Measuring Technical Change: A Simple Dual Approach†

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In this paper a remarkably simple, yet economically consistent, framework is developed which yields measures of technical change in terms of cost savings not attributable to changes in output levels and input and output prices. Our approach does not require specification or estimation of any production, cost or transformation functions. Furthermore, no assumptions on the properties of technical progress (e.g., exogeneity, embodiment, neutrality, etc.) are needed. Empirical results based on Solow's data (Solow 1957) are very supportive.

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1 Introduction

The literature on technical change is huge and still growing. Despite notable contributions of many prominent economists (e.g., Solow 1957; Stigler 1961; Kendrick and Sato 1963; Hicks 1965; Samuelson 1965; Beckman and Sato 1969; Binswanger1974; Stevenson 1980; Kopp and Smith 1985; Färe et al. 1989 and Chavas and Cox 1990, just to name a few), theoretical as well as empirical problems surrounding this issue remain. In particular, the factor or total factor productivity index approach has been heavily criticised due to its inability to incorporate causal explanation on the movement of factor productivity (Jorgenson and Griliches 1967). This criticism can be readily extended to the non-parametric programming approaches recently developed by Kopp and Smith (1985) and Färe et al. (1989). Conversely, the production function approach pioneered by Tinbergen (1942) and Solow (1957) inherits the usual problems associated with model specification and estimation. Another shortcoming of the conventional production function approach, perhaps the most serious one unnoticed so far, is its inconsistency with economic theory. As is commonly known, technological change, in economics, is equivalent to changes in the underlying production function (functional form and/or parameters of a given function). However, most previous studies (e.g., Jorgenson et al. 1987, Baumol et al. 1989) have explicitly assumed the same production function, which implies the same technology, for all the sample observations. It is illogical, to say the least, to measure technical change when technology has been assumed to be the same or unchanged in the first place.

In this paper, we develop measures of technical change, which are free of all the problems discussed above. In our approach, each input–output observation may be generated by a different function and no assumptions on the character of technical change are required at all. Contrary to the popular primal approach where technical progress is measured as changes in output not attributable to changes in cost, in this paper dual measures are derived where technical change is measured as savings in cost not attributable to changes in input and output prices and output levels (see Chavas and Cox 1990). The analytical framework and measures derived are remarkably simple.

2 Theoretical Underpinning

Within a dual framework, technical change is measured in terms of cost savings. However, changes in production cost can be attributed to three sources: (a) factor substitution in response to relative input price changes; (b) economies of scale as production expands; and (c) technical progress. Following conventional wisdom by assuming constant returns to scale (CPTS), we intend to measure technical change free of substitution effects. This is one advantage of our approach over previous ones. As we do not think that merely identifying a trend of technical chnage is good enough, measures yielding concrete values of cost savings due to technical change is our pursuit in this paper. This is another advantage of our approach.

Although the focus of this paper is on measurement of technical change, discussions on the characters or properties of technical change can not be left out. This is because how to represent technical change in a production model determines the measures to be resulted in as well as outcomes of empirical studies. In fact, the most significant contribution of this paper lies in the assumption-free treatment of technical change in a production function.

Despite the fact that it has been predominantly taken as being exogenous (Solow 1957, 1967, Jorgenson et al. 1987, Baumol et al. 1989, Chavas and Cox 1990), technical change, in my opinion, is largely endogenous in reality. This is because nobody on the earth would do anything for nothing; producers adopting new technologies must be driven by economic or other incentives. Conversely, intuition may suggest existence of exogenous technical change. In particular, the possibility of spillover effects of technology advance can not be excluded. The embodiment and disembodiment classification is also arbitrary. Improvement in management skills may be embodied or equally likely disembodied in various inputs; reorganisation of existing materials can hardly be viewed as embodied technical change. Denison (1985) objects embodiment and Solow (1988) agrees. However, there is no theoretical or empirical evidence favouring Solow's factor-augmentation, or for that matter output-augmentation assumption (see Burmeister and Dobell 1969). The same thing can be said about assuming either neutral or biased technical change. Neutrality is very difficult to comprehend given the interactions between technical invention and innovation and market forces (Blaug 1963, Binswanger 1974); but precluding its existence without proper qualification is not recommended.

All in all, any assumptions on properties of technical change are open to criticism and may be groundless theoretically or empirically. This, of course, do not necessarily imply that studies imposing such assumptions are of little value. One has to start somewhere in order to obtain any results and to promote further works gradually removing those assumptions.

Unfortunately, we have not advanced much since Solow (1957, 1967) in terms of relaxing assumptions on properties of technical change. The most general specification of a production function with technical change has been

$$Y = f(X;T), \tag{1}$$

where Y = output; X = input vector and T = technical change index. It is a common practice to use time as a proxy for T. Main assumptions implied in such a practice include: (a) f is not a concave function unless technical innovations are non-productive (See Romer 1990); (b) technical change is a smooth, continuous and in some cases monotonic function of time; (c) technological progress is exogenous; and (d) parametric specification and estimation of the production function are usually required. None of these assumptions are sensible and some are fatally unrealistic. Specifically, the sensitivity of analytical results and conclusions to parametric model specification and estimation is notorious (Chavas and Cox 1990). Worse still, the formulation in (1) is claimed to be too general to be useful (Solow 1967, Burmeister and Dobell 1969). Hence, further assumptions, such as neutrality, factor-augmenting, are imposed. It is important to emphasise that the formulation in (1) does not allow for changes in functional forms (i.e., changes of f itself).

