Functional Forms and Economic Surplus Changes[+]

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Abstract

Equilibrium displacement modelling (EDM) is now a popular procedure to estimate the extent and distribution of the benefits from new technology or demand enhancement. Alston and Wohlgenant showed empirically that choice of functional form had only a small impact on the measurement of economic surplus changes. However many recent studies have continued to assume global linearity, which implies the quite stringent restriction that the own-price supply elasticity exceeds unity. In this paper it is shown analytically, using the Taylor Expansion Theorem, that if parallel shifts are assumed and the research-induced shift of supply (or demand) is small, only a local linear approximation of any functional form is required to accurately measure economic surplus changes. Thus no assumption of functional form is required, nor is the restriction that the supply elasticity exceeds unity. The analytical approach allows an assessment of the magnitude and direction of errors made in estimating changes in prices, quantities and economic surplus. Alternative surplus formulae are also compared for situations involving multiple markets and endogenous shifts.

Keywords: economic surplus measurement, functional form, equilibrium displacement modelling

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1. Introduction

Following Buse (1958), Muth (1964) and Gardner (1979), there has been a considerable literature applying comparative static analysis to structural models of commodity markets to estimate small finite changes in prices and quantities and consequent changes in economic surplus, from new technology or demand enhancement. This approach has been termed 'equilibrium displacement modelling'(EDM) by Piggott (1992). In empirical applications of this method, changes in prices and quantities have been estimated from linear approximations of the demand and supply curves and shifts in demand and supply curves have been assumed to be parallel. Both assumptions are likely to be a source of error in addition to errors in estimating the extent of the supply or demand shift.

Lindner and Jarrett (1978) pointed to the potential errors from making an incorrect assumption about the nature of the supply shift. No doubt similar errors can arise from assuming that promotion and changes in product quality can be modelled as a parallel shift in demand. Rose's (1980) arguments about the difficulty of forming expectations a priori about the nature of a supply shift from new technology have been well accepted and the custom has been to assume parallel shifts in demand and supply in the absence of strong evidence to the contrary. In this paper parallel demand and supply shifts are maintained and attention is focussed on errors from using linear approximations of the true functional forms of demand and supply.

The issue of functional form and linear approximation has been a continuing source of confusion in the EDM literature, despite the efforts of Alston and Wohlgenant (1990) and Alston, Pardey and Norton (1995). The main point of confusion concerns whether linear approximation of demand and supply based on point estimates of demand and supply elasticities necessarily requires the global imposition of either a linear or constant elasticity functional form. Alston and Scobie's (1983) application of EDM was criticised by Freebairn, Davis and Edwards (1983) as actually using "a global linear ... but ...a constant elasticity model as a local approximation". From their empirical experiment using linear and constant elasticity demand and supply functions, Alston and Wohlgenant (1990) have provided some evidence that for small parallel supply shifts, the errors in estimating changes in prices, quantities and economic surplus using the linear approximation methods of EDM are small when the 'true' demand and supply curves are of constant elasticity form. They therefore concluded that functional form is not a critical issue in EDM applications. However many studies have continued to assume global linearity, which, to some, also implies a quite stringent restriction that supply is elastic in order to have a positive supply intercept (for example, Piggott, Piggott and Wright 1995).

In this paper, the issue of functional form is addressed analytically. This examination confirms the empirical results of Alston and Wohlgenant (1990) that no assumption about functional form is needed either to approximate changes in prices and quantities or to estimate changes in economic surplus using standard formulae (as in Mullen, Alston and Wohlgenant 1989, and Piggott, Piggott and Wright 1995). It is shown that the errors that arise from making local linear approximations of the true functional form are small as long as the supply (or demand) shift is parallel and the amount of the shift, λ, is small. Thus there is no need to assume that supply is elastic or that demand and supply are globally linear. The contribution of this paper over that by Alston and Wohlgenant (1990) is that the analytical approach used here allows more general conclusions to be drawn than is possible from their empirical approach. In particular some of their empirical conclusions are specific for the constant elasticity function and do not hold with generality.

Using the mathematical concept of 'order of magnitude' (Ledermann and Vajda 1982, Chapter 2), the errors in estimating price, quantity and consumer and producer surplus changes are calculated (the formal definition and properties of the mathematical notation $O(\,)$ are given in Appendix 1). These errors are shown to be of the infinitesimal order of $O(\lambda^2)$ when $\lambda \to 0$. The error in the total surplus change measure is even smaller, being of the order of $O(\lambda^3)$ when $\lambda \to 0$. The exact expressions and the upper bounds of these errors are derived as an indication of direction of the errors and the accuracy of the EDM results. In Section 3 the paper also explores the appropriate formulae for measuring surplus change areas for a multimarket situation when the supply and/or demand shift is endogenous, and similar results concerning EDM errors are given as for a single market model in Section 2.

2. A Simple Model

Consider a simple model of a market for a commodity. Assume that the true demand and supply curves for the considered commodity are not known but can be represented in general form as:

(1) S_1 $Q = S(P)$ *initial supply curve*

(2) D_1 $Q = D(P)$ *initial demand curve*

The intersection of the above curves, $E_1(Q_1, P_1)$, is the initial equilibrium point with Q_1 and P_1 as the initial equilibrium quantity and price (Figure 1). Assume that a new technology will cause the supply curve to shift down and, following Rose, that the shift is parallel in the price direction. The new supply curve will be:

(3) S_2: $Q = S(P-\Delta S)$ *new supply curve*

where $\Delta S = S_2 - S_1 = \lambda P_1$ is the constant change in cost on a per unit basis, λ is the cost change as a percentage of the initial price level P_1, and $\Delta S<0$ and $\lambda<0$ for a downward supply shift[1]. λ is exogenous to the model. The new equilibrium point is the intersection of D_1 and S_2 and denoted as $E_2(Q_2, P_2)$ in Figure 1.

The aim of using the EDM is to employ knowledge of the current equilibrium price and quantity (point E_1), the demand and supply elasticity values at E_1, and the amount of the percentage supply shift, λ, to approximate the changes in price and quantity associated with moving to the new equilibrium point E_2 and thus estimate the economic surplus changes resulting from the displacement from E_1 to E_2. These

[1] $\Delta S>0$ and $\lambda>0$ for an upward supply shift.

market parameters are the basis of a local linear approximation of the true but unknown demand and supply functions. Below we will illustrate geometrically and prove analytically that, as long as the supply shift is parallel and the amount of the shift λ is small, the errors involved in the EDM procedure are very small in both solving for the new equilibrium point E_2 and calculating surplus changes. The sizes of the errors are expressed in the 'order of magnitude' of the percentage shift, λ, when $\lambda \to 0$

2.1 Solving For Price and Quantity Changes

Price and quantity changes are estimated by applying comparative static analysis to the demand and supply functions above. Differentiating the logarithms of equations (2) and (3) gives

(4) $\quad dQ/Q = \eta(dP/P)$

(5) $\quad dQ/Q = \varepsilon(dP/P-\lambda)$

where η and ε are the demand and supply elasticities at point E_1. Approximating the infinite small change d( ) with a finite change Δ( ) from point E_1 to E_2, ie.

$$\Delta Q = Q_2 - Q_1 \approx dQ \text{ and}$$

$$\Delta P = P_2 - P_1 \approx dP,$$

we have the following linear relationship between the percentage change of price (EP^*), the percentage change of quantity (EQ^*) and the percentage supply shift λ.

(4)' $\quad EQ^* = \eta EP^*$

(5)' $\quad EQ^* = \varepsilon(EP^* - \lambda)$

Solving (4)' and (5)' jointly gives an estimate of the relative price and quantity changes resulting from the research-induced supply shift:

(6) $\quad EP^* = \lambda\varepsilon/(\varepsilon-\eta)$

(7) $\quad EQ^* = \lambda\varepsilon\eta/(\varepsilon-\eta)$

Thus the new equilibrium point E_2 is approximated by $E_2^*(Q_2^*, P_2^*)$, where

(8) $\quad Q_2^* = Q_1(1+EQ^*) \quad$ or $\quad EQ^* = (Q_2^*-Q_1)/Q_1$

(9) $\quad P_2^* = P_1(1+EP^*) \quad$ or $\quad EP^* = (P_2^*-P_1)/P_1$

where the superscript '*' represents the EDM estimates of the true variables.

Figure 1 illustrates geometrically how an EDM approximates the new equilibrium point E_2 and the error involved. With the EDM approach, we use the tangent to the demand curve D_1 at initial point E_1, denoted by D_1^*, to locally approximate D_1. Similarly, the tangent to the initial supply curve S_1 at point E_1, denoted by S_1^*, is used to locally approximate S_1. S_1^* is then shifted down by $\Delta S=\lambda P_1$ in the price direction to obtain S_2^*, which is the tangent to S_2 at point $B(Q_1, P_1+\Delta S)$. S_2^* is used as a local approximation of the new supply curve S_2. The intersection point of D_1^* and S_2^*, denoted by E_2^*, is then used to approximate the new equilibrium point E_2.

