HEDGING WITH CROP YIELD INSURANCE FUTURES

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I. Introduction

Crop farmers as well as many other companies whose business is tied to the grain production and marketing face both price and yield risk. Traditionally, futures price contracts and options on futures are used to manage price risk. However, similar market based instruments for managing yield risk have not been available. Instead, federal agricultural support programs such as deficiency payments and non-recourse loan programs along with subsidized crop yield insurance programs have served as alternatives to market based risk management mechanisms. Now, in an important new development, the Chicago Board of Trade (CBOT) has launched its Crop Yield Insurance (CYI) Futures and Options contracts. The first contract of the CYI complex that began trading on June 2, 1995 was Iowa Corn Yield Insurance Futures and Options. Other yield contracts are scheduled to follow.

The behavior of the firm under joint price and output uncertainty has been previously studied in the literature. In a pioneering work, McKinnon (1967) showed that since the correlation between individual yield and local price is typically negative, the risk minimizing hedge against price risk is less than expected output. Extensions of McKinnon's risk-minimization analysis have generally focused on attempts to cast the problem into the expected utility maximization framework. Since the general solution to the expected utility maximization problem is not analytically tractable, most authors have embraced some assumptions regarding the form of the utility function and the distribution

of the random variables. Standard examples include the joint normality of profits and prices (Grant, 1985), or constant absolute risk aversion (CARA) and normally distributed profits/revenues which reduces the expected utility maximization approach to the analytically simpler mean-variance framework (Rolfo, 1980). Problems with assuming the normality of profits, which is the product of two random variables (price and quantity), were addressed in Lapan and Moschini (1994). They derived an exact solution for the optimal futures hedge for the CARA utility function and jointly normally distributed price, basis and yield, but not profits.

In this paper we show that even with two instruments (price and yield futures contracts) for managing price and yield risks and no price basis and yield basis risks, the firm still cannot generally eliminate all uncertainty. However, a risk minimizing firm can reduce its variance of revenue by hedging in both markets rather than just using the price futures market. The analytical results are illustrated by examining the effectiveness of the dual hedging strategy with price and yield futures for hypothetical corn producers located in the three major corn producing counties in North Carolina.

II. Expected Utility of Profit Approach

For the expected utility of profit maximizing producer facing both price and yield risk and having only one instrument (forward pricing or price futures contract) to hedge risk, Grant (1985) has shown that the production decision cannot be determined in general. The optimal scale of production depends on the relationship between price and yield and the producer's utility function. Similarly, there is no general solution for the

optimal forward (futures) position either. The inclusion of the new crop yield futures contract into a farmer's risk management strategy does not dramatically alter previous results. Holthausen's (1979) separation result in which the production decision is independent of the agent's utility function and the distribution of the random price cannot be obtained. Even if there are no price and yield basis risks, the covariance between price and output creates a situation where risk cannot be completely eliminated and hence a perfect hedge does not exist. Additional assumptions about the form of the utility function and the distribution of random variables are needed to derive further results.

Consider a two period problem. In the first (planting) period all decisions regarding cash and futures positions are made, and in the second (harvesting) period all uncertainties are resolved and all outstanding positions are closed and proceeds collected. Consider further that the scale of production X (the number of acres planted) is exogenously determined. For many agricultural producers, especially those participating in various government commodity programs, this may not be an overly restrictive assumption. The total output is therefore Xy, where y denotes stochastic yield. The cost of output c(X) is assumed to depend on the scale of production, with c'(X)>0, and there are no transactions costs related to trading futures contracts. Assuming an increasing, strictly concave, twice differentiable von Neumann-Morgenstern utility function, U, with terminal profit π as the sole argument, the problem can be defined as:

$$
\begin{aligned}
& MaxE[U(\pi)] \\
& s.t. \\
& \pi = pXy + (F - f)h_f + K(Z - z)h_z - c(X)
\end{aligned}
\tag{1}
$$

where p is local cash price, F is the futures price at planting, f is futures price at harvest, h_f

is the price futures market position (positive if short, negative if long), y is the individual farm's yield (bushels/acre), Z is the contract underlying yield at planting, z is the contract underlying yield at harvest, h_z is the yield futures market position (positive if short and negative if long), and K is a multiplier. In the case of corn yield contracts, K equals $100 for each bushel per acre harvested. The lower case symbols p, f, y, and z denote stochastic variables, and the upper case letters F, Z, X, and K denote known constants. The formulation of the problem in (1) captures the presence of both price and yield basis risks. The first order conditions for an extremum are

