A Bio-economic Model of Perennial Production with Deficit Irrigation

Bradley Franklin
Department of Economics, University of California, Riverside
International Water Management Institute

Kurt Schwabe
Department of Environmental Sciences
University of California, Riverside

Keith C. Knapp
Department of Environmental Sciences
University of California, Riverside

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Contents

Introduction

The potential impact of climate variability and climate change on agricultural productivity is a well-researched subject. The validity of the economic impacts derived from such studies depends crucially on the accuracy of the characterization of the underlying production processes. Accordingly, significant advances in the representation of the biophysical aspects of agricultural production have been made in studies such as Letey, Dinar, and Knapp (1985), and Kan, Schwabe, and Knapp (2002), and Schlenker and Roberts (2009). Unfortunately, similar advances in the representation of perennial production are not well captured by the existing literature; consequently, the economics of perennial agriculture are poorly understood. Adequately modeling perennial crop production involves recognizing that it is inherently dynamic due to several salient physical traits including an establishment period of multiple years before marketable yields are produced, a long life in commercial production of up to 50 or more years, and the long-lasting impact of the pattern and timing of input use and other exogenous factors such as weather on the productivity of the crop over its life. Furthermore, the hump-shaped age-yield relationship characteristic of most perennial crops means that perennial production is essentially non-linear. Due to these factors, perennial crop production is best represented as an investment under uncertainty characterized by non-linear dynamics, a characterization not reflected by the current literature.

This study is concerned with perennial production in arid and semi-arid regions where irrigation is common and, therefore, focuses on the effects of water supply variability on farm management decisions which affect perennial stocks. Although wine grapes are the crop of interest here, the model is sufficiently general that it should be applicable to a wide variety of perennial crops. During periods of drought, one adaptation available to irrigated perennial producers is to reduce water application levels, a practice known as deficit irrigation. Studies have shown that while moderate deficit irrigation of mature vines can be an effective way to manage scarce water resources, it can also result in decreased current season yields as well as losses in permanent biomass structure and carbohydrate

reserves thereby diminishing future season yields. Equally concerning, deficit irrigation of immature vines can result in delayed maturity or crop death. Grape growers are aware of such interseasonal trade-offs and manage their grapevines accordingly; however, there is very little formal modeling that would allow economists to understand how these micro-level decisions impact production at the farm and regional levels.

We develop a field-level, bio-economic model capable of describing the interseasonal dynamics of water applications to perennial crops. Few economic studies make any attempt to model deficit irrigation of perennial crops; those that do are limited to two-stage Dantzig-style models with ad hoc future yield penalties and therefore have difficulty accounting for the effects of deficit irrigation in a realistic manner. While Bellman and Hartley (1985) describe a theoretical model of tree crops in which the entire history of input use and exogenous factors is tracked, a model that does so in practice is likely to be computationally infeasible. For that reason, the current study attempts to encapsulate the effects of crop age, soil salinity, and irrigation history on yield potential by using an unobserved biomass state variable. The biomass variable, which represents vine capacity, captures the irrigation history of the crop in a single state variable, thus allowing us to use a stochastic dynamic programming framework to analyze optimal management decisions over the life of the crop given stochastic water supplies. Yields are a function of plant age, biomass, current season water applications and salinity. The biomass law of motion is a stylized representation of findings in the viticultural science literature and is calibrated to reproduce observed yield effects, both within season and across seasons, from varying irrigation quantities. Salinity affects yields in the current season via water uptake as well as in the future by changing root zone salinity levels.

While the focus of our study is on the interseasonal effects of varying seasonal water applications due to scarce and variable water supplies over time, some consideration of the intraseasonal dynamics is required in order to create a realistic and flexible model of irrigated wine grape production. Since several factors affect how water applications translate into water uptake by grapevines, it is more accurate to develop a model that

reflects the changing agronomic conditions throughout the growing season rather than treating the seasonal irrigation water as a single quantity of water applied once per season. Plant transpiration depends on the soil moisture and salinity which are constantly changing over the course of the season. Depending on soil conditions, only a portion of the water applied at any point in time will become available to the grapevines as some water will inevitably be lost to deep percolation into the water table, evaporation from the soil surface, or run-off into other bodies of water. These processes are inherently non-linear and have thresholds, which imply that models which depend solely on seasonal averages may give misleading results. Successful irrigators are keenly aware of the condition of their fields throughout the year and seek to maintain soil moisture over the course of the season such that yields are not inhibited by water deficits. Moreover, the phenological stages of vine growth require the irrigator's discrimination as to when deficit irrigation is acceptable. Using a detailed intraseasonal model of hydrological and soil processes as a data-generating mechanism results in a realistic interseasonal model and leaves open the possibility of analyzing the impacts of different deficit irrigation techniques such as regulated deficit irrigation (RDI), sustained deficit irrigation (SDI), and partial root zone drying (PRD).

Because of the long-lived nature of perennial crops, the effects of deficit irrigation may be felt across multiple seasons. By characterizing the optimal management rules that recognize these effects and the implicit trade-offs, we are able to better characterize agricultural water demand and, by extension, to get a better estimate of the possible benefits from water trade. While the emphasis here is on attempting to represent bio-economic considerations more realistically, the model can be readily used to assess the effects of changing market and climatic conditions over time on perennial production. The model may be used to better understand the effects of climate change in arid and semi-arid regions where perennial crop production is important, such as in California and the Murray-Darling Basin of Australia. In addition, water quality, including water salinity, is important in such regions and the detailed intraseasonal model used here can shed light on how perennial irrigation management decisions may affect farm run-off and

deep percolation into groundwater aquifers.