We can see from Figure 1 that the difference between E_2^* and E_2 is very small as long as the amount of the shift ΔS (or λ) is small. E_2^* will coincide with E_2 when D_1 and S_1 are exactly linear around the neighbourhood of E_1 (ie. when $S_2 = S_2^*$ and $D_1 = D_1^*$ locally). In other words, the EDM price and quantity changes are exactly correct for local linear demand and supply functions. We can also see from Figure 1 that we always overestimate the size of EP for a downward shift in supply. EQ can be over or under estimated depending on the relative distance from S_2 to S_2^* and from D_1 to D_1^*.

Analytically, through a Taylor expansion of the demand and supply functions, we can show that the error involved in approximating point E_2 with E_2^* is of the order of $O(\lambda^2)$ when $\lambda \to 0$, ie

(10) $\quad |EP-EP^*| = O(\lambda^2)$

(11) $\quad |EQ-EQ^*| = O(\lambda^2)$

The mathematical proof of this result is given in Proposition 2 of Appendix 2. Equations (10) and (11) are strong mathematical results which imply that, if a percentage shift λ is small, the approximation errors are roughly of the magnitude of λ^2. But, for a given small value of λ in a particular problem, it is also useful to know the exact expression for the error terms in terms of the parameter values in a particular model. Examining the higher order terms in the Taylor expansion, we can derive a relationship between the magnitude of the error and the first and second order parameters of demand and supply curves (elasticity and curvature). The upper bounds of the errors can then be estimated as (proofs in Proposition 4 in Appendix 2):

(12) $\quad |EP - EP^*| \leq |2Q_1(\varepsilon-\eta)|^{-1}(P_1^2\lambda^2)\, |D^{(2)}(c1) - S^{(2)}(c2)|$

(13) $\quad |EQ - EQ^*| \leq |2Q_1(\varepsilon-\eta)|^{-1}(P_1^2\lambda^2)\, \max(|\varepsilon D^{(2)}(c_1)|, |\eta S^{(2)}(c_2)|)$

where $D^{(2)}(c_1)$ and $S^{(2)}(c_2)$ are the second derivatives of demand and supply with respect to price and $P_2 \leq c_i \leq P_1 (i=1,2)$. Therefore, if we are willing to make assumptions of ranges of the first (elasticities) and second derivatives of demand and supply, we can estimate the size of these errors. We can also see from equations (12) and (13) that the errors are zero when the second derivatives are zero. In other words, the EDM estimates for price and quantity changes are exactly correct when demand and supply are strictly linear around the neighbourhood of the current equilibrium point.

Some interesting results are also obvious from these error expressions. For example, the more inelastic ($|\varepsilon|$ and $|\eta|$, smaller) and curved ($|D^{(2)}(c_1)|$ and $|S^{(2)}(c_2)|$, bigger) are the demand and supply functions, the bigger are the errors in EP*. Additionally, if we assume that the supply curve is increasing and concave and the demand curve is decreasing and concave in the vicinity of the equilibrium point, ie

$$(14) \quad \varepsilon>0, \quad S^{(2)}(P)<0, \text{ and}$$

$$(15) \quad \eta<0, \quad D^{(2)}(P)>0,$$

where $P \in (P_2, P_1)$, we can also show that (proof in Remark 1 of Proposition 4, Appendix 2)[2]

$$(16) \quad EP \geq EP^*$$

In other words, we always overestimate the size of a price decrease (when EP<0 in the case of a downward supply shift) and underestimate the size of a price increase (when EP>0 in the case of an upward supply shift)[3]. This analytically proves the empirical result of Alston and Wohlgenant (1990) but also demonstrates that their finding is conditional on the nature of the curvature of demand and supply functions (as can be seen from equation A.37). The sign of the error term for the quantity change $EQ\text{-}EQ^*$ depends on the relative sizes of the demand and supply elasticities and curvatures. It can be positive or negative. The empirical result from Alston and Wohlgenant (1990) that quantity change in the constant elasticity case is always overestimated does not hold with generality.

2.2 Calculating Surplus Changes

The displacement from the initial equilibrium point E_1 to the new equilibrium point E_2 will cause changes in the producer, consumer and total surplus measures. Referring to Figure 1, the initial consumer surplus (CS_1) and producer surplus (PS_1) with respect to E_1 are the areas P_1E_1C and $A_1E_1P_1$. The new consumer surplus (CS_2) and producer surplus (PS_2) after the shift are the areas P_2E_2C and $A_2E_2P_2$. Thus the changes in the consumer surplus (ΔCS), producer surplus (ΔPS) and total surplus (ΔTS), when assuming a parallel shift from S_1 to S_2, are the areas $P_2E_2E_1P_1$, FBE_2P_2 and $FBE_2E_1P_1$, respectively. Analytically, these "true" surplus change areas can be expressed by the integrals of the demand and supply functions along the price axis.

$$(17) \quad \Delta CS = \text{Area}(P_2E_2E_1P_1) = \int_{P2}^{P1} D(P)dP$$

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[2] The same result holds for a demand shift.

[3] For a demand shift, the price decrease will be overestimated for an upward demand shift and the price increase will be underestimated for a downward demand shift.

(18) $\Delta PS = Area(FBE_2P_2) = \int_{P1+\Delta S}^{P2} S(P-\Delta S)dP$

(19) $\Delta TS = Area(FBE_2E_1P_1) = \int_{P2}^{P1} D(P)dP + \int_{P1+\Delta S}^{P2} S(P-\Delta S)dP$

If we know the exact functional forms of the demand and supply curves, these surplus changes can be calculated exactly either analytically or numerically.

Figure 1 illustrates the errors involved in measuring the above surplus areas using an EDM. Since we use D_1^* and S_2^* to locally linear approximate the "true" demand and supply curve D_1 and S_2, we are using areas $P_2^*E_2^*E_1P_1$, $FBE_2^*P_2^*$ and $FBE_2^*E_1P_1$ to approximate the true surplus change areas in equations (17)-(19). These approximations can be analytically represented as (Alston 1991):

(20) $\Delta CS^* = Area(P_2^*E_2^*E_1P_1) = -P_1Q_1\, EP^*(1+0.5EQ^*)$

(21) $\Delta PS^* = Area(FBE_2^*P_2^*) = P_1Q_1(EP^* - \lambda)(1+0.5EQ^*)$

(22) $\Delta TS^* = Area(FBE_2^*E_1P_1) = -\lambda P_1Q_1(1+0.5EQ^*)$

where $\lambda<0$ for a downward supply shift[4].

Geometrically we can see from Figure 1 that the error in measuring ΔTS with ΔTS^* is extremely small (with triangle $BE_2^*E_1$ approximating BE_2E_1). The errors in estimating ΔCS and ΔPS with ΔCS^* and ΔPS^* are also relatively small, depending on the magnitude of the error in EP^*.

Analytically, in Appendix 2 (Proposition 3), using the Taylor Expansion Theorem we proved the following results:

(23) $|\Delta CS - \Delta CS^*| = O(\lambda^2)$

(24) $|\Delta PS - \Delta PS^*| = O(\lambda^2)$

(25) $|\Delta TS - \Delta TS^*| = O(\lambda^3)$

In other words, the error in measuring the total surplus change is of a higher order of "infinite small" (when $\lambda \to 0$) than that of measuring consumer and producer surplus changes. This is the mathematical reasoning behind the "striking" empirical results of Alston and Wohlgenant (1991) that for the case of constant elasticity demand and supply, the error in ΔTS^* is much smaller than that in ΔPS^*, ΔCS^*, EP^* and EQ^*.

Another interesting analytical result is that the level of the errors in measuring producer and consumer surplus changes (ΔPS and ΔCS) is largely due to the error in

[4] $\lambda>0$ for an upward supply shift.

estimating the price and quantity changes (especially EP). We can prove that if the new equilibrium point is known exactly, ie. $E_2^* = E_2$, the errors in measuring ΔPS and ΔCS, which now only arise from assuming that the curves joining E_1 and E_2 and B and E_2 are linear, are much smaller. If we define:

(26) $\Delta CS^{**} = -P_1 Q_1 EP(1+0.5EQ)$

(27) $\Delta PS^{**} = P_1 Q_1 (EP-\lambda)(1+0.5EQ)$

(28) $\Delta TS^{**} = -\lambda P_1 Q_1 (1+0.5EQ)$,

we can show (Remark 1 of Proposition 3, Appendix 2):

(29) $|\Delta CS - \Delta CS^{**}| = O(\lambda^3)$

(30) $|\Delta PS - \Delta PS^{**}| = O(\lambda^3)$

(31) $|\Delta TS - \Delta TS^{**}| = O(\lambda^3)$

In other words, the errors in measuring surplus change areas are to a large extent due to the errors in estimating price and quantity changes (EP mainly). The errors in using local linear approximations of nonlinear curves in measuring surplus areas are trivial compared to the errors caused by not accurately locating the new equilibrium point. This can be seen from Figure 1.

