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MATHEMATICAL PROGRAMMING MODELS EMPLOYED IN THE ANALYSIS OF THE COMMON AGRICULTURAL POLICY

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Abstract

The goal of this paper is to provide information about the progress, over the last 10 years, of mathematical programming for analysis of the behaviour of farm producers under the European Union Common Agricultural Policy. We show in detail the evolution of mathematical programming and how this methodology can be applied at different levels (regions or sectors) for the main CAP policies. The evolution of mathematical programming is shown by the emergence of a new group of supply and equilibrium models, "Positive Mathematical Programming" (PMP) and "Symmetric Positive Equilibrium Problems". (SPEP) SPEP was developed in order to solve problems posed by the lack of data and the need for methodologies to represent different farm typologies in different environments. This paper thus analyses the main contributions to current developments in mathematical programming, different models used in Europe and how the main CAP tools are included in these models.

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1. Introduction

The aim of this work is to describe the main features and potential uses of a group of models which simulate the behaviour of farm producers through mathematical programming in order to estimate the main effects of changes in the EU Common Agricultural Policy.

Until recently mathematical programming models have mainly been used to help producers optimise the management of their farms. Here, the same methodology is applied to homogeneous sector or area groups of farmers with the aim of supplying policy makers with information on the possible effects of applying specific agriculture policies in clearly defined areas or sectors.

The renewed interest in mathematical programming for analysing agriculture policy has several reasons. Not only is the CAP evolving, but there is also an increase in the information available that can help meet the construction requirements of models. Another crucial factor is the theoretical evolution of mathematical programming in recent years. This has moved from traditional linear and quadratic programming to Positive Mathematical Programming (PMP) and Symmetric Positive Equilibrium Problem (SPEP). PMP and SPEP aim at analysing explicitly the effects of agriculture policies at regional or sector levels using information sets that were considered insufficient for earlier methodologies.

These instruments are extremely useful in that they are much better suited than other methodologies to supplying information for the planning of regional and national agricultural policies. They give policy makers real analytical tools for drawing up the regional planning documents that are increasingly required by central government, such as the Rural Development Plans which came out of Agenda 2000. Unlike other methodologies t MP can can enable the construction of models yielding economic production and financial information at regional or lower levels, thus facilitating the assessment of the efficacy of EU regulations.

This section looks at models selected on the basis of criteria of methodological approach and the intervention policies analysed. Unlike other models described elsewhere in this volume, the MP models in this section do not have a distinctive “name”, while the specific methodology does. Taking into account the chronology of development of this category of models, we shall first describe Linear Programming (LP) models and the basic methodological principles underlying them. This is followed by a discussion of more recent and complex models based on Howitt and Paris’s PMP and the same authors’ SPEP. There is a discussion of the main applications of MP in analysing the effects of the CAP and the main problems involved.

As the models share various elements as regards basic hypotheses, internal architecture, functional forms, data typology and parameter construction , the second section contains a brief introduction to the main elements of MP. The third section presents individual models with a focus on different methodologies and issues in agricultural policy. This aspect is discussed in more detail in section four, which looks at the interpretative efficacy of different models’ treatment of individual agricultural policy measures. As with the other chapters in this volume, the investigative framework focuses on the following policy areas: price support, trade policies, output restrictions, partially decoupled measures and voluntary policies. The conclusion contains a number of general reflections on the methodologies proposed.

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2 Common characteristics of MP models

2.1 Basic theory underlying LP models

Linear programming was one of the main instruments used to analyse the effects of agricultural policy measures applied at the farm level, in that it was used in agriculture as a support instrument for farming choices aimed at identifying the "optimum" production combination to maximise entrepreneur income given existing policies[1]. Linear programming started and is used as a microeconomic instrument based on the principle of maximisation with constraints. The following definition is given by De Benedictis and Cosentino (1979) "*Linear programming is theory and methods for solving problems of maximisation (or minimisation) of a linear mathematical function (the objective function) subordinated to a certain number of constraints which are also linear*"[2].

Linear programming models[3] are thus based on the theory of fixed output and coefficients, postulating a finite number of techniques to obtain a given good. It is assumed both that only a limited number of goods can be produced, and also that the number of potential techniques which can be used to produce them is also limited. This hypothesis is justified by the fact that the level of knowledge of the development in production techniques is frequently insufficient, and in most cases is limited to the assessment of production corresponding to the quantity of factors used in each process. In practice, it is postulated that the entrepreneur or entrepreneurs in the model have perfect knowledge of the relation between production factors, the amount of output obtained and the level of unit costs of each production activity and future market prices.

Fixed coefficient production theory of course postulates that there is a proportional relation between production factors and goods and / or services obtained in each process, which is expressed in technical coefficients. These characterise the individual production processes since they express the quantity of each factor needed to obtain product units and show the technology available to the farm. This technological framework is called the technology matrix.

Thus, the technical matrix includes all the technical possibilities subject to constraints on the entrepreneur in producing a good or goods, expressed in linear equations composed of matrices. The technical coefficients inserted into the matrix equations clearly show the link between the factor or factors required and the carrying out of a single process or processes. Each production factor can be used in the process until it is exhausted, which constitutes the constraint on carrying out the process.

In practice, the structure of linear programming models consists of a row vector (c') representing the coefficients of the objective function to maximise (or minimise) the technical matrix (A), a column vector called the constraints vector (b) which defines the limiting factors and yields the same number of relationships of equality (=) or inequality ($\geq$, $\leq$) as the equations of the problem, showing equilibrium or disequilibrium between the requirement for each factor and its availability or production capacity. The unknown variables (x) are the quantity of goods and factors to put onto the market or to purchase respectively.

The above notation allows us to formulate a generic problem of (primary) maximisation as follows:

Maximise	$Z = c'x$
Subject to	$Ax<b$
with	$x>0$

[1] More information is found in Giacomini and Arfini (1993)

[2] In M. De Benedictis, V. Cosentino (1979), p 638.

[3] For a discussion of the theory and applications of linear programming see specific manuals such as: Panattoni, Campus, (1969), Cosentino, De Benedictis (1979), Chiang (1984), Hazell, Norton (1986), Paris (1991).

In the same way, the expressed problem as a problem of (primary) minimisation is formulated as follows:

Maximise	$Z =$	$c' x$
Subject to		$Ax \le b$
with		$x \ge 0$

where Z and C are the objective functions corresponding to the total earnings and total cost respectively.

Both these problems are defined as primary in that they aim to solve the entrepreneur's first problem: identifying the unknown quantity (x) of goods and factors that maximise (or minimise) a profit or cost function. There is a symmetric problem able to analyse the 'hidden'- less obvious-part of the economic problem in a primary approach as follows. Once the quantities of factors available are known, the aim of the dual problem is to identify the cost of limiting factors in order to minimise the overall cost for the farm of maintaining economic equilibrium. The meaning of this constraint is that unit cost is higher than or equal to the market price. The values expressing unit cost of processes are usually indicated by (y) and represent (dual) variables of the dual problem, unlike x which represents the variable of the primary problem. The dual variables of the dual problem also yield indications on increases of income if availability of the limiting factor increases by one unit. For this reason the dual variables are also called *shadow prices* of thelimiting factors[4] . As we shall show in subsequent sections, they give useful information for the measuring of several parameters, which are difficult to fix and which depend closely on the farm's production strategy and its geographical location, like the production costs of processes and the marginal cost of limiting factors such as milk quotas and land.

Although formulating LP problems is relatively simple, it is possible to formulae fairly complex problems through a suitable use of binding equations linking the requirement for factors to their availability. Moreover, as a direct result of the formulation's simplicity , it is possible to refer the same type of analysis to an individual farm or a group of farms aggregated on the basis of a geographical area or on the basis of their techno-economic orientation (Farm Type).

What changes in the model is not its structure, but its size, the number of activities considered and the number of equations necessary to describe the relationships between each activity and production factors. The most important change, however, is the capacity of technical coefficients (matrix A) and economic coefficients (vector c') to show the group of farms the model describes effectively.

In this work it is important to emphasise that the difficulty in the construction of LP models for the analysis of agricultural policy is not the representation of relations between processes or between activities by the model itself. The difficulty is rather the availability of information and data which can give as true a picture as possible of the strategy of the group of "representative" entrepreneurs. It needs to be pointed out that farm enterprises even within the same region have very different structures, outputs and specialisation levels which means there are extremely different types of farm. Widening the area of action of a model naturally makes it more difficult to show the technologies used in each type of farm correctly and thus the relative costs of production factors.

[4] It is important to note that the term 'dual' in MP (dual problem and dual solution) is used differently from in econometrics, as will be clear from the chapters written by Paolo Sckokai.

There are specialised data banks like FADN/RICA[5] for variable costs, but for technical coefficients, which cannot be directly measured, the relation between factors used and products obtained needs to be estimated with suitable econometric models, with the help of experts or through the use of scientific literature.

These last aspects become more important when we move from LP models of farm management to models analysing agricultural policy. This is because possible mistakes in measuring technical and economic coefficients, especially on the cost side, compromise the model's capacity to represent the group of farms in the statistical sample, and consequently assess agricultural policies accurately.

2.2 From Linear Programming to PMP

In order to take account of the problems of giving the LP model greater capacity to represent the phenomenon observed for homogenous groups of farms, MP and particularly LMP was developed theoretically and methodologically to ensure greater analytical capacity for agricultural policy problems.

