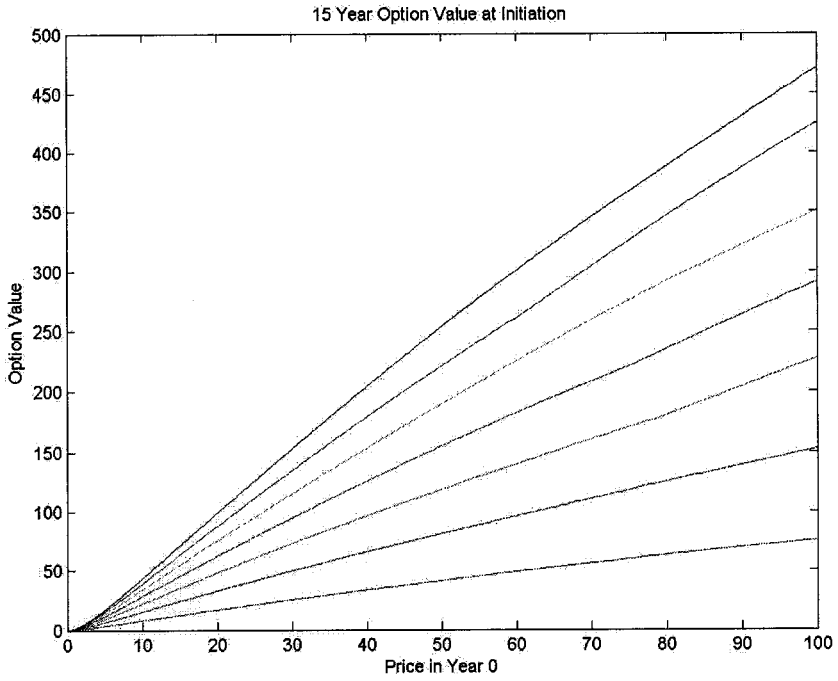

Figure 6.6: Baseline Option Contract Values Under Geometric Brownian Motion

The range of contract values is much larger under geometric brownian motion than under mean reversion. A 7/15 option contract's value ranges from a minimum of $35.54 to a maximum of $116.90 under mean reversion, but falls between $24.94 and $425.51 under geometric brownian motion, for the displayed prices. Again, this is due to the difference in variability of prices inherent in the two stochastic processes. Mean reversion is more predictable than geometric brownian. Thus, we expect option prices to be lower under mean reversion to reflect the relatively low level of addition flexibility the buyer gains.

Sensitivity analysis of the results also yields varying conclusions depending on the stochastic process in use. Following the same procedures I outlined in the mean reversion scenario, I examined the responsiveness of multiple-exercise option contract values while changing the discount rate and then the price process volatility. Figure 6.7 displays the value of a 7/15 option contract, for each possible starting price for water, as the discount rate rises from 0.0136 to 0.1086. The top curve on the graph corresponds to a discount rate of 0.0136; the lowest curve is for a discount rate of 0.1086. The level of discount rate becomes more important as the initial price of water becomes smaller.

The difference in contract value between the minimum and maximum discount rates is $78.66 (51%) for a water price of $25.00 in period zero but $74.08 (17%) when the initial water price is $100.00 This finding is in sharp contrast to the sensitivity analysis of discount rate under mean reversion. Recall from Figure 6.5 and Table 6.5 that changing the discount rate had a more muted effect on option contract values, regardless of the starting water price.

Varying the volatility parameter generates still more interesting results, as seen in

