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Terms of trade, capital accumulation and the macro-economy in a developing country: a theoretical analysis

This paper attempts to explain the terms of trade adjustment and the process of capital accumulation in a monetary model of sectoral interlinkage under rational expectation. The paper utilises a very standard dual economy framework in which industry and agriculture are two distinct sectors of production. Agriculture production is supply constrained. This may arise due to fixed endowment of land, weather condition etc. On the other hand, employment and output in the industrial sector are determined on the basis of profit maximisation in the presence of wage indexation. The asset structure of the economy includes the stock of primary commodities as one form of asset holding. Since the stock of agricultural commodities is one of the financial assets, its demand is subject to speculation which may entail fluctuating agricultural prices. Many factors have effects on fluctuating agricultural prices. These include monetary shock, parametric changes in agricultural production, changes in government expenditure etc. In this paper we discuss the comparative static effects of parametric changes of these factors. The paper shows that the short run and long run effects of any particular shock are quite different, not only quantitatively, but also qualitatively. Accordingly, the policy message of the paper is that the short run response may not be a reliable guide to the design of policy.

Keywords: industry-agriculture interlinkage, capital accumulation, terms of trade dynamics

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Introduction

In recent years there has been increasing interest in macroeconomic models of sectoral interlinkage in which commodity price fluctuation is put at the centre of all concerns. This concern is easily understandable, since fluctuations in commodity prices induce fluctuations in real national income and pose problems for macroeconomic management. These models have been seen as a way to formulate policy guidelines to reduce these fluctuations. A key element in many of these models is overshooting of agricultural prices[1] caused by unanticipated monetary expansion. The point is obvious. If a stock of primary good is an asset and its price adjusts instantaneously, while industrial price adjusts slowly, agricultural price overshoots in response to monetary expansion.

The academic interest in the possible impacts of monetary policy on agricultural markets is of recent origin. Frankel (1986) set up a closed economy model to explain overshooting of agricultural prices following an unanticipated expansion in money stock. He emphasised the distinction between fixed price sectors (manufacturing), where prices adjust slowly and a flex price sector (agriculture), where prices adjust instantaneously in response to a change in the money supply[2]. Lai *et al*. (1996) employed Frankel's framework to investigate how anticipated and unanticipated monetary shocks influence commodity prices. Moutos and Vines (1992) also examined the role of commodity prices in the macroeconomic process of output and inflation determination. The econometric models of Orden and Fackler (1989) and Ghironi (2000) show that an increase in money supply raises agricultural prices relative to the general price level for more than a year, indicating persistence of the effect of monetary changes on relative agricultural prices both in the short run and in the long run. Saghaian *et al*. (2002) also explained short run and long run effects on agricultural prices in response to unanticipated monetary shocks. Bakucs and Fertő (2005) tested the overshooting hypothesis with reference to Hungarian agricultural prices. Saghaian *et al*. (2002) studied the overshooting of agricultural prices in four Asian countries (Korea, Philippines, Thailand and Indonesia).

Once we get a story of commodity price fluctuations, however, there arise some immediate questions. Is this fluctuation solely attributable to monetary shock? Do the existing monetary macro models provide an appropriate framework to predict commodity prices and more generally terms of trade behaviour? And, what are the macroeconomic implications of fluctuations in terms of unemployment and output?

We develop a monetary macro model specially designed for a developing country. In particular, this paper makes a theoretical attempt to find answers to these questions. Firstly, we introduce the medium term dynamics through adjustment of capital stock[3]. Secondly, the paper addresses the issue of unemployment of industrial labour. In the model of the existing literature, full employment of labour is assumed and money is neutral in the long run. On the contrary, we incorporate unemployment through wage indexation. Specifically, we assume that industrial wages are indexed to the consumer price index. Thirdly, the paper pays explicit attention to both monetary and real shocks as sources of volatility of terms of trade. The model in this paper is a structuralist

[1] Overshooting of prices is defined as a temporary change in its value beyond its long run equilibrium.

[2] Frankel's model can be seen as an application of Dornbusch's model of exchange rate dynamics.

[3] There exists a literature on the medium-term adjustment in a dual economy. Cardoso (1981) applied a dual economy framework to capture the relationship between agricultural inflation and wage formation. The dynamics in the Cardoso model are located in structural disequilibrium, namely the mismatch between wage goods constraint and non-fulfilment of aspirations of different groups. Taylor (1985) extended the model to include the capital accumulation process in an analysis of steady-state dynamics. Rattsø (1989) offered a similar investigation of the medium-run dynamics for a dual economy. However, these works left aside the role of speculation in inducing volatility in agricultural price.