Literature Review

Because of the heterogeneity of vines cultivars, micro-climate, soil quality and other factors, some simplifying assumptions based on the viticultural literature are required to make modeling the biology of wine grape production practical. With regard to the commercially productive life of a grapevine it is safe to assume a 4-5 year vine establishment period before maximum yields are attainable (Gutierrez et al., 1985). Grapevines may live to be well over 100 years old and continue to bear yields sufficient to produce boutique wines. However, the bulk of vines in commercial production are much younger since yields begin to decline beyond a certain age. Depending on the region in question, a reasonable upper bound on the age of vines in commercial production can be assumed to be 40 years (Mullins et al., 1992) or perhaps as much as 50-60 years (Gutierrez et al., 1985).

Seasonal potential evapotranspiration (PET) for mature vines in the Southwest US and Australia is about 650-800 mm (Williams and Matthews, 1990) and the midpoint of this range (725 mm) will be assumed for the purposes of this study. During establishment, seasonal PET is assumed to be 300 mm in the first year, 400 mm in the second, and 590 mm in the third year before reaching maturity in year 4 (Mullins et al., 1992). Grimes and Williams (1987) derived a formula describing the relationship between the same season actual yield relative to maximum yield and actual crop ET relative to PET for Thompson seedless grapes which is approximately linear from a relative ET of 0.4 to 1:

$$\text{Relative Yield} = 0.976 * \text{Relative ET}^{0.409} \tag{1}$$

These findings imply that a 50% reduction in ET results in a 26% decrease in yields. Whereas deficit irrigation reduces yields, it also increases water use efficiency (tons of

yield per ML of water applied) non-linearly. Williams (2010) found in a 5 year field study of Cabernet Sauvignon in central coast of California deficit irrigation always resulted in increased water use efficiency[1]:

Relative ET	Water Use Efficiency (tons per mL)
.25	15.8
.50	8.1
.75	6.2
1	4.8
1.25	4.4

Table 1: Water use efficiency findings from Grimes and Williams (1987)

Interpolating over the data points reveals that water use efficiency is an approximately quadratic decreasing function of relative ET. Holding yield quality considerations aside, this is the essence of the trade-off encountered in deficit irrigation; when the cost of irrigation water is high relative to output prices then a profit-maximizing irrigator will sacrifice yields for decreased water costs and, in so doing, increase water use efficiency. The severity of the deficit irrigation will be determined by current season water and output prices, expectations of future prices, and expectations of future yield losses that may result from passing certain irrigation thresholds.[2]

It is important to note that while deficit irrigation affects both current and subsequent season yields for mature grapevines, for immature vines the result may be that maturity is delayed for one or more seasons (Williams and Matthews, 1990). Based on discussions with grape growers, it is evident that while deficit irrigation of mature vines is not ideal, it is done in practice on vines under 4 years of age but never for newly planted vines pre-

[1]It should also be noted that the trial was conducted using 5 different rootstocks and found no significant interaction between irrigation amounts and rootstocks used. The implication for the current study is that we may safely ignore the issue of the rootstock used.

[2]In the last several years in South Australia, for example, grapes prices have been very low and water relatively scarce such that many producers have chosen to "mothball" their vines- to apply the minimum amount of water deemed necessary to avoid severe damage of the vine with the intent of not harvesting during the current season and waiting for grape prices to rise in subsequent seasons.

sumably because the vines are so young that irreparable damage, and possibly premature death, may occur. For this reason, it is justifiable to exclude deficit irrigation of vines less than one year old by assuming that doing so results in the immediate death of the vine.

While Bellman and Hartley (1985) describe a theoretical model of perennial tree crops in which the entire history of input use and exogenous factors is tracked, in practice a model that does so is cumbersome at best and computationally infeasible at worst. For that reason, the current study attempts to encapsulate the effects of age, soil salinity, and irrigation history on yield potential by using a single, unobserved biomass state variable. The use of biomass as a state variable is justifiable in the sense that there is "an annual increase in dry matter of permanent structures" (Mullins et al., 1992) which includes the roots, trunk, and cordons. Studies have shown that the dry mass in permanent structures decreases from one season to the next in response to deficit irrigation. For instance, Williams and Grimes (1987) found that in a 4 year trial of Thomson seedless grapes a deficit irrigation treatment of 52% of required ET resulted in a decrease of 26%, 17% and 31% to the cordons, trunk and roots respectively over the period of the trial relative to the control given 100% of required ET.

A potential complication with the interpretation of the use of biomass as a state variable is that the permanent biomass of grapevines, and woody perennials generally, differs from the parts of the plant that transpire; they provide the infrastructure for the seasonal growth of shoots and leaves which, in turn, transpire. Within the growing season, the canopy is typically pruned heavily to ensure both the quantity and quality of the resulting yield. Thus, intraseasonal biomass is a quantity to be managed whereas interseasonal, or permanent, biomass represents the productive potential of the vine across seasons. It is this abstract quantity which is of interest to the farmer maximizing returns on a long term investment. The fact that vine roots and trunk may continue to increase in biomass long after yields have reached a plateau can be dealt with by assuming the existence of a maximum productive biomass level.

