AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Environmental Tradeoffs in Resource Management: Fish, Power and Carbon on the Columbia River

Shepard C. Buchanan[1]
Bonneville Power Administration
Portland, Oregon, USA

40th Annual Conference of the
Australian Agricultural and Resource Economics Society
Melbourne, 13-15 February 1996

As resource conflicts grow, resource managers are increasingly faced with tradeoffs among environmental assets. The management of the Columbia River system in the US Pacific Northwest is illustrative of such conflicts.

The Columbia River was developed to provide benefits of flood control, irrigation, municipal and industrial water use, navigation, recreation, and electric power. The river has long been managed to maximise its power benefits. However, the river's development has adversely affected one of its greatest resources--salmon--bringing several species to the brink of extinction.

Today, the river is managed to accord salmon the highest priority. Consequently, the power system has lost much of the flexibility it formerly enjoyed, and because of those changes replacement power sources are needed. Simultaneously, concern with greenhouse gases, principally carbon dioxide has grown.

This paper illustrates the conflicts resource managers face when attempting to develop cost-effective new power resources, avoid further carbon emissions, and save the salmon from extinction. It shows that constraints imposed by efforts to save salmon make it much more difficult to achieve another environmental goal, the reduction of carbon emissions.

---

[1] The author is Manager for Power Business for Bonneville Power Administration, Portland, Oregon, USA. The views expressed herein reflect the author's views only, and do not necessarily reflect the views or policies of Bonneville Power Administration.

Introduction

The Columbia River Basin is the largest and most complex hydro-electric system in the world. The river is the lifeblood of the United States' Pacific Northwest[2], providing economic and environmental benefits for millions of people.

Significant development on the river began with the building of giant dams in the 1930's in response to the economic malaise of the Great Depression. As envisioned then, and carried through by later generations, the system was developed for power, irrigation, navigation, recreation and flood control.[3] Measured in those terms, the system has been a resounding success, providing uncountable billions of dollars of benefits to the region.

As development proceeded, however, environmental assets were variously ignored, dismissed, discounted, or misunderstood. The greatest environmental asset was the once spectacular runs of salmon and steelhead. At the dawn of the 20th century, salmon runs numbered in the tens of millions of fish each year. At the dawn of the 21st century, the salmon are heading towards extinction, numbering less than two million each year.

As the region's power needs continue to grow while the salmon runs continue to decline, conflicts between the power community and the environmental community appear inevitable. These conflicts involve two significant and closely related issues. The first is how to operate the river. Storage and releases of water from reservoirs affect both the availability and timing of water to produce electric power as well as the survivability of migrating fish. Significant operational changes have been instituted to benefit fish.

The second issue is what new power resources should be developed to replace lost hydropower and to meet growing loads. Not surprisingly, power interests and environmental interests disagree on which resources are best. Power interests advocate resources that can be operated to complement the new river operations that have been instituted to improve fish survival. Environmental interests advocate power resources that are renewable and don't emit greenhouse gases, such as carbon dioxide, which contribute to the risks of climate change.

As will be shown in this paper, the renewable power resources advocated by environmental interests are not compatible with a hydro system that is operated for fish. Such resources could add billions of dollars to the costs of electric power in the Pacific Northwest while providing minimal environmental benefits in the form of reduced emissions of greenhouse gases.

A Look at the Columbia River System

The Columbia River rises in British Columbia, Canada and, after a journey of 1955 kilometres (km), empties into the Pacific Ocean in Oregon. Along the way it is joined by numerous tributaries, comprising a watershed of 670,000 km^2. Its average annual volume of runoff is 244,000 gigaliters (GL). (By comparison, the Murray River averages 11,000 GL annually.) The Columbia's value as a hydro-electric system results from that huge volume of water falling over an elevation of 795 meters along the way. The system provides 75 percent of the electric power generated in the Pacific Northwest. It has a peak

---

[2] The U. S. Pacific Northwest region consists of the states of Montana, Idaho, Washington, and Oregon.