(2.1) $$\frac{\partial E(U)}{\partial h_f} = E[U'(\pi)(F - f)] = 0$$

(2.2) $$\frac{\partial E(U)}{\partial h_z} = E[U'(\pi)(Z - z)] = 0$$

and since F and Z are known constants, (2.1) and (2.2) can be written as:

(3.1) $$Cov[U'(\pi), f] = E[U'(\pi)][F - E(f)]$$

(3.2) $$Cov[U'(\pi), z] = E[U'(\pi)][Z - E(z)]$$

Assuming that the distribution of profits and futures price and the distribution of profits and futures yield are bivariate normal[1], Rubinstein's (1976) result is applicable so that cov[U'(π), f] = E[U''(π)] cov(π, f); and cov[U'(π), z}] = E[U''(π)] cov(π, z), and the first order conditions (3.1) and (3.2) become:

(4.1) $$E[U''(\pi)][Cov(pXy, f) - Var(f)h_f - Cov(z, f)Kh_z] = E[U'(\pi)][F - E(f)]$$

[1] The joint normality of py and f, required to obtain (4.1), and the joint normality of py and z, required to obtain (4.2), does not require cash price (p) or observed yield (y) to be normal, but rather their product must be normal. As seen from (1) futures price (f) and futures yield (z) enter the profit function additively.

(4 2) $$E[U''(\pi)][Cov(pXy,z) - Cov(z,f)h_f - Var(z)Kh_z] = E[U'(\pi)][Z - E(z)]$$

Dividing the above equations by E[U'(π)] and defining λ = -E[U''(π)]/ E[U'(π)], (4.1) and (4.2) reduce to

(5 1) $$\lambda(\sigma_f{}^2 h_f + \sigma_{zf} Kh_z - \sigma_{Rf}) = [F - E(f)]$$

(5 2) $$\lambda(\sigma_{zf} h_f + \sigma_z{}^2 Kh_z - \sigma_{Rz}) = [Z - E(z)]$$

where: $\sigma_f^2 = \mathrm{var}(f), \sigma_z^2 = \mathrm{var}(z), \sigma_{Rf} = \mathrm{cov}(pXy, f), \sigma_{zf} = \mathrm{cov}(z, f), \sigma_{Rz} = Cov(pXy, z)$. It can be shown under the assumption of a constant absolute risk averse (CARA) utility function and normally distributed profit, λ equals the Pratt-Arrow measure of absolute risk aversion The solution to the system of first order conditions (5) determines the firm's hedging strategy where the scale of production is exogenously determined Using Cramer's rule the optimal positions in the price futures market h_f^* and yield futures market h_z^* are given by:

(6.1) $$h_f^* = \frac{1}{(1-\rho_{fz}^2)}\left\{\frac{\sigma_{Rf}}{\sigma_f^2} - \frac{\sigma_{fz}\sigma_{Rz}}{\sigma_f^2\sigma_z^2} + \frac{F - E(f)}{\lambda\sigma_f^2} - \frac{[Z - E(z)]\sigma_{fz}}{\lambda\sigma_f^2\sigma_z^2}\right\}$$

(6.2) $$h_z^* = \frac{1}{K(1-\rho_{fz}^2)}\left\{\frac{\sigma_{Rz}}{\sigma_z^2} - \frac{\sigma_{fz}\sigma_{Rf}}{\sigma_f^2\sigma_z^2} + \frac{Z - E(z)}{\lambda\sigma_z^2} - \frac{[F - E(f)]\sigma_{fz}}{\lambda\sigma_f^2\sigma_z^2}\right\}$$

where, $\rho_{fz}^2 = \frac{[Cov(f,z)]^2}{Var(f)Var(z)}$ is the square of the correlation coefficient between futures price and contract underlying yield. Notice that the existence of optimal solutions h_f^* and h_z^* requires that the contract underlying yield and futures price are not perfectly correlated, i.e., $(1-\rho_{fz}^2) \neq 0$.