There was a move away from *normative* LP models showing the optimum production combination the farm producer could follow, if the starting situation was not binding for production choices towards *positive* models. Here the main aim is to give as true a picture as possible of the situation and then simulate the behaviour of farm producers as parameters in which the object of agricultural policy intervention is shifted.

This work was started by Heady (1964, 1978) and Howitt (1995) and continued by Paris and Arfini (1995) and Paris and Howitt (1998). It has given rise to a new typology of Positive Mathematical Programming models stimulated by issues in EU agricultural policy. More recently Howitt and Paris (2000) have further developed the original idea of PMP into a new Symmetric Positive Equilibrium Problem model (SPEP). These two models are very important in analysing agricultural policy and are described below.

2.2.1 Why Positive Mathematical Programming?

Briefly, since the early applications of LP models to an individual farm or aggregate groups of farms for the analysis of agricultural policy, researchers have had to face the problem of excessive specialisation and the need to make models reproduce as accurately as possible observed reality. For this reason, some researchers (Heady et al, 1978) increased the set of structural constraints (**Ax £ b**) in their LP models with a another set of constraints on possible levels of activity, using the level realised and observed, called $\mathbf{x_R}$ as a constraint. The structure of this restriction can conveniently be shown as **x £** $\mathbf{x_R}$, indicating that the amounts produced (**x**) should be lower or equal to those realised output ($\mathbf{x_R}$) thus obliging the model to reflect farmers decisions faithfully.

[5] The accounting information network (RICA) or Farm Accountancy Data Network (FADN) was created as an instrument to provide more information to support the Commission's proposals concerning prices and, at the same time, to observe profitability trends at farm level under the CAP measures.
The purposes of the FADN are laid down clearly in theirrules (EEC Regulations 79/65, art.1): " The agricultural accounting information network is created for the purposes of the common agricultural policy and, in particular, for: a) the annual verification of incomes of agricultural holdings of a well-defined observation field; b) the analysis of the economic operation of agricultural holdings." FADN collect data at farm level across all the EU country.

The basic idea of Positive Mathematical Programming is that it is easier to collect information on output of a farm ($\mathbf{x_R}$) than information on production costs, and to use this information to construct models that can correctly represent the entrepreneur's observed behaviour. In other words, output levels are the result of complex decisions mainly based on a total cost function known only to the entrepreneur, which is difficult to measure externally. Observing the production function can, however, lead to an estimate of the cost function because this function can be considered the dual function of the previous one. On the other hand, constructing models for the analysis of agricultural policy for homogenous groups of farms rather than individual farms leads researchers to use data banks of agricultural accounts such as RICA, which are unlikely to contain all necessary information or the quantitative or qualitative construction requirements of the model.

PMP methodology aims to use these elements taking as a working hypothesis the dual theory of MP models and the theory of production costs which shows them in a semi-definite positive symmetrical matrix.

Before looking in detail at the early phases of PMP, we need once more to consider one very important aspect for the analysis of agricultural policy; the relation between the structure of the model and representation of the sample. Two possible solutions could be to use an 'average farm' as suggested by Hazell and Norton (1986) or to use a 'frontier' approach as suggested by Paris and Arfini (2000).

This approach recognises that even where farms are homogeneous as regards techno-economic orientation (FT), not all individual entrepreneurs activate the same processes but rather choose the set of crops for their farm on the basis of precise elements of cost in a process of "*self-selection*". Consequently any appropriate methodology for constructing a model reflecting the behaviour of all entrepreneurs in the homogeneous sample needs to leave the entrepreneur free not to activate a certain process, even if it is present in the region, by reproducing the self-selecting behaviour of the farm producer. As is shown below, this result can be obtained by using a frontier cost factor compared to the farms in a homogenous sample to compare the function with the cost level of the individual crops of each farm in the sample.

2.2.2 PMP methodology

PMP methodology comprises three stages. The first stage is defined by N LP models, one for each farm in the sample, and by a supplementary LP model for the whole sample. The n-th model for the n-th farm $n = 1, \ldots, N$ uses all the available information to find the shadow price vector for limiting production factors that can be allocated, such as land, **y**, and the differential marginal cost vector corresponding to the vector of output levels constructed for each activity, $\boldsymbol{\lambda}$. The LP model of the n-th farm has the following structure:

(1) $$\max\ (\mathbf{p}'_n \mathbf{x}_n - \mathbf{c}'_n \mathbf{x}_n)$$

subject to

(2) $$\mathbf{A}_n \mathbf{x}_n \leq \mathbf{b}_n$$

(3) $$x_{nj} \leq x_{Rnj}\,, \quad \text{per} \quad x_{Rnj} > 0\,, \quad j = 1, \ldots, J_n$$

$$x_n \geq 0$$

where $\boldsymbol{p}_n$ is the vector of prices imposed by the n-th farm, $\boldsymbol{c}_n$ its vector of "*accounting*"[6] variable costs per unit of output from the data bank, $\boldsymbol{A}_n$ the matrix of technical coefficients fixed relative to limiting production factors, $\boldsymbol{b}_n$ is the vector of constraints relating to the availability of limiting production factors, and $\boldsymbol{x}_{Rn}$ is the vector of actual output levels. The vector $\boldsymbol{x}_n$ is obviously non-negative. Each farm presents *I* limiting production factors and J_n products. The vector on the use of the factor of land for each individual process is shown by $\boldsymbol{h}_{Rn}$. In this model the land is the only limiting production factor and the n-th matrix of technical coefficients is defined as $\boldsymbol{A}_n = [a_{nij}]$, where $a_{nij} = h_{Rni} / x_{Rnj_n}$

The model has a very simple structure as there are none of the constraints present in 'classical' LP models applied to farms described in manuals, such as the constraints of crop rotation, output, sales etc.. The model has two other types of constraint; "structural" (2) and "calibration" (3). The first is to respect the overall availability of the land factor for the farm, and the second is to respect the production choices made by the farmer in terms of output quantities. Each restriction is associated with a corresponding shadow price (or dual value): the constraints of the allocable production factor (2) are associated with the shadow price vector $\boldsymbol{y}_n$, and the calibration vector (3) is associated with the vector of differential marginal costs $\boldsymbol{\lambda}_n$.

The dual of the model (1) - (3) thus appears as:

(4) $$\min\ (\boldsymbol{b}'_n \boldsymbol{y}_n + \boldsymbol{\lambda}'_n \boldsymbol{x}_{Rn})$$

subject to

(5) $$\boldsymbol{A}'_n \boldsymbol{y}_n + \boldsymbol{\lambda}_n + \boldsymbol{c}_n \geq \boldsymbol{p}_n$$

$$y \geq 0,\ \lambda \geq 0$$

where vectors $\boldsymbol{y}_n$ and $\boldsymbol{\lambda}_n$ are non-negative [7].

The supplementary LP model (*N*+1) for the whole sample is defined by taking into account all the resources of the farms in the sample and the production activities carried out, as the information related to a single large farm where the availability of land of all the farms is added together in vector $\bar{\boldsymbol{b}}$, the output levels are added together in vector $\bar{\boldsymbol{x}}_R$, the prices and costs are calculated as averages, restricted of course to farms with the same crops. The matrix of technical coefficients for the whole sample is defined by the ratio of total hectares for each process divided by the total quantity of output of that crop. For the sake of clarity and completeness, the primary LP model for the whole FT sample is as follows:

(6) $$\max\ (\boldsymbol{p}'\,\boldsymbol{x} - \boldsymbol{c}'\,\boldsymbol{x})$$

subject to

(7) $$\boldsymbol{A}\boldsymbol{x} \leq \bar{\boldsymbol{b}}$$

(8) $$\boldsymbol{x} \leq \bar{\boldsymbol{x}}_R$$

$$\boldsymbol{x} \geq \boldsymbol{0}$$

Where the primary variable (*x*) is non-negative, the dual specification of the model is formulated as follows

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[6] These costs are only a part of the total variable costs, and are considered specific variable costs.

[7] For a detailed interpretation of the two models, dual and primary, see Paris and Howitt (1998: p 126-127)

(9) $$\min_{y\geq 0,\lambda\geq 0} = (\bar{\boldsymbol{b}}'\bar{\boldsymbol{y}} + \bar{\boldsymbol{\lambda}}'\bar{\boldsymbol{x}}_R)$$

subject to

(10) $$\boldsymbol{A}'\bar{\boldsymbol{y}} + \bar{\boldsymbol{\lambda}} + \bar{\boldsymbol{c}} \geq \bar{\boldsymbol{p}}$$

$$\boldsymbol{y} \geq \boldsymbol{0},\ \boldsymbol{\lambda} \geq \boldsymbol{0}$$

where the dual variables are non-negative.

It is important to remember that the only objective at this first stage of PMP methodology is to obtain a coherent and precise measurement of the marginal cost associated with the vector of the output level $\boldsymbol{x}_{Rn}$ for each activity. From the above specification of primary and dual LP models the vector of the marginal cost of the n-th farm is given by $(\boldsymbol{\lambda}_n + \boldsymbol{c}_n)$, while the marginal cost vector for the whole sample is $(\bar{\boldsymbol{\lambda}} + \bar{\boldsymbol{c}})$.