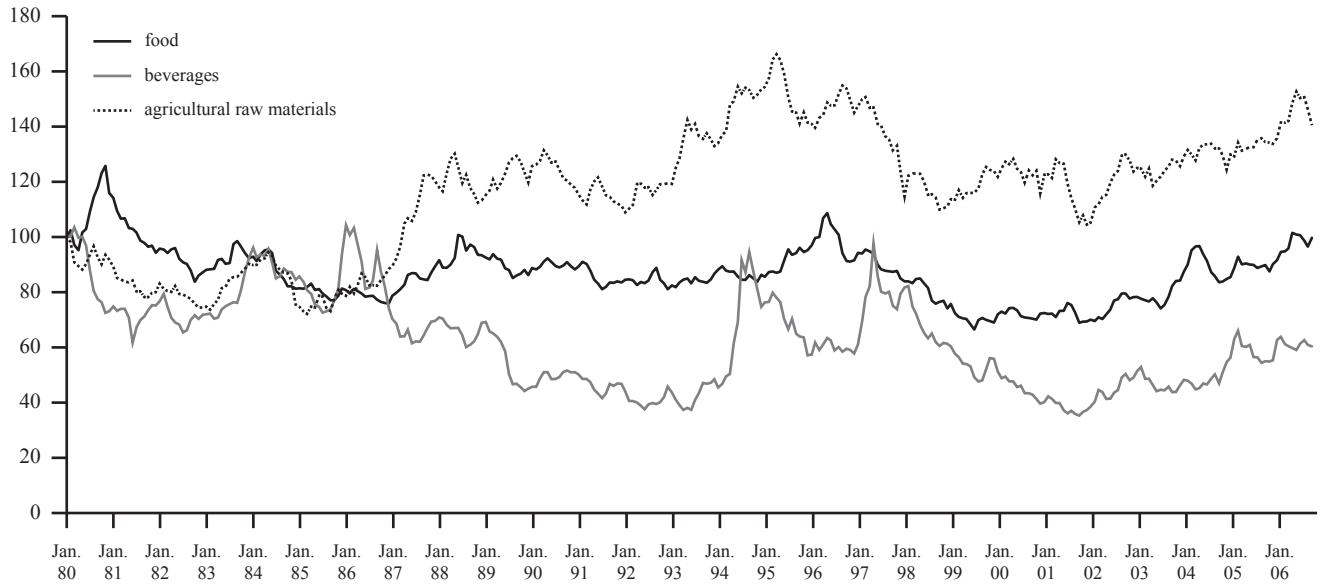

Figure 1: Price developments of agricultural commodities, 1980-2006 (index, January 1980 = 100).
Source: Böwer *et al*. (2007)

monetary model focusing on terms of trade volatility, capital accumulation, employment and output determination in the industrial sector under rational expectation and supply constraints imposed by the agricultural sector. Our model is more complex than the existing models due to two major modifications which are prompted by structural features in a developing country, namely unemployment and medium term dynamics through capital accumulation.

The paper is organised as follows. In the next section we build up the model and then we examine dynamic adjustment and examine saddle path stability of the system. Consideration of a few comparative static exercises follows, while the final section concludes the paper.

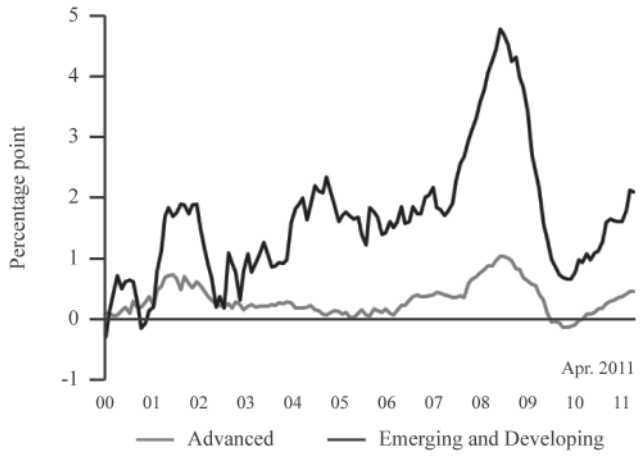

Figure 2: Contribution of food to headline inflation.
Source: Loungani (2012)

The model

Here we attempt to develop a monetary macro model of sectoral interlinkage. The model combines elements from recent research on commodity price fluctuations in a rational expectation framework with (somewhat old) structuralist macroeconomic models of sectoral interlinkage in a dual economy structure. We integrate dual economy interlinkage in a developing country with full stochastic rational expectations equilibrium in a monetary framework. Such integration and use of the model to study the macroeconomic implications of commodity price fluctuations induced by a variety of shocks are the purposes of this paper.

The goods produced are manufactures (produced in the industrial sector) and primary commodities (produced in the agricultural sector). We assume wage indexation for industrial workers, which allows for unemployment in the economy. Food price is a jump variable, which instantaneously guarantees asset market equilibrium. In fact, prices of many agricultural commodities show a high degree of volatility caused by time lags between production decisions and delivery to the market; delayed and inappropriate responses by producers to price signals; inelastic supply and natural shocks. This volatility aspect of commodity prices is evident from Figure 1.

Furthermore, the contribution of surging prices of primary commodities in overall inflation is quite substantial for developing countries compared to the advanced countries (Figure 2).

On the other hand, industrial price is sticky which leads to disequilibrium in the market for industrial goods. Moreover, we assume that the economy is closed. Thus, the model in this paper is a disequilibrium, dual economy model with unemployment and perfect foresight.

This section is subdivided into three subsections. The first contains a description of the supply side and this is followed by a description of asset structure with portfolio choice. The demand side is analysed in the third subsection along with an inflation mechanism for industrial goods.

The supply side

Agricultural output: Agricultural output (F) is fixed in the time frame of our model[4]. Thus, we get the following supply function of the agricultural output:

$$F = \overline{F} \tag{1}$$

Industrial output: Labour (L) and capital (K) are used to produce the industrial output (Y). The production function for industrial production takes the following form:

$$Y = f(L,K),\ f_L > 0,\ f_{LL} < 0,\ f_{LK} > 0 \tag{2}$$

Employment in the industrial sector is derived from the condition of profit maximisation, namely equality between marginal product of labour and real product wage. Thus we get the labour demand function as:

$$L = L\left(\frac{W}{P_Y}\right),\ L' < 0 \tag{3}$$

where W is the money wage and P_Y is the price of industrial output.

Next we consider money wage determination. Instead of assuming flexible adjustment in money wage, we take money wage to be determined as an outcome of a bargaining process. Specifically, we assume that money wage is determined by a social pact which protects the real consumption wage. In other words, money wage is linked to the consumer price index:

$$W = P_Y^{\alpha} P_F^{1-\alpha}\ 0 < \alpha < 1 \tag{4}$$

where α and $1-\alpha$ are constant expenditure shares of industrial goods and agricultural goods respectively and P_F is the price of agricultural output.