The optimal hedge in futures price contracts (6.1) consists of the risk minimizing component (the first two terms in parentheses) and the speculative component (the last two terms in parentheses). Alternatively, the optimal futures price hedge (6.1) consists of the price risk component (the first and third terms) and the yield risk component (the second and fourth terms). The first term of the optimal price hedge reflects the position in price futures contracts required to minimize the variability of profit associated with the fluctuation of local price at harvest. The second term results from the presence of yield uncertainty and the availability of the second instrument. The speculative components of the optimal futures price hedge consist of a price speculative part (the third term) and a yield speculative part (the fourth term), both inversely related to the producer's degree of risk aversion. If the producer is infinitely risk averse ($\lambda = \infty$), the speculative components become zero. If the producer is risk neutral ($\lambda = 0$), a small deviation in either expected futures price or expected yield at harvest from the current quotes will induce an infinite speculative position. If the futures price and futures yield at planting are unbiased estimators of the futures price and contract underlying yield at harvest, the speculative terms become zero. If the futures price at harvest is expected to be greater than the current quote, the price speculative term draws the optimal hedge toward a long futures price position. An expected increase in the yield futures, i.e. $Z - E(z) < 0$, implies a reduction in the futures price position since $Cov(f,z) < 0$, and the yield speculative term thus draws the optimal hedge toward a long futures price position.

The intuition for the above results can best be developed by gradually introducing more restrictive assumptions into the model thereby tracing out some of the well known

results from the earlier literature. Assume first that yield futures are not available. Under such circumstance, $\rho^2_{fs} = 0$, and the second and fourth terms in (6.1) drop out. The formula for the optimal hedge becomes (where the subscript 0 indicates that no yield futures position is taken)

$$h_{f0} = \frac{\sigma_{pf}}{\sigma^2_f} + \frac{F - E(f)}{\lambda \sigma^2_f} \tag{6.1'}$$

which is the result developed by Grant (1985). In addition to no yield futures, one can further assume no yield uncertainty ($y = \mu_y$). Then, the first component of (6.1') becomes $\frac{\mu_q Cov(p,f)}{Var(f)}$, where $\mu_q = X\mu_y$ denotes the certain output, and expression (6.1') can therefore be rewritten as

$$h_{f0\mu_y} = \mu_q \frac{\sigma_{pf}}{\sigma^2_f} + \frac{F - E(f)}{\lambda \sigma^2_f} \tag{6.1''}$$

which is a familiar expression for the optimal hedge under price and basis risk (Vukina and Anderson, 1993) The second term in (6.1'') is a speculative component. If the producer is infinitely risk averse, or the futures price is unbiased, F = E(f), the speculative component becomes zero The remaining term μ_q[Cov(p,f)/Var(f)] is the familiar risk minimizing hedge from Kahl (1983).

Further assuming forward contracting as a hedging device (i.e., F = b, and b is a forward price) eliminating the basis risk from the analysis (i.e., f = p), (6.1'') becomes :

$$h_b = \mu_q + \frac{b - E(p)}{\lambda \sigma^2_p} \tag{6.1'''}$$

which is Holthausen's (1979) formula for optimal forward contracting.

The interpretation of the optimal position in yield futures (6.2) is similar to that of

the optimal hedge in price futures (6.1). The first two terms are the risk minimizing hedge, and the last two terms represent the speculative position. Alternatively, the optimal yield futures hedge consists of a yield risk component (the first and third terms) and a price risk component (the second and fourth terms). To develop better understanding, we present two simpler cases.

First, assume a nonstochastic price and no futures price contracts. Then, the price risk component terms and ρ^2_{fz} become zero, the third term remains unchanged, while the first term in (6.2), $\frac{Cov(pXy,z)}{Var(z)}$, can be rewritten as $\frac{\mu_y XCov(y,z)}{Var(z)}$. The optimal position in yield futures in the presence of yield risk and yield basis risk but no price uncertainty is:

$$(6.2') \qquad h_{0z|p} = \frac{pX}{K}\frac{\sigma_{yz}}{\sigma^2_z} + \frac{Z - E(z)}{\lambda\sigma^2_z}$$

An optimal hedge in yield futures (6.2') is the sum of the risk minimizing hedge and the speculative hedge. This result is parallel to the optimal hedge in futures price contracts under price and basis risk assuming nonstochastic yields (6.1'').