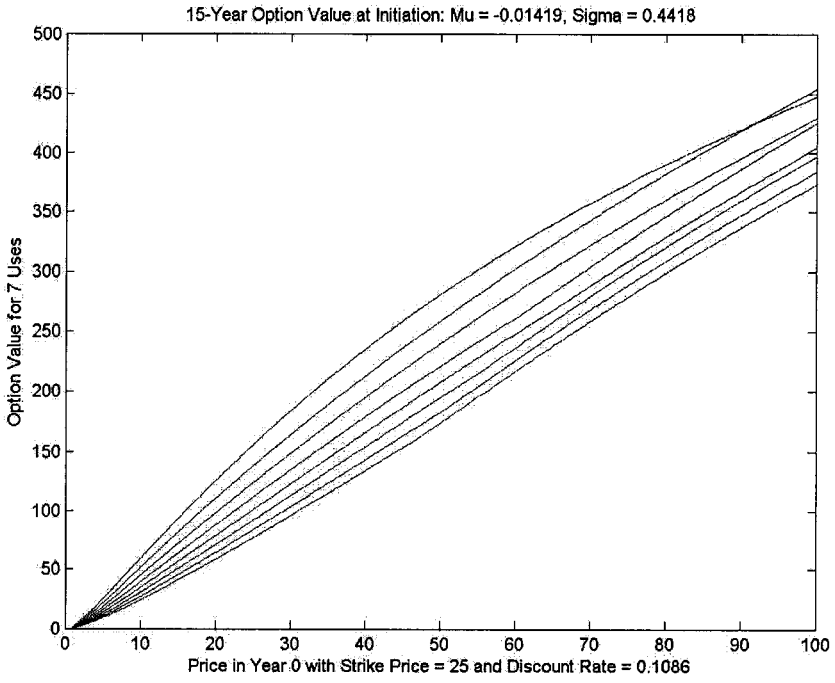

Figure 6.7: Option Contract Values as Discount Rate Varies, Under Geometric Brownian Motion

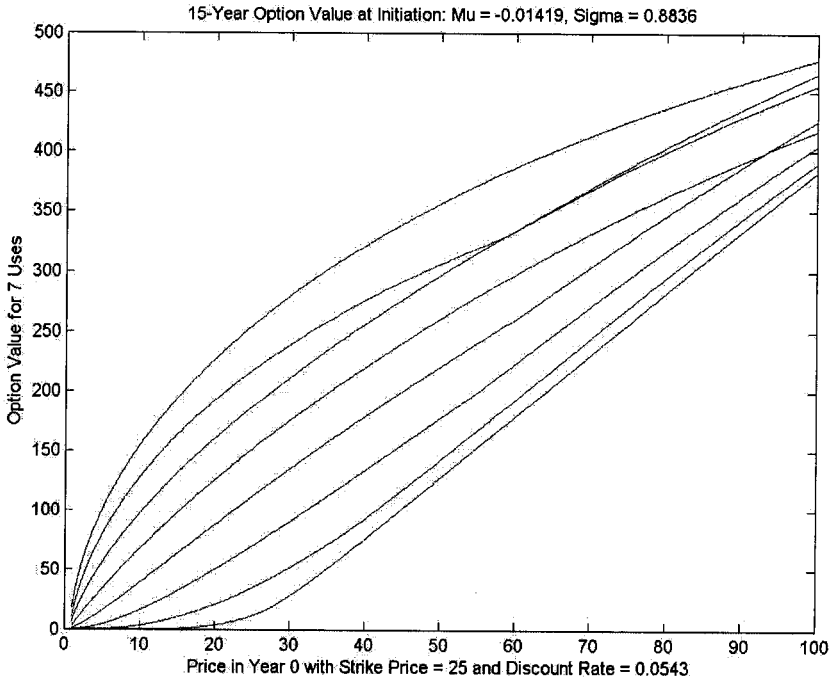

Figure 6.8: Option Contract Values as Volatility Varies, Under Geometric Brownian Motion

Figure 6.8. Figure 6.8 displays the value of a 7/15 call option for the full range of water prices, as volatility increases from 0.1105 to 0.8836. The highest curve in the graph is associated with the highest volatility; the bottom curve is generated by the lowest volatility. The numerical value of this price parameter becomes more important as the initial price of water approaches the strike price of the option contract. To see this, consider three possible water prices in period zero: \$7.04 per acre-foot, \$25.00, and \$100.00. The differences in value of the multiple-exercise option contract from the maximum to the minimum volatility for these three cases are \$125.57, \$243.62, and \$94.95, respectively. Option contract values are more responsive to changes in volatility than to changes in discount rate.

Are model results more sensitive to changes in price parameters under geometric

brownian motion or mean reversion? The answer to this question depends on the initial price of water. Referring to Figures 6.3 and 6.8, note that the speed to reversion factor η becomes more influential as water price increases. In contrast, changes in volatility have less effect on option contract value as the starting price of water increases, for levels beyond the strike price.