The upper bounds for the errors in measuring surplus changes are estimated as functions of demand and supply parameters by (proofs in Proposition 5, 6 and 7, Appendix 2):

(32) $|\Delta CS - \Delta CS^*| \leq |2(\eta-\varepsilon)|^{-1} P_1^3 |D^{(2)}(c_1) - S^{(2)}(c_2)| \lambda^2$

$+ \max\left(|[2(\eta-\varepsilon)]^{-1} \eta P_1^3 [S^{(2)}(c_2) - D^{(2)}(c_1)]|,\ |(1/6) P_1^3 D^{(2)}(c_1)| \right) \lambda^3$

$+ |[8Q_1(\eta-\varepsilon)^2]^{-1} \eta P_1^5 [S^{(2)}(c_2) - D^{(2)}(c_1)]^2| \lambda^4$

(33) $|\Delta PS - \Delta PS^*| \leq |2(\eta-\varepsilon)|^{-1} P_1^3 |D^{(2)}(c_1) - S^{(2)}(c_2)| \lambda^2$

$+ \max\left(|[2(\eta-\varepsilon)]^{-1} \varepsilon P_1^3 [S^{(2)}(c_2) - D^{(2)}(c_1)]|,\ |(1/6) P_1^3 S^{(2)}(c_2)| \right) \lambda^3$

$+ |[8Q_1(\eta-\varepsilon)^2]^{-1} \varepsilon P_1^5 [S^{(2)}(c_2) - D^{(2)}(c_1)]^2| \lambda^4$

(34) $|\Delta TS - \Delta TS^*| \leq \max\left(|(5\eta-2\varepsilon)|,\ |3\varepsilon| \right) |12(\eta-\varepsilon)|^{-1} P_1^3 |S^{(2)}(c_2) - D^{(2)}(c_1)| \lambda^3$

where $P_2 \leq c_i \leq P_1 (i=1,2)$. Equations (32)-(34) can be used to estimate the sizes of the errors if we are willing to make assumptions about the elasticities and second derivatives of the demand and supply curves. As for the price and quantity changes, the surplus measures are exactly correct for local linear demand and supply curves, when all the second derivatives in the above error terms are zero. When λ is very small, the $\mathbf{O}(\lambda^3)$ and $\mathbf{O}(\lambda^4)$ terms in equations (32) and (33) are much smaller than the $\mathbf{O}(\lambda^2)$ terms. Thus we can use the $\mathbf{O}(\lambda^2)$ terms in (32) and (33) as rough estimates of consumer and producer surplus errors, ie.

$$(32)' \quad |\Delta CS - \Delta CS^*| \lesssim |2(\eta-\varepsilon)|^{-1}P_1^{\,3}|D^{(2)}(c_1)-S^{(2)}(c_2)|\,\lambda^2$$

$$(33)' \quad |\Delta PS - \Delta PS^*| \lesssim |2(\eta-\varepsilon)|^{-1}P_1^{\,3}|D^{(2)}(c_1)-S^{(2)}(c_2)|\,\lambda^2$$

From the expression for the error terms (Remark 2 of Proposition 5 and 6, Appendix 2), we can also show that, under the assumptions in equations (14) and (15)[5],

$$(35) \quad \Delta CS \lesssim \Delta CS^*$$

$$(36) \quad \Delta PS \gtrsim \Delta PS^*$$

This implies that, for a research-induced downward supply shift, we will almost always overestimate consumer surplus gain and underestimate producer surplus gain[6]. The total surplus change can be over or under estimated (Remark of Proposition 7), though the error in the total surplus measure is much smaller. These results are consistent with the empirical evidence from Alston and Wohlgenant (1990).

The results in this section are for a supply shift. Similar results hold when the initial shift occurs in the demand curve, as described in the footnotes.

3. A Multimarket Situation

Now we consider the use of EDM in multimarket situations. When products are related in demand or inputs are related in supply, a new technology or demand enhancement which initially has an impact in one market may induce shifts in demand and supply in other related markets.

Thurman (1990) has pointed out the difficulty of interpreting welfare measures when there are feedback effects or induced shifts in supply and demand in multimarket situations where netputs are substitutes or complements to each other. This issue needs to be addressed in empirical applications. In addition to this problem of the welfare significance of surplus areas, there arises a problem of measuring these areas correctly when there are induced changes in demand and supply. In section 3.1 we show that the standard formulae for changes in economic surplus, as in equations (20)-(22), are not suitable for markets with endogenous shifts. We go on to show that the measures used by Piggott, Piggott and Wright (1995) are more appropriate for

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[5] These results also hold for upward and downward shifts in demand.

[6] For an upward supply shift, the size of the consumer surplus loss will be underestimated and the size of the producer surplus loss will be overestimated. Similarly, for demand shifts, it can be shown that we almost always overestimate the consumer surplus gain and underestimate the producer surplus gain for an upward demand shift, and underestimate the consumer surplus loss and overestimate the producer surplus loss for a downward demand shift.

both exogenous and endogenous demand and supply shifts. In fact, as demonstrated above, because these measures do not require an assumption of elastic supply as presumed by Piggott, Piggott and Wright, they are quite general in nature Analytical and geometrical insights into the use of EDM methodology in multimarket situations are presented in Section 3.2.

3.1 Economic Surplus Formulae

Before repeating the analysis above for the accuracy of EDM in multimarket situations, we first look at the appropriate formulae for measuring surplus change areas in the multimarket situation. For each individual market in the model, a number of situations are possible with respect to whether demand and/or supply curves shift and whether these shifts are generated endogenously or exogenously.

Consider a particular market i in the model. If we use λ_i and τ_i to represent the percentage shifts of supply and demand respectively (exogenous or endogenous), the commonly-used economic surplus change formulae are

(37) $\Delta PS_i^* = P_{1,i}Q_{1,i}(EP_i^* - \lambda_i)(1+0.5EQ_i^*)$

(38) $\Delta CS_i^* = P_{1,i}Q_{1,i}(\tau_i - EP_i^*)(1+0.5EQ_i^*)$

(39) $\Delta TS_i^* = \Delta PS_i^* + \Delta CS_i^* = P_{1,i}Q_{1,i}(\tau_i - \lambda_i)(1+0.5EQ_i^*)$.

These formulae are only useful if we know the values of λ_i and τ_i. When the shift is exogenous and known (eg. λ_i= -1%) or we know that a curve does not shift (eg. λ_i= 0), equations (37)-(39) are appropriate. However if there are endogenous shifts where the amounts of the shifts λ_i and τ_i are unknown but nonzero (they can be endogenously solved from the results of price and quantity changes), using zero for the values of λ_i and/or τ_i will result in incorrect measures of the surplus areas.

Figure 2 illustrates how we can mismeasure the surplus changes if we use the formulae (37)-(39) inappropriately. If for example we are dealing with new technology that reduces the cost by 1%, then the producer surplus change can be correctly measured using (37) with λ_i =-1%. But, if in a multimarket situation, where there is also an induced demand (endogenous) shift in market i, setting τ_i to zero in (38) will mismeasure the change in consumer surplus. In this case the area being measured with τ_i =0 will be $P_2^*E_2^*E_1P_1$, while the actual EDM measure of consumer surplus change should be area $P_2^*E_2^*AB$ with $\tau_i \neq 0$ but unknown.

For the general situation including the case when both demand and supply curves have shifted and some of the shifts are endogenous, it can be shown that the following formulae are more appropriate for measuring the conventional surplus areas:

(40) $\Delta PS_i^* = P_{1,i}Q_{1,i}(EQ_i^*/\varepsilon_i)(1+0.5EQ_i^*)$

(41) $\Delta CS_i^* = P_{1,i}Q_{1,i}(-EQ_i^*/\eta_i)(1+0.5EQ_i^*)$

(42) $\Delta TS_i^* = \Delta PS_i^* + \Delta CS_i^* = P_{1,i}Q_{1,i}EQ_i^*(1/\varepsilon_i - 1/\eta_i)(1+0.5EQ_i^*)$

where ε_i and η_i refer to the supply and demand elasticities for market *i* at the initial equilibrium point E_1. Formulae (40)-(42) are shown to be equivalent to (37)-(39)(Appendix 3), but are preferred since they do not explicitly require the values for λi and τ_i, which are unknown for endogenous shifts.

Note that, in the multimarket situation, the underlying assumption for the above surplus change measures is that the new equilibrium point E_2 in each market is reached through parallel exogenous or endogenous demand or/and supply shifts.