We challenge the claim by basing our analysis on a even more general formulation:

(2) $$Y_h = f_h(X_h),$$

where h = observation subscript. Unlike in (1) where technical change is a quantifiable input 'T', here it is represented as a 'force', rightly unspecificed, which causes the production function to change across observations. The change of function may be in the functional form and/or in the parametrs of the function. The change may have been accompanied with introduction of new equipment (embodiment) or it may have not (disembodiment). The force or technical change may be internal (endogenous) and/or external (exogenous). As well, the force may affect the effectiveness of inputs (factor augumenting) or on the effectiveness of all inputs combined together (effectiveness of individual inputs remains the same). Both neutral and biased technical change are permissible in our specification. In brief summary, formulation (2) is more general in every sense (probably as general as it could be) than (1) and it does not have any of the four assumptions implied in the latter.

As is known, a production function in essence represents a particular state or combination of technologies, thence technological change should be, as is defined in economics, modelled as change of the production function. A change of a function necessarily means variation in the form of the function and/or in parameter values of the function. Changes in the values of a variable like T in a function like (1) only result in movements along, not shift of, the function. In this sense, postulating (1) and quantifying technical change by letting T change are theoretically inconsistent and logically confusing.

3 The Simple Dual Approach

In this section, we derive dual measures of technical change based on (2). Needless to say, specification of technology and specification of technological change are two different matters. While we impose no restrictions on the properties of technical change, it is impossible to measure technical change in the complete absence of any assumptions on the structure of technology and producer's behaviour (see Diamount et al. 1978). As a consequence, it is assumed that the technologies under study are characterised as displaying constant returns to scale (CRTS) and that factors are paid their marginal revenues (profit maximisation). It should be pointed out that these two assumptions are standard ones prevailing in the literature on production function approach to technical progress (see Solow 1957; Romer 1990).

Now, define the following notations: $X_h = (x_{h1}, x_{h2}, \cdots, x_{hK})$ is an input vector in the h-th time period or from the h-th firm ($h = 1, 2, ..., H$); Y_h is the corresponding output. The technology transforming X_h into Y_h is represented by f_h (noting that fs can be different or same for different observations), so $Y_h = f_h(X_h)$. Let $P_h = (p_{h1}, p_{h2}, \cdots, p_{hK})$ be the real price vector (prices of inputs divided by the output price) corresponding to the input vector X_h, then the actual production cost for producing Y_h is $c_h = \sum_{k=1}^{K} p_{hk}x_{hk} = P_h^t X_h$.

To illustrate our approach, a simplified two-factor, two- firm/time-period production process is depicted in Figure 1, where $X_r (r = 1, 2)$ denote observed input vectors, $I(Y_r)$ denote isoquants with observed output levels Y_r, and C_r denote isocost lines with actual costs $c_r = P_r^t X_r$. Relative prices are assumed to be constant in this section; C_1 is thus parallel to C_2.

Given the assumption of linearly homogenous production functions (i.e., CRTS), the expansion path under technology f_r is a straight line from the origin which passes through X_r. Clearly, assuming constant relative prices, output would expand along a given expansion path without technological progress and any departure from the path implies technical change (this, of course, does not necessarily mean that technical change can not occur along a given expansion path. We shall discuss this case shortly). Therefore, in

Figure 1, $Y_1 = f(X_1)$ must have been produced under a different technology (i.e., f_1) than that for producing $Y_2 = f(X_2)$, namely f_2, where $f_1 \neq f_2$. In the absence of technical change, production expansion from Y_1 to Y_2 would be alone the old expansion path and result in a point, say X_s, rather than X_2 being observed. It is clear that output Y_2 can be produced either under technology f_1 which requires input X_s or under technology f_2 which requires input X_2. Therefore, a dual measure of technical change in terms of differences in costs with which the same output can be produced is

$$TE_{12} = P^t X_s - P^t X_2,$$

where P denotes any appropriate price vector. We use base period prices in this paper, thus $P = P_1$. By so doing, we effectively remove any effects on TE, which can be produced by changes in input prices.[§]

From Figure 1, it becomes very clear that technical change in our framework is considered as a shift of the isoquant with output level Y_2 from $I'(Y_2)$ to $I(Y_2)$. When there is no technical change, no such a shift occurs, hence $X_s = X_2$ and $TE = 0$.