Now, $$\frac{W}{P_Y} = \theta^{1-\alpha} \tag{5}$$

where $\theta = \frac{P_F}{P_Y}$, is the terms of trade.

From equations (1), (3) and (5) we get the supply function of industrial goods:

$$Y_s = Y_s(\theta) \text{ with } Y_s' = \frac{dY_s}{d\theta} = \left(\frac{f_L}{f_{LL}}\right)(1-\alpha)\theta^{-\alpha} < 0$$

Aggregate output: Let Z denote aggregate output (or real income) measured in units of industrial goods:

$$Z = \theta F + Y_s(\theta) \tag{6}$$

In what follows we take $\frac{dZ}{d\theta} > 0$ [5]

The financial sector

In the conventional dual economy models, food price equates to the flow supply and flow demand for agricultural output. Here, by contrast, we focus on the role of stock of primary goods as financial assets and we study the effect of forward-looking food price expectation[6]. We assume that stocks of food grains and government bonds are perfect substitutes. The financial sector is represented by the following equations:

$$\frac{M}{P_Y} = aZ - lr\ a,l > 0 \tag{7}$$

$$\frac{\dot{P}_F}{P_F} + s = r \tag{8}$$

where s is the difference between the convenience yield and the storage cost of holding primary commodities and is taken to be positive.

Equation (7) represents the money market equilibrium where M is the nominal money balance, which is deflated by the price of manufactured goods. We choose a linear money demand function where demand for the real balance depends negatively on the interest rate and positively on real income.

Unlike manufactured goods, primary commodities are used as one form of asset along with money and bonds. Equation (8) reflects the assumption of perfect substitutability between stocks of primary commodities and bonds such that returns on these two assets are brought into equality through arbitrage. The term $\frac{\dot{P}_F}{P_F}$ represents expected change (and actual change under the assumption of perfect foresight) in food price. Hence, return on stocks of primary commodities is $(\frac{\dot{P}_F}{P_F} + s)$. The return on bonds is the interest rate (r)[7].

Demand side and industrial price inflation

We choose a linear consumption function such that total private consumption expenditure is $C = \beta Z$, where β is the marginal propensity to consume. Since α is the constant share of expenditure on industrial goods, the consumption demand for industrial goods is $\alpha\beta Z$. Investment expenditure on the industrial goods depends on the real interest rate $(r-\pi_y)$ where r is the nominal interest rate and π is the industrial inflation rate and we choose a linear investment function, namely $a - v(r-\pi)$ ($a>0$, $v>0$). Government expenditure (G)

[4] Availability of land emerges as a binding constraint on agricultural growth. The potential of further expansion of the net sown area is practically non-existent and, given the environmental commitments, it may further decline. Moreover, a feeble price response is attributable not only to a reasonably fixed land supply, but also to lower initial input intensity and crop pattern governed by agro-climatic factors. One can introduce capital accumulation for the agricultural sector as well. However, agriculture in most developing countries has not experienced any major surge in private investment. In any case, industrial investment occurs more quickly and at a faster rate, compared to investment in the agricultural sector.

[5] The response of Z in response to θ is calculated as follows:

$$\frac{dZ}{d\theta} = F + Y_s' = \frac{P_F F - WLe_L}{\theta}$$

where e_L is the elasticity of demand for labour in the industrial sector.

Clearly, $\frac{dZ}{d\theta} > 0$, if $\frac{P_F F}{WL} > e_L$

This condition is more likely to be satisfied in a dual economy with a significant share of agricultural output in national income.

[6] We note that the price of manufactured goods is used as the deflator to express real income measured in units of industrial goods. However the model abstracts from adjustment in the stock of agricultural goods. Though this is a limitation of the model, this is a purposive abstraction which we make to keep the model within tractable limits. The similar abstraction is also made by Frankel (1986) and Moutos and Vines (1992).

[7] Thus we use an asset approach to examine food price dynamics.

is parametrically given. Thus the aggregate demand for the industrial goods is $(\alpha\beta Z + a - v(r-\pi) + G)$.

Next, we consider inflation dynamics. Instead of allowing instantaneous market clearing of industrial goods, we assume the industrial price to be sticky. However, the inflation rate can change in response to excess demand for the industrial goods. Now a few comments on the industrial price are in order. Since the price of industrial goods cannot jump, the industrial sector can remain in disequilibrium. The form of disequilibrium determines the inflation rate. Thus, the current industrial price level is determined by the past rates of inflation and the current inflation rate determines the future price level. At any particular point in time the industrial price is predetermined. Now the inflation rate of industrial price is:

$$\frac{\dot{P}_Y}{P_Y} = \pi_y = \delta\{(\alpha\beta Z + a - v(r-\pi) + G) - Y_S\}, \delta > 0 \qquad (9)$$

Dynamic adjustment and stability

Firstly we consider dynamic adjustment of the terms of trade. Equations (8) and (9) can be combined together to produce the dynamic adjustment in the terms of trade. Noting that $\theta = \frac{P_F}{P_Y}$, we get:

$$\frac{\dot{\theta}}{\theta} = \frac{\dot{P}_F}{P_F} - \frac{\dot{P}_Y}{P_Y} = r - s - \pi\{(\alpha\beta Z + a - v(r-\pi) + G) - Y_S\} \quad (10)$$