6.7 Option Contract Values and Water Price Process in the Texas Dataset

The results of my analysis clearly show that the distinction between mean reverting water prices and water prices that exhibit geometric brownian motion has important implications for the value of option contracts in the Texas Rio Grande. The two price processes yield notably different dollar values for multiple-exercise option contracts. Sensitivity of contract values to changes in key parameters also varies depending on the underlying price process. Correctly assessing the stochastic nature of the price path for water is therefore a critical consideration for applying this valuation model to actual option transactions.

Empirical analysis of the Texas dataset casts doubt on the assertion that short-term water prices in this region follow geometric brownian motion. The variance ratio test of section 6.2.1 in particular suggests that the prices in the dataset do not follow a random walk. However, limitations of the timespan of the data make it difficult to diagnose mean reversion with confidence. Despite the concern of a relatively short time series, mean reversion is indeed the more relevant model for the underlying stochastic process in the price data from the Texas Rio Grande. I rest this assertion on two pieces of evidence: 1) strong

and consistent rejection of RW1 through the variance ratio test, and 2) the observations of the Watermaster of the Rio Grande. Carlos Rubinstein's first-hand experience with water allocation in southwest Texas and his formal role in administering water transfers have led him to conclude that water prices tend to follow a somewhat predictable pattern, generally centering near $25.00 acre-foot (Rubinstein, 2003).

The fact that option contracts for water have not yet emerged in the Texas Lower Rio Grande may also confirm my conclusion of mean reversion in the water price process. Recall from Table 6.3 that a 7/15 call option is worth less than $53.00 when the price of water is $50.00 per acre-foot or less. The low value of the contract may thus be overwhelmed by the transaction costs of individually negotiating and formalizing the contracts. The higher values for option contracts that result under geometric brownian motion would make it more likely that such contracts would be appearing as a transfer mechanism in southwest Texas.

Chapter 7

Policy Implications and Extensions

In its 2001 assessment of needed research on water resources for the coming century, the National Resource Council cites a need for more extensive research on economic institutions for managing water scarcity (Water Science and Technology Board, 2001). Included in this category of priority research is "further research on the development of efficient markets and market-like mechanisms for allocating water" (p. 37). In formulating, implementing, and testing a valuation algorithm for multiple-exercise option contracts for water, the methodology and empirical findings of this thesis represent a modest but notable contribution to the area noted by the National Research Council.

The research presented here also expands the small but growing intersection between financial economics and natural resource economics. Drawing on both the Black-Scholes option pricing framework and on water transfer pricing approaches based in agricultural models (such as Michelsen and Young, 1993) yields an innovative methodology with applications in finance and in resource economics.

7.1 Primary Findings

The main contributions of this research fall into three broad categories: the financial engineering problem, the data issues, and the empirical results. My methodology for valuing an option contract that may be exercised more than one time in its lifespan is a new financial engineering technique that can be used and may have applications in a wide range of settings. Though water markets were my motivation, there is nothing in the algorithm of Chapter 4 that restricts the model's use to this specific market context. The electricity industry, with an active derivatives market, is a likely alternate application of this option-valuation algorithm.

The process of preparing raw data from the Rio Grande Watermaster's office so that it was appropriate as input to the option valuation computer program was also instructive. Future users of similar datasets should be alert to the complications that arise from nonsynchronous trading, for example. The short timeframe of available data is also an important consideration that may reduce confidence in empirical analysis of price process parameters. Aggregation techniques and their implications for reducing variability in the data also merit attention in markets that are thinly-traded relative to financial markets.

The empirical results of this thesis are shown in detail in Chapter 6. To summarize, I found:

- The choice of geometric brownian motion or mean reversion as the stochastic process driving water prices has a large impact on the value of multiple-exercise option contracts. This implies the great value of a dataset that is sufficiently long and complete to assure confidence in the ultimate diagnosis of which price process to use.

- Under a simple mean-reverting process, the value of the option contract is particularly sensitive to the parameters that capture the speed of reversion to the mean and the standard deviation of the price shocks. Again, this underscores the need for confidence in parameter estimates.

- In contrast to the speed of reversion parameter, results under mean reversion are relatively insensitive to changes in the value of the discount rate. Therefore, it may not be worthwhile to expend much research effort trying to pinpoint the "correct" discount rate.