The second stage of the PMP approach sees the construction of the marginal cost function using linear specification of the parameters, an aspect that becomes important when there is a high number of farms. The supplementation of the marginal cost function into the variables in the model (shown by the quantities of output) included within the admissible domain yields the total variable cost function for the whole sample. Our hypothesis is that the cost function presents a quadratic functional form for quantities[8] $\boldsymbol{C(x) = x'Qx\ /2}$, where the matrix $\boldsymbol{Q}$ is symmetrical, positive and semi-defined.

Given the structure of the LP model described above, the marginal cost function associated with the FT can be shown as $mc(\boldsymbol{x}) \equiv \bar{\boldsymbol{\lambda}}_{LP} + \bar{\boldsymbol{c}} = \boldsymbol{Q}\bar{\boldsymbol{x}}_R$.

As the cost function is a frontier function for the FT as a whole, the cost function for each individual farm is expressed by a non-negative deviation. So the marginal cost of the n-th farm is shown by $mc(\boldsymbol{x}_n) \equiv \boldsymbol{\lambda}_{LPn} + \boldsymbol{c}_n = \boldsymbol{Q}\boldsymbol{x}_{Rn} + \boldsymbol{u}_n$, where the non-negative vector $\boldsymbol{u}_n$ takes on the role of indexing the FT cost function with the specific characteristics of the n-th farm. With this specification the cost for any farm of producing a given level of output for any activity is not lower than the same cost for the FT.

As was mentioned above, not all the farms in the sample choose to cultivate every crop present in the region. In order to take into account the self-selection process, the marginal costs of each farm need to be further refined in order to distinguish between activities actually carried out by the farm and those not. This distinction can be made by formulating two sets of constraints for the n-th farm. The first group of constraints concerns crops cultivated in which the relation of the marginal cost can be described as follows:

(11) $$\mathrm{cm}_{nk}|\ x_{Rk} > 0: \quad \lambda_{nk} + c_{nk} = Q_k \boldsymbol{x}_{Rn} + u_{nk},\ \text{if activity } k \text{ - is carried out},\ k = 1,\ldots,J_n.$$

The second group of constraints concerns the activities not carried out by the n-th farm. In this case the relation of marginal cost is a weak inequality compared to the level of FT marginal cost for that output:

(12) $$\mathrm{cm}_{nk}|\ x_{Rk} = 0: \quad \bar{\lambda}_k + \bar{c}_k \leq Q_k \boldsymbol{x}_{Rn} + u_{nk},\ \text{if activity } k \text{ is not carried out},\ k = 1,\ldots,J - J_n.$$

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[8] Input prices are not available and are assumed to be fixed.

Lastly, for the requisite of $\boldsymbol{Q}$ matrix of the quadratic cost function to be positive, the symmetrical and semi-defined is met using Cholesky factorisation

(13) $$\boldsymbol{Q} = \boldsymbol{LDL}'$$

where $\boldsymbol{L}$ is the lower triangular unit of the matrix, $\boldsymbol{L}'$ is its transposition and $\boldsymbol{D}$ is a diagonal matrix in which the elements are non-negative. This last aspect is particularly important in that it is possible to show that ***LDL'*** is a semi-defined positive matrix if and only if all the diagonal elements of $\boldsymbol{D}$ are non-negative.

Two different approaches have been taken for construction of the marginal cost function. The first approach uses minimum least squares and the second uses the principal of maximum entropy[9] . Both approaches allow matrix $\boldsymbol{Q}$ to be estimated and its cost function constructed, but they lead to different results applied to diverse agricultural policy problems (Paris , Arfini, 2000).

When estimates $\boldsymbol{Q}$ and $\boldsymbol{u}_n$ are used, obtained through least squares or maximum entropy, the third stage of PMP is to assemble a non-linear model using the variable estimated variable cost function and able to reproduce the primary and dual solutions of the first stage LP models. There are N models of quadratic programming, $n = 1, \ldots, N$, one for each farm of the sample:

(14) $$\max_{\boldsymbol{x}_n \geq 0} (\boldsymbol{p}_n'\boldsymbol{x}_n - \boldsymbol{x}_n'\hat{\boldsymbol{Q}}\boldsymbol{x}_n / 2 - \hat{\boldsymbol{u}}_n\boldsymbol{x}_n)$$

subject to

(15) $$\boldsymbol{A}_n\boldsymbol{x}_n \leq \boldsymbol{b}_n$$

$$x_n \geq 0$$

The following dual model is associated

(16) $$\min_{\boldsymbol{y}_n \geq 0, \boldsymbol{x}_n \geq 0} (\boldsymbol{b}_n'\boldsymbol{y}_n + \boldsymbol{x}_n'\hat{\boldsymbol{Q}}\boldsymbol{x}_n / 2)$$

subject to

(17) .

$$y_n \geq 0,\ x_n \geq 0$$

This model reproduces exactly the use of the land resource and output levels observed in the base period of the n-th farm. In other words, the primary and dual solution of the quadratic programming model is exactly the same as the primary and dual solution of the initial LP model which in its turn reproduces the results obtained in the base period. This is the meaning of calibration in PMP methodology.

Formally, for the whole sample of farms within the same FT, the level of total output and entrepreneur decisions are obtained from the following model of quadratic programming:

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[9] See Howitt and Paris (1998) and Paris and Arfini (2000) for methodology of maximum entropy

(18)

subject to

(19)

$x ³ 0$

with the following dual model associated

(20)

subject to

(21)

$y ³ 0, x ³ 0$

This last primary model (18-19) can be used to analyse various agricultural policy scenarios, particularly the CAP, allowing price variation, compensation and limitations in resources. We shall discuss below agricultural policy problems using PMP methodology.

2.3 From PMP to SPEP

Paris and Howitt (2000) have further developed PMP as described above, taking into consideration not only variable costs but also fixed costs sustained by farms. In effect, PMP in its initial formulation (Paris and Arfini, 1995; Howitt, 1995) was the object of criticism precisely for the information used to construct models, which was judged too limited to develop a suitable methodology.

From the start, the aim of PMP was to construct a link between traditional econometric approaches, always considered positive, and mathematical programming, which is always normative. There is a simple difference between the two approaches. Econometrics uses decisions by economic agents as an instrument to infer the structure of their decision-making processes, while MP, once the structure of the production unit is known, outlines the decisions the economic agents should take. The approaches cannot be defined as exclusively positive or exclusively normative.

As we have seen above, ‘conventional’ MP does not take into consideration decisions by economic agents, while PMP, thanks to its positivity restraints incorporates the behaviour of entrepreneurs and allows the model to reflect observed reality. On the bass of this hypothesis, PMP does not require a “time series” of observations but theoretically could use a single observation. This is because the first measurement for the sample contains most of the necessary information for constructing the model, while the subsequent information becomes increasingly less important.

Although it has the same objectives and has the same theoretical basis, SPEP is substantially different from PMP. In the first stage of the LP model a problem of economic equilibrium is introduced, while in the third stage (calibration) the structural constraints linked to fixed factors are substituted by a further problem of economic equilibrium between the functions of demand and supply of factors and the respective costs and marginal earnings corresponding to the activities. SPEP’s biggest innovation is to postulate that the ‘fixed’ factor of LP models can be “unlinked” from their “fixedness” incorporating their shadow price in the cost function. This is no longer the “traditional” cost function comprising only variable marginal costs, but it comprises the total marginal costs. So that once shadow price for limiting factors "*y*" is used in estimating the cost function, the technical matrix "*A*" is incorporated into the same function thanks to its "duality" with respect to output function. Using the SPEP approach, the marginal cost vector thus becomes $(\lambda + c + A'y)$.

The SPEP model hypothesis is that each farm in each region or sector is subject to different limiting factors such as land, water and capital, and each of these has a specific market price in the region. So the aim becomes to estimate the latent variables identified as the marginal cost of output for each process and the limiting factors.

This first result is obtained in the "first stage" of the SPEP model, using the definition of a problem of economic equilibrium as follows:

(22) $\boldsymbol{Ax + b}$	$\le \boldsymbol{b}$	$y \ge 0$
(23) $\boldsymbol{x}$	$\le \boldsymbol{x_R}$	$\lambda \ge 0$
(24) $\boldsymbol{A'y + l + c}$	$\ge \boldsymbol{p}$	$x \ge 0$
(25) $\boldsymbol{y}$	$\ge \boldsymbol{r}$	$\beta \ge 0$

where "**r**" is the market price vector of limiting factors and the others are the LP model constraints described for stage 1 of the PMP model.

Model (22-25) has a clearly symmetrical form. The first two constraints are quantity constraints and show the demand and supply of limiting factors and products. They are thus considered "primary constraints" of the problem. The other two constraints are marginal cost and marginal earnings of output and input respectively and are considered "dual constraints" of the problem. Within the model, the two symbols $(\boldsymbol{l})$and $(\boldsymbol{b})$ play important roles which represent vectors of dual variables of the corresponding constraints. When the dual variable $(\boldsymbol{l})$ appears in the restriction of marginal cost together with vector (**c**) it signifies variable cost. The dual variable $(\boldsymbol{b})$ appearing in constraint (22) defines effective supply of limiting factor $(\boldsymbol{b}\text{-}\boldsymbol{b})$ differently from the usual representation of fixed factors represented by vector $\boldsymbol{b}$. Vector $(\boldsymbol{b})$ prevents the generalisation of supply of limiting factors allowing a more realistic and flexible representation of fixed factors. Vector $(\boldsymbol{b})$ can be thought of as an upper limit of allocable fixed factors, which does not imply an implicit marginal cost of zero as happens in the usual notation $\boldsymbol{Ax}$ £ $\boldsymbol{b}$. Moreover, vector $(\boldsymbol{b})$ is not considered as a slack variable. The symbols to the right of the four constraints are the corresponding dual variables.