In an implicit form the terms of trade dynamics can be expressed as:

$$\frac{\dot{\theta}}{\theta} = g(\theta, K, F, G, M) \qquad (11)$$

with $g_1>0$, $g_2>0$, $g_3>0$, $g_4<0$, $g_5<0$ where:

$$g_1 = A + \theta\frac{\delta r}{\delta\theta} - \theta\left[\delta\left\{\alpha\beta\frac{\delta Z}{\delta\theta} - v\frac{\delta r}{\delta\theta} + v\frac{\delta\pi}{\delta\theta} - Y_s'\right\}\right] > 0\left(\frac{\delta r}{\delta\theta}, \frac{\delta Z}{\delta\theta}, \frac{\delta\pi}{\delta\theta}\right) > 0$$

and we assume that food price inflation is greater than industrial price inflation and A is a constant.

$$g_2 = \theta\left[\frac{\delta r}{\delta K} - \delta\left\{-v\frac{\delta r}{\delta K} + v\frac{\delta\pi}{\delta K} + \frac{\delta Y_s}{\delta K}\right\}\right] > 0\left(\frac{\delta r}{\delta K} > 0, \frac{\delta\pi}{\delta K} < 0, \frac{\delta Y_s}{\delta K} > 0\right)$$

$$g_3 = \theta\left[\frac{\delta r}{\delta F} - \delta\left\{\alpha\beta\frac{\delta Z}{\delta F} - v\frac{\delta r}{\delta F} + v\frac{\delta\pi}{\delta F}\right\}\right] > 0\left(\frac{\delta r}{\delta F} > 0, \frac{\delta Z}{\delta F} < 0, \frac{\delta\pi}{\delta F} > 0\right)$$

and we assume that food price inflation is greater than industrial price inflation.

$$g_4 = -\theta\delta < 0$$

$$g_5 = \theta\left[\frac{\delta r}{\delta M} - \delta\left\{-v\frac{\delta r}{\delta M} + v\frac{\delta\pi}{\delta M}\right\}\right] < 0\left(\frac{\delta r}{\delta M} < 0, \frac{\delta\pi}{\delta M} > 0\right)$$

The intuitive explanation of sign restrictions are as follows:

We begin with an initial steady state value θ such that $\frac{\dot{\theta}}{\theta} = 0$ and examine the effect on $\dot{\theta}$ following any change in variables that appear in equation 11. A rise in θ leads to an increase in the real income and hence the demand for money goes up which leads to an increase in interest rate and causes food price inflation. This follows from equations (7) and (8). On the other hand, a rise in θ leads to a fall in the industrial output. On the demand side, private consumption expenditure on industrial good rises, but private investment falls. For reasonably low interest elasticity of private investment, we can assume that there is an increase in the excess demand for industrial goods and a consequent increase in the rate of industrial price inflation. However, we assume that the rate of food price inflation exceeds that of industrial price inflation in response to the rise in θ such that $g_1>0$.

Next, we consider a rise in K. This causes industrial output to rise. Since aggregate income rises for any given value of θ, the interest rate rises to maintain money market equilibrium, which leads to food price inflation (this follows from equation 8).

On the other hand, a rise in the industrial output and fall in investment reduce the rate of industrial price inflation. Hence, $g_2>0$. Now, a rise in F increases the real income, which causes interest rate to rise. As a result the rate of food price inflation increases. However, the effect on the inflation rate of industrial price is ambiguous. On the one hand, we have an increase in consumption expenditure on the industrial output and on the other we have a fall in investment expenditure. However, we assume that the rise in food price inflation exceeds any possible increase in the rate of industrial price inflation such that $g_3>0$. The explanation for $g_4<0$ is simple. A rise in government expenditure increases the rate of industrial price inflation and hence makes $\frac{\dot{\theta}}{\theta} < 0$, i.e. $g_4<0$.

Lastly, we consider the effect of increase in money supply on $\frac{\dot{\theta}}{\theta}$. A rise in money supply entails a fall in the interest rate such that the rate of food price inflation falls and a rise in investment leads to an increase in the rate of industrial price inflation. This leads to $\frac{\dot{\theta}}{\theta} < 0$, i.e. $g_5<0$.

Next, we consider the dynamics of capital accumulation. Beyond the short run, investment alters the size of the capital stock: $\dot{K} = a - v(r-\pi) - \delta(K), \delta' > 0$, where δ is the rate of depreciation and is positively related to the stock of industrial capital. In an implicit form, the dynamics of capital accumulation can be expressed as:

$$\dot{k} = \phi(\theta, K, F, M) \qquad (12)$$

With the following restrictions on the signs of partial derivatives: $\phi_1 < 0, \phi_2 < 0, \phi_3 < 0, \phi_4 > 0$.

$$\phi_1 = -v\frac{\delta r}{\delta\theta} + \frac{\delta\pi}{\delta\theta} < 0\left(\frac{\delta r}{\delta\theta} > 0, \frac{\delta\pi}{\delta\theta} > 0 \text{ and we assume that } \frac{\delta r}{\delta\theta} > \frac{\delta\pi}{\delta\theta}\right)$$

$$\phi_2 = -v\frac{\delta r}{\delta K} + \frac{\delta\pi}{\delta K}\left(\frac{\delta r}{\delta K} > 0, \frac{\delta\pi}{\delta K} > 0 \text{ and we assume that } \frac{\delta r}{\delta K} > \frac{\delta\pi}{\delta K}\right)$$

$$\phi_3 = -v\frac{\delta r}{\delta F} + \frac{\delta\pi}{\delta F}\left(\frac{\delta r}{\delta F} > 0, \frac{\delta\pi}{\delta F} > 0 \text{ and we assume that } \frac{\delta r}{\delta F} > \frac{\delta\pi}{\delta F}\right)$$

$$\phi_4 = -v\frac{\delta r}{\delta M} + \frac{\delta\pi}{\delta M}\left(\frac{\delta r}{\delta M} < 0, \frac{\delta\pi}{\delta M} > 0\right)$$

Intuitive explanations for these sign restrictions are as follows:

A rise in θ, K and F lead to an increase in the interest rate

and hence reduce investment, while a rise in M leads to a fall in the interest rate and hence a rise in investment.