Closely related to permanent biomass structures is the reserve of nutrients such as non-structured carbohydrates that are stored within them during dormancy. The carbohydrate reserves are what is actually required to start new seasonal growth in grapevines and are even more important in other perennial crops because the order of development in grapes (grow shoots and then flowers) is reversed necessitating bigger reserves (Mullins et al., 1992). Similarly, Gutierrez et al. (1985) highlight the interseasonal effect of these reserves, stating that the "[e]xcessive reduction of reserves imposed by overcropping not replenished within season may result in reduced yields or quality in the following season." In the study by Grimes and Williams (1987), similar to their dry mass findings, the non-structured carbohydrates were reduced by an average of 34%, 30% and 32% in the cordons, trunk and roots respectively. Yet another way of interpreting the problem is to recognize that in grapes and other perennials the "differentiation of reproductive structures is initiated in the season prior to the season in which those structures mature fruit" (Williams and Matthews, 1990). This means that the incipient buds that grow at the end of the season determine the number of potential clusters per vine and this, in conjunction with the stored reserves and vine water status, limits the following season's potential yield. In this sense, the incipient buds constitute the biomass that it of interest. Also, this provides the linkage between seasons since the water deficit in one season may limit bud formation which therefore limits the following season's yield. While this is theoretically true, Williams and Matthews (1990) state that different studies have found both positive and negative effects of water deficits in one season on the next season's yield. They hypothesize that this may result from a failure to adequately quantify vine water status and also note that the timing of the water deficits within the season is critical to determining interseasonal effects. Matthews and Anderson (1989) found that, relative to continuous irrigation, deficits early in the season, late in the season, and throughout the season (full deficit) all had a negative impact on yields both within the season and for the next season. The authors conducted 3 different deficit irrigation treatments over 3 consecutive years on Cabernet Franc vines in Napa Valley, CA. A summary of the relevant results below shows that deficits late in the season had the least detrimental effect,

followed by early season and then full season. The effects in the first year were relatively mild compared to the second year and in the third year both early and late deficit vines bounced back while full deficit vines did not (relevant datum not published but shown in yield graph).

Treatment	Year 1	Year 2	Year 3
Late Deficit	9	26	2
Early Deficit	16	50	28
Full Deficit	22	54	-

Table 2: Percentage Decrease Relative to Same Year Maximum Yield from Williams and Matthews(1990)

To summarize, the long-lived nature of perennial crops means that the pattern of input use over time affects yield potential in future seasons. The physical linkage between seasons is expressed in permanent biomass structures, nutrient reserves stored in those structures, and incipient bud formation at the end of season. Water deficits at any point in the season have the potential to affect any or all of these aspects of the vine and, via these mechanisms, affect future yield levels. For the purposes of the present study, we propose a stylized model of a permanent biomass index which is calibrated to reflect several of the key findings in this section regarding the effects of water deficits on future season yields.

Interseasonal Model

We assume a profit-maximizing firm that manages a single field of the perennial crop. The choice variables are the amount of water applied (w_t) and crop removal (z_t), which is binary variable. The state variables of the model are biomass (b_t), crop age (k_t), and soil salt level (s_t). The biomass level is an index that ranges from 0 to 100, which is calibrated to replicate some key results from the viticultural science literature. The initial biomass level for a newly planted crop ($k = 0$) is represented by $b_0 = 20$, which serves as a threshold

level. If the crop is damaged such that it falls below this level, then it is considered to be dead and must be replanted. The crop ages possible range from 0 to $k_{max} = 40$ years old. The soil salt level is determined by the intraseasonal model as detailed in the appendix and is adopted from Muralidharan and Knapp (2009). The firm's profit is given by

$$\pi_t = py_t - \gamma_w w - \gamma_k - \gamma_z z_t \tag{2}$$

where p is the crop price, y_t is the crop yield, γ_w is the water cost per cm, γ_k is the (possibly age-dependent) non-water production cost term, and γ_z is the removal cost.

Water-yield Relationship

Potential evapotranspiration (pet_t) varies by crop age (in years) and is given by

$$pet_t = \begin{cases} 30 \text{ cm}, & 0 \leq k_t < 1 \\ 40 \text{ cm}, & 1 \leq k_t < 2 \\ 59 \text{ cm}, & 2 \leq k_t < 3 \\ 72.5 \text{ cm}, & 3 \leq k_t \leq k_{max} \end{cases} \tag{3}$$

The amount of actual evapotranspiration (e_t) corresponding to a given amount of water applied is a function of soil moisture, soil salinity, and water salinity as is detailed in the appendix. Relative evapotranspiration (r_t) gives us a measure of the proportion of actual evapotranspiration (e_t) relative to PET and is adjusted for the efficiency of the irrigation system (ie) in place, which we assume to be a high-efficiency system in which $ie = 0.85$. The term $full_t$ represents the full water requirement given irrigation efficiency such that $full_t = \frac{pett}{ie}$.

$$r_t = \begin{cases} \frac{e_t}{full_t}, & w_t < full_t \\ 1, & e_t \geq full_t \end{cases} \tag{4}$$

We assume linearly increasing yields with age during establishment ($k < 3$) and then linearly declining yields between $k_{dec} = 30$ to $k_{max} = 40$ years of age. The resulting age-yield relationship forms a plateau, which we show mathematically and graphically below:

$$y_{max} = \begin{cases} 0 & k_t < 1 \\ 6 & 1 \leq k_t < 2 \\ 12 & 2 \leq k_t < 3 \\ 20 & 3 \leq k_t \leq k_{max} \end{cases} \tag{5}$$