In order to complete the equilibrium the corresponding deviation variables need to be associated:

(26)	$\boldsymbol{y'(b - Ax - b)}$	$= 0$
(27)	$\boldsymbol{l'(x_R - x)}$	$= 0$
(28)	$\boldsymbol{x'(A'y + l + c - p)}$	$= 0$
(29)	$\boldsymbol{b'(y - r)}$	$= 0$

The specification of the problem (in 22-29) no longer allows an LP problem where the objective function can be maximised; it needs to be solved with a model of economic equilibrium. The model is thus no longer normative in maximising profit, but rather takes on a positive connotation of the equilibrium model.

The solution of the SPEP (symmetrical) model given by the set of constraints (22-29) leads to the estimation of output levels ($\mathbf{x}$), the actual supply of limiting factors ($\mathbf{b}$-$\mathbf{b}$), the total marginal cost of activities ($\mathbf{A'y} + \mathbf{l} + \mathbf{c}$) and the marginal cost of the limiting factors ($\mathbf{y}$).

In the "second stage" of the SPEP model, the total cost function is estimated. This is unlike the PMP model which develops the total variable cost function. Paris and Howitt (2000) use Leontief "generalised" function to show input, and a quadratic functional form to show output in order to avoid linearity constraints imposed by technology. In order to ensure sufficient flexibility with respect to starting data, Paris and Howitt add an intercept, while the fact that the cost function is homogenous to input prices is guaranteed [10].

In the "third stage" of SPEP, once the parameters of the total cost function have been established, a "calibration model", which also has the structure of an equilibrium problem is used. The aim of the third stage is, however, to yield results for an "base year" for output quantities, supply of limiting factors and their shadow prices, without using linear technology. Once this result is obtained it is possible to analyse agricultural policy as described for PMP, in other words, taking price or compensation levels as parameters and observing the effects on the supply of products, the demand for limiting factors, and on their shadow prices.

It is important to note that Howitt and Paris's SPEP model does not exclude optimisation behaviour by entrepreneurs. Moreover, the non-linear cost function allows the the construction of a hypothesis on non-linear technology, eliminating the constraints imposed by the use of fixed production coefficients in the technical matrix.

So SPEP is a more flexible instrument than PMP for analysing agricultural policy. It gives a represents reality more completely in that it allows for the incidence of fixed factors, especially financial ones, on farmers' decisions. This aspect is particularly important in those sectors or FTs where the fixed costs linked to technology or structures are decisive. It ensures an interpretation which is certainly nearer to the actual behaviour of farmers and meets real agricultural policy scenarios.

3 Main Mathematical programming models used for analysing CAP Problems

There are numerous models using mathematical programming to analyse agricultural policy problems. Many were developed primarily for company planning and were subsequently adopted for generic agricultural policy problems. Only recently have they been applied to Common Agricultural Policy problems.

Consequently, many of these models have the same matrix, which retains a microeconomic view of agricultural policy problems. In other words, at the centre of the model is the agricultural enterprise or the individual entrepreneur who has to adapt to the changes in agricultural policy.

Moving on from these important observations, the models can be classified according to two main elements. The first is the methodology and the second is the number of farms or the aggregates of farms comprising the sample. This gives us the following schema:

[10] See Paris and Howitt (2000) for details of the functional form of the total cost function and the methods used to estimate it.

Schema 1 - **Main groups of models for agricultural policy analysis.**

	Linear Programming	***LP and econometric estimates***	***PMP***	***SPEP***
Farm models	Hazell-Norton Dono Arfini Cesaro Garoglio e Mosso Iotti e Setti Angeli, Carbone, Severini			
Regional models	Giacomini-Arfini-Cesaro Cesaro, Ferro, Povellato. Hazell-Norton	AROPAJ LUAM	Paris e Arfini Howitt Barqaui e Boutald Judez , Hackelei e Britz Paris, Montresor	Paris-Howitt
Sector models	Hazell-Norton	AROPAJ LUAM	Paris-Howitt Hackelei e Britz	Paris-Howitt

3.1 Farms models

Linear Programming models which reproduce the characteristics of a single farm and simulate farmers' behaviour faced with possible agricultural policy scenarios can be considered "company" or "farm models".

Such models were originally farm management instruments used in technical assistance to help farmers better plan their activities in line with European Community policy from 1988 onwards. In construction, these are classical LP models.

Farm models have been developed by many researchers to study the effects of CAP measures on individual farms. Angeli, Carbone and Severini (2000) use LP models to estimate the effects of the Berlin seed agreement at farm level. Another example is Dono (1996) who develops a model to analyse agro-environment problems in a set of farms in Central Italy. Arfini (1995) uses LP to simulate the behaviour of a group of families in different activities on the application of EU Reg. 2078/92. Cesaro (1993) uses dynamic LP to study the effects of introduction of compulsory set-aside. Garoglio and Mosso (1992) use LP to study the effects of milk quotas on a farm in Piedmont (Northern Italy). Many other researchers have published similar work in sector literature.

Farm models developed for technical assistance or for studying the impact of CAP measures have the undoubted advantage that they are simple to construct as they show observed reality. They also yield the information needed to construct the specific technical matrix for the company under study, and thus greatly reduce the risk of error in assessing the farmer's behaviour. But these models do not represent an area or sector because it is not possible to apply statistical inference of the results to farms as a whole.

In conclusion, these models which describe case histories are extremely useful for technical assistance and to estimate impacts on individual farms, but they are much less useful for public decision makers who require information on the effects over an entire area or production sector.

3.2 Regional models

In order to make a wider assessment of agricultural policy scenarios, and to make the model more statistically significant (i.e. better able to interpret results), different methodological paths have been followed for the different groups of models.

Using models very similar to farm models, the first attempt to make the models more widely applicable was made before the CAP reform. The *n* farms in the sample for a geographical area were reduced to a single representative farm using simple and weighted averages for the parameters needed to construct the model. The biggest problem is establishing the criteria for aggregating farms into a sample which is truly representative. It is especially difficult to assess how the sample performs with regard to farm statistics on structure, economics and output.

The following works describe such attempts: Heady (1978) on American agriculture, Hazell and Norton (1986) describing the techniques of constructing a model representing the area studied, Hazell and Scandizzo (in Hazell and Norton 1986) on the agriculture of North-east Brazil and Paris and Ester (1995) on Australian agriculture.

Territorial linear programming models have been applied to CAP problems in Italy by researchers working ad hoc on agricultural systems in several regions. Particularly important is the research by Giacomini, Cesaro and Arfini (1992) which develops a model for Emilia Romagna, and the research by Ferro and Cesaro (1992) which develops a model for the Veneto region. Both of these use a "average farm" to represent the whole for technical, economic, and structural parameters of the model. Price and cost, economic parameters and structural parameters can be obtained from Regional data banks such as the FADN/RICA data bank in Emilia Romagna. However, the technical coefficients can only be estimated.

Clearly, the most delicate aspect and the biggest problem with this group of models is obtaining parameters to describe the technology used by types of farms, corresponding to size or the main type of output, even if they are in the same geographical region. It also needs to be underlined that if farms are aggregated solely on the basis of structural characteristics, their output orientation and different degrees of specialisation tend to be overlooked.

The risk of developing models that do not correctly represent the technology used and thus the costs of the different production processes is considerable. This means that any estimation of entrepreneurs' behaviour represented by the model does not necessarily correspond to reality and consequently can supply policy makers with wrong information.

Thus, both models, the elements of constraint (UAA used for the process and work force used by type) are supplied by the national census. This, on the one hand, allows dimension to be shifted as a function of the level of desegregation of the data (municipality or the smallest unit of local government, groups of councils, provinces and regions). On the other hand, it leads to a problem of representativeness, because as the area examined decreases in size, the number of farms in the RICA decreases as well. There is also the problem that the census is carried out only every ten years and the information is, therefore, not necessarily up to date.

From the point of view of "modelling" CAP agricultural policy instruments, these models are extremely flexible, as they allow researchers to simulate the effect of different measures on an "average farm". But they are also extremely difficult to construct, not so much methodologically but from the point of obtaining information on the type of farm to be studied, and in constructing the technical matrix and the statistical treatment of the basic information.

The next two sections discuss two important attempts to overcome these limitations and problems. The attempts are models of regional linear programming, AROPAJ for French agriculture and LUAM for British agriculture.

3.2.1 AROPAJ: the French model

The ARJOPAL model (Jayet, 1990) was developed by the INRA Agricultural Research Centre at Grignon in order to use linear programming to study the problems of CAP in France and, with an adequate data base, in the rest of Europe, the United Kingdom, Italy, Spain, Portugal and Germany. The model can be considered regional in that national identity is found by adding together different regions.The model allows researchers to consider the main CAP instruments, such as guaranteed prices, subsidies, set aside and quotas. It also allows new processes to be introduced, so researchers can assess how new economic production systems are working, as well the effect and the effectiveness of environmental policies that modify the objective function of farming.