- When the future price of water is modeled using geometric brownian motion, the discount rate becomes more influential in the determination of contract values than it is under mean reversion.

- Multiple-exercise option contract values are less sensitive to changes in the volatility parameter of a random walk process than they are to changes in the discount rate, for water prices that are high relative to the strike price in the initial period. For low initial water prices, however, contract values are more responsive to changes in volatility than to changes in the discount rate.

7.2 Future Extensions and Refinements

The multiple-exercise option contract valuation program I devised and implemented for this research is a significant advancement in the use of financial economics to inform natural resource decision-making. Still, this model will be even more useful

with modifications and extensions that enable the model to incorporate the physical and economic complexity of water markets. I propose three possible future avenues of research with this model: examining alternative stochastic price processes, accounting for transaction costs, and linking water quality to water quantity considerations.

Thus far, I have generated option contract values when water prices evolve according to geometric brownian motion and a simple mean-reverting process. The variance ratio tests I conducted, though, provide evidence that price may follow a random walk that includes a mean-reverting component. Summers (1986) and Fama and French (1988) suggest that this type of price process may apply to some stock market data. Under this stochastic process, data exhibit a variance ratio of less than one for two-period returns and rejection of the random walk hypothesis becomes more emphatic as the timespan of returns increases.

I have not rigorously investigated this possibility with the Texas dataset. Preliminary analysis, however, shows that the percentage of water price series that fail to reject the null hypothesis of random walk falls as the timespan for returns increases.[1] For two-period returns, 87% of the price series rejected the random walk hypothesis. Considering four-period and six-period returns resulted in 98% and 99.6% rejection, respectively. Furthermore, the average percentage of variance ratios of less than 1 was 93% for seven iterations of 10,000 price series each. Exploring my algorithm's response to a combined random walk and mean-reverting process will be the first task on my post-dissertation research agenda.

In Chapter 2 I expressed my reservations about the validity of applying the Black-

[1]Recall that I generated price series in sets of 10,000 each.

Scholes assumption of no transaction costs to a water markets setting. Clearly, as I argued earlier, research has shown that transaction costs can accumulate quickly depending on legal and institutional obstacles to water transfer. Adding transaction cost considerations to my theoretical formulation and computer program would not be technically challenging. Estimating the functional form and value of transaction-cost parameters might prove difficult given the fragmented nature of data in most water markets.

This thesis has focused solely on water quantity issues, to the exclusion of water quality concerns. Still, in Chapter 1 I emphasized the importance of considering water quality, timing of delivery, and water quantity when making allocation decisions. One of my long-term goals for this option valuation methodology is to draw on existing methods for integrating quality and quantity in water market models, and to allow water quality measures to influence contract values in my program. Weber (2001), Giannias (1999), and Sabbaghi and Spulber (1995) present interesting models that introduce quality and environmental constraints into water market approaches.

7.3 Policy Implications

The central policy implication of this research is that option contracts for water have value and could be a useful addition to the range of available market-based mechanisms for water allocation. Institutional and legal restrictions should be revised to allow option contracts to be used for short-term water transfers, as they are now in California. The valuation methodology presented here could play a role in facilitating option contracts by helping buyers and sellers estimate the value of multiple-exercise contracts.

In order to ensure that this model's results present an accurate assessment of option contract values, governmental agencies and other institutions involved in water transactions should follow the Rio Grande Watermaster's example by recording price and volume data. The parameter estimates for discount rate and price process variables depend on time series data, which are not readily accessible in most areas. Improving record-keeping and data-sharing would, over time, generate high-quality data that cover a long timeframe. This would enable a more definitive assessment of the nature of the underlying stochastic process driving water prices.

More frequent use of option contracts will spur the need to monitor and examine their third-party effects. One-time, short-term water transfers are generally thought to have fewer third-party effects than permanent rights transfers. Multiple-exercise option contracts that allow for repeated water diversion over a period of years may not be so benign. If such contracts become widespread, their impact on instream flows, water quality, and community economies may require further study.