[3] This paper focuses on conflicts with power uses. The other benefits mentioned here are also affected, although not as significantly. However, many of the same findings with regard to power would apply similarly to these other uses.

capability of 31,000 Megawatts (MW)[4] and supplies more than 150,000 gigawatt-hours (GWh) of energy annually.

There are 11 dams on the mainstem of the Columbia in the U.S., including Grand Coulee. Grand Coulee is the largest power producing plant in the U.S., with 6,700 MW of capacity, the equivalent of six large nuclear plants. In the entire Columbia Basin, there are more than 100 hydroelectric projects.

Three Competing Goals

Operating a large and complex river system while attempting to satisfy three competing goals--power system needs, fish needs, and minimising emissions of greenhouse gases-- means that tradeoffs are inevitable. Allocating the limited river resources among these competing objectives is a classic economic problem.

Power

Hydropower has one simple equation: Water equals power. Power system planning is based on a determination of how much firm energy the hydrosystem can generate. Firm energy is produced on a guaranteed basis, determined by the amount of energy that can be generated given the region's worst historical water conditions. In most years, however, precipitation and snow pack exceed historical lows so that the system produces a significant amount of secondary or nonfirm energy

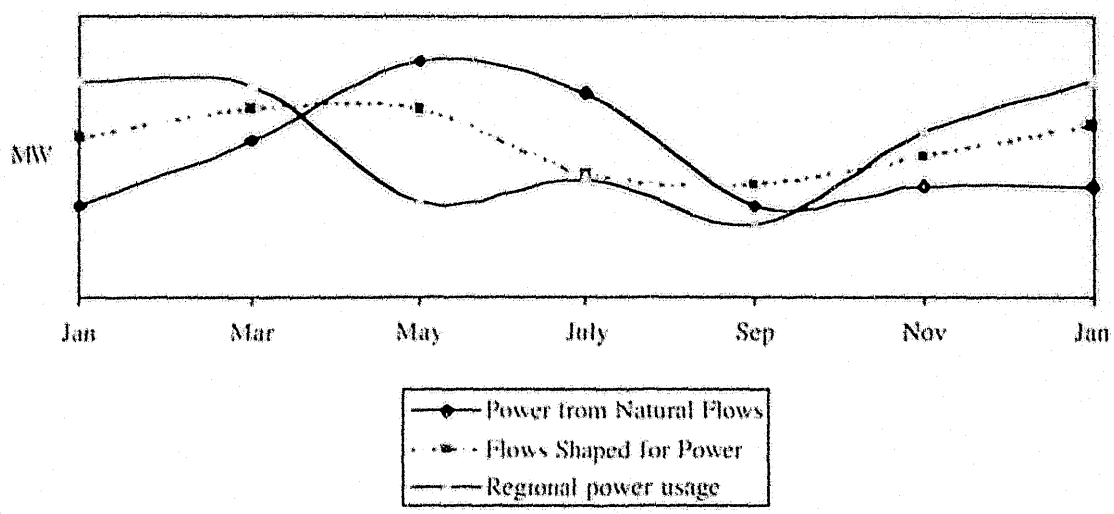
But stream flows in the region do not follow the same pattern as electric energy use. Consumers in the Pacific Northwest require more electricity in the winter than in summer to meet winter heating needs. The Columbia River, however, is driven by snow melt, with high runoff in the late spring and early summer. Natural flows are low in the fall and winter, when demand for power is high.

Figure 1 shows both the energy that the hydrosystem's natural flows would produce in a typical year, and the electric energy that regional consumers actually use over the same period.

Storage reservoirs are the key to matching the region's plentiful water resources with electricity use patterns. Energy, in the form of water, is held in reservoirs when natural stream flows exceed power generation requirements. Water is released for generation when it is needed to produce electricity. Altering flows to more closely match loads is called "shaping". Figure 1 shows the production of energy from flows shaped for power.

[4] Electric power is measured in Watts. Because the amounts are so large, it is common to use prefixes designating different amounts such as kilo (thousand) and Mega (million). The letters "k" and "M" respectively are used as abbreviations. Electric energy results from applying electric power over time. Thus, energy terms have a time dimension, typically hours or months, which are abbreviated "h" or "m", respectively.