3.2 Errors for Multimarket EDM

We have shown in Section 2 that when using EDM in a single closed market, the errors in estimating EP, EQ, ΔCS and ΔPS are of the order $O(\lambda^2)$ (or $O(\tau^2)$) and in estimating ΔTS are of the order $O(\lambda^3)$ (or $O(\tau^3)$). With the multimarket situation, assume that there are k commodities (markets) involved in the model, and P_i, Q_i are the price and quantity for market i (i=1, 2, ..., k). Define a price and quantity vector Y = $(P_1, ..., P_k, Q_1, ..., Q_k)'$ and an exogenous shifter vector A. The structural model characterising the relationship among different markets can be described by a 2k equation vector function

(43) $F(Y, A)=0$

Applying comparative static analysis to each equation and converting it to finite relative changes before-and-after the shift:

(44) $A(EY^*) = BA$ or

(45) $EY^* = A^{-1}BA$

where $EY^* = (EP_1^*, ..., EP_k^*, EQ_1^*, ..., EQ_k^*)'$ are the EDM estimates of price and quantity changes and A and B are parameter matrixes consisting of the first order parameters of the demand, supply and production functions. Because (45) involves a local linear approximation of F(Y, A), it can be shown that

(46) $EY=A\text{-}1BA+O(A^2)=EY^*+O(A^2)$

In other words, if we use λ to represent the maximum of the exogenous percentage shift in the model, it can be shown that :

(47) $EP_i = EP_i^* + O(\lambda^2)$ (i=1,...,k)

(48) $EQ_i = EQ_i^* + O(\lambda^2)$ (i=1,...,k)

(49) $\Delta CS_i = \Delta CS_i^* + O(\lambda^2)$ (i=1,...,k)

(50) $\Delta PS_i = \Delta PS_i^* + O(\lambda^2)$ (i=1,...,k)

$$(51)\quad \Delta TS_i = \Delta TS_i^* + O(\lambda^3) \qquad (i=1,\dots,k)$$

which is a similar result to the single market model.

It is also possible to derive, using a Taylor expansion with remainder, the exact expressions of the above error terms, in terms of the first order and second order parameters of the demand, supply and production functions as we do for the single market situation. Analytical expressions of the error terms can be very complicated when the size of the multimarket model is large. For a particular model, if we are willing to make assumptions about the first and second order parameters for all the demand, supply and production functions and work through the Taylor expansion, the upper bounds of the errors can be estimated.

Figure 2 shows geometrically the situation when both demand and supply curves shift in a particular market i. The 'true' new equilibrium point is E_2 and the EDM estimate is E_2^* which is solved through equation (45). Assuming that E_2^* is reached through parallel shifts and linear approximations of the initial and new demand and supply curves, as for the simple case in Section 2, the resulting 'true' surplus changes and their EDM approximations for market i are:

$$(52)\quad \Delta CS_i = \text{Area}(P_2E_2AB) = \int_{P2}^{B} D_2(P)dP$$

$$(53)\quad \Delta CS_i^* = \text{Area}(P_2^*E_2^*AB) = P_{1,i}Q_{1,i}(-EQ_i^*/\eta_i)(1+0.5EQ_i^*)$$

$$(54)\quad \Delta PS_i = \text{Area}(P_2E_2DC) = \int_{C}^{P2} S_2(P)dP$$

$$(55)\quad \Delta PS_i^* = \text{Area}(P_2^*E_2^*DC) = P_{1,i}Q_{1,i}(EQ_i^*/\varepsilon_i)(1+0.5EQ_i^*)$$

$$(56)\quad \Delta TS_i = \text{Area}(BAE_2DC) = \int_{P2}^{B} D_2(P)dP + \int_{C}^{P2} S_2(P)dP$$

$$(57)\quad \Delta TS_i^* = \text{Area}(BAE_2^*DC) = P_{1,i}Q_{1,i}EQ_i^*(1/\varepsilon_i - 1/\eta_i)(1+0.5EQ_i^*)$$

Note from Figure 2 that, for the case of endogenous shifts, the amount of the shifts λ_i and τ_i can be estimated from the position of E_1, demand and supply elasticities at E_1 (from which slopes of D and S at E are derived) and the position of the EDM solution of E_2^*. This is exactly what formulae (40)-(42) do analytically to measure the surplus areas.

4. Conclusion

Equilibrium displacement modelling has become a popular procedure to estimate the extent and distribution of the benefits from new technology or product promotion. As the basis of this procedure is the linear approximation of demand and supply curves, there have been concerns about the extent of errors that arise when the true functional forms are not linear. Additionally there has been concern as to whether it is necessary to assume global linearity, which implies undesirable restrictions such as supply must be elastic.

Alston and Wohlgenant (1990) have shown empirically that the errors are very small when the true demand and supply functions are of constant elasticity rather than linear form. In this paper, we have been able to generalise their results to any functional form by taking an analytical rather than an empirical approach. In general our results confirm their findings but the analytical approach enables us to identify some of the underlying assumptions of their empirical results Some of their empirical conclusions are shown not to hold with generality.

We also confirmed their finding that EDM methodology is exactly correct for *local*, rather than global[7], linear demand and supply functions, and hence, that the restriction of elastic supply is not necessary.

Two steps are involved in the EDM procedure: (1) estimating price and quantity changes, EP and EQ, and (2) calculating surplus changes ΔCS, ΔPS and ΔTS. Local linear approximation of the unknown 'true' functional form is used in both steps. It was shown that the major source of errors comes from the first step. When the amount of the percentage shift $\lambda \to 0$, the errors resulting from the first step are of the infinite small order $O(\lambda^2)$, while those from the second step only cause errors of the order $O(\lambda^3)$, if EP and "Q are exact. But since we do not know the exact values of EP and EQ and have to use the results from the first step to calculate ΔCS, ΔPS and ΔTS, the accuracy of the consumer and producer surplus measures ΔCS and ΔPS is reduced to $O(\lambda^2)$. Fortunately, the error in total surplus change is still $O(\lambda^3)$.

The actual expressions for the error terms in estimating price, quantity and surplus changes are given in terms of the demand and supply parameters. Determinants of the size and direction of the errors can be seen from these expressions. For example, the size of the errors increases as demand and supply become more inelastic and curved. Also, for a downward supply shift, we will always overestimate the price decrease and consumer surplus increase and underestimate the producer surplus increase. The upper bounds of all the errors can actually be estimated using these expressions if we are willing to assume values for the elasticities and second derivatives.

We also noted from the analytical derivation that, theoretically, the elasticity values η and ε in the EDM must refer to the initial equilibrium point E_1 on the demand and supply curves for the results to be exactly correct. The empirical implication of this is that, when we suspect the elasticity values have been changing over time, values calculated using the price and quantity at E_1 should be preferred to an average estimated over a long time series. Thus for an explicit functional form, the values of

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[7] The use of Taylor's expansion with remainder enable us to analytically prove this.

elasticities should be taken at the base price and quantity values used in the model. Note that care must be taken to avoid atypical situations such as extreme drought.

Finally a reminder of an earlier qualification, that while in this paper we have focussed on the mathematical or geometrical measurement of the conventional surplus areas, we note Thurman's (1990) concerns that in a multimarket situation when there is more than one source of feedback in the model, some surplus areas in some markets may not have a clear welfare interpretation.This issue has to be addressed in each situation.

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Appendix 1. Order of Magnitude

Mathematical tool of the 'order of magnitude' is used to examine the sizes of the errors of EDM estimates when λ is near zero. We first introduce the notation of **O**(.).

Definition. Let f(x) and g(x) be two functions. If

$$\lim_{x \to 0} |f(x)/g(x)| = c,$$

where $0<c<+\infty$ is a constant, then we write

$$f(x) = \mathbf{O}(g(x)).$$

Examples.

(i) $2x^2 + x^3 = \mathbf{O}(x^2)$ as $x \to 0$;

(ii) $\sin(x) = \mathbf{O}(x)$ as $x \to 0$, ie. sin(x) turns to zero with the same rate as x when $x \to 0$;

(iii) $\ln(1+x) = \mathbf{O}(x)$ as $x \to 0$, ie. ln(1+x) turns to zero with the same rate as x when $x \to 0$.

Properties. As $x \to 0$,

(i) $\mathbf{O}(1)$ = constant;

(ii) $c\mathbf{O}(g(x)) = \mathbf{O}(g(x))$ where $0<c<+\infty$ is a constant;

(iii) If $f(x) = \mathbf{O}(g(x))$, then $[f(x)]^k = \mathbf{O}([g(x)]^k)$, k = 1, 2, ...;

(iv) $[\mathbf{O}(x^m)]\,[\mathbf{O}(x^n)] = \mathbf{O}(x^{m+n})$, m, n = 1, 2 ...;

(v) $[\mathbf{O}(x^m)]\,/\,[\mathbf{O}(x^n)] = \mathbf{O}(x^{m-n})$, m, n = 1, 2, ... and m > n;

(vi) $\mathbf{O}(x^m) \pm \mathbf{O}(x^n) = \mathbf{O}(x^{\min(m,n)})$, m, n = 1, 2, ...

Appendix 2. Derivation of Errors in a Single Market Model

Refer to Figure 1, consider the initial supply and demand with general functional forms

(A.1) S_1: $Q = S(P)$ *initial supply*

(A.2) D_1: $Q = D(P)$ *initial demand*

The intersection of S_1 and D_1 is assumed to be point $E_1(Q_1, P_1)$, where Q_1 and P_1 are initial equilibrium quantity and price. Assume that a new technology has resulted a paralleld downward shift of S_1 to S_2

(A.3) S_2: $Q = S(P-\Delta S)$ *new supply*

where $\Delta S = S_2 - S_1 = \lambda P_1$ is the amount of shift along the price direction, λ is the percentage shift with respect to the initial price P_1 and $\Delta S < 0$ and $\lambda < 0$ for downward shift. The new equilibrium point after the shift is reached as the intersection of S_2 and D_1, denoted as $E_2(Q_2, P_2)$. Refer to Figure 1 in the following derivation

Define relative price and quantitity changes from point E_1 to point E_2 as

(A.4) $EP = (P_2 - P_1)/P_1$

(A.5) $EQ = (Q_2 - Q_1)/Q_1$

The EDM approximation of the relative price and quantity changes are

(A.6) $EP^* = (P_2^* - P_1^*)/P_1^* = \lambda\varepsilon/(\varepsilon-\eta)$

(A.7) $EQ^* = (Q_2^* - Q_1^*)/Q_1^* = \lambda\varepsilon\eta/(\varepsilon-\eta)$

where η and ε are the demand and supply elasticities at the initial equilibrium point E_1.