This research holds lessons for other research in the broader realms of natural resource economics and financial economics. While these two fields may seem far removed from each other, my efforts demonstrate that they can make use of and learn from each other. Certainly the models and computational techniques used in finance can inform specific applications in natural resource economics. But the relationship is not a one-way street: natural resource scenarios and the use of financial models with non-financial data can also highlight areas for improvement in financial economics research. The multiple-exercise option-valuation algorithm described in this thesis is not limited to use in water markets;

it is a general model that can be applied to other markets where multiple-exercise option contracts may emerge. Thus, natural resource applications have generated a financial engineering result.

Finally, as the National Research Council (Water Science and Technology Board, 2001) also noted, this investigation points to the need for continued research on mechanisms for efficient water allocation. As one of our planet's most precious resources, and as a resource subject to increasing scarcity, water deserves serious attention from researchers and policy-makers alike. The research must be grounded in data from actual transactions and enhanced by the institutional, legal, and cultural details of those transactions. To accomplish this, researchers must enlist the support and cooperation of the participants in water markets. This cooperation, and the trust it will foster, will improve the quality and relevance of research and facilitate its implementation in water markets and policy arenas.

Appendix A

Acre-Foot Conversion Factors

The following relationships are useful when converting between different measures of water volume.

1 acre-foot = 43,560 cubic feet

1 acre-foot = 1613 cubic yards

1 acre-foot = 1233.5 cubic meters

1 million cubic meters = 810 acre-feet

Appendix B

Approximating the Standard Normal CDF

Hull (1997) recommends this polynomial method for approximating the cumulative standard normal distribution function, N. Let

$$
\begin{aligned}
k &= \frac{1}{1+\gamma x} \\
\gamma &= 0.33267 \\
a_1 &= 0.4361836 \\
a_2 &= -0.1201676 \\
a_3 &= 0.9372980, \text{ and} \\
N'(x) &= \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}.
\end{aligned}
$$

Then when $x \geq 0$,

$$N(x) = 1 - N'(x)(a_1k + a_2k^2 + a_3k^{3)}, \text{ and}$$

and when $x < 0$,

$$N(x) = 1 - N(-x).$$

This gives values for $N(x)$ that are generally accurate to four decimal places. This appendix follows Hull's presentation (1997; p. 243). See also Abramowitz and Stegun (1972).

Appendix C

Selected References from the Water Markets Literature

The body of existing research on water markets is massive in both range and depth. Here are a few selected references.

C.1 General Overviews

Anderson and Snyder (1997), Easter (1996), Easter *et al.* (1999), Frederick (1998), Hartman and Seastone (1970), Horbulyk and Adamowicz (1997), Howe (1997), MacDonnell and Rice (1994), and Shupe *et al.* (1989).

C.2 Empirical Studies and Simulations

Booker and Young (1994), Charney and Woodard (1990), Dinar *et al.* (1998), Gardner and Miller (1983), Houston and Whittlesey (1986), Howitt (1994), Ise and Sunding

(1998), Keith *et al.*(1989), Michelsen (2000), Turner and Perry (1997), and Vaux and Howitt (1984).

C.3 Theoretical or Conceptual Approaches

Frohlich and Oppenheimer (1995), Jordan (1999), Michelsen (2000), and Rosegrant and Binswanger (1994).

C.4 Legal and Institutional Analysis

Burton (1992), Colby (1999), Colby and Destree (2000), Curie (1985), DuMars (1985), Easter and Feder (1997), Easter and Tsur (1995), Gould (1989), Howe and Goodman (1995), Huffman (1997), Michelsen (1994), Rice and MacDonnell (1993), Smith (1989), and Thompson (1997).

C.5 International Perspectives

Becker *et al.* (1996) - Middle East, Diao and Roe (1998) - Morocco, Rosegrant and Gazmuri (1994) - Chile and Mexico, Schleyer and Rosegrant (1996) - Chile, Schmidt and Plaut (1993) - Israel, Sturgess (1997) - Australia, and Teerink and Nakashima (1993) - Japan.

C.6 State-Specific Analyses in the United States

C.6.1 Arizona

Charney and Woodard (1990), Fuller (1997)

C.6.2 California

Burton (1992), Howitt (1994), Israel and Lund (1995), Parker (1997), Schmidt and Plaut (1993), Vaux and Howitt (1984), Wahl and Davis (1986), Watters (1995), Weinberg (1997), Zilberman *et al.* (1994), and many others.