FIGURE 1 - COLUMBIA RIVER POWER PRODUCTION



The ability to shape the river system is of considerable value because it means the region can meet loads more often, thereby reducing the amount of investment in additional generating resources that would otherwise be required.

The variation of river flows within a year is significant but the variation from year to year can be just as significant. For example, in the May-June period the highest flows ever observed were more than 34 megaliters (ML) per second, while the lowest were less than 3 ML per second. The amount of power the system can produce in a single year can vary between 11,000 MW and 20,000 MW depending on water availability. The difference is equivalent to the amount of energy produced by 10 large nuclear plants.

The nuclear and coal plants that serve the region are operated in a baseload manner, meeting a constant, stable load, 24 hours a day, week in and week out. Because these plants are not easily switched on and off, they are less flexible than the hydropower plants which can be ramped up and down quickly and easily to produce more or less power at any given time. Thus, hydro plants can follow ups and downs in demand very efficiently and are the key to meeting peak power loads.

Meeting Fish Needs

The Columbia River Basin is a world-renowned producer of salmon and steelhead. But the abundance of these fisheries is not what it used to be. Irrigation, timber harvesting, commercial fishing, mining, pollution, power production, flood control, and other factors have contributed to the decline of the basin's anadromous[5] fish population. In 1991 several species of salmon came under the protection of the United States' Endangered Species Act (ESA) which requires that, when a species of plant or animal is declared endangered, all parties whose actions could affect the survival of the species must do everything necessary to restore the species to a viable state. Because of the ESA, the region must now act to ensure, as far as biologically possible, the survival of those species.

[5] Anadromous fish species are born in freshwater and, as juveniles, migrate to saltwater. They reach adulthood at sea, and return to their freshwater birthplace to reproduce.

In recent years, efforts to restore the once magnificent fishery have focused on the dams. Blockage of the upstream passage of adult fish was a recognised problem even before the dams' construction began. To reduce this effect, fish ladders were built at most dams. Other dams, however, have blocked access to hundreds of miles of spawning and rearing areas. Grand Coulee dam, for example, is too high for fish ladders. When built in 1941, it permanently eliminated salmon and steelhead for 500 miles upstream to the river's source.

Dams also adversely affect juvenile fish in their downstream migration. Many fish are killed when they pass through turbines as they migrate downstream, or they may be stunned by the pressure drop across the turbines, making them easy prey for predators. In addition, the slower flows caused by the series of reservoirs may delay their arrival at the ocean, resulting in death.

Fish restoration programs have provided bypass facilities, primarily to aid downstream migration of juvenile fish, and led to the construction of more fish hatcheries, habitat improvements, and screening of irrigation diversions. The region also has changed the way the river is operated to better protect fish and wildlife. One major change is to spill water over the dams during most of the spring and summer juvenile migration to pass fish over the spillways instead of through the turbines.

Another significant modification is to increase river flow during the spring migration period. The increased flow helps "flush" fish down the river and reduces their exposure to predators and other hazards in reservoirs. Providing these flows means that power generation sometimes is reduced during the winter as water is stored in reservoirs to ensure that sufficient water is available in the spring.

All told, more than $2 billion has been spent over the past 10 years to restore the Columbia River's anadromous fish. But potentially the most costly actions are yet to come with the spills and flow modifications described above that are intended to improve downstream migration of juvenile fish. Costs result from several factors.

First, water that is stored during the fall and winter to provide higher flows during the spring migration is no longer available to meet power needs when those needs are the greatest. As a result, the region must acquire replacement power to meet winter needs either from developing new resources or purchasing on the open market.

Second, the tremendous amount of water released in the spring means that power production is at its greatest in the spring. However, needs are typically at their lowest then. Therefore, the combination of diminished demand for power and surplus[6] generation yields the lowest possible prices.