In the steady state $\frac{\dot{\theta}}{\theta} = 0$ and $\dot{K} = 0$

The slope of $\dot{\theta} = 0$ is $\frac{d\theta}{dK}\Big|_{\dot{\theta}=0} = -\frac{g_2}{g_1} < 0$ and the slope of $\dot{K} = 0$ is $\frac{d\theta}{dK}\Big|_{\dot{k}=0} = -\frac{\phi_2}{\phi_1} < 0$

Equations (11) and (12) constitute a system of differential equations in the terms of trade and the capital stock. The terms of trade is free to jump in response to news, which includes unanticipated current and future changes in exogenous variables and policy instruments. However, the capital stock in the industrial sector is a predetermined variable which can change only on a flow basis in response to investment. We concentrate on a stable saddle path because it gives an economically meaningful result. In the presence of perfect foresight the existence of a unique convergent saddle path requires that there must be one positive and one negative characteristic root such that the determinant:

$$\Delta = \begin{vmatrix} \phi_2 & \phi_1 \\ g_2 & g_1 \end{vmatrix} < 0 \text{ i.e.} -\frac{g_2}{g_1} < -\frac{\phi_2}{\phi_1}$$

This condition is satisfied if the $\dot{K} = 0$ locus is steeper than the $\dot{\theta} = 0$ locus (Figure 3). The saddle path SS is downward sloping and flatter than the $\dot{\theta} = 0$ locus. The equation of the saddle path is

$$(\theta - \bar{\theta}) = \left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)(K - \bar{K}) = \left(\frac{g_2}{\lambda_1 - g_1}\right)(K - \bar{K})$$

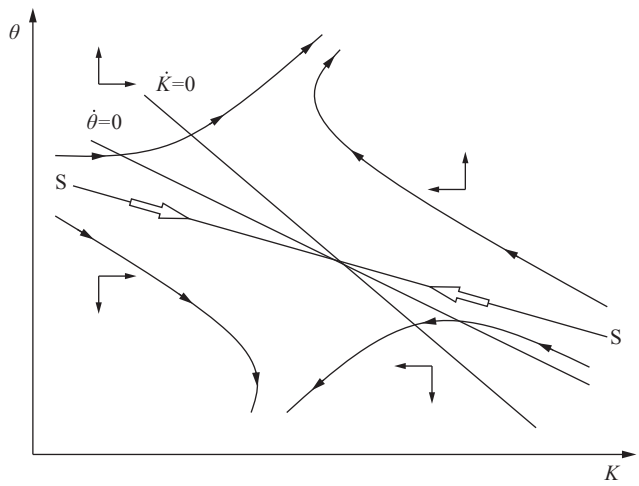

Figure 3: Phase diagram and saddle path stability with θ being the jump variable and K being the slow moving variable.
Source: own composition

Comparative statics

In this section we consider the effect on terms of trade and capital stock in response to a variety of shocks. The specific shocks examined are: (a) increase in money supply, (b) increase in government expenditure and (c) a rise in the agricultural output of food.

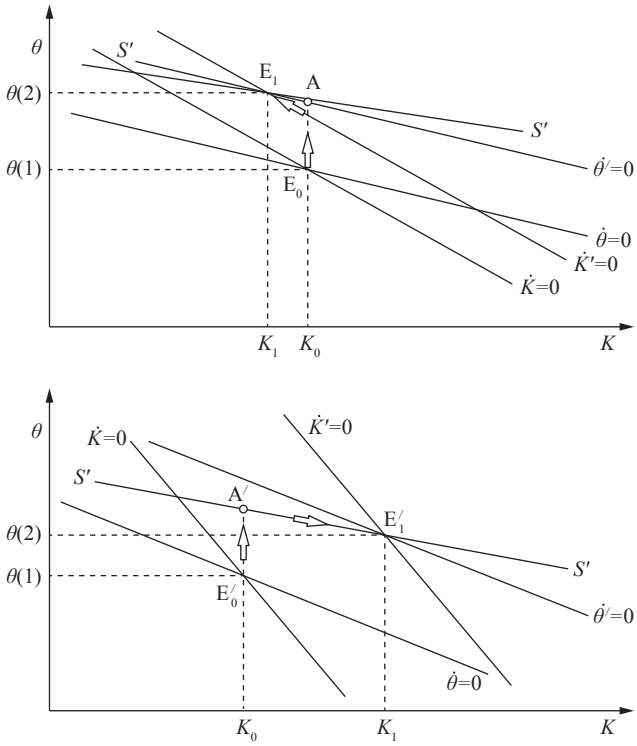

Figure 4: (a) Undershooting and (b) overshooting of θ associated with monetary expansion.
Source: own composition

Monetary expansion

In our model the real money supply in units of industrial goods is fixed. Thus money supply adjusts in response to inflation of industrial goods. In other words, money supply is proportional to the price of industrial goods. By monetary policy we mean change in this proportion[8]. An increase in real money balance may generate a contractionary effect on industrial output along with industrial price inflation. The explanation of the effect of monetary expansion is straightforward.

Let us consider an increase in real balance. This leads to a fall in the interest rate and hence it causes a fall in the rate of food price inflation. Again, private investment rises due to the fall in the interest rate, which entails a rise in the rate of industrial price inflation. On both counts, we have a fall in the rate of change in θ, that is, we have $\dot{\theta} < 0$, starting from $\dot{\theta} = 0$. Hence, θ will rise to maintain $\dot{\theta} = 0$. This is represented by an upward shift of the $\dot{\theta} = 0$ curve. Since interest rate falls, private investment rises and the $\dot{K} = 0$ curve shifts to the right. For any given value of the stock of industrial capital, the terms of trade increases leading to a rise in the real wage rate and a consequent fall in the industrial output. A possible long run outcome is a fall in the stock of industrial capital at the end the adjustment process. In the case of a fall in the capital stock, the terms of trade undershoots its long run equilibrium value. This will exacerbate the initial industrial recession. On the other hand, if capital stock increases, terms of trade overshoots in the long run (Figures 4a and 4b).