The 'true' and EDM approximation of the economic surplus changes resulting from displacement from E_1 to E_2 are

(A.8) $\Delta CS = \text{Area}(P_2E_2E_1P_1) = \int_{P2}^{P1} D(P)dP$ *'true' consumer surplus change*

(A.9) $\Delta PS = \text{Area}(A_1BE_2P_2) = \int_{P1+\Delta S}^{P2} S(P-\Delta S)dP$ *'true' producer surplus change*

(A.10) $\Delta TS = \text{Area}(A_1BE_2E_1P_1)$

$$= \int_{P2}^{P1} D(P)dP + \int_{P1+\Delta S}^{P2} S(P-\Delta S)dP$$ *'true' total surplus change*

(A.11) $\Delta CS^* = \text{Area}(P_2{}^*E_2{}^*E_1P_1)$

$= -P_1Q_1EP^*(1+0.5EQ^*)$ *estimated consumer surplus change*

(A.12) $\Delta PS^* = \text{Area}(A_1BE_2{}^*P_2{}^*)$

$= P_1Q_1(EP^*-\lambda)(1+0.5EQ^*)$ *estimated producer surplus change*

(A.13) $\Delta TS^* = \text{Area}(A_1BE_2{}^*E_1P_1)$

$= -\lambda P_1Q_1(1+0.5EQ^*)$ *estimated total surplus change*

(a). Proof of the Order of Magnitudes for the Errors (Equation (10), (11), and (21)-(23))

Using the mathematical tool of the 'order of magnitude', we examine the sizes of the errors of EDM estimates when λ is near zero.

Proposition 1. The percentage changes of price and quntity are of the same order of 'infinite small' as the percentage shift λ when $\lambda \to 0$, ie.

(A.14) $\quad O(EP) = O(EQ) = O(\lambda).$

Proof. Refer to figure 1, connect points E_1 and E_2 with a straight line and consider the triangle E_1E_2B. Side $E_1B=\Delta S$, height $E_2D=\Delta Q$ and $E_1D=\Delta P$. Thus it is obvious using Property (ii) that

$$O(\Delta P)=O(\Delta Q)=O(\Delta S)$$

Since $\quad EP=\Delta P/P_1, \quad EQ=\Delta Q/Q_1 \quad$ and $\quad \lambda=\Delta S/P_1,$

using Property (ii) again we have $\quad O(EP) = O(EQ) = O(\lambda)$ #

Proposition 2. The errors in EDM estimation of price and quantity changes are of the order of λ^2 when $\lambda \to 0$, ie.

(A.15) $\quad |EP-EP^*| = O(\lambda^2)$ and $|EQ-EQ^*| = O(\lambda^2).$

Proof. Taylor expanding demand function D_1 at point P_1 and taking the value at point P_2, we have

$$D(P_2) = D(P_1) + D^{(1)}(P_1)(P_2-P_1) + O(|P_2-P_1|^2)$$

ie. $\quad Q_2 = Q_1 + D^{(1)}(P_1)P_1EP + P_1{}^2O(|EP|^2)$, or

$$EQ = (Q_2\text{-}Q_1)/Q_1 = D^{(1)}(P_1)P_1/Q_1\ EP + O(|EP|^2) \qquad \textit{using Property (ii)}$$

(A.16) $\qquad = \eta EP + O(|EP|^2)$

where $D^{(1)}(P_1)$ is the first derivative of demand function at point P_1. Similarly, Taylor expanding new supply function S_2 in equation (A.3) at point P_1-ΔS and evaluating at point P_2, we have

$$Q_2 = S(P_2\text{-}\Delta S) = S(P_1) + S^{(1)}(P_1)(P_2\text{-}P_1\text{-}\Delta S) + O(|P_2\text{-}P_1\text{-}\Delta S|^2)$$

where $S^{(1)}(P_1)$ is the first derivative of supply function at point P_1. Thus

$$EQ = (Q_2\text{-}Q_1)/Q_1 = S^{(1)}(P_1)P_1/Q_1\ (EP\text{-}\lambda) + O(|EP\text{-}\lambda|^2), \text{ or}$$

(A.17) $EQ = \varepsilon(EP\text{-}\lambda) + O(|EP\text{-}\lambda|^2)$

From Proposition 1 and Property (iii), we also have

$$O(|EP|^2) = O(\lambda^2) \quad \text{and} \quad O(|EP\text{-}\lambda|^2) = O(\lambda^2)$$

Thus equating (A.16) and (A.17):

$$\eta EP + O(\lambda^2) = \varepsilon(EP\text{-}\lambda) + O(\lambda^2), \text{ or}$$

(A.18) $EP = \lambda\varepsilon/(\varepsilon\text{-}\eta) + O(\lambda^2) = EP^* + O(\lambda^2)$ *(using (A.6))*

(A.19) $EQ = \lambda\varepsilon\eta/(\varepsilon\text{-}\eta) + O(\lambda^2) = EQ^* + O(\lambda^2)$ *(using (A.7))*

gives $|EP\text{-}EP^*| = O(\lambda^2)$ and $|EQ\text{-}EQ^*| = O(\lambda^2)$.

Proposition 2 is therefore proved. #

Proposition 3. The errors of the EDM measurements of consumer and producer surplus changes are of the order $O(\lambda^2)$ when $\lambda \to 0$. The total surplus measure are of the order of $O(\lambda^3)$ as $\lambda \to 0$. ie.

(A.20) $|\Delta CS - \Delta CS^*| = O(\lambda^2)$,

(A.21) $|\Delta PS - \Delta PS^*| = O(\lambda^2)$ and

(A.22) $|\Delta TS - \Delta TS^*| = O(\lambda^3)$

Proof.

(i) Proof of (A.20)

Taylor expanding D_1: $Q = D(P)$ at point P_1:

$$D(P) = D(P_1) + D^{(1)}(P_1)(P\text{-}P_1) + O(|P\text{-}P_1|^2)$$

Thus $\Delta CS = \int_{P2}^{P1} D(P)dP = -Q_1 (P_2 - P_1) - (1/2) D^{(1)}(P_1)(P_2-P_1)^2 + \mathbf{O}(|P_2-P_1|^3)$

$= -P_1Q_1EP - (1/2)\eta P_1Q_1(EP)^2 + \mathbf{O}(|EP|^3)$

From equation (A.16), $\eta EP = EQ + \mathbf{O}(\lambda^2)$.

$\Rightarrow \quad \Delta CS = -P_1Q_1EP - (1/2)P_1Q_1EP(EQ+ \mathbf{O}(\lambda^2)) + \mathbf{O}(\lambda^3)$

(A.23) $= -P_1Q_1EP(1+0.5EQ) + \mathbf{O}(\lambda^3)$ *using Property (iii)*

$= -P_1Q_1(EP^* + \mathbf{O}(\lambda^2))\,[1+0.5(EQ^* + \mathbf{O}(\lambda^2))] + \mathbf{O}(\lambda^3)$ *(using (A.18) and (A.19))*

$= -P_1Q_1EP^*(1+0.5EQ^*) + \mathbf{O}(\lambda^2)$ *using Property (iii) and (v)*

$= \Delta CS^* + \mathbf{O}(\lambda^2)$ *using Property (iii -d (v)*

(A.20) is thus proved. #

(ii) Proof of (A.21).

Taylor expanding S_1: $Q=S(P-\Delta S)$ at point $P_1+\Delta S$:

$S(P-\Delta S) = S(P_1) + S^{(1)}(P_1)(P-P_1-\Delta S) + \mathbf{O}(|P-P_1-\Delta S|^2)$

$\Rightarrow \quad \Delta PS = \int_{P1+\Delta S}^{P2} S(P - \Delta S)dP$

$= Q_1(P_2-P_1-\Delta S) + (1/2)S^{(1)}(P_1)(P_2-P_1-\Delta S)^2 + \mathbf{O}(|P_2-P_1-\Delta S|^3)$

$= P_1Q_1(EP-\lambda) + (1/2)P_1Q_1\varepsilon(EP-\lambda) + \mathbf{O}(\lambda^3)$

$= P_1Q_1(EP-\lambda) + (1/2)P_1Q_1(EQ+ \mathbf{O}(\lambda^2)) + \mathbf{O}(\lambda^3)$ *(using (A.17))*

(A.24) $= P_1Q_1(EP-\lambda)(1+0.5EQ) + \mathbf{O}(\lambda^3)$

$= P_1Q_1(EP^* + \mathbf{O}(\lambda^2)-\lambda)[1+0.5(EQ^* + \mathbf{O}(\lambda^2))] + \mathbf{O}(\lambda^3)$ *(using (A.18) and (A.19))*

$= P_1Q_1(EP^* - \lambda)(1+0.5EQ^*) + \mathbf{O}(\lambda^2)$

$= \Delta PS^* + \mathbf{O}(\lambda^2)$ #

(iii) Proof of (A.22).