Finally, the spill program extends into the summer when power prices climb, but the water that is spilled generates no power and therefore no revenues. The combination of increased purchases, reduced revenues, and paying for fishery restoration programs costs up to $500 million each year and could go higher. Saving the region's valuable salmon and steelhead fisheries at a cost acceptable to the region's electric power consumers will continue to require a skilful and often contentious balancing act.

Meeting the Environmental Agenda--Reducing Greenhouse Gas Emissions

As the Pacific Northwest's population and associated economic activities continue to grow, so do its electric power needs. New power resources will continue to be developed to meet those needs. What resources should the region target for development?

Environmental groups have been strongly supportive of efforts to protect and restore the salmon and steelhead resources of the Pacific Northwest. Recognising that those efforts, by reducing the availability

[6] Throughout this paper the term "surplus" refers to a surplus of generation available compared to load demands. "Deficits" refer to a deficiency of generation available compared to load demands.

and timing of power from the hydroelectric system, may hasten the need for new generating resources, those same groups are actively trying to influence investments to meet growing power needs. At the same time, concern with the risks of global warming is rising, and that concern directly affects the kinds of new power resources advocated by the environmental community.

Global warming is linked to emissions of "greenhouse gases", most notably carbon dioxide (CO_2). Carbon is emitted whenever fossil fuels are burned. Coal-fired power plants emit the most carbon per unit of electric energy. Natural gas-fired plants emit only half as much as coal for the same amount of electric power, while renewable resources such as wind and geothermal emit no greenhouse gases.

The concern with emissions of carbon has led to a strong advocacy for renewables in the Pacific Northwest.[7] The environmental community believes that the region should meet any new generation needs with combinations of the various renewable resources. The renewables are viewed as being environmentally benign as well as having the virtue of producing no greenhouse gases.

Evaluated over their lifetime, renewable resources cost between 4¢ and 5¢ per kWh.[8] This compares to new gas-fired generation costing between 2.0¢ and 3¢ per kWh. Power purchases[9] are even cheaper--about 2¢ per kWh.

The proponents of renewables argue that if all costs such as pollution, consumption of irreplaceable assets, and emissions of greenhouse gases are taken into account, the renewables are actually cheaper. Most studies of power generation technologies do indicate somewhat higher environmental costs associated with non-renewables, but not by an amount anywhere near sufficient to overcome the direct financial cost advantage of the conventional resource types.[10]

Moreover--and this is the crux of the problem for renewables in the Pacific Northwest--the reduced flexibility of the power system resulting from operating the river to meet fish needs, and the variability and unpredictability of the water availability in the system mean that flows can no longer be shaped to meet loads. Operational changes to enhance the fishery resource conflict with the environmental community's goal of reducing carbon emissions from new resources.

The large fish flow requirements in the spring, including spill, have limited the region's ability to draft reservoirs in the fall and winter months to meet loads. Just since 1991, nearly 8,000 MW-mo. from September through March have been lost--an amount of power equivalent to a large nuclear power plant operating around the clock over that period. In the worst water years, losses of more than 4,000 MW-mo. in a single month are possible. Conversely, in an average year an additional 3,200 MW-mo. of surplus power is available during the spring months.

In the past, a net loss of 4,800 MW-mo. of energy (the 8,000 MW loss less the 3,200 MW gain) would have meant adding 400 MW of year-round energy. (Applying 400 MW over 12 months would yield

---

[7] Interestingly, although hydropower is also a renewable resource--and in most parts of the world is included in any categorisation of "renewables"--in the Pacific Northwest, the term "renewables" is used almost exclusively for wind, solar, and geothermal power. Hydropower is viewed with disfavour by environmental activists because of its linkage with the demise of salmon and steelhead.

[8] All prices cited in this paper are in US dollars.

[9] Power purchases consist of power bought and sold on the open market. That power may actually be produced by any kind of resource, and much of it is produced by coal or nuclear power plants. However, at the margin, the bulk of such purchases are produced by gas-fired generation. Purchase costs typically reflect only variable operating costs of power generation.

[10] For estimates of the environmental costs of various generating resources, see Pace University, Environmental Costs of Electricity, Oceana Publications, New York, 1990.