C.6.3 Colorado

Gardner and Miller (1983), Howe (1998), Howe and Goodman (1995), Howe *et al.* (1986b), Michelsen (1994), Michelsen (2000), Rice and MacDonnell (1993), Saliba *et al.* (1987), Tregarthen (1983).

C.6.4 Other states and regions

Brajer *et al.* (1989) - New Mexico, Easter *et al.* (1983) - North Central Region, Frerichs and Easter (1990) - Great Lakes, Houston and Whittlesey (1986) - Pacific Northwest, Ise and Sunding (1998) - Nevada, Keith *et al.* (1999) - Utah, Saliba (1987) - Southwest, Supalla (2000) - Nebraska, Thorson (1989) - Montana, Turner and Perry (1997) - Oregon.

Appendix D

Map of Texas Watermaster Areas

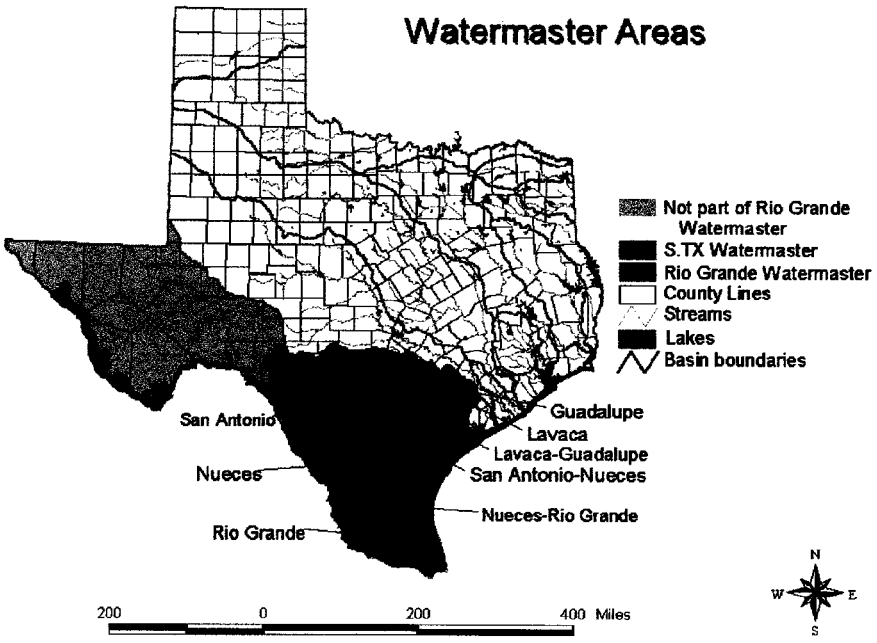

Figure D.1: Texas Watermaster Jurisdictions (TNRCC, 2002).

Appendix E

Matlab Code for Variance Ratio Tests

VRQ computes a variance ratio test for time series data in accordance with Campbell *et al.* (1997; p.50). If teststat is outside the interval [-1.96,1.96], reject the null hypothesis that the prices follow a random walk type 1.

SYNTAX:

[muhat,sigbara,sigbarc,vrqbar,teststat]=vr2(price,q)

INPUTS

price : column vector of raw price data

q : number of periods in the return

OUTPUTS:

muhat : sample mean estimator

sigbara : sample variance estimator

```
sigbarc : sample variance estimator using even numbered observations

vrqbar : the ratio of the two variances

teststat : standardized test statistic to test the null hypothesis

lnprice=log(price);

rr1=length(price);

n=(rr1-1)/q;

returns=lnprice(2:(n*q+1))-lnprice(1:(n*q));

muhat=(lnprice(end)-lnprice(1))/(n*q);

sigbara=[sum((returns-muhat).^2)]/(n*q-1);

mm=q*(n*q-q+1)*(1-(q/n*q));

qreturns=lnprice((q+1):(n*q+1))-lnprice(1:(n*q+1-q));

sigbarc=[sum((qreturns-q*muhat).^2)]/mm;

vrqbar=sigbarc/sigbara;

d1=2*(2*q-1)*(q-1);

d2=3*q;

denom=d1/d2;

teststat=(vrqbar-1)*(sqrt(n*q))/sqrt(denom);

function [muhat,sigbara,sigbarc,vrqbar,teststat]=vrq(price,q)
```

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