[8] We consider the cases where monetary changes are both unanticipated and implemented as soon as they are announced.

In Figure 4a (4b) the equilibrium initially jumps from point E_0 ($E_0^{/}$) to point A (A$^{/}$) along the new saddle path. Since K cannot jump instantaneously, point A (A$^{/}$) is the only transitional position. The transition towards the new steady state is characterised by the path between A (A$^{/}$) and E_1 ($E_1^{/}$), along which K declines (rises) and θ increases. In fact, a decrease (increase) in K causes a rise (fall) in θ causing undershooting of θ. The final and initial changes in θ are represented by following two equations respectively:

$$d\theta = \alpha_2 dM = \frac{-g_5\phi_2 + g_2\phi_4}{\Delta} dM \begin{array}{l} > 0 \; if - \frac{g_5}{g_2} > \frac{\phi_4}{\phi_2} \\ < 0 \; otherwise \end{array}$$

The initial change in θ is given by:

$$\theta(0) - \bar{\theta}(1) = \left(\alpha_2 - \alpha_1\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)\right) dM$$

Subtracting the second equation from the first equation we obtain the value of undershooting or overshooting of θ and this is given by:

$$\bar{\theta}(2) - \bar{\theta}(0) = \alpha_1\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) dM$$

If $\alpha_1 > 0$ i.e. capital stock increases, then the terms of trade overshoots. On the other hand, if $\alpha_1 < 0$ i.e. capital stock decreases the terms of trade undershoots[9].

Rise in government expenditure

An increase in government spending on manufactured goods leads to a rise in the rate of industrial price inflation. This in turn causes $\dot{\theta} < 0$, starting from $\dot{\theta} = 0$. Thus fiscal expansion shifts the $\dot{\theta} = 0$ curve upwards. For any given value of the stock of industrial capital, the terms of trade rises on impact and the economy is placed at point A. Over a period of time the capital stock begins to decline and the terms of trade rises further. Thus, the model shows undershooting of the terms of trade following fiscal expansion. The rise in terms of trade causes an initial contraction of the industrial sector, which is aggravated by a fall in the capital stock.

Our result needs to be contrasted with the general price overshooting model (Frankel, 1986; Moutos and Vines, 1992). In those papers, there was an unambiguous expansion of industrial output and an ambiguous effect on the terms of trade. Moreover, in their papers the terms of trade may either overshoot or undershoot. In our model, the terms of trade unambiguously move in favour of the agricultural sector and, given wage indexation in the industrial sector, a rise in the terms of trade invariably leads to a fall in industrial output. Moreover, capital stock decumulation in the industrial sector causes undershooting of the terms of trade.

This is illustrated in Figure 5 where the equilibrium initially jumps from point E_0 to point A along the new saddle path. Since K cannot jump instantaneously, point A is the only transitional position. The transition towards the new steady state is characterised by the path between A and E_1 along which K declines and θ increases. In fact, a decrease

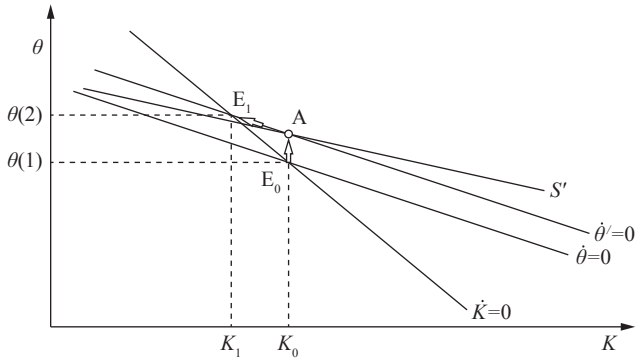

Figure 5: Undershooting of θ on account of a rise in government expenditure.
Source: own composition

in K causes a rise in θ causing undershooting of θ. The initial and final changes in θ can be captured by following equations:

$$d\theta = \alpha_4 dG = \frac{-g_4\phi_2}{\Delta} dG > 0$$

The initial change in θ is given by:

$$\theta(0) - \bar{\theta}(1) = \left(\alpha_4 - \alpha_3\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)\right) dG$$

Subtracting the second equation from the first equation we obtain the value of undershooting of θ and this is given by:

$$\bar{\theta}(2) - \bar{\theta}(0) = \alpha_3\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) dG$$

Since $\alpha_3 < 0$ i.e. capital stock decreases, the terms of trade undershoots.[10]

Rise in agricultural output and relaxation of supply constraint

Rising agricultural production is of great importance in promoting economic development. Since land is a major constraint, agricultural expansion requires significant technological progress. In this paper a rise in agricultural output may lead to an expansion in industrial output. This is amenable to easy economic interpretation.

Suppose we consider an increase in agricultural output. This leads to an increase in aggregate income and hence the demand for money rises. It follows from the money market equilibrium the interest rate rises, leading to $\frac{\dot{P}_F}{P_F} > 0$. On the other hand, the effect on excess demand for the industrial output is ambiguous. It has already been explained in the context of dynamic adjustment of the system, $\frac{\dot{\theta}}{\theta} > 0$ following an increase in the agricultural output. Hence, the $\frac{\dot{\theta}}{\theta} = 0$ curve shifts downwards (Figure 6). Again, a rise in interest

[9] See the mathematical appendix for derivation of both steady state effects and transitional dynamics.