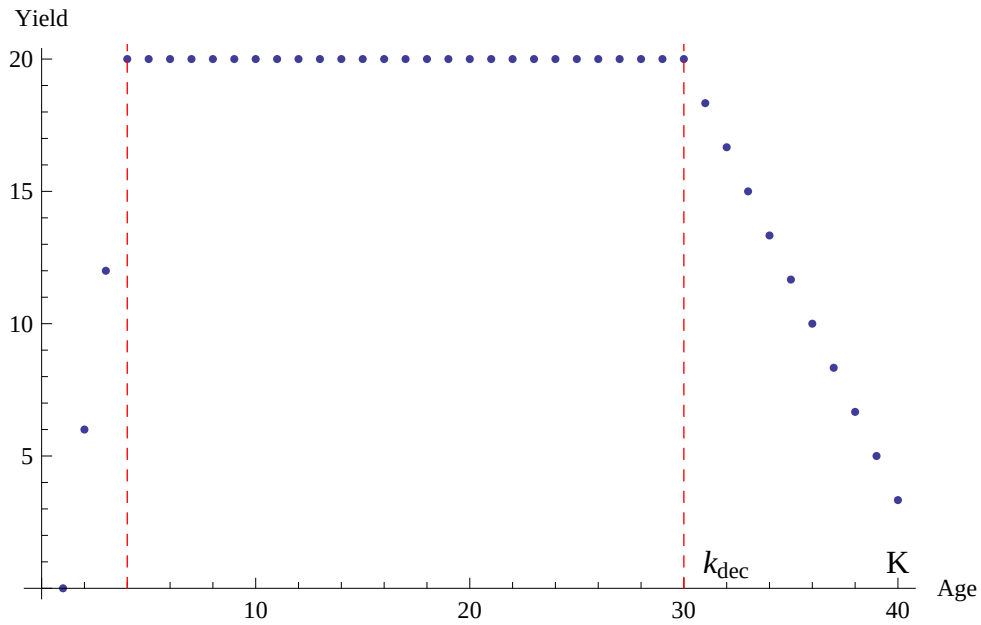

Figure 1: Maximum yield by age (tonnes/hectare)

Aside from age, the other determinants of yields are water applications and crop biomass. We model yields as the product of maximum yield by age multiplied by a function f_0 that represents the effect of current season water applications and f_1 which signifies the effect of biomass on yields.

$$y_t = y_{max} * f_0(r_t) * f_1(b_t) \tag{6}$$

The sub-function f_0 is given by Equation 1 and f_1 is a Hill function chosen for its flexible form:

$$f_1 = \frac{b^n}{b_{mid}^n + b^n} \tag{7}$$

where the parameters $n = 6$ and $b_{mid} = 50$ are chosen to calibrate the function to results from the viticultural science literature. We show both f_0 and f_1 graphically below:

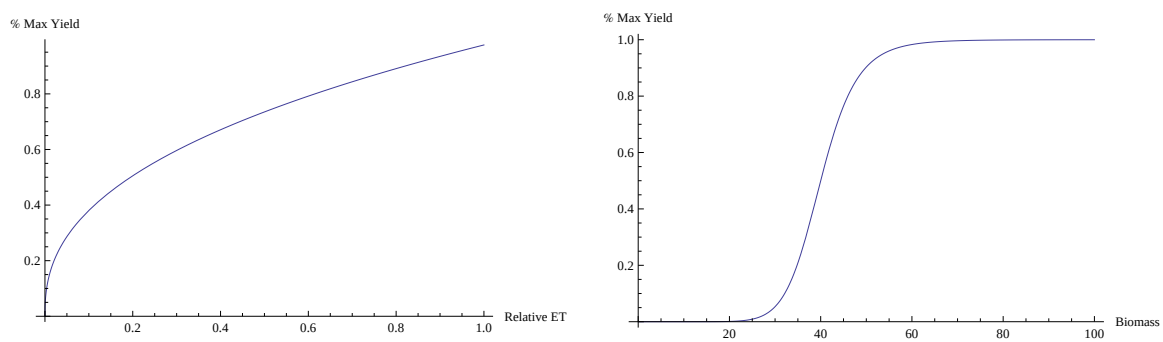

Figure 2: Relative yield as a function of (i) Relative ET and (ii)Biomass

Model Dynamics

There are three laws of motion corresponding to the three state variables of the model. The law of motion for salt balance is given by the intraseasonal model (see appendix) assuming that water applications are evenly distributed within the growing season. The key features of this law of motion are that soil salt is increasing in existing soil salt and first increases and then decreases in the amount of water applied as for relatively low irrigation water amounts salt builds up in the soil whereas higher amounts result in salt being flushed from the soil via deep percolation. The crop age advances each period except for when the crop becomes so damaged that it needs to removed ($b_t < b_0$), it is voluntarily removed ($z_t = 1$), or it has reached the maximum age allowed by the model ($k_t = k_{max}$). Hence, the law of motion for crop age is

$$k_{t+1} = \begin{cases} k_t + 1 & k_t < k_{max},\ b_{t+1} \geq b_0 \\ 0 & k_t = k_{max} \text{ or } b_{t+1} < b_0 \text{ or } z_t = 1 \end{cases} \tag{8}$$

The biomass law of motion is a function of current period biomass, relative evapotranspiration (r_t), and crop age. Next period biomass (b_{t+1}) is increasing in both current biomass and relative ET. Howver, the effect of crop age is that it makes biomass growth more sensitive to deficit irrigation for young crops. This is captured by the parameter $\rho(k_t)$, which takes on values that make it costly to deficit irrigate crops during establishment.