$\Delta TS = \Delta CS + \Delta PS = -P_1Q_1EP(1+0.5EQ) + \mathbf{O}(\lambda^3)$

$+ P_1Q_1(EP-\lambda)(1+0.5EQ) + \mathbf{O}(\lambda^3)$ *(using (A.23) and (A.24))*

(A.25) $= -\lambda P_1Q_1(1+0.5EQ) + \mathbf{O}(\lambda^3)$

$= -\lambda P_1Q_1\,[1+0.5(EQ^* + \mathbf{O}(\lambda^2))] + \mathbf{O}(\lambda^3)$ *(using (A.19)*

$= -\lambda P_1Q_1\,[1+0.5EQ^*] + \lambda\mathbf{O}(\lambda^2) + \mathbf{O}(\lambda^3)$

$= \Delta TS^* + \mathbf{O}(\lambda^3)$ #

Remark 1. If the price and quantity changes could be found exactly, ie. $EP=EP^*$ and $EQ=EQ^*$, the errors in the surplus measure itself would be of the order of $\mathbf{O}(\lambda^3)$ when $\lambda \to 0$, ie if we define

(A.26) $\Delta CS^{**} = -P_1Q_1EP(1+0.5EQ)$

(A.27) $\Delta PS^{**} = P_1Q_1(EP-\lambda)(1+0.5EQ)$

(A.28) $\Delta TS^{**} = -\lambda P_1Q_1(1+0.5EQ)$,

then

(A.29) $|\Delta CS - \Delta CS^{**}| = \mathbf{O}(\lambda^3)$

(A.30) $|\Delta PS - \Delta PS^{**}| = \mathbf{O}(\lambda^3)$

(A.31) $|\Delta TS - \Delta TS^{**}| = \mathbf{O}(\lambda^3)$

The implication of this result is discussed in the paper.

Proof. These are very obvious from equations (A.23)-(A.25) in the proof. #

(b) Derivation of the Error Expression and Upper Bounds

Proposition 2 and 3 showed some very strong mathematical results that when the amount of the initial parallel shift $\lambda \to 0$, the errors of EDM procedure are of the order $\mathbf{O}(\lambda^2)$ or $\mathbf{O}(\lambda^3)$. But empirically λ is a finite small value, and therefore the error will not be exactly λ^2 for a particular model. To obtain some knowledge of the signs and magnitudes of the errors for a given problem, it is neccessary to estimate the sizes of the errors with a particular λ and the parameters of the demand and supply. Below we use the Taylor expansion with reminder to derive the exact expression and the upper bounds of these errors.

Assumptions. Demand function is always increasing and concave. Supply function is always decreasing and convex. ie.

(A.32) $\varepsilon > 0,\ S^{(2)}(P) < 0$

(A.33) $\eta < 0,\ D^{(2)}(P) > 0$ #

Proposition 4. $|EP - EP^*| \le |2Q_1(\varepsilon-\eta)|^{-1}(P_1^2\lambda^2)\,|D^{(2)}(c1) - S^{(2)}(c2)|$

and $|EQ - EQ^*| \le |2Q_1(\varepsilon-\eta)|^{-1}(P_1^2\lambda^2)\max(|\varepsilon D^{(2)}(c1)|, |\eta S^{(2)}(c2)|)$

Proof. Repeating the first half of the proof of Proposition 2 but using the Tayor expansion formula with remainder, we have, in places of (A.16) and (A.17)

(A.34) $EQ = \eta EP + (2Q_1)^{-1}D^{(2)}(c_1)(P_2-P_1)^2$

(A.35) $EQ = \varepsilon(EP-\lambda) + (2Q_1)^{-1}S^{(2)}(c_2)(P_2-P_1-\Delta S)^2$

where $P_2 \le c_i \le P_1$ (i=1,2). EP can then be solved through (A.34) and (A.35):

(A.36) $EP = \lambda\varepsilon/(\varepsilon-\eta) + [2Q_1(\eta-\varepsilon)]^{-1}[S^{(2)}(c_2)(P_2-P_1-\Delta S)^2 - D^{(2)}(c_1)(P_2-P_1)^2]$, ie.

(A.37) $EP - EP^* = [2Q_1(\eta-\varepsilon)]^{-1}P_1^2\,[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$

Since $|EP-\lambda| \le |\lambda|$ and $|EP| \le |\lambda|$ (obvious from Figure 1), and $S^{(2)}(c_2)(EP-\lambda)^2 \le 0$ and $D^{(2)}(c_1)(EP)^2 \ge 0$ (from assumptions (A.32) and (A.33), we have

$$|EP - EP^*| \le |2Q_1(\varepsilon-\eta)|^{-1}(P_1^2\lambda^2)\,|D^{(2)}(c1) - S^{(2)}(c2)|$$

EQ can be solved through (A.34) and (A.36):

$$EQ = \lambda\eta\varepsilon/(\varepsilon-\eta) + [2Q_1(\eta-\varepsilon)]^{-1}\eta[S^{(2)}(c_2)(P_2-P_1-\Delta S)^2 - D^{(2)}(c_1)(P_2-P_1)^2]$$

$$+ (2Q_1)^{-1}D^{(2)}(c_1)(P_2-P_1)^2, \text{ or}$$

(A.38) $EQ - EQ^* = [2Q_1(\eta-\varepsilon)]^{-1}P_1^2\,[\eta S^{(2)}(c_2)(EP-\lambda)^2 - \varepsilon D^{(2)}(c_1)(EP)^2]$

Assume assumptions (A.32) and (A.33) are satisfied, then $\eta S^{(2)}(c_2)(EP-\lambda)^2$ and $\varepsilon D^{(2)}(c_1)(EP)^2$ are both positive. Therefore

$$|EQ - EQ^*| \le |2Q_1(\varepsilon-\eta)|^{-1}(P_1^2\lambda^2)\max(|\eta S^{(2)}(c_2)|, |\varepsilon D^{(2)}(c_1)|) \quad \#$$

Remark 1. From assumptions (A.32) and (A.33), the error term in equation (A.37) $\Delta_p=[2Q_1(\eta-\varepsilon)]^{-1}P_1^2\,[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2] > 0$. Thus

(A.39) $EP \ge EP^*$

The empirical implication of this result is given in the paper. #

Remark 2. The sign in the error term in (A.38) is indeterminate depanding on the relative sizes of $\eta S^{(2)}(c_2)(EP-\lambda)^2$ and $\varepsilon D^{(2)}(c_1)(EP)^2$. Hence EQ can be over or under estimated. #

Proposition 5. $\left| \Delta CS - \Delta CS^* \right| \leq |2(\eta-\varepsilon)|^{-1} P_1^3 |D^{(2)}(c_1) - S^{(2)}(c_2)| \lambda^2$

$$+ \max\left(|[2(\eta-\varepsilon)]^{-1}\eta P_1^3 [S^{(2)}(c_2) - D^{(2)}(c_1)]|,\ |(1/6)P_1^3 D^{(2)}(c_1)| \right) \lambda^3$$

$$+ |[8Q_1(\eta-\varepsilon)^2]^{-1}\eta P_1^5 [S^{(2)}(c_2) - D^{(2)}(c_1)]^2| \lambda^4$$

Proof. In the proof of Proposition 3, (i), if we use the Taylor expansion with remainder, we have

$$\Delta CS = \int_{P2}^{P1} D(P)dP = -P_1Q_1EP - (1/2)\eta P_1Q_1(EP)^2 - (1/6)P_1^3 D^{(2)}(c_1)(EP)^3$$

From (A.37):

(A.40) $EP = EP^* + [2Q_1(\eta-\varepsilon)]^{-1}P_1^2 [S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2] = EP^* + \Delta_p$

Substitute EP for the above $EP^* + \Delta_p$, we have

$$\Delta CS = -P_1Q_1(EP^* + \Delta_p) - (1/2)\eta P_1Q_1(EP^* + \Delta_p)^2 - (1/6)P_1^3 D^{(2)}(c_1)(EP)^3$$

$$= \Delta CS^* - P_1Q_1\Delta_p - \eta P_1Q_1EP^*\Delta_p - (1/6)P_1^3 D^{(2)}(c_1)(EP)^3$$

$$= \Delta CS^* - [2(\eta-\varepsilon)]^{-1}P_1^3[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$$

$$- [2(\eta-\varepsilon)]^{-1}\eta P_1^3[S^{(2)}(c_2)(EP-\lambda)^2EP^* - D^{(2)}(c_1)(EP)^2EP^*]$$

$$- [8Q_1(\eta-\varepsilon)^2]^{-1}\eta P_1^5[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]^2$$

$$- (1/6)P_1^3 D^{(2)}(c_1)(EP)^3$$

Consider the signs of $O(\lambda^2)$, $O(\lambda^3)$ and $O(\lambda^4)$ terms in the above error under the assumptions in equations (A.32) and (A.33), and note that $|EP-\lambda| \leq |\lambda|$, $|EP| \leq |\lambda|$ and $|EP^*| \leq |\lambda|$, we have

$$\left| \Delta CS - \Delta CS^* \right| \leq |2(\eta-\varepsilon)|^{-1} P_1^3 |D^{(2)}(c_1) - S^{(2)}(c_2)| \lambda^2$$

$$+ \max\left(|[2(\eta-\varepsilon)]^{-1}\eta P_1^3 [S^{(2)}(c_2) - D^{(2)}(c_1)]|,\ |(1/6)P_1^3 D^{(2)}(c_1)| \right) \lambda^3$$

$$+ |[8Q_1(\eta-\varepsilon)^2]^{-1}\eta P_1^5 [S^{(2)}(c_2) - D^{(2)}(c_1)]^2| \lambda^4 \quad \#$$

Remark 1. When λ is very small, the error in estimating ΔCS can be roughly estimated by the $O(\lambda^2)$ term, ie.