Here too three elements, which could compromise the results, can play a decisive role: the quality and availability of data, aggregation mistakes and the drawing up of parameters to describe the technology used.

AROPAJ was applied to estimating the effects of the McSharry proposals for seeds and set-aside in analysing the effects of introducing more extensive production techniques in breeding and long term forecasting of CAP effects on individual sectors of agriculture in France. More recently (Hofstetter, Jayet and Marzocchi 1999), AROPAJ has been applied to the likely effects of Agenda 2000 for all EU countries.

To put it briefly, it can be defined as a regional model based on the supply system for the main fkinds of fruit and vegetable and livestock production, where all the parameters define a production set and where the coefficients of the objective function are estimated on the basis of microeconomic data obtained from RICA.

Each linear model has the standard structure of LP models described above. The aim is to show the different typologies of producers and to simulate the behaviour of each type of farm, assuming that the farmer wants to maximise net income, subordinate to a series of linear restraints. If there is a set of farms for each type rather than an individual farm, then the same number of models as the number of farms in the sample are constructed. Subsequently, from the individual farm results, the aggregate results are obtained for groups of farms homogenous in terms of structure or geographical area. In practice, AROPAJ aggregates individual activities between different types of farms, giving the model much more validity than the exclusively micro-economic view of average LP models.

The choice of farm type is thus the characterising aspect of the model in that this choice, made a priori by the researcher, gives the model a precise identification with regard to subsequent agricultural policy analyses. The criteria used by AROPAJ to identify typologies are geographical division and techno-economic orientation identified by the starting FT. A third criteria is added to these two general aggregation criteria of:this is specifically linked to agricultural policies in that it takes into account regional planning which defines homogenous areas with regard to expected EU subsidies.

In each model, the technical matrix is divided into different sections, each of which corresponds to the technological requirements of the most important vegetable and animal production processes. The following processes are included: annual crops (soft wheat, barley, sugar beet, sunflowers, maize, rice, soybean) and fodder crops. The model also looks at 34 species of animals. Several specific inequality constraints have been introduced for some kinds of crops, especially cereals and oilseeds which enable the model to take into account agronomic practices, especially rotation. As regards cattle breeding there are various constraints with respect to internal resources and feeding balance that reflect the nutrition requirements of the type of animal farmed. Animal output is tied to plant output through constraints which mean that forage can be either produced and consumed on the farm or purchased externally. Milk quotas are also taken into account.

The main difference between AROPAJ and Italian regional models is not so much in the quantity of plant and animal processes and their links, as in the fact that the AROPAJ approach provides a complete description of the technological framework.

Although AROPAJ considers labour, animal feed and chemical products as important production factors, it cannot completely define the technological set for all the processes. This is because the information source, the European RICA, neither supplies itemised cost figures for individual activities nor technical coefficients linked to the individual factors.

In order to overcome this serious problem, AROPAJ uses a procedure to estimate technical coefficients based on European RICA figures using a multivariate technology which takes into account the measurements made over different years of the constant sample of farms for prices and yields, and a regression model to estimate the variable costs per process.

These figures supply a complete first set of measurements for all the parameters. Next, an adjustment phase is required to calibrate the model to observed reality. This is undertaken by experts and is the most important and delicate phase in the model's development.

But very often the solution of the model does not coincide with the "observed solution" that is the overall result does not correspond with the results for each type of farm.

In order to find an acceptable "calibration" between the actually observed solution and the solution suggested by the model, a procedure must be activated to minimise the distance between the estimates and observed measurement for each farm type. The calibration parameters are selected on the basis of a parametric analysis of the farms examined. When AROPAJ was applied to Italy, France and Britain it was found that the parameters are mainly linked to animal feed and rotation practices showing that different farm types based on thecriteria outlined above adopt different technologies.

As has been pointed out, the structure and elements of AROPAJ are not dissimilar to the area models developed in Italy. The greatest differences lie in the measurement of technical coefficients, production costs and in the calibration process. Moreover, the calibration is partly carried out by econometric instruments which give the model greater "objectivity" and greater precision in describing observed reality.

On the other hand, AROPAJ has a relatively complex procedure and requires greater experience on the part of the researchers using it. It also requires a large statistical base fora detailed description of the different types of farms, using up considerable resources in the construction and the maintenance of the model.

3.2.2 The British model: LUAM (Land Use Allocation Model)

LUAM, like AROPAJ, aims to analyse the effects of agricultural policy measures country-wide. LUAM too is a linear programming model developed to assist public agricultural policy makers in England and Wales. It was created in 1985 at the Farm Management Unit at Reading University and was gradually implemented with the help of the Ministry of Agriculture in order to assess the effects of the CAP at regional levels.

It is based on the Land Classification System (LCS) which divides the whole of England and Wales into 15 levels as regards quality. The classification is based on a multivariate cluster of approximately 200 quality and quantity variables describing the geography, geology and climate of a network of points 1km apart over the whole of the two countries. Each quality level is an amalgam of different the types of qualities of land and contains different land uses associated with soil characteristics. Detailed information is available including the plant species grown and the specific uses of the ground for each square kilometre. There is, however, no overall classification of types of land use. There is rather a combination of physical-geographical characteristics and land use, and this is considered a proxy of the production capacity of the land in that particular area.

An important aspect of the model is that each level of quality contains a mix of processes or activities in the area which can be re-aggregated to supply the exact assessment of the effects of agricultural measures on individual activities.

The disaggregation obtained through the LCS ensures significant accuracy in the subsequent inference of the results of the different agricultural policy scenarios for an area . The area can also be linked to geo-referencing instruments such as GIS.

LUAM makes it possible to reproduce the whole agricultural system of England and Wales as if it were a single farm executing all production activities. For each quality level, different inputs and resources can be used and the respective output can be obtained.

LUAM is, in fact, basic to the whole structure of the model and aims to highlight agriculture in England and Wales through a model of linear programming using a defined number of processes and land uses along with labour and capital. Figures on aggregate use of land, on animals and labour are obtained from the Census, and figures on output associated with the use of each factor are supplied by the Ministry of Agriculture.[11]

Figures on the methods used by farmers and costs in the different areas of the country come from the Farm Business Survey (FBS) and figures on the local agronomic characteristics due to environmental differences from the LCS.

The structure of the LUAM model remains that of a traditional LP model where the net income of the firm is subject to the technological matrix and to a set of restraints relating to the resources available, the agronomic methods, and the environmental constraints from the LCS.

Each class of quality is considered an ecological subsystem containing different nationally defined sub sectors. The option is thus left open to modify land use by changing from one process to another, while the quality class of the land itself cannot be changed. So in LUAM each quality class is identified as a single sub-matrix defining all the ways land is used.

Since land resource cannot be transferred from one class to another, the structure of each farm defined according to quality is shown in the remaining fourteen classes, thus creating a series of matrices in a ‘scale’, which, put together, define the LUAM matrix. The links between various classes of quality are made through equations linking activities and factors. As was shown above, the output from every quality class constitutes the total supply for England and Wales for a whole series of activities such as milk, meat, wool, lamb, soft grain wheat, barley, potatoes, wood, etc. Moreover, the input equations enable an economic assessment of factors that need to be purchased to produce the goods in the various quality classes so that the objective function of the model becomes the maximisation of net income for England and Wales. The LUAM model, moreover, does not allow for trade between the various regions in the model, for example intermediate goods such as animal feed, and it implicitly assumes there is no variation in company structure in terms of size, type of output and number of employees.

The LUAM model also poses the problem of fixing technical coefficients which constitute a technical sub-matrix, one for each class of quality. The solution is to calculate technical coefficients from Farm Business System data using a suitable econometric procedure. The authors themselves (Harvey and Reheman, 1988) admit that, although the FBS sample is not statistically representative, it is the only source of information on production costs in different areas of Great Britain and different land qualities. As in the Italian models and AROPAJ, this phase is the most

[11] The information is from Department Net Income Calculation (DNIC)

delicate and important in the calibration of the LUAM model to observed reality and its reliability clearly depends on the correct calculation of the technical coefficients [12].

Once calibrated to observed reality the LUAM model can be used for two different ends. The first is to forecast the structure of supply and land resource uses after market price variations or specific agricultural policy measures. These might be CAP measures such as price reduction, quota systems, set-aside included or any other “exogenous” variations in the structure of the model. The second aim of LUAM is to make forecasts of “endogenous” variations in the model following the modification of ties or relationships in its structure, priveleging the information given in the results, for exmple the shadow prices of binding factors.

LUAM has recently been further improved in order to maximise its coverage of CAP agricultural policy scenarios.

LUAM is thus a very complete instrument for analysing agricultural policy at both area and sector levels. Yet, as will be made clear from this brief description, the various sources of the data and organisations supplying information need to be integrated. LUAM, in effect, gives a high return to policy makers, but it also absorbs a lot of resources, particularly financial ones, in keeping the model and the statistics up to date.

These limitations are, however, common to AROPAJ and Italian models. LP has significant advantages, notably its simplicity in modelling agricultural policy. But it runs into difficulties when representing areas wider than a single farm, such as a whole region or European country.