4,800 MW-mo.) The flexibility of the system would have allowed that much to be shaped to meet the same need. But that flexibility is gone.

In the constrained system, resources with high fixed costs cannot economically meet the new shaping requirements necessitated by the change in system operations. Under most water conditions, adequate nonfirm hydropower is available to displace other forms of generation. Resources with proportionally high operating costs and low fixed costs are best suited for this purpose. This is because the operating costs can be avoided, or displaced, when the resources are not needed whereas the fixed costs can not. So, during times of excess hydropower availability, displaceable resources are much more economic. High fixed cost, low variable cost resources do not provide significant savings when displaced.

Purchases consist almost entirely of variable costs while about two-thirds of the cost of new gas-fired resources are variable costs. Such resources fit well with the new operating conditions of the hydrosystem.

Renewables are, without exception, proportionally high fixed cost, low variable cost resources, so there is no economic benefit associated with their displacement. Consequently, additional renewables are the least preferable new resources, from an economic point of view. Therefore, the operational changes necessitated by fish restoration measures are inconsistent with another important environmental goal, the development of renewable generating resources.

The Consequences of Resource Choices

The U.S. west coast power system is a highly interconnected system which, while not operated by a single entity, is nevertheless subject to economic dispatch. This means that the lowest cost generating resources are run, or dispatched, first, and, as loads require, increasingly more expensive resources are dispatched. Resources having relatively higher variable costs than fixed costs are operated to meet swings in loads.

There are three major types of generating resources in the western system: nuclear, coal, and gas-fired. Nuclear and coal plants are run as base load plants; that is, they are run around the clock, except during periods of maintenance or during forced, unplanned outages. Typically, they are not dispatched for economic reasons. Displaceable gas-fired plants are operated during the day but shut down at night when loads drop off, and shut down entirely for months at a time when loads or prices drop significantly.

The interconnectedness of the entire system means that a robust wholesale power market exists. The region buys and sells surplus power from as far north as Canada, as far south as Mexico, and as far east as Texas. Therefore, conditions in the Pacific Northwest affect and are affected by conditions throughout the western U.S. The existence of a large, competitive wholesale power market means there is a ready supply of energy to meet regional needs when necessary and a ready market to sell energy which is surplus to needs.

Consider the changes in river operations just since 1991 which are equivalent to an annual loss of 400 MW of energy. As discussed, however, the annual average is misleading: in some months the loss is over 2000 MW, while in some months the system gains over 1500 MW. Let's compare an alternative of adding renewable resources to meet that overall loss to the alternative the region actually plans to follow.

Acquire 400 MW of Renewables

Acquiring 400 MW of renewable generating resources would reduce deficits by 400 MW in each month that deficits occur while adding 400 MW to surpluses in other months. In 2 months, this would just about exactly offset the average deficit. In most months when the system is deficit, however, it is deficit by more than 1000 MW, so the 400 MW addition of renewables would not eliminate the overall deficit.

What would be the economic and environmental effects? The 400 MW would cost about $175 million per year.[11] Over a 20-year planning period, this amounts to about $1.7 billion present value. If the region chose to meet all deficits with renewables, it would take an annual investment of nearly $900 million, an $8.5 billion present value.

The additional costs would be incurred regardless of flow levels on the hydropower system. During periods of surplus power availability, as happens nearly every spring, the renewables' costs could not be avoided. Nor would the large deficits be reduced significantly during the fall and winter months. Therefore, it isn't just the higher life-cycle costs of renewables that make them economically unattractive. If a greater portion of their costs were variable, rather than fixed, then at least some of their costs could be avoided.

Virtually all of the resources that would be displaced by the renewables would be gas fired. Using plausible assumptions about the efficiency of those plants and the CO_2 emissions that would thereby be avoided, a total of 1.7 million tons of CO_2 would be reduced annually. In addition, there would be a small (2,500 tons per year) reduction in nitrogen oxide emissions, the only other significant pollutant associated with natural gas. These are clear environmental benefits.