(A.41) $\left| \Delta CS - \Delta CS^* \right| \lessapprox |2(\varepsilon-\eta)|^{-1} P_1^3 |D^{(2)}(c_1) - S^{(2)}(c_2)| \lambda^2$

Remark 2. When λ is very small,

(A.42) $\Delta CS \lessapprox \Delta CS^*$.

This is because, from the above proof, $\Delta CS - \Delta CS^* = - P_1Q_1\Delta_p + O(\lambda^3)$ where $- P_1Q_1\Delta_p = -[2(\eta-\varepsilon)]^{-1} P_1^3[S^{(2)}(c_2)(EP+\lambda)^2 - D^{(2)}(c_1)(EP)^2] \le 0$. The implication of this result is in the main text.

<u>**Proposition 6**</u>. $|\Delta PS - \Delta PS^*| \le |2(\eta-\varepsilon)|^{-1}P_1^3|D^{(2)}(c_1)-S^{(2)}(c_2)|\lambda^2$

$$+ \max\left(|[2(\eta-\varepsilon)]^{-1}\varepsilon P_1^3[S^{(2)}(c_2) - D^{(2)}(c_1)]|,\ |(1/6)P_1^3 S^{(2)}(c_2)| \right)\lambda^3$$

$$+ |[8Q_1(\eta-\varepsilon)^2]^{-1}\varepsilon P_1^5[S^{(2)}(c_2) - D^{(2)}(c_1)]^2|\lambda^4$$

<u>**Proof.**</u> In the proof of Proposition 3, (ii), if we use the Taylor expansion with remainder, we have

$$\Delta PS = \int_{P1+\Delta S}^{P2} S(P-\Delta S)dP$$

$$= P_1Q_1(EP-\lambda) + (1/2)\varepsilon P_1Q_1(EP-\lambda)^2 + (1/6)P_1^3 S^{(2)}(c_2)(EP-\lambda)^3$$

$$= P_1Q_1(EP^*-\lambda+\Delta_p) + (1/2)\varepsilon P_1Q_1(EP^*-\lambda+\Delta_p)^2 + (1/6)P_1^3 S^{(2)}(c_2)(EP-\lambda)^3$$ *(using (A.40))*

$$= P_1Q_1(EP^*-\lambda) + (1/2)\varepsilon P_1Q_1(EP^*-\lambda)^2 + P_1Q_1\Delta_p$$

$$+ (1/2)\varepsilon P_1Q_1[2(EP^*-\lambda)\Delta_p + \Delta_p^2] + (1/6)P_1^3 S^{(2)}(c_2)(EP-\lambda)^3$$

(A.43) $= \Delta PS^* + [2(\eta-\varepsilon)]^{-1}P_1^3[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$

$$+ [2(\eta-\varepsilon)]^{-1}\varepsilon P_1^3(EP^*-\lambda)[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2] + (1/6)P_1^3 S^{(2)}(c_2)(EP-\lambda)^3$$

$$+ [8Q_1(\eta-\varepsilon)^2]^{-1}\varepsilon P_1^5[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]^2$$

Consider the signs of the above $O(\lambda^2)$, $O(\lambda^3)$ and $O(\lambda^4)$ terms according to assumptions in equations (A.32) and (A.33), and note that $|EP-\lambda| \le |\lambda|$, $|EP| \le |\lambda|$ and $|EP^*-\lambda| \le |\lambda|$, we come to the result of Proposition 6. #

<u>**Remark 1.**</u> When λ is very small, the error in ΔPS^* can be roughly estimated by the $O(\lambda^2)$ term, ie.

(A.44) $$|\Delta PS - \Delta PS^*| \lessapprox |2(\eta-\varepsilon)|^{-1}P_1^3|S^{(2)}(c_2) - D^{(2)}(c_1)|\lambda^2$$ #

<u>**Remark 2.**</u> We almost always underestimate producer surplus change. From (A.43), $\Delta PS - \Delta PS^* = P_1Q_1\Delta_p + O(\lambda^3)$ where $P_1Q_1\Delta_p = [2(\eta-\varepsilon)]^{-1} P_1^3[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2] \ge 0$. Therefore when λ is very small

(A.45) $$\Delta PS \gtrapprox \Delta PS^*$$

Proposition 7. $|\Delta TS - \Delta TS^*| \le$

$$\max(|(5\eta-2\varepsilon)|,\ |3\varepsilon|)\,|12(\eta-\varepsilon)|^{-1}P_1^{\,3}|S^{(2)}(c_2) - D^{(2)}(c_1)|\,\lambda^3$$

Proof. $\Delta TS = \Delta CS + \Delta PS$

$$= -P_1Q_1EP - (1/2)\eta P_1Q_1(EP)^2 - (1/6)P_1^{\,3}\,D^{(2)}(c_1)\,(EP)^3$$

$$+ P_1Q_1(EP-\lambda) + (1/2)\varepsilon P_1Q_1\,(EP-\lambda)^2 + (1/6)P_1^{\,3}\,S^{(2)}(c_2)\,(EP-\lambda)^3$$

$$= \lambda\,P_1Q_1 - (1/2)\eta P_1Q_1(EP^*+\Delta_p)^2 + (1/2)\varepsilon P_1Q_1(EP^*+\Delta_p-\lambda)^2$$

$$+ (1/6)P_1^{\,3}\,[S^{(2)}(c_2)(EP-\lambda)^3 - D^{(2)}(c_1)(EP)^3]$$

Using $EQ^* = \eta EP^*$ and $EQ^* = \varepsilon(EP^*-\lambda)$, we have (details omitted):

$$\Delta TS = \lambda P_1Q_1 + (1/2)\lambda P_1Q_1EQ^* - (1/2)\eta P_1Q_1EP\Delta_p$$

$$+ (1/6)P_1^{\,3}\,[S^{(2)}(c_2)(EP-\lambda)^3 - D^{(2)}(c_1)(EP)^3] + (1/2)\varepsilon P_1Q_1\Delta_p(EP-\lambda)$$

$$= \Delta TS^* - [4(\eta-\varepsilon)]^{-1}\eta P_1^{\,3}EP[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$$

$$+ (1/6)P_1^{\,3}\,[S^{(2)}(c_2)(EP-\lambda)^3 - D^{(2)}(c_1)(EP)^3]$$

$$+ [4(\eta-\varepsilon)]^{-1}\varepsilon P_1^{\,3}(EP-\lambda)[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$$

$$= \Delta TS^* + E_1 + E_2 + E_3,$$

where $E_1<0$, $E_2<0$ and $E_3>0$ according to the assumptions (A.32) and (A.33), and $|EP-\lambda| \le |\lambda|$, $|EP| \le |\lambda|$. Thus

$$|\Delta TS - \Delta TS^*| \le \max(|E_1 + E_2|, |E_3|)$$

$$\le \max(|(5\eta-2\varepsilon)|,\ |3\varepsilon|)\,|12(\eta-\varepsilon)|^{-1}P_1^{\,3}|S^{(2)}(c_2) - D^{(2)}(c_1)|\,\lambda^3 \quad \#$$

Remark. ΔTS can be over or under estimated depanding on the relative sizes of $|E_1 + E_2|$ and $|E_3|$. #

Appendix 3. Equilvalence of Surplus Change Formulae

Consider two sets of surplus change formulae, the first widely used (Alston, Norton and Pardey (1995):

(B.1) $\Delta PS_1^* = P_1Q_1(EP^*-\lambda)(1+0.5EQ^*)$

(B.2) $\Delta CS_1^* = P_1Q_1(\tau-EP^*)(1+0.5EQ^*)$

and the second recently applied by Piggott, Piggott and Wright (1995):

(B.3) $\Delta PS_2^* = P_1Q_1(EQ^*/\varepsilon)(1+0.5EQ^*)$

(B.2) $\Delta CS_2^* = P_1Q_1(-EQ^*/\eta)(1+0.5EQ^*)$.

We want to prove that under appropriate conditions these formulae are equilvalent.