3.2.3 Positive Mathematical Programming and regional models.

The above description of difficulties in the use of LP helps us to see why so many researchers use other methodologies, as can be seen from the various contributions to this volume. PMP and SPEP, in particular, allow researchers to use mathematical programming to analyse the effects of agricultural policies . PMP, in particular, shortens the calculating phase in setting the technical matrix parameters, as it is possible to use data directly from banks such as the European RICA and British FBS. It avoids the need for manipulation or estimates that may sometimes include subjective assessments.

Since 1995, in fact, many works have appeared using PMP to analyse the effects of CAP sub regionally, regionally, nationally and at the European level. The success of PMP is also clear from the fact that it is used in two EU funded research projects (CAPRI and EUROTOOLS[13]) that develop models to analyse CAP.

But although these models both use the RICA data bank and PMP as basic methodologies, there are various differences between them which derive from the different interpretations of PMP as it was originally developed by Howitt and Paris.

At this point it is important to note that PMP is still being developed and, although it is able to make detailed assessments on the main lines of agricultural policy, it has to be used with caution in order to avoid errors in the measurement phase.

The application made by Paris and Arfini (2000) isa case in point. These authors emphasise that calculating the matrix “***Q***” with minimum least square or with maximum entropy gives extremely different results when estimating the effects of agricultural policy. A comparison between the two

[12] See Harvey and Reheman, 1988 for more details on the method of estimating technical coefficients.
[13] CAPRI (the Common Agricultural Policy Regionalised Impact Analysis) is a research project coordinated by the University of Bonn. EUROTOOLS (Tools for Evaluating EU Agricultural Policies at Different Decision Levels) is coordinated by the University of Bologna.

approaches, in fact, in an overall assessment of agricultural policy, suggests that the results obtained with maximum entropy are more plausible than those with minimum least square. This is because the results of minimum least square is too sensitive to price variation, and involves a degree of substitution and / or complementarity between crops which appears to be unreasonable. On the other hand, the results of maximum entropy are more conservative and appear to take into account the inertia traditionally found in the agricultural sector.

It is, however, still difficult to find a criterion for aggregating the different models, since each researcher adds something new to the theoretical basis developed by Howitt in 1995 and by Paris and Howitt in 1998.

Only Paris uses self selection (Paris and Arfini, 1999, Paris and Arfini, 2000) to describe the behaviour of different entrepreneurs within the same typology in the model. All the other researchers develop a model of "single" PMP, or rather one for each typology, with farms being represented by a "*typical average*" farm by the use of appropriate averages.

The following is a list of important works using PMP.

- The INRA model developed in Nancy by Barkoui, Boutault and Rousselle, 1999. This simulates the impact of Agenda 2000 for seeds up to the year 2005 for 12 EU countries subdivided by Region.

- The model developed at the University of Madrid (Judez et al, 1998, 1999 and 2000) which extensively revises the Howitt Paris model. It does not use the dual information produced by the model but market prices of limiting factors. This model has been applied to estimate the effects of Agenda 2000 measured in various regions of Spain to define a production scenario compared to year one for several typologies of farm corresponding to the most important classes of FT (technical orientation) in each region. This model uses data from RICA as well, and each model shows the results of a representative farm for each FT obtained from the average of farms present in the corresponding sample group of farms in RICA.

- The model developed at the University of Galway (Garvey and Steele, 1998) applies PMP procedure and maximum entropy and estimates the effects of Agenda 2000 for single representative farms selected on the basis of FT physical size.

- The models developed at the University of Bonn in the CAPRI programme in 1997. (Heckelei and Britz, 1997; Lohe Heckelei, Britz and Lohe, 1998; Heckelei and Britz, 1998; Heckelei and Britz, 1999a) PMP is used to develop a model of regional supply for each EU country on the basis of forecast prices for each year. It was necessary to construct a model of aggregate supply for each member country linked with a subsequent demand model, and PMP was used to develop the supply model for each country. This involved additional analysis, as the approach developed by Paris and Howitt is a supply model which does not consider the production of intermediate goods such as forage used in breeding. For this reason Hackeley and Britz (199b) developed a model using PMP and maximum entropy which are able to calibrate forage requirements for agriculture, creating the equilibrium conditions between demand and supply within different regions of EU countries[14] .

Nevertheless, the main criticism made by the Bonn group is that PMP, according to Howitt and Paris's model, (1996) uses only one dual observation concerning the marginal cost of land for fixed factors, and introduces "support values" to ensure the correct curvature of the function of estimated cost. The Bonn researchers, rather than using a single observation, suggest the adoption of a historical series so as to make the calculation of parameters constructing the technical matrix and the cost function more statistically robust.

Howitt and Paris reply to this criticism with SPEP which, although based on a single observation, calculates the total cost function by increasing the number of constraint factors in the model.

[14] The supply of forage for agriculture is also included in a PMP model in Paris and Arfini (1999 and 2000).

The Bonn PMP model is used to estimate the effects of Agenda 2000 agricultural policies in EU countries (Hackeley and Britz, 2000), divided into 200 regions according to the NUTS II classification on "COP" seeds and milk and meat. It is used in addition to the models currently used by Bonn University Agricultural Policy department and supports DG AGRI policymaking [15].

The last PMP model for regional analysis to be looked at here is the work of a research group (Paris, Montresor, Arfini and Mazzocchi, 2000) which combines PMP with a multivariate analysis method, with the aim of increasing the model's overall capacity for interpretation.

The starting point is the observation that most studies now available, as we have seen in this overall review, aim to supply information on variations in both the supply of agricultural products for a defined area and in the income of farming households and the cost of policies borne by government and / or elasticity coefficients to use for other analyses. All Mathematical Programming models, LP, PMP and SPEP meet these requirements. But today's policy makers for Rural Development Plans at regional level need to be able to see how agricultural policy, and particularly the CAP, interact with other sectors, with society as a whole and, last but not least, with environmental variables in the area.

In an attempt to meet this need, the model developed by Paris Montresor, et al. (2000) consists of various phases. In the first, statistical analysis is made by multivariate analysis (MSA) composed of analysis by main components and a cluster analysis, to identify homogenous areas in terms of socio-economic levels and environments. The second phase is PMP, which measures the impact of policies and supply indices to be used in the third phase. The third phase again uses MSA and assesses whether the scenarios of agricultural policy shown by the PMP model modify the aggregation of homogenous areas carried out in the first phase.

3.3 Sector models

Sector models give policy makers an assessment of agricultural policy effects for a single good , for example soft wheat, or a single sector , for example cereals. In comparison with the models described above, it is clear that, although many are useful at area level, they also approximate as sector models given that they are built with the precise aim of analysing supply variations in output as a result of CAP intervention at regional, national and / or EU level.

MP models have a greater propensity to simulate entrepreneur options, in the same way as those models which supply indices of elasticity. But MP is still important; supply is measured by taking into account the different production capacities of the different Regions and the different types of farm aggregated according FT.

Sector models are distinguished from area models by the aggregation criterion, which in this case gives more privileges FT and hence output. Hazell and Norton (1986) clearly describe the technique for building sector models using MP, and as the AROPAJ and LUAM models show, it is possible to estimate aggregate supply for large groups of products. However, one of the biggest problems of aggregating firms by output and not by structure is excessive specialisation in output. Where farms are very specialised, with a limited set of processes, LP, in fact, shows significant rigidity in finding 'corner'[16] solutions as economic variables, prices, costs and subsidies vary.

[15] It is important to emphasise that the Bonn framework links mathematical programming to methods and models studied elsewhere in this volume, particularly in the chapter on partial multiproduct and multiregion equilibrium models.

[16] 'Corner' solutions are obtained near the points of a geometrical shape, a triangle in a bidimensional space or a tetrahedron in a three-dimensional space, called a simplex. It is an optimum efficient solution in that it guarantees the

Consequently, it is difficult to exploit the whole model for agricultural policy analysis, and in particular to assess the effects of CAP measures.

It was the problem of excessive specialisation of farms that encouraged Howitt and Paris (1988) to develop PMP and SPEP (Howitt and Paris 2000), moving towards non-linear objective functions which offer greater sensitivity to variations in agricultural policy, particularly prices and compensation. Moreover, given that PMP and SPEP can 'estimate' technology through dual information obtained from 'positivity' and structural constraints, it is possible to construct models which are extremely close to the level of specialisation in single typologies of farm. It is not necessary to use empirical estimates or researchers' subjective assessments as the data can come from data banks of agricultural statistics such as RICA.

There are explicit examples of this in Paris and Arfini (1999), where the supply of COP seeds and of milk and meat in the Emilia Romagna region are calculated. Hekelei and Britz (2000) calculate the supply of COP seed sand main agricultural animal output, milk, meat and chicken for the whole EU. Gohin et al (1999) analyse the consequences of reform on the seed sector in France within Agenda 2000.

To conclude this section, it is important to note that whatever methodology is used, LP, PMP or SPEP , the mathematical model is only one aspect of the 'estimation procedure'. Other parts are essential: a) an adequate data base, b) an aggregation criterion functional to agricultural policy objectives, c) an adequate methodology for calculating technical coefficients, d) a link from results to the reality. These properties are found in only a handful of the models examined (AROPAJ, LUAM, CAPRI). But it is these very properties that allow continuity in application and give the models the versatility that enables them to tackle any agricultural policy problem posed by the CAP. The above models have thus become a point of reference for the other models engaged in agricultural policy analysis, such as FAPRI, CAPMAT and SPEL/EU-MFSS.