Actual Regional Strategy--Rely on Purchases

Because of the robust and reliable purchase power market which exists now, and is likely to exist for several more years, at least, the region intends to rely on purchases to meet deficits in those months when its own resources are insufficient to meet loads. Purchases cost an average of about 2¢ per kWh. These costs can be avoided entirely when sufficient power is available from the region's own resources; for example, during good water years.

Under this strategy, the region can expect to pay an additional $70 million each year to make up for the losses resulting from hydropower operational changes. This compares to the $175 million annual cost of the renewables strategy.

There are two significant drawbacks to this strategy. First, the $70 million annual cost is an average. Because of the significant variations in water availability from year to year, costs will vary widely. This means that power rates may also vary widely.

Second, this strategy will clearly result in more emissions--an additional 1.7 million tons of CO_2 annually. As concerns about the risks of climate change are growing, any increase in emissions of CO_2 must be taken seriously.

Discussion of Resource Choices

Purchased power, and even new gas-fired resources, can be displaced when not needed, thus saving significant economic costs. Renewables, however, can not be displaced economically. This, combined with much higher life-cycle costs, means that renewables are poor additions to the power system. From the power system's point of view, purchases would be the most attractive additions, followed by new gas-fired generators.

Renewables do produce significant environmental benefits, but those benefits come at a high cost. First, emissions of nitrogen oxides from new gas-fired plants are relatively easy and cheap to control. Those controls add only a few percent to the cost of a new plant. (The 2Ω¢ to 3¢ per kWh cost for new gas-fired generators cited previously includes the cost of advanced nitrogen oxide controls.)

[11] See Appendix A for additional details on economic calculations in this section.

More importantly, the CO_2 emissions reductions achievable by investing in new renewable resources would be extremely costly. The difference in costs between the two alternatives is $105 million per year. Averaging that cost over the total emissions reductions from the renewables alternative yields a cost of emissions reductions of about $60 per ton. This compares to standard cost estimates for CO_2 ranging from less than $1 per ton, up to $20 per ton.[12] Thus, as a strategy for reducing CO_2 emissions, the renewables strategy is extremely costly. Were the region to embark on a tree planting program instead, it could achieve similar reductions in CO_2 for about 5 - 10% of the renewables cost.

Conclusion

It is ironic that the imposition of stringent river operation requirements to benefit one environmental resource, the precious salmon and steelhead fisheries of the Pacific Northwest, makes it more difficult to achieve another significant environmental benefit, the reduction in emissions of CO_2.

Renewables are disadvantaged, generally, by their higher costs. But their high fixed costs compared to their variable costs further disadvantage them in the Pacific Northwest where the hydrosystem puts a premium on flexibility of generating resources. The tremendous variability of the system means that those resources whose costs can be avoided when water is plentiful are more valuable.

The region now spends about $500 million per year in its effort to save the salmon and steelhead from extinction. This is not a trivial sum. Proponents of adding an additional several hundred million dollars per year to the region's power costs to induce a shift to renewables face an uphill battle. This is especially true when the western U.S. is awash in low cost power and is expected to be so for the next several years.

With abundant purchased power available, connections to the rest of the west, tremendous amounts of water available in some years, and huge supplies of natural gas known to be available, it appears likely that the region will continue its efforts to save the fish. It appears equally unlikely that it will embark on costly new efforts to reduce CO_2 emissions by making significant investments in renewable generating resources.

[12] These costs are based on the costs of planting trees to sequester carbon, a fairly standard costing technique. These costs are cited in Pace, id. at pp. 127 - 191.

Appendix A

Derivation of Economic Costs of Renewables and Carbon Reductions

Renewables Cost

400 MW * 8,760 hours/year * 1000 kW/MW * $0.05/kWh = $175,200,000/year

$1.00/year, 20 Years, 8% Interest = Present worth factor of 9.82.