Refer to Figure 2. The EDM estimates of the surplus changes are:

(B.5) $\Delta PS^* = \text{Area}(CDE_2^*P_2^*)$

(B.6) $\Delta CS^* = \text{Area}(ABP_2^*E_2^*)$

In other words, we want to prove.

Proposition.

(B.7) (i) $\Delta PS_1^* = \Delta PS_2^* = \Delta PS^*$

(B.8) (ii) $\Delta CS_1^* = \Delta CS_2^* = \Delta CS^*$

Proof.

(i) $\Delta PS^* = \text{Area}(CDE_2^*P_2^*) = (1/2)(Q_1+Q_2^*)\,|GD|$

$|GD| = |E_1D| - |E_1G| = |\lambda P_1| - |P_2^*-P_1|$

For the situation of downward supply shift and upward demand shift illustrated in Figure 2, $\lambda<0$ and $P_2^*-P_1<0$. Thus $|\lambda P_1| = -\lambda P_1$ and $|P_2^*-P_1| = -(P_2^*-P_1)$, and

$$\Delta PS^* = (1/2)(Q_1+Q_2^*)(P_2^*-P_1-\lambda P_1)$$

$$= (Q_1+0.5(Q_2^*-Q_1))(P_2^*-P_1-\lambda P_1) = P_1Q_1(EP^*-\lambda)(1+0.5EQ^*) = \Delta PS_1^*$$

On the other hand, looking at triangle GDE_2^* in Figure 2,

$$|GD| = |E_2^*G|\,|\text{slope}(S_2^*)| = |Q_2^*-Q_1|\,|\text{slope}(S_2^*)|.$$

Also, $|slope(S_2^*)| = |slope(S_1^*)| = (P_1/Q_1)/\varepsilon$,

where $\varepsilon>0$ is the supply elasticity at E_1, and $Q_2^*-Q_1>0$, therefore

$$\Delta PS^* = (1/2)(Q_1+Q_2^*)\,|GD| = Q_1(1+0.5EQ^*)\,|Q_2^*-Q_1|(P_1/\varepsilon Q_1)$$

$$= P_1Q_1(EQ^*/\varepsilon)(1+0.5EQ^*) = \Delta PS_2^*$$

ie. $\Delta PS_1^* = \Delta PS_2^* = \Delta PS^*$

(ii) $\Delta CS^* = Area(ABP_2^*E_2^*) = (1/2)(Q_1+Q_2^*)\,|AG|$

and $|AG| = |AE_1| + |E_1G| = |\tau P_1| + |P_2^* - P_1|$,

where $\tau>0$ and $P_2^* - P_1<0$

$\therefore |\tau P_1| = \tau P_1$ and $|P_2^* - P_1| = -(P_2^* - P_1)$.

Thus $\Delta CS^* = (1/2)(Q_1+Q_2^*)\,|AG| = Q_1(1+0.5EQ^*)(\tau P_1-(P_2^* - P_1))$

$$= Q_1(1+0.5EQ^*)\,P_1(\tau-EP_2^*) = \Delta CS_1^*.$$

Considering triangle AGE_2^*,

$$|AG| = |E_2^*G|\,|slope(D_2^*)| = |Q_2^*-Q_1|\,|slope(D_2^*)| = |Q_2^*-Q_1|\,|slope(D_1^*)|$$

Also, from the definition of demand elasticity at point E_1,

$$|slope(D_1^*)| = (P_1/Q_1)/|\eta| = -(P_1/Q_1)/\eta,$$

where $\eta<0$ is the demand e isticity at E_1. Therefore

$$\Delta CS^* = (1/2)(Q_1+Q_2^*)\,|AG| = -Q_1(1+0.5EQ^*)|Q_2^*-Q_1|(P_1/\eta Q_1)$$

$$= P_1Q_1\,(-EQ^*/\eta)(1+0.5EQ^*) = \Delta CS_2^*$$

where $|Q_2^*-Q_1|=Q_2^*-Q_1$ since $Q_2^*-Q_1>0$. This proved

$$\Delta CS_1^* = \Delta CS_2^* = \Delta CS^*.$$

The above proof is for the case of upward demand shift ($\tau>0$), downward supply shift ($\lambda<0$), $EP^*<0$ and $EQ^*>0$ as illustrated in Figure 2. The same results as in (B.7) and (B.8) can be shown in the same way for other situations of the shifts with the appropriate signs of λ, τ, EP^* and EQ^*. #

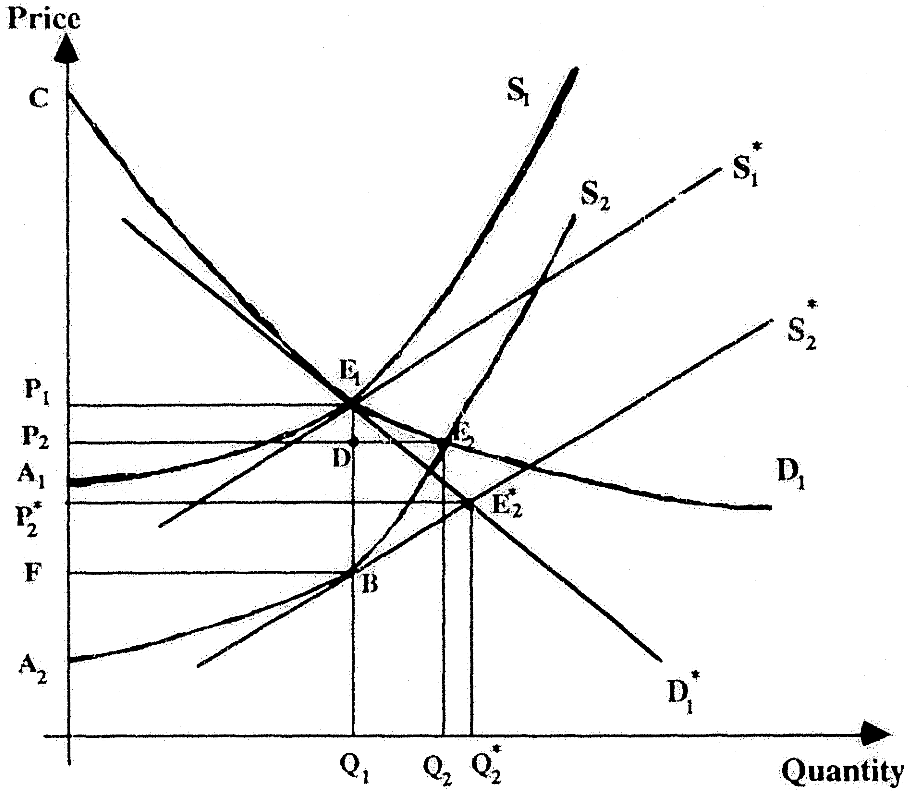

Figure 1

Geometrical Demonstration of the EDM Approximation in a Single Market Model

$E_1B=\lambda P_1$: exogenous shift

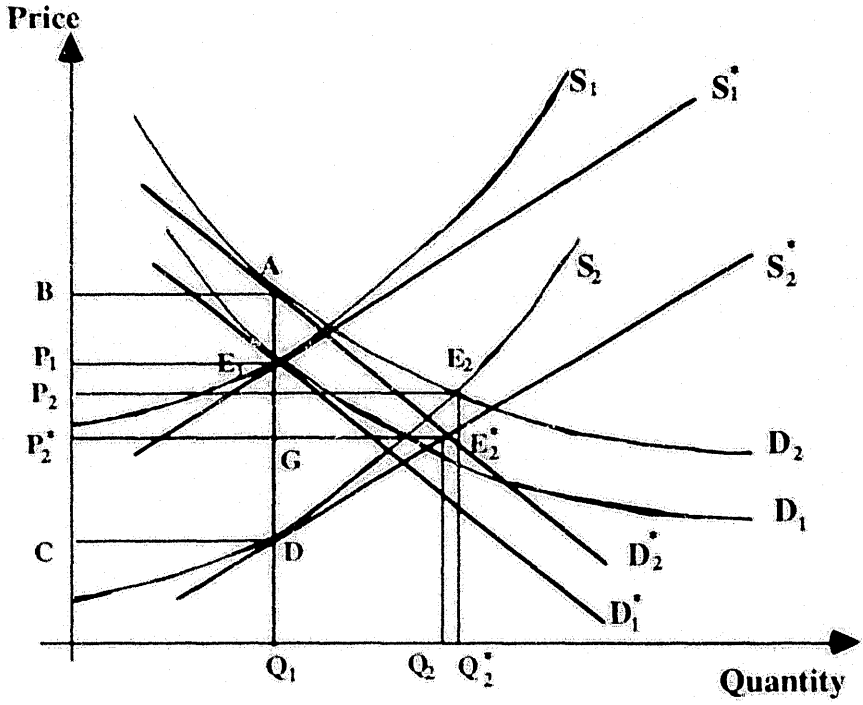

Figure 2

Geometrical Demonstration of EDM Approximation for an Individual Market in a Multimarket Model

$AE_1 = \tau P_1$: exogenous or endogenous demand shift

$E_1D = \lambda P_1$: exogenous or endogenous supply shift