4 Representing CAP measures

This section describes how various economists have introduced the main agricultural policy measures into their models. We shall place particular emphasis to CAP measures and critically analyse the technical solutions, results and their usefulness for agricultural policy analysis.

It is crucial to note at this point that a lot of work pays little attention to the structure of the constraint of the specific agricultural policy instrument and tends to concentrate rather on the general framework of the model, or for PMP models on the method of calculating the cost function. In fact, as was mentioned previously, CAP problems can be fairly easily inserted into MP models as the techniques are very general and are fully described in LP manuals such as Hazell and Norton (1986) or Cosentino and De Benedictis (1979).

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maximum value of the objective function and respects technological constraints. In the phase of searching for a solution, each itinerary developed by the algorithm of the simplex generates a simplex in a space which varies according the number of constraints in the problem.

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4.1 Direct price support

One of the main agricultural measures dealt with by MP models is what is called price policy, or more accurately, the relation between institutional and market prices.

All the models we have seen so far use market prices for farmers given in data banks such as RICA to describe the starting situation. They postulate a direct link between institutional price variation (indicated price, threshold price and intervention price) and the prices used by farmers. This, moreover, implies that prices are always exogenous to the model and that there is a 'pull' effect between institutional and market prices. In other words, they discuss how a reduction in market price will correspond to a price reduction in cereal prices, for example the 15% laid down by Agenda 2000, and whether this reduction will be transferred in a linear manner to all cereals or in varying degrees for different cereals.

Most researchers hypothesise that prices fall according to the regulations in a uniform way for all cereals. Only Paris and Arfini (1999) hypothesise that price reduction is not transmitted homogenously to all cereals and that there is a relation between the price of soft grain wheat and COP crops (hard-grain wheat, barley, maize and soya) depending on the precise situation. In other words, a price reduction of 10% in the market price of soft wheat becomes a differentiated variation in prices for other COP seeds. This hypothesis implies an increase in the number of different policy scenarios which need to be described. Paris and Arfini, in fact, show one scenario with uniform price transmission between institutional and market prices, as well as alternative scenarios where only soft wheat varies in price according to the CAP regulation, and the price of other COP products varies in a differentiated way, which exactly reproduces the relation between prices and soft wheat recorded in the past.

In none of the works described up to here have researchers considered the risk factor in farmers' behaviour empirically. Although the bibliography contains many references to applications of quadratic programming on the price side, only Paris (1997b) formalises the introduction of the risk element associated with product prices in the objective function of the PMP model. Paris uses the methodological structure originally suggested by Freund (1956) according to the following hypotheses: 1) the use of the Von Neumann-Morgensitern forecast utility approach 2) the existence of a negative exponential utility function, 3) that prices are distributed as normal causally occurring variables, 4) a priori knowledge of price averages and variance. But the most delicate phase of the model is calculating the risk-aversion-coefficients for each entrepreneur, obtained indirectly from the dual solutions of the problem through *chance-constraint programming*[17].

Paris (1997) formalises the MO model in the calibration phase, with risk factor, with n firms in the sample, as follows:

(30)

subject to

(31)

[17] For more details in calculating risk coefficients through chance constrained programming, which is outside the goal of the present work, see Paris (1997b), also Paris and Arfini (2000).

Obviously, this model is associated with the entire sample of farms where the deviation vector, ***u***, is a null vector[18] .

4.2 Trade policies

Because of the aims of the models described up until now, trade policy models have not been considered by MP models. The preceding models are all supply based as their main objective is to estimate supply factors (land) and the main types of agricultural output affected by EU intervention. But there are two exceptions to this: LUAM and CAPRI. These include demand for factors and goods within trade relationships.

LUAM is not yet fully operational, but it has three aims: a) simulate effects on a large part of agricultural output in EU countries; b) calculate the trade relationships within EU countries and between the EU and the rest of the world, c) supply a model providing a more detailed regional analysis including different economic and trade scenarios.

LUAM, originally developed for England and Wales, is now amplified dividing the EU into 65 regions, as divided by European RICA, and looks at 5 types of farm for each region. The trade model, with submatrices for demand and supply, is inserted onto this base. The model also distinguishes between products with fixed domestic prices and demand, where export constraints are comparatively simple, and products where demand and prices are fixed endogenously. In the latter case, various hypotheses are required such as a) the price estimated for a region is considered constant, b) each region can decide whether to produce or import a product, c) prices for the same product may vary between regions, d) the model is assumed to simulate the supply curve and e) the demand curve is calculated outside the model through econometric estimates or estimates taken from the literature on demand elasticity.

CAPRI has the main aim of building a model to analyse agricultural policy combining regional analysis with sectors across Europe. This structure requires simultaneous analysis of market effects for each product and of the regional effects of agricultural policy insofar as these reflect domestic and EU markets.

The market and the activities linked to agricultural policy instruments require a very explicit level of disaggregation of the model for products, but a simultaneous system solution to optimise producers' and consumers' surplus for 200 regions for 50 products cannot be computed. Consequently, the model is divided into two parts, one for demand and one for supply. The supply part consists of individual MP models for approximately 200 regions (NUTS II). The market module has the usual multi-product structure. Aggregated supply is obtained from regional models and on this basis the model supplies market equilibrium prices. This iterative process between supply and the market component finds equilibrium conditions of the market and subsequently comparative static operations can be applied to the demand or supply side.

Looked at in more detail, the methodology of the market model is based on standard concepts of multi-commodity models (Britz, 1998) developed along the lines of the SWOPSIM[19] models. In order to model demand and supply for regional and international markets, logarithm functions are used. These start from regional production and consumer prices, and are united by price transmission functions, and lead to a uniform world market. Demand parameters are not calculated, but are calibrated on the basis of estimated elasticity and / or levels reported in the literature (Wike

[18] represents the measurement of forecast utility compared to benefits is the risk premium associated with the utility function and forecast price. The other terms are as usual.

[19] SWOPSIM is fully described in the chapter of Piero Conforti .

and Vritz, 1998). The price transmission function considers tariffs and includes mobile duties which in turn depend on the evolution of international prices.

As regards products, which are important for the CAP, the processing of oil-seeds is explicitly shown in the model on the assumption of a fixed rate of transformation for "cakes" and the by-products of pressing. For milk products (dried skimmed milk, butter and others), equilibrium constraints are introduced between fat and protein levels and with products obtained from milk processing. The price of fresh milk and dairy products is derived uniformly from the price of the fat and protein content, weighed as a percentage plus the cost of unit transformation which is assumed to be constant.

4.3 Quantity constraints on output.

Quantity limits on output are part of two types of agricultural policy instruments. One type is a limit on output of certain products subject to EU intervention, for example milk quotas. The other type is an enforced reduction of the use of certain production factors, like land, with the aim of reducing the supply of certain products such as seeds.

4.3.1 Milk quotas

Two different approaches can be used to analyse the problem of milk quotas. The first simulates farm reorganisation when quotas are applied. The second estimates the "price" of the quota and hence the farmer's interest in acquiring or selling quotas.

In both cases it is simple to insert milk quotas into LP models, because a quota is considered in the same way as a production factor available in a limited quantity, as a structural constraint on the business.

From the point of view of writing the constraints, the part of the model relating to milk quotas contains only three lines of constraints as shown in the table. This is because milk production has two distinct phases, the 'sold milk' phase (x_1) (measured in quintals with price p), and milk production through the number of cows (**x_2**) (measured in number or animals with cost **c** per head).

Table 2 **Milk quota constraint**

F.O. Max RL	p x_1	- c x_2	
Subject to :			
Structure constraint		1	<= farm capacity
Quota constraint	1		<= milk quota
Yield constraint	1	- yield	= 0

The full utilisation of this factor, milk quotas, as yield per cow increases, leads to a reduction in the number of cows per farm and hence a reorganisation of breeding. It also makes explicit the relative marginal value represented by the market price up to which the farmer will go to acquire output rights from other producers so as to increase his own output quota.

The bibliography contains studies of both types,but it needs to be clear that even if they are concerned with agricultural policy, on the whole these models are similar to “company models” whose aim is that of optimising company strategy. Examples are Garoglio and Mosso (1992), Iotti and Setti (1993), Arfini (1997) and Barkooui and Boutauld (1998b).

These works show milk quotas introduced into the model in the way described above and in many LP manuals such as Hazell and Norton (1986), Paris (1991) Cosentino and De Benedictis (1979).

The area and sector models described in previous sections do not, however, mention milk quotas. There are two main kinds of reasons for this; one “technical” and one concerning research aims. The technical reason is that milk quotas may be equally well considered as an ordinary constraint. As far as research aims are concerned, it is often difficult, at least in Italy, to establish the aggregate milk quota per region. Moreover, in some regions it is possible to purchase the milk quota “factor” where market conditions make it advantageous, which would entail a regional or national constraint but not a constraint for the individual farmer. A third reason is that an increase in demand is an important indication for the policy maker of the output orientation of farmers and the size of quotas.

4.3.2 Set-aside

Set-aside, where farmers are obliged not to use part of their production factors, has a different role in LP or PMP models. This policy is shown as compulsory and involves compensation paid per hectare of COP seed surface.