Introducing an additional assumption that the individual farm's yield is identical to the contract underlying yield, i.e., y = z, expression (6.2') simplifies to:

$$(6.2'') \qquad h_{0z|y=z} = \frac{pX}{K} + \frac{Z - E(z)}{\lambda\sigma^2_z}$$

which parallels (6.1'''). The second term in (6.2'') is a speculative demand. If the producer perceives yield futures as unbiased, i.e., Z- E(z) = 0, he will hedge routinely. As mentioned in Section II, the routine hedge is a strategy where yield futures position value equals anticipated revenue, i.e., $KZh_z = pXZ$, and hence $h_z = pX/K$. If the expected yield

is less than the current estimate, i.e., Z - E(z) > 0, the producer's short hedge in yield futures will exceed the anticipated revenue from the planted acreage.

III. Risk Minimizing Hedge

The traditional role of hedging is risk reduction (e.g. McKinnon, 1967). With the presence of joint price and output risks and the availability of two hedging instruments, the problem facing the agricultural firm is one of selecting the optimal position in both price and yield futures markets that will minimize the variance of profits. Of course, minimizing the variance of profits makes sense only for a given scale of production, because otherwise the farmer could always minimize risk by producing nothing. After calculating the variance the minimization problem translates into

$$\text{(1')} \qquad \underset{h_f\, h_z}{Min}\, Var(\pi) = \sigma_R^{\,2} + h_f^{\,2}\sigma_f^{\,2} + K^2 h_z^2 \sigma_z^{\,2} - 2h_f\sigma_{Rf} - 2Kh_z\sigma_{Rz} + 2Kh_f h_z \sigma_{fz}$$

where σ^2_R = Var(pXy) = Var(R) is the variance of profit without hedging (cash marketing only) and other symbols have been previously defined. The solution to (1') gives the formulae for the risk minimizing hedges in price and yield futures:

$$\text{(7.1)} \qquad \hat{h}_f = \frac{\sigma_{Rf}\sigma_z^{\,2} - \sigma_{fz}\sigma_{Rz}}{\sigma_f^{\,2}\sigma_z^{\,2} - \sigma_{fz}^{\,2}}$$

$$\text{(7.2)} \qquad \hat{h}_z = \frac{\sigma_{Rz}\sigma_f^{\,2} - \sigma_{Rf}\sigma_{fz}}{K\,(\sigma_f^{\,2}\sigma_z^{\,2} - \sigma_{fz}^{\,2})}$$

The signs of the risk minimizing hedges (7.1) and (7.2) are ambiguous. The denominators are positive because of the second order conditions for a strict local minimum of Var(π). Thus, both price and yield hedges can be either short or long depending on the signs and

relative magnitudes of the covariance terms. This result is the consequence of the presence of two sources of risk rather than two hedging instruments. This can be illustrated by evaluating the sign of the risk minimizing hedge under the joint presence of price and yield risk and the availability of only a price futures instrument. Under these assumptions, the risk minimizing hedge in price futures (7.1) reduces to:

$$\hat{h}_{f0} = \frac{\sigma_{Rf}}{\sigma_f^2} \tag{7.1'}$$

The risk minimizing hedge (7.1') can be either long (-) or short (+) since Cov(pXy, f) may be positive or negative

In the risk minimizing framework, hedging effectiveness is measured by the reduction in the variance of profits/revenues. One can compare the reduction in the variance of revenue between various hedging strategies and the cash marketing strategy, or between two different hedging strategies. Under the joint presence of price and yield risk and the availability of only a price futures market, the risk minimizing hedge is given by (7.1'). Let $R_{f0} = pXy + (F - f)\hat{h}_{f0}$ be the risk minimizing revenue resulting from hedging using (7.1') whose variance can be calculated as $Var(R_{f0}) = \sigma_R^2 - \frac{\sigma_{Rf}^2}{\sigma_f^2}$. The variance eliminated by hedging in price futures compared to the variance of the cash marketing strategy under joint price and yield risk is given by:

$$\Delta_1 = Var(R) - Var(R_{f0}) = \frac{\sigma_{Rf}^2}{\sigma_f^2} \geq 0 \tag{8}$$

Since (8) is always non-negative, engaging in hedging with price futures contracts in the presence of joint price and yield uncertainty guarantees the reduction in the variance of revenue over cash marketing strategy.