$$b_{t+1} = \begin{cases} (1-\sigma)b_t + r_t^{\rho(k_t)}\sigma b_t + r_t^{\rho(k_t)} b_t \left(1 - \frac{b_t}{b_{max}}\right) & k_t < k_{max} \\ b_0 & k_t = k_{max} \text{ or } z_t = 1 \end{cases} \tag{9}$$

where $1-\sigma$ represents a full deficit irrigation penalty and $\sigma = 0.3$ is chosen to calibrate the model. Note that when relative ET $= 1$ the first piece of this function simplifies to a logistic function:

$$b_{t+1} = b_t + b_t \left(1 - \frac{b_t}{b_{max}}\right) \qquad k_t < k_{max} \tag{10}$$

Dynamic Programming Model

The model is formulated as a Dynamic Programming problem with $V(\cdot)$ representing the value function, α being the discount rate, and $\bar{q}$ signifying the water allocation, which we assume to be constant.

$$V(b_t, k_t, s_t) = \underset{w_t, z_t}{\text{Max}}\ \pi_t + \alpha V(b_{t+1}, k_{t+1}, s_{t+1}) \tag{11}$$

s.t.

$$w_t \leq \bar{q},\ b_{t=1} = b_0,\ k_{t=1} = 0,\ s_{t=1} = s_0 \tag{12}$$

The amount of water applied is constrained to be less than the allocation amount. The firm starts with a new crop; hence the initial level of biomass corresponds to that of a new crop and the age starts at zero. The initial salt mass is given also. The value function resulting from the model parameters is shown below with age held constant for a mature crop. Note that it is increasing in biomass and decreasing in the salt level.

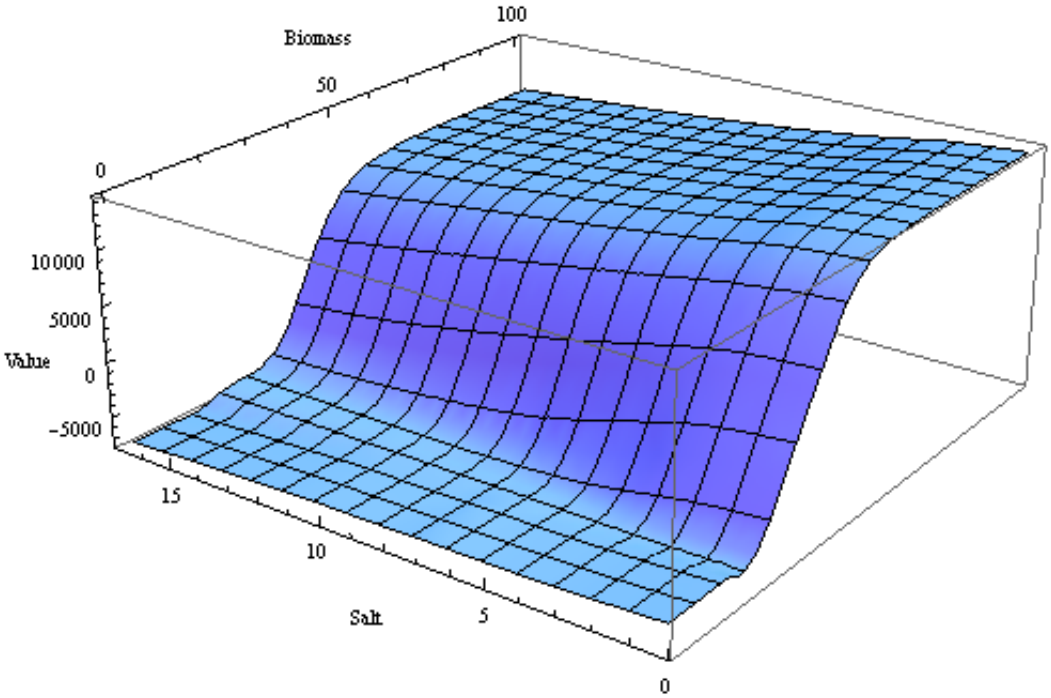

Figure 3: Value function holding age constant

The 100 year time paths for key variables are depicted below. Note that biomass levels increase initially to a plateau where they remain over the life of the crop before removal in year 39. Since removals occur at the beginning of the time period and the time path plots the biomass at the end of the time period, we see that the minimum biomass level is only reported for the first year in the time path. Upon removal in later periods, the end of period biomass is given by the biomass law of motion in Equation 9 with $b_t = b_0$. Similar to the biomass levels, the time path of salt accumulation shows an initial increase up until a plateau which remains until a new planting occurs, at which time the amount of water is enough to fully water the new crop and flush the salt out of the soil. The water applied is equal to the allocation level of 80 cm for all periods except for once when the

crop is one year old. At that point, it appears that the penalty from deficit irrigation is too low to prevent the model from saving on irrigation costs by under-watering the crop. The time path of profits follows the age-yield relationship as shown previously. Note that there are fixed costs for planting and removals which is result in large losses when new plantings occur.