[10] Long run and transitional dynamics of fiscal expansion can be calculated very easily by similar methods as in increase in money supply.

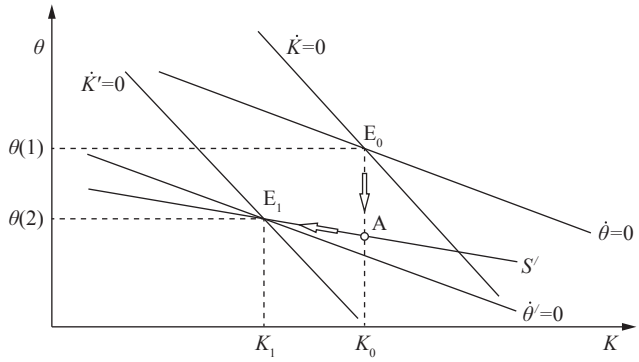

Figure 6: Overshooting of θ associated with an increase in agricultural output.

Source: own composition

rate reduces investment and the $\dot{K} = 0$ shifts to the left. The effect on the stock of industrial capital is ambiguous. In case of a rise (fall) in the stock of industrial capital the terms of trade undershoots (overshoots) its long run equilibrium value. However, it is unambiguous that the terms of trade decline on impact, leading to an expansion of industrial output. The final effect on industrial output is, however, ambiguous. If the stock of industrial capital rises, the initial favourable effect on the industrial output is reinforced. In the case of a decline in the industrial capital the initial effect may be short-lived and industrial recession may follow, despite an increase in the agricultural output.

The final change in θ can be given as:

$$d\theta = \alpha_6 dF = \frac{-g_3\phi_2 + \phi_3 g_2}{\Delta} dF \begin{array}{l} < 0 \; if - \frac{g_3}{g_2} > \frac{\phi_3}{\phi_2} \\ > 0 \; otherwise \end{array}$$

The initial change in θ is given by:

$$\theta(0) - \bar{\theta}(1) = \left(\alpha_6 - \alpha_5\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)\right) dF$$

Subtracting the second equation from the first equation we obtain the value of undershooting or overshooting of θ and this is given by:

$$\bar{\theta}(2) - \bar{\theta}(0) = \alpha_6\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) dF$$

If $\alpha_6 < 0$ i.e. capital stock decreases, then the terms of trade undershoots. On the other hand, if $\alpha_6 < 0$ i.e. capital stock increases, then the terms of trade overshoots.

Discussion

In this paper we have examined the role of commodity prices in shaping the macroeconomic process of output, inflation and terms of trade determination. Our aim has been to show the important role which commodity prices can play in overall macroeconomic developments. The short run and long run impacts of different macroeconomic policy add to price and income instability, and influence financial viability of farmers tremendously.

The empirical relevance of such a model is also very robust. But we depart from these existing literatures in terms of two fundamental differences in the characterisation of a dual economy. The specific features of the model in this paper are completely suitable for a developing economy. One basic difference is the nature of determination of industrial output. The paper addresses the issue of unemployment of industrial labour. In the existing literature, most of the models assume full employment of labour and accordingly these models obtain long run neutrality of money. We depart from the existing literature in the sense that we incorporate unemployment through wage indexation.

The second difference is the medium run adjustment through the process of accumulation of industrial capital, which our paper focuses on. All medium run macro models are non-monetary macro models. On the other hand, the existing monetary macro models which explain the volatility of agricultural price is typically a short-run model in the sense that adjustment in capital stock is ignored. We try to integrate both approaches by incorporating adjustment of capital stock in a monetary, dual economy framework. The adjustment in the stock of industrial capital also leads to a difference in the short run and the long run effects of different macroeconomic shocks.

One possible extension of the paper is to introduce open economy dimensions into the model. One can realistically assume that the agricultural sector produces not only for the home market but also for the external market, particularly in the post-WTO situation and the industrial sector uses an imported input. There is hardly any literature on dual economy interlinkage in an open economy, monetary macro model[11]. Clearly the content of our paper can be suitably modified to accommodate relevant open economy issues.

Mathematical appendix

Saddle path derivation

From equation (11) and (12) we can write:

$$\dot{K} = \phi(\theta, K) \quad (1)$$
$$\dot{\theta} = g(\theta, K) \quad (2)$$

Taking Taylor series linear approximation around the initial steady state values $\bar{\theta}_1$ *and* $\bar{K}_1$ for period $t \leq T$,[12] we get:

$$0 = \phi_2(K - K_1) + \phi_1(\theta - \theta_1) \quad (3)$$
$$0 = g_2(K - K_1) + g_2(\theta - \theta_1) \quad (4)$$

Arranging (3) and (4) in matrix form, we get:

$$\begin{bmatrix} \dot{K} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \phi_2 & \phi_1 \\ g_2 & g_1 \end{bmatrix} \begin{bmatrix} K - \bar{K}_1 \\ \theta - \bar{\theta}_1 \end{bmatrix} \quad (5)$$

Similarly for period $t \geq T$,[13] we get:

$$0 = \phi_2(K - K_2) + \phi_1(\theta - \theta_2) \quad (6)$$

[11] Orden and Fackler (1989), Ghironi (2000), Saghaian *et al.* (2002), Nag end Goswami (2005) and Asfaha and Jooste (2007) develop an open economy macro model which examines the issue of volatility of agricultural price.

[12] Here T denotes the time period at which a change in exogenous variable occurs.