The availability of the crop yield futures enables further reduction in the variance of revenue by simultaneous hedging in price and yield futures according to formulae (7.1) and (7.2). Let $Var(R_{f0})$ be the variance of revenue resulting from cash sales and hedging in price futures defined earlier and $Var(R_{fz})$ is the variance from (1') after risk minimizing hedges have been employed After cumbersome but straightforward algebra, it can be shown that the magnitude of variance eliminated by entering the crop yield futures market in addition to being hedged in the price futures becomes:

$$\Delta_2 = Var(R_{f0}) - Var(R_{fz}) = \frac{1}{(1-\rho_{fz}^2)\sigma_z^2}\left(\frac{\sigma_{Rf}\sigma_{fz}}{\sigma_f^2} - \sigma_{Rz}\right)^2 \geq 0 \tag{9}$$

Expression (9) is always non-negative If Δ_2 equals zero, hedging in crop yield futures does not contribute to the variance reduction. By construction, Δ_2 equals zero only if the risk minimizing yield futures position given by (7.2) equals zero.

Notice that the risk minimization results obtained so far do not depend on any assumptions about the distribution of the random variables. Further analysis, however, requires additional assumptions. Assuming the stochastic variables p, f, y, and z are multivariate normal[2], it can be shown that:

[2] Notice that what we assume here is the joint normality of prices and yields but not profits. These results are independent of the results obtained earlier in the paper with the expected utility approach where the joint normality of profits and futures prices and the joint normality of profits and futures yields were required.

(10.1) $$\sigma_{Rf} = X\mu_y\sigma_{pf} + X\mu_p\sigma_{yf}$$

(10.2) $$\sigma_{Rz} = X\mu_y\sigma_{pz} + X\mu_p\sigma_{yz}$$

where $\mu_y = E(y)$ and $\mu_p = E(p)$. We should point out that the assumption of normally distributed yields is not usually supported empirically. However, the additional insights gained through the comparative statics analysis is probably sufficient to justify the use of this assumption. Substituting (10.1) and (10.2) into (7.1) and (7.2) yields risk minimizing hedges under normality:

(11.1) $$\hat{h}_f = \frac{X\mu_y\sigma_z^2\sigma_{pf} + X\mu_p\sigma_z^2\sigma_{yf} - X\mu_y\sigma_{pz}\sigma_{fz} - X\mu_p\sigma_{fz}\sigma_{yz}}{\sigma_f^2\sigma_z^2 - \sigma_{fz}^2}$$

(11.2) $$\hat{h}_z = \frac{X\mu_y\sigma_{pz}\sigma_f^2 + X\mu_p\sigma_{yz}\sigma_f^2 - X\mu_y\sigma_{pf}\sigma_{fz} - X\mu_p\sigma_{yf}\sigma_{fz}}{K(\sigma_f^2\sigma_z^2 - \sigma_{fz}^2)}$$

Substituting (10.1) into (8) and differentiating Δ_1 with respect to σ_{pf} explains the impact of the basis on hedging effectiveness:

(12) $$\frac{\partial\Delta_1}{\partial\sigma_{pf}} = \frac{2X\mu_y(X\mu_y\sigma_{pf} + X\mu_p\sigma_{yf})}{\sigma_f^2}$$

The expression in the parentheses of (12) is equal to the covariance between revenue and futures price (10.1) which is positive only if the risk minimizing hedge from (7.1') is short. Hence, the increase in the covariance between cash price and futures price increases the hedging performance of the risk minimizing short hedge in futures price contracts compared to the cash marketing strategy. Contrary to that, if the risk minimizing hedge is

long, the decreasing covariance between cash price and futures price improves hedging effectiveness.

Similarly, expressions (10.1) and (10.2) can be used to simplify (9) into:

$$(9') \qquad \Delta_2 = \frac{1}{(1-\rho_{fz}^2)\sigma_z^2}\left(\frac{-X\mu_y\sigma_{pz}\sigma_f^2 - X\mu_p\sigma_{yz}\sigma_f^2 + X\mu_y\sigma_{pf}\sigma_{fz} + X\mu_p\sigma_{yf}\sigma_{fz}}{\sigma_f^2}\right)^2.$$