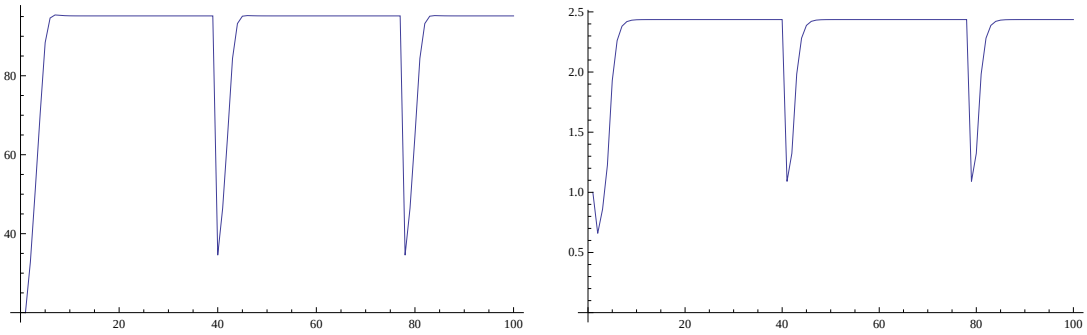

Figure 4: Baseline time paths for (i) Biomass index and (ii) Salt (t/cm)

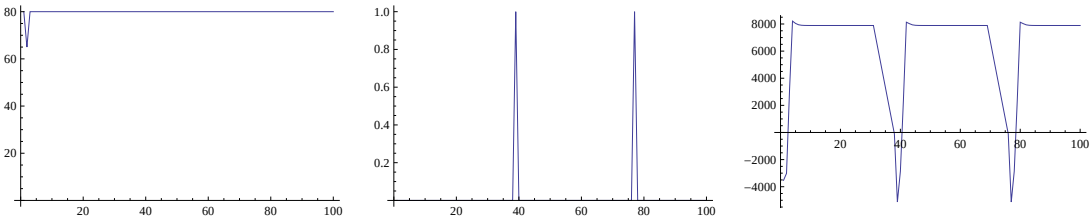

Figure 5: Baseline time paths for (i) Water applications (cm) (ii) Removals (years) and (iii) Profits ($A\$/ha.$)

Below we see the effects of lowering the allocation level from 80 cm to 50 cm. Water applications are always the full amount of 50 cm and, as a consequence, the amount of water applied is never enough to flush the salt out to the extent that occurs under the baseline of 80 cm allocations. The biomass levels and yield levels are also lower as a result of deficit irrigation. Furthermore, the removal age changes from year 39 to year 40, the maximum age allowed for in the model.

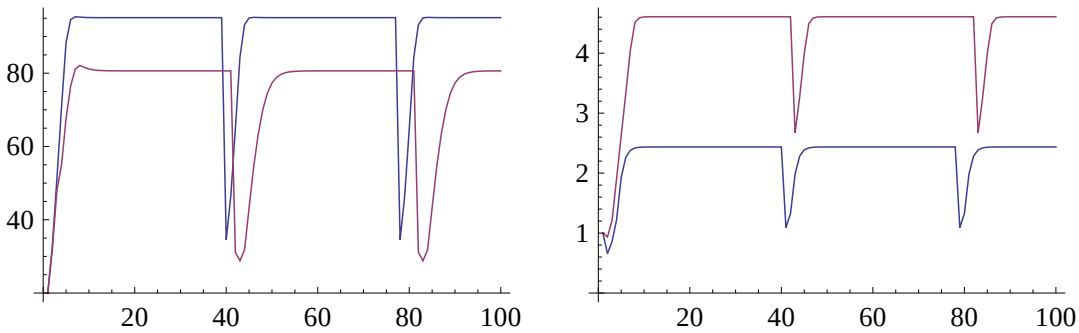

Figure 6: Time paths for (i) Biomass and (ii) Salt with $\bar{q} = 80$ (blue) and $\bar{q} = 50$ (purple)

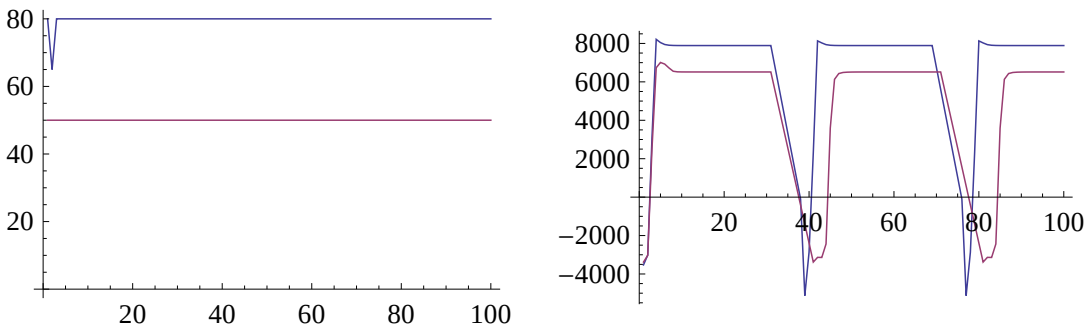

Figure 7: Time paths for (i) Water applications (ii) Profits with $\bar{q} = 80$ (blue) and $\bar{q} = 50$ (purple)

Stochastic Dynamic Programming Model

We obtained modeled historical annual irrigation diversions under current water entitlement rules and levels of development for a 110 year period in the South Australian MDB obtained from Connor et al. (2011). Based on that data, we fit the cumulative distribution function below:

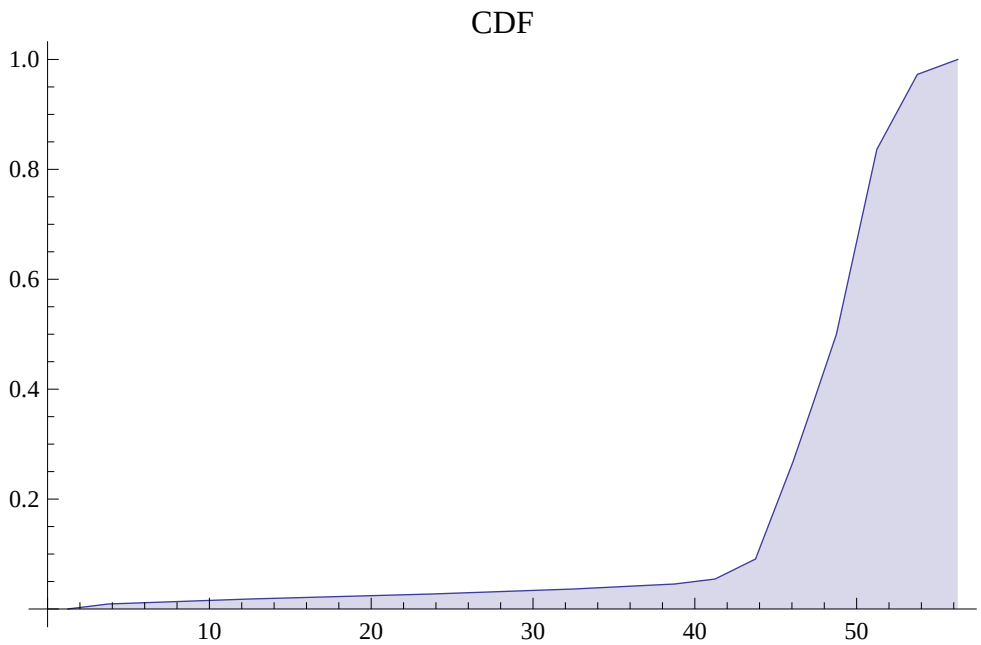

Figure 8: CDF for water allocations

We then use the fitted distribution to generate stochastic water allocations for the Dynamic Programming model. The value function becomes

$$V(q_t, b_t, k_t, s_t) = \underset{w_t,\, z_t}{\text{Max}}\ \pi_t + \alpha\ \mathbb{E}_t\left[V(\tilde{q}_{t+1}, b_{t+1}, k_{t+1}, s_{t+1})\right] \tag{13}$$

where q_t is the allocation level given at time t, $\tilde{q}_{t+1}$ is the stochastic water allocation for time period $t+1$, and $\mathbb{E}_t$ is the expectations operator.

Conclusions

Increasing water scarcity and variability is a concern for farmers in many regions of the world. Many arid and semi-arid areas have substantial land area devoted to long-lived perennial crops. The fixed investment in perennials affects how farmers react to changes in water supplies. In the short run, farmers may seek to obtain additional water markets or may shift water from one land use to another. One very common practice is deficit irrigation, which allows for water savings at the expense of current, and possibly future, yields. For perennial crops, deficit irrigation over one or more consecutive years may decrease the productivity of the crop. Depending on the health and age of the crop, these effects can be long-lasting or even permanent. To our knowledge, there exist no economic studies of perennial crops that model these interseasonal effects in a rigorous manner. This paper represents an attempt do so, taking wine grapes as the crop of interest and calibrating the model to replicate some key results from the viticultural science literature. However, the model structure is general and could be applied to any number of perennial crops, including citrus and nut crops. The preliminary model results show how farmers manage the interseasonal effects of water applications via deficit irrigation as needed. The model shows how the amount of water available affects yields, soil salt levels, and removal decisions over time. Incorporating stochastic water supplies will shed light on how farmers manage water variability and allow us to develop a fuller understanding of agricultural water demand in regions with significant perennial crop production.

Appendix

Intraseasonal Model

Plant transpiration is a function of not only the quantity of water used in irrigation but the timing of those irrigations in the context of constantly changing soil (moisture and salinity) and weather (potential ET, rainfall, etc.) conditions. Since these processes are inherently non-linear and have thresholds, models of irrigated agriculture based solely on seasonal averages are likely to provide misleading results. Moreover, since irrigators use both their knowledge of the phenological development of vine growth and monitoring of soil conditions during the growing season to decide when to irrigate, a model that includes some description of intraseasonal dynamics is both more flexible and representative of the realities faced by farmers. To address such concerns, an intraseasonal model of hydrological and soil processes based on Muralidharan and Knapp (2009) is used here as a data generating mechanism in order to relate field management over the course of the season to the carry-over effects on the health of the grape vines and the potential impacts on future yields. ET is modeled as a function of maximum ET which depends on age as described above as well as matric potential and soil salinity:

$$e_t = \frac{\bar{e}(k_t)}{1 + \left(\frac{\phi_{e1} c_t + h_t}{h_{50}}\right)^{-\phi_{e2}}} \tag{14}$$

where $\bar{e}(k_t)$ is maximum ET, c_t is soil salinity, h_t is matric potential, h_{50} is soil water potential that would result in a 50% reduction in ET, ϕ_{e2} is a crop-dependent parameter and ϕ_{e1} is a parameter with no physical interpretation. Note that ET depends on maximum ET which varies by age of crop. This means that separate regressions are required to determine the coefficients for each age class. All the results that follow for the seasonal model assume mature vines with a maximum ET of 725 mm.