$175,200,000 * 9.82 = $1,720,000,000

$175,200,000/year/400 MW * 2000 MW = $876,000,000/year

$876,000,000 * 9.82 = $8,600,000,000

Purchase Power Cost

400 MW * 8,760 hours/year * 1000 kW/MW * $0.02/kWh = $70,080,000/year

$70,080,000 * 9.82 = $688,200,000

Carbon Emissions from Natural Gas-Fired Generation

117 lbs. CO_2/MMBtu *.0085 MMBtu1/kWh * 400 MW * 8,760 hours/year *

1000 kW/MW * 1 ton/2000 lbs. = 1,742,000 tons CO_2/year

Annual Cost of Carbon Reductions

($175,000,000/year - $70,000,000/year) = $105,000,000/year

$105,000,000/1,700,000 tons CO_2 = $61.76/ton CO_2

From equation 8, the rate of change in the value of an additional unit of water at dam i ($\dot{\lambda}_i$) is a declining function. The volume of water held at the dam, as an increase in the electricity generated at dam i is associated with a reduction in the effective hydraulic head and the volume of water held at that dam.

Solving for λ in 7, differentiating λ with respect to time and equating with terms in equation 8 gives:

$$\frac{\partial F}{\partial s} = \left(\mathbf{A}^T\right)^{-1} \frac{\partial^2 F}{\partial \mathbf{v} \partial t} \tag{10}$$

From equation 9:

$$\mathbf{A}^T \frac{\partial F}{\partial s} = e^{-rt}\left\{\frac{1}{2} a \mathbf{A}^T \mathbf{s}^{-1/2} \alpha \mathbf{v} - b \mathbf{A}^T \mathbf{s}^{-1/2} \alpha \mathbf{v} \mathbf{v}^T \left(\alpha \mathbf{s}^{1/2} \mathbf{1} - \delta\right)\right\} \tag{11}$$

Combining common terms in equations 10 and 11 gives a matrix equation composed of a constant, terms linear in v and quadratic terms in v:

$$\begin{aligned} 0 &= e^{-rt}\left(\frac{\partial^2 F}{\partial \mathbf{v} \partial t} - \mathbf{A}^T \frac{\partial F}{\partial s}\right) \\ &= \frac{1}{2} a \mathbf{G}\mathbf{x} + \left(\dot{a} - ra\right)\mathbf{Q} + \frac{1}{2} a\left(\mathbf{G}\mathbf{A} - \mathbf{A}^T \mathbf{G}\right)\mathbf{v} - (b\mathbf{G}\mathbf{x} - 2br\mathbf{Q})\mathbf{Q}^T \mathbf{v} - b\mathbf{Q}\mathbf{x}^T \mathbf{G}\mathbf{v} \\ &\quad + b\left(\mathbf{A}^T \mathbf{G} - \mathbf{G}\mathbf{A}\right)\mathbf{v}\mathbf{v}^T \mathbf{Q} - b\mathbf{Q}\mathbf{v}^T \mathbf{G}\mathbf{A}\mathbf{v} \end{aligned} \tag{12}$$

where:

$$\mathbf{G} = \alpha \mathbf{S}^{-1/2} = diag\left(\alpha_1 S_1^{-1/2}, \ldots\ldots, \alpha_n S_n^{-1/2}\right)$$
$$\mathbf{Q} = \alpha \mathbf{S}^{1/2} \mathbf{1} - \delta = \left(\alpha_1 S_1^{1/2}, 1 - \delta_1, \ldots\ldots, \alpha_n S_n^{1/2} \mathbf{1} - \delta_n\right)$$

which solved for v by numerical approximation. The algorithm used here was a globally convergent Newton's method adapted from Press, Tevkolsky, Vetterling and Flannery 1992.

Equation 12 is an interior solution which may not meet the inequality or boundary constraints at each dam (maximum and minimum flow rates and storage volumes). These constraints are met if: outflow rates are less than hydraulic capacity and do not draw storage levels below minimum operating levels; and; forced outflows, when storage capacity is exceeded, are used for power generation before storage at any other dam is drawn down for power generation. To determine the interior and boundary components of the solution, the following algorithm is implemented. First, an interior solution is computed for all dams. Second, flow rates which are above maximum levels are exogenised at their maximum and a new interior