We see that simultaneously hedging in price futures and crop yield futures versus hedging only in price futures will not significantly reduce the variance of profit if the contract underlying yield is very volatile ($\Delta_2 \to 0$, if $\sigma_z^2 \to \infty$). To evaluate the impact of the price basis risk (i.e. correlation between cash and futures prices) on the hedging effectiveness, differentiate Δ_2 with respect to σ_{pf}.

$$(13.1) \qquad \frac{\partial\Delta_2}{\partial\sigma_{pf}} = \frac{-2X\mu_y\sigma_{fz}}{(1-\rho_{fz}^2)\sigma_z^2\sigma_f^2}\left(\frac{X\mu_y\sigma_{pz}\sigma_f^2 + X\mu_p\sigma_{yz}\sigma_f^2 - X\mu_y\sigma_{pf}\sigma_{fz} - X\mu_p\sigma_{yf}\sigma_{fz}}{\sigma_f^2}\right)$$

The expression $(X\mu_y\sigma_{pz}\sigma_f^2 + X\mu_p\sigma_{yz}\sigma_f^2 - X\mu_y\sigma_{pf}\sigma_{fz} - X\mu_p\sigma_{yf}\sigma_{fz})$ is identical to the numerator of the risk minimizing hedge in yield futures (11.2), and is positive for a short hedge and negative for a long hedge in crop yield futures. Since σ_{fz} is negative, the sign of the entire partial derivative (13.1) is positive for a short hedge. Hence, an increase in the covariance between cash price and futures price increases the hedging performance of the two instruments hedging strategy compared to hedging in price futures only if the crop yield futures hedge is short. Contrary to that, if the risk minimizing crop yield futures hedge is long, a decreasing covariance between cash and futures price improves hedging effectiveness of the two instrument hedging strategy over price futures hedging only.

Finally, the effect of yield basis risk (i.e. the correlation between an individual farm's yield and the contract underlying yield) on hedging effectiveness of the yield futures can be examined by evaluating the sign of the following partial derivative:

$$(13.2) \qquad \frac{\partial \Delta_2}{\partial \sigma_{yz}} = \frac{2X\mu_p}{(1-\rho_{fz}^2)\sigma_z^2}\left(\frac{X\mu_y\sigma_{pz}\sigma_f^2 + X\mu_p\sigma_{yz}\sigma_f^2 - X\mu_y\sigma_{pf}\sigma_{fz} - X\mu_p\sigma_{yf}\sigma_{fz}}{\sigma_f^2}\right)$$

The partial derivative (13.2) is positive if the expression in parentheses, which is identical to the numerator of the risk minimizing hedge in yield futures (11.2), is positive. Thus, if the risk minimizing hedge is a short hedge, then an increase in the covariance between an individual farm's yield and the contract underlying yield improves the effectiveness of the two instrument hedge over the price futures hedge only. The expression (13.2) is negative, if the risk minimizing hedge in crop yield futures is a long hedge. In this case, the smaller the covariance between an individual farm's yield and the contract underlying yield (σ_{yz}) the larger the reduction in the variance of profits generated by simultaneously hedging in price and yield futures compared to hedging in price futures only.

IV. Effectiveness of Dual Hedging for North Carolina Corn Producers

To calculate the theoretical reduction in the variance of revenue under three different marketing scenarios from Section III, we have selected three leading corn producing counties in North Carolina. One local spot price in each county was used for county level estimates: Elizabeth City for Pasquotank County, Greenville for Pitt County, and Lumberton for Robeson County. Since most of the corn in North Carolina is harvested in the period between September 1 and October 15, the annual observations

used are the averages of weekly observations (Thursdays) within this six-weeks period. The cash prices are No. 2 yellow shelled corn prices paid to producers for grain delivered in bulk to elevators and were taken from various issues of Weekly Grain Report of the North Carolina Department of Agriculture. The futures prices are settlement prices for the CBOT December corn futures. Farm level yields were approximated with the average county yields. Since yield contracts started trading only recently, historical series of trading data on yield futures does not exist, and hence was approximated with the realized Iowa state average yield. The available data set covers the 1951-1994 period, for the total of 44 observations. The scale of production was fixed at 500 acres.

In order to compute the risk minimizing hedges given by (7.1) and (7.2) as well as the reduction in the variance of revenue under various marketing scenarios given by expressions (8) and (9) we used the historical sample estimates of the variance-covariance matrix of the joint distributions of unhedged revenue ($R = pXy$), harvest period futures price (f) and the futures contract underlying yield (z) for the period 1951-1994. Table 1 summarizes the obtained results.