Matric potential is defined to be the amount of work that must be done to overcome the attractive forces of water molecules to soil particles and is expressed as:

$$h_t = \frac{1}{\phi_{h1}} \left(\left(\frac{m_t/m_{rz} - \theta_{res}}{\theta_{sat} - \theta_{res}} \right)^{\frac{-1}{\phi_{h2}}} - 1 \right)^{\frac{1}{\phi_{h3}}} \tag{15}$$

where m_t is soil moisture, m_{rz} is rootzone depth, and the other terms are parameters (van Genuchten, 1978). Note that, given the functional form and parameter values, ET increases in matric potential and decreases in soil salinity while matric potential increases in soil moisture. Deep percolation (d_t) is defined as the difference between irrigation water applied w_t and the water storage capability of the soil given the field capacity m_{fc}, current soil moisture m_t, and current ET:

$$d_t = \text{Max}[0, w_t - (m_{fc} - (m_t - e_t))] \tag{16}$$

Soil moisture is equal to the previous period's soil moisture plus rainfall ($rain_t$) and effective irrigation water ($ie * w_t$) less evapotranspiration and deep percolation:

$$m_{t+1} = m_t + rain_t + ie * w_t - e_t - d_t \tag{17}$$

Note that the irrigation efficiency coefficient, ie, is assumed to be 0.85 in accordance with the dominant use of drip and micro spray irrigation systems in South Australia. Rainfall data is taken from average monthly rainfall as reported for Loxton, South Australia[3]. The law of motion for rootzone salt mass is defined as:

$$s_{t+1} = s_t + c_{wt} w_t - c_{dt} d_t \tag{18}$$

where s_t is salt mass, c_{wt} is the salinity of water applied, and c_{dt} is the salinity of deep percolation water. Soil salinity is simply the ratio of salt to moisture in the soil: $c_t = \frac{s_t}{m_t}$.

[3]Data downloaded from www.bom.au.

Data Generating Mechanism

We assume uniform weekly irrigation events over a 22 week growing season from November to March (Biswas et al.). In reality, irrigation events are unlikely to be uniform throughout the season given that grapevines need more or less water depending on the stage of the growing season and that there are constraints on when water deliveries can be made. For a full model of intraseasonal dynamics, such considerations would be important; however, for the present model some consideration of intraseasonal dynamics are useful only because they ensure a realistic approximation of the hydrological processes involved.

Denoting weeks during the growing season with τ and years with t, for a feasible range of seasonal water applications the intraseasonal model is used to generate $e_t = \sum_{\tau} e_{\tau t}$ and analogously s_t, m_t, d_t. The data is generated using every possible combination of starting soil moisture and salt values with upper bounds set to the field capacity for moisture storage and the salt mass level that would ensure zero yields respectively. Using the model as specified above, mappings are created between the initial values, water application levels, and the resulting variables of interest. Interpolating over the mappings gives the seasonal ET and end of season salt mass levels that result from a given seasonal water application level.

Alternatively, rather than using interpolation one can regress the variables of interest calculated by the model against the initial values and water application amounts specified. In that case, one possible specification is to regress seasonal ET and end of season salt levels on linear and quadratic terms for initial soil moisture, initial soil salt mass, and seasonal water applications. The regression coefficients are then used to form the seasonal relative ET and salt mass law of motion equations in the interseasonal model.

Comparison of the interpolation and regression methods showed that interpolating provided more accurate estimates of the data generated by the intraseasonal model and was therefore used in the interseasonal model. However, the regression results may be of in-

terest to the reader and are therefore reported here. Table 1 shows the regression results with seasonal ET as the dependent variable. Initial soil moisture at the beginning of the growing season is denoted as m_0, initial soil salt mass is s_0, and total seasonal water applied in cm is w_{tot}. Note that the ratio of m_0 and s_0 gives soil salinity. The adjusted r-squared of the model is .992. Table 2 reports the results of the regression in which ending salt mass is the dependent variable. The adjusted r-squared is 0.845.

	Estimate	Standard Error	t-Statistic	P-Value
m0	1.38866	0.00526997	263.503	$5.195168094 \times 10^{-11567}$
s0	−1.87022	0.00746691	−250.468	$2.670304359 \times 10^{-10675}$
wtot	0.703139	0.000850644	826.595	$2.557225702 \times 10^{-45868}$
$m0^2$	−0.0218387	0.000208908	−104.537	$6.939533849 \times 10^{-2260}$
$s0^2$	0.0215297	0.00044327	48.5701	$4.446144760 \times 10^{-509}$
$wtot^2$	−0.00221754	5.71232×10^{-6}	−388.204	$2.982682809 \times 10^{-20271}$

Table 1: Seasonal ET Regression (Mature Vines)

	Estimate	Standard Error	t-Statistic	P-Value
m0	0.405332	0.00290444	139.556	$2.596266739 \times 10^{-3881}$
s0	0.963578	0.00411523	234.149	$1.5023205055 \times 10^{-9575}$
wtot	−0.0397818	0.000468815	−84.856	$7.476336466 \times 10^{-1515}$
$m0^2$	−0.0143302	0.000115135	−124.464	$7.679208467 \times 10^{-3140}$
$s0^2$	−0.0368756	0.000244299	−150.945	$4.843723511 \times 10^{-4479}$
$wtot^2$	−0.0000160098	3.14822×10^{-6}	−5.08533	3.67623×10^{-7}

Table 2: End of Season Salt Mass Regression (Mature Vines)

The number of data points used to generate the above results was 104,615 representing all possible combinations of the initial soil moisture and salt mass levels in conjunction with all possible levels of seasonal water applications. The large amount of data accounts for the generally very high t-statistics reported. One should use caution in interpreting the above regression results; all data generated is by definition non-random and heterogeneous units apply to each of the dependent and independent variables. Therefore, the magnitude of the coefficients reported is meaningless independent of their ability to be used to concisely summarize the data generating mechanism. One may note however that the signs of the coefficients make sense intuitively.

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