Compulsory set-aside can be treated in different ways according to the research aim . One example is the dynamic LP analysis of slippage made on two different types of set-aside in farms in the Veneto region by Cesaro (1993). The model applies to a typical big producer, and compares three different scenarios of constraints, prices and other conditions. The interesting aspect of the model’s construction is that the typical farm has been divided into classes of output, so that compulsory set aside options can be simulated on the place (the type of output substituting which crop) and on the time (the year).

It is hypothesised that the farmer aims for the highest possible gross income, calculated as the sum of gross income of the processes activated including any crop or set-aside compensation. The objective function is multi-period and the gross margin is the sum of the gross margins of all the years considered. The crop mix for each class of output that of the RICA sample and can only be modified by introducing compulsory set-aside into each period considered. The model has an objective function, a constraint which fixes the set-aside percentage, which means that there is a maximum area of land available for each class of crop, one constraint on use and three rotation constraints. These rotation constraints mean that in each output class only one surface area can be set aside for each production class over a six-year period. Set aside is thus imposed first on less efficient areas and gradually on the more productive ones. The model thus simulates the options for set aside on fixed crop areas, so that in practice the farmer can choose between each crop and set-aside, and still respect the percentage limits for set aside and rotation.

LP and PMP area and sector models incorporate compulsory set-aside in different ways. Few of them actually make set-aside explicit.

Area and sector models such as LUAM, AROPAJ, Barkaoiu and Boutauld (1998a), Gohin (1999) Judez et al (1999) CAPRI consider set-aside part of the CAP but do not explain the structure of the constraint. Paris, Montresor et al (2000) describe in detail how it can be introduced.

Here too the objective function of the model is to maximise total gross income obtained as the difference between the income from each process including compensation received for set-aside and the fall in seed prices, minus crop costs. Set aside naturally comes into the objective function as a constant percentage, which multiplies the compensation for each of the COP seeds by the rate of compulsory set-aside. Subsequently, in the phase of developing agricultural policy, any variation in surface area deriving from a variation in the set-aside rate is added or subtracted to the constraint of the total land surface of the farm, showing that this can be exploited for all processes.

In each case set-aside is decoupled from output in the model, and as various simulations show, variation in the set-aside rate has repercussions on the overall organisation of the business. All the area models, therefore, are not only able to calculate the area destined to be set aside for a region or a European country, but also supply the total cost of this agricultural policy instrument for the public sector.

4.4 Partially decoupled policies

These include some of the main EU polices and guarantee a large part of farm income. They cover animal husbandry as well as seeds. Governments, therefore, need to know a priori the cost of these policies, the effects on supply and their effectiveness in guaranteeing income particularly for smaller farms.

It is clear that MP is helpful here; it brings out the geographical detail more clearly than other methodologies and differentiates between " regionalisation programmes" and calculates the effects on the entire supply chain. It also reveals possible distortions and undesirable consequences.

It is, moreover, simple to incorporate these polices into MP models. They appear as a modification of the objective function where income from a crop includes unit compensation received per hectare or per animal. The input, incorporated into the objective function, is thus linked to the production factor, a "cost process" and not to the output, an "income process". The policies are only apparently decoupled in that income levels per hectare, including both processes, are fixed and compensation is transferred entirely to output. Nevertheless, the model does simulate farmer behaviour correctly in the sense that for many farmers, compensation does not justify modifying production policy in terms of intensifying output. It can, in fact, lead them to modify output towards those processes which have a higher level of compensation.

This procedure is not explicitly described by the area and sector models described above, but it is the simplest formulation and the most effective in making models flexible and responsive to new scenarios. It allows a comparison with an observed "basic" reality. Moreover, the very ease of inserting these modifications encourages researchers to try out scenarios of contemporaneous variations in compensation levels for different outputs where it is difficult to see which measures have the most effect on farmers' options.

In fact, it is more difficult to incorporate these policies into PMP models because to do so it would be necessary to make some variables more explicit, such as the unknowns of surface area or the number of animals (for each process) since compensation is linked to this variable[20].

4.5 Voluntary policies

Voluntary policies are those which leave farmers free to adopt the measures they want. In exchange for financial compensation, or partial payment, the farmer may accept a new type of output or a new crop technique for his farm. The twenty-year set–aside (Reg. 2078/92) is one of

[20] See Paris, Montresor et al (2000) on the structure of this constraint particularly for animal farms

these policies, as is to a certain extent Reg. 1765/92 in that the farmer may accept or reject as he chooses the payment and organisation, general or simplified, to adopt.

Here too MP models are more useful than other methodologies as they can reproduce the farmer's behaviour in his working environment more clearly. This becomes particularly important as the CAP leaves regional authorities a margin of choice in fixing compensation levels, because they relate to the environmental and geographical characteristics of the areainvolved.

We shall now take a look at the different ways in which voluntary policies are represented in models.

- Voluntary set-aside features in ttthe model as an alternative "new process";it requires a certain amount of factors (labour) and EU money as the only earning. The model's optimisation process shows the level of the process in terms of numbers of hectares. The problem is posed in these terms in all LP analyses of land . It can be found in Giacomini, Cesaro and Arfini (1992) and the LUAM and AROPAJ models. In PMP models, voluntary set-aside is not mentioned- as there is no real output, there is no comparable dual figure. It is thus not possible to estimate the cost of the process within the cost function ***Q*** constructed from PMP models.
- Voluntary policies based on Reg. 2078/92 can also be studied with LP models informing policy makers of their spread. From the point of view of the model, such measures can be considered alternative processes competing with other land uses already carried out by the farmer. All new processes are inserted into the objective function specifying the selling price, the compensation and unit costs, and the technical matrices have to show the technical coefficients for the new production techniques.

Examples can be found in Giacomini, Cesaro and Arfini (1992) and in the LUAM and AROPJ models, which moreover allow intervention of the objective function. This means that the technology parameters can be modified and the degree of benefit in adopting Reg. 2078/92 measures can be ascertained for different types of farm and for different areas. Having said that, the PMP models covered in this chapter do not include the possibility of voluntary measures. The main difficulty for PMP model is the case in which the farmer, at the beginning, does not activate them, so that there is no positivity information on which to estimate relative production costs. Subsequently, in the agricultural policy part of the model a new cost matrix ***Q*** is necessary for the new processes. This involves formulating a new matrix ***Q*** representing both the new and old processes. But this rather complex situation has not been developed in any of the models studied here.

There does not appear to be any specific MP studies of the benefit of adopting Reg. 1765/92 or of compensation options. However, the approach would be similar to those described above.

5. Conclusions

We would like to conclude by making a number of observations on the use of MP in analysing CAP effects by region and/or sector. The first important point is the extreme versatile nature of MP in showing the"many" forms of agriculture and the "many" kinds of farmers in the EU agricultural system. MP can thus be used specifically to analyse farm, region or sector level.

MP has advantages in terms of simplicity in construction and interpretation of results, but several elements severely limit its application. The main problem is the need for technical and economic information in order to represent different types of farm correctly, and the characteristics of the region or sector to be analysed. This lack of adequate data in the development of regional or sector models has led some researchers to construct *"farm models"* and others to create highly complex and detailed models, such as LUAM and AROPJ, in order to make them applicable to an area.

Only recently has "classical" LP been developed with the aim of overcoming obstacles to their macro use in studying the CAP. Such is the work by Paris and Howitt on PMP and SPEP. Even in the absence of all the information which has been considered essential, it is now possible to develop models that can not only represent farmers' behaviour correctly but also model the whole range of CAP measures.

The most important aspect, however, is that even today there are very few MP models in Europe which can be considered as points of reference. What is required is agricultural policy analyses using all the various types of MP models (LP, PMP and SPEP etc.), what we have instead is the widespread use of applications which are little more than case studies.

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Table 1. **Summary of models**

	Farm models	*AROPAJ*	*LUAM*	*PMP Paris / Arfini*	*PMP Barqaui / Boutauld*	*PMP Judez*	*CAPRI*	*PMP Ghoin*	*SPEP*
Objective of analysis	Farm management, optimisation resource use and simulations	Forecasts and simulations on CAP and environment policies	Forecasts and simulations on CAP	Forecasts and simulations on CAP / Management	Forecasts and simulations on CAP	Forecasts and simulations on CAP / Company management	Forecasts and simulations on CAP	Forecasts and simulations on CAP	Forecasts and simulations on CAP
Target	Farm	Area / Sector	Area / Sector	Area / Sector	Area / Sector	Sector	Area / Sector	Sector	Area / Sector
Type of model	Supply	Supply	Demand Supply	Supply	Supply	Supply	Demand Supply	Supply	Supply
Year	According to the analysis								
Time horizon	According to the policy								
Sectors	According to objectives	Seeds and animal husbandry	Seeds and animal husbandry	Seeds and animal husbandry permanent crops	Seeds	Seeds and permanent crops	Seeds and animal husbandry	Seeds	Seeds
Methodology	LP, PLD.	LP	LP	PMP	PMP	PMP	PMP	PMP	SPEP
Parameters	Exogenous and from Interview	Econometric estimate	Econometric estimate	Endogenous estimates on basic information	Endogenous estimates on basic information	Exogenous	Endogenous estimates on analysis of historical series	Endogenous estimates on	Endogenous estimates on basic information
Source of data	Direct interviews/ CensusIntervist e dirette / Censimenti / RICA	RICA	Census / FBS /	RICA / Census	RICA	RICA	RICA / Census	RICA	RICA