The results for all three counties are fairly similar. The introduction of hedging in price futures would reduce the variance of cash marketing revenue by 84% in Pasquotank county, by 72% in Pitt county and by 74% in Robeson county. Further reduction in the variance of revenue generated by the inclusion of the second hedging instrument is less dramatic: an additional 5% in Pasquotank, 10% in Pitt, and 9% in Robeson county. Relatively modest reduction in the variance of revenue generated by the dual hedging strategy over the single price futures hedging strategy can be explained by two factors.

Table 1: Comparison of various hedging scenarios for 500 acres of corn

Pasquotank County

	Price Futures Only	Price and Yield Futures
Number of Price Contracts	10.8	9.6
Number of Yield Contracts	0	4.4
% Variance reduction over no hedge	83.64%	89.45%

Pitt County

	Price Futures Only	Price and Yield Futures
Number of Price Contracts	7.4	6.2
Number of Yield Contracts	0	4.2
% Variance reduction over no hedge	71.91%	82.02%

Robeson County

	Price Futures Only	Price and Yield Futures
Number of Price Contracts	8.4	7.1
Number of Yield Contracts	0	4.6
% Variance reduction over no hedge	74.02%	83.47%

First, the historical volatility of the contract underlying yield (Iowa state average) is high causing the lower effectiveness of the two instrument hedge relative to hedging in the price futures market only For the period under consideration the average Iowa yield was 93.3 bushels per acre with the standard deviation of 27.6, whereas in the same period the means and standard deviations of Pasquotank, Pitt, and Robeson counties corn yields were 84.7 bpa and 24.2, 63.2 bpa and 20.6, and 59.6 bpa and 24.0

Second, the correlation between individual county level yields in North Carolina and contract underlying yield (Iowa state average) is relatively low, which in cases where risk minimizing hedge is short, leads toward a reduced hedging performance of the two instruments strategy compared to hedging in price futures only. For example, the correlation coefficients between Pasquotank, Pitt, and Robeson counties yields and Iowa state average yield are 0.75, 0.69, and 0.75 while the correlation coefficient between

Pasquotank, Pitt, and Robeson counties local cash prices and CBOT December futures price is 0.98 in all three cases. That explains a significant reduction in the variability of cash marketing revenues caused by hedging in price futures and a relatively modest reduction in variability of revenues caused by the inclusion of the yield hedging.

Finally, the results may suffer from the fact that no historical futures trading data for the new yield futures is available and therefore the series had to be approximated with the Iowa realized yield

V. Conclusions

We have shown how the yield futures contract recently introduced by the Chicago Board of Trade can be used along with a standard price futures contract to manage the risk faced by a producing firm confronted with both price and yield risk. Although there are two instruments for managing risk (the yield contract and the price contract), even if there are no price basis and yield basis risks, the firm generally cannot eliminate all uncertainty.

We have derived the optimal hedges in both the price futures market and the yield futures market under varying assumptions about the objective of the firm and the existence of the bases risks. A comparison of hedging effectiveness between price futures hedge and simultaneous price futures and yield futures hedge was then performed. We found that a risk minimizing firm can reduce its variance of profit by hedging in both markets rather than just using the price futures market, and that the effectiveness of a two instrument hedge depends on the volatility of the contract underlying yield. The greater

the variance of the underlying yield, the less effective the two instrument hedge will be relative to hedging in the price futures market only. The results also show that the hedging effectiveness of using the new crop yield insurance futures depends critically on the price and yield bases and that the direction of the effect depends on the established crop yield futures position. If the crop yield risk minimizing hedge is short, the increase in the covariance between cash price and futures price increases the hedging performance of the two instruments strategy compared to hedging in price futures only. The increase in the covariance between an individual farm's yield and the contract underlying yield has the same effect

The usefulness of the dual hedging in price and yield futures in the presence of both price and yield risks is illustrated using the data for North Carolina corn producers. Relatively modest reduction in the variance of revenue generated by the dual hedging strategy over the single price futures hedging strategy can be explained by the high historical volatility of the contract underlying yield and the low correlation between individual county level yields in North Carolina and contract underlying yield (Iowa state average).

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