[13] Here $t \geq T$, denotes the time period after a change in exogenous variable takes place.

$$0 = g_2(K - K_2) + g_1(\theta - \theta_2) \tag{7}$$

The dynamics in period $t \geq T$ are specified by

$$\begin{bmatrix} \dot{K} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \phi_2 & \phi_1 \\ g_2 & g_1 \end{bmatrix} \begin{bmatrix} K - \bar{K}_2 \\ \theta - \bar{\theta}_2 \end{bmatrix} \tag{8}$$

Clearly eigen values λ_1 *and* λ_2 of equation 5 and 8 are identical. For the saddle path stability we require that $\lambda_1\lambda_2 = \Delta = \begin{bmatrix} \phi_2 & \phi_1 \\ g_2 & g_1 \end{bmatrix} < 0$. Let us assume that $\lambda_1 < 0$, $\lambda_2 > 0$.

Now, over the period $0 < t \leq T$, the solutions for K and θ are given by

$$K(t) = \bar{K}_1 + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \tag{9}$$

$$\theta(t) = \bar{\theta}_1 + \left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) A_1 e^{\lambda_1 t} + \left(\frac{\lambda_2 - \phi_2}{\phi_1}\right) A_2 e^{\lambda_2 t} \tag{10}$$

Again, for the period $t \geq T$, the solutions for K and θ are:

$$K(t) = \bar{K}_2 + A_1' e^{\lambda_1 t} + A_2' e^{\lambda_2 t} \tag{11}$$

$$\theta(t) = \bar{\theta}_2 + \left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) A_1' e^{\lambda_1 t} + \left(\frac{\lambda_2 - \phi_2}{\phi_1}\right) A_2' e^{\lambda_2 t} \tag{12}$$

It is noted that $\theta(t)$ and $K(t)$ do not diverge as $t \to \alpha$ when $A_2' = 0$ and hence the solutions would be:

$$K(t) = \bar{K}_2 + A_1' e^{\lambda_1 t} \tag{13}$$

$$\theta(t) = \bar{\theta}_2 + \left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) A_1' e^{\lambda_1 t} \tag{14}$$

The remaining constants A_1, A_2 and A_1' are obtained by solving the equations:

$$A_1 + A_2 = 0 \tag{15}$$

$$A_1 - A_1' e^{\lambda_1 T} + A_2 e^{\lambda_{12} T} = dK \tag{16}$$

$$\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)(A_1 - A_1')e^{\lambda_1 T} + \left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) A_2 e^{\lambda_{12} T} = d\theta \tag{17}$$

Here $d\theta$ and dK are shifts in the steady state corresponding to particular shift parameter. Now the stable saddle paths after time T are described by equations (11) and (12). Eliminating $A_1' e^{\lambda_1 T}$ from these equations we get:

$$(\theta - \bar{\theta}_2) = \left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)(K - \bar{K}_2) = \left(\frac{g_2}{\lambda_1 - g_1}\right)(K - \bar{K}_2) \tag{18}$$

Comparative statics

Increase in money supply

<u>Long run effects:</u>

From equation (11) and (12) we get:

$$\dot{K} = \phi(\theta, K, M); \phi_1 < 0, \phi_2 < 0, \phi_4 > 0, \tag{11a}$$

$$\dot{\theta} = g(\theta, K, M); g_1 > 0, g_2 > 0, g_5 < 0 \tag{12a}$$

Differentiating equations (11a) and (12a) with respect to M and setting $\dot{K} = 0$ and $\dot{\theta} = 0$ respectively we get:

$$\begin{bmatrix} \phi_2 & \phi_1 \\ g_2 & g_1 \end{bmatrix} \begin{bmatrix} \frac{dK}{dM} \\ \frac{d\theta}{dM} \end{bmatrix} = \begin{bmatrix} -\phi_4 \\ -g_5 \end{bmatrix} \tag{19}$$

Applying Cramer's Rule we get:

$$\frac{d\theta}{dM} = \alpha_2 = \frac{-g_5\phi_2 + g_2\phi_4}{\Delta} > 0 \; if - \frac{g_5}{g_2} > \frac{\phi_4}{\phi_2} \; < 0 \; elsewhere \tag{20}$$

$$\frac{dK}{dM} = \alpha_1 = \frac{-g_1\phi_4 + g_5\phi_1}{\Delta} > 0 \; if - \frac{g_1}{g_4} > \frac{\phi_1}{\phi_4} \; < 0 \; elsewhere \tag{21}$$

Transitional details

To obtain the initial jump in θ after increase in M, we set $t = 0$ and using $A_1 = -A_2$ in equation 10 we get:

$$\theta(0) = \bar{\theta}_1 + \left(\frac{\lambda_2 - \lambda_1}{\phi_1}\right) A_2 \tag{22}$$

Solving equations (16) and (17) we get:

$$A_2 = \frac{\left(\alpha_2 - \alpha_1\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)\right)}{\frac{\lambda_2 - \lambda_1}{\phi_1}} dM \tag{23}$$

Substituting the value of A_2 in equation (22) we get:

$$\theta(0) = \bar{\theta}(1) + \left(\alpha_2 - \alpha_1\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)\right) dM$$

$$\Rightarrow \theta(0) - \bar{\theta}(1) = \left(\alpha_2 - \alpha_1\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)\right) dM \tag{24}$$

Equation (24) represents initial jump in θ.

The value of overshooting or undershooting of θ we subtracting equation 24 from equation 20 and this is given by $\bar{\theta}(2) - \theta(0)$ ($\bar{\theta}(2)$ is the final steady state value of θ)

$$= \alpha_2 dM - \left(\alpha_2 - \alpha_1\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right)\right) dM$$

$$= \alpha_1\left(\frac{\lambda_1 - \phi_2}{\phi_1}\right) dM$$

Since $\frac{\lambda_1 - \phi_2}{\phi_1} < 0$ from the slope saddle path, we get clearly undershooting of θ when capital stock decreases i.e. $\alpha_1 < 0$. On the other hand, if capital stock increases the terms of trade overshoots in the long run.

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