The measure is not operational unless $X_s = (x_{s1}, x_{s2}, \cdots, x_{sK})$ can be obtained. It is important to note that with linearly homogenous production functions, $x_{sk} = \lambda x_{1k}$ $(k = 1, 2, \cdots, K)$, where λ is the proportionate change in inputs assuming there is no technical change. Given CRTS, λ must be equal to the ratio of Y_2 to Y_1. Thus, an operational measure of technical change is

$$\begin{aligned} TE_{12} &= P_1^t \frac{Y_2}{Y_1} X_1 - P_1^t X_2 \\ &= P_1^t (\lambda X_1 - X_2) \end{aligned} \quad (3)$$

where $\lambda = Y_2/Y_1$.

Up till now, we have assumed that technical change is signalled by production expansion being deviated from the initial expansion path. One may

[§]For example, if $Y_2 = Y_3$, $X_2 = X_3$, and there is no technical change from period 2 to period 3, one would expect $TE_{12} = TE_{13}$. Clearly, this can be achieved if P_1 is used. If the current prices P_2, P_3 are used, it is most likely that $P_2 \neq P_3$, which leads to $TE_{12} \neq TE_{13}$.

well ask whether or not TE_{12} is still applicable if X_2 is on the initial expansion path. The answer is yes, as can be seen easily from Figure 2. However, since X_1 and X_2 are on the same expansion path, technical change between X_1 and X_2 (if there is any) must be neutral. In this case, $X_2 = \lambda' X_1$, where λ' represent the proportion at which inputs are altered from X_1 to X_2. Therefore, the measure becomes $TE_{12} = P_1^t X_1 (\lambda - \lambda')$. In the absence of technical change, λ' must be identical to λ, $TE_{12} = 0$. Conversely, if technology progressed, X_2 must be closer to X_1 than X_s does (as is shown in Figure 2), i.e., $\lambda' < \lambda$ and $TE_{12} > 0$.

4 Consideration of Relative Price Changes

So far, it has been maintained that relative input prices are constant. This is unlikely to be the case in reality. When the relative prices change, a rational producer will adjust his/her resource allocation by moving along his/her old production function (i.e., use the old technology). This movement leads to what has been called price-induced technical change. Such a technical change is different to the one commonly discussed and, according to Griliches (1958), its effects (we shall call them substitution effects) should be removed.

Figure 1 can be modified to describe the production process with relative price changes. This is shown in Figure 3, where all notations have their earlier definitions. Now, after allowing for price effects, X_* is the input vector needed to produce Y_1 under P_2, and X_p is the input vector needed for producing Y_2 under P_2 assuming no technical change occurred. X_r and $Y_r (r = 1, 2)$ are still the observed input vectors and output levels.

It can be seen that removing substitution effects is equivalent to finding X_* or X_p and substituting X_* for X_1 or X_p for X_s in the measure of technical change. In other words, Y_2 can now be produced by using either X_p under the old technology or X_2 under the possibly new technology. Thus, the measure of technical change in the case of relative input price changes becomes

$$TE_{12}^p = P_1^t (X_p - X_2)$$

Finding X_* or X_p for the calculation of TE^p could be very difficult even with parametric specification of the underlying production function being given. However, noting that $X_p = (Y_2/Y_1)X_* = \lambda X_*$ and technical change under relative price changes can then be represented by

$$(4) \qquad TE^p_{12} = \lambda P_1^t X_* - P_1^t X_2,$$

it is thus sufficient if $P_1^t X_*$ can be obtained.

Giving up parametric specification implies the impossibility of calculating the exact X_* or $P_1^t X_*$. However, the production response surface between two points, say X_1 and X_*, can be approximated¶. According to Varian (1984, p. 317), the best linear approximation is given by

$$\nabla f_1(X_1)\cdot(X_* - X_1) = f_1(X_*) - f_1(X_1),$$

where $\nabla f_1(X_1)$ is the gradient vector of the underlying production function evaluated at X_1. A better alternative, which is adopted in this paper, is to use the quadratic approximation (Theil 1967; Diewert 1976):

$$(5) \qquad \frac{1}{2}[\nabla f_1(X_1) + \nabla f_1(X_*)]\cdot(X_* - X_1) = f_1(X_*) - f_1(X_1).$$

Following Christensen, et al. (1971), it can be shown that the above equation provides a second-order approximation to any arbitrary twice-continuously-differentiable linearly homogenous function.

Using Euler's Theorem, equation (5) reduces to

$$(6) \qquad \nabla f_1(X_1)\cdot X_* - \nabla f_1(X_*)\cdot X_1 = 0.$$

Since X_1 and X_* are at the equilibrium points under prices P_1 and P_2, respectively, one immediately obtains

$$(7) \qquad P_1^t X_* = P_2^t X_1.$$

¶Approximation is a norm in economics, particularly in the economic theory of measurement

Referring to (4), we have

$$TE^p_{12} = \lambda P^t_2 X_1 - P^t_1 X_2 \tag{8}$$
$$= \frac{Y_2}{Y_1} P^t_2 X_1 - P^t_1 X_2$$

which can be used to measure technical change if relative prices are altered across observations. Obviously, when there is no price change, $P_2 = P_1$, TE^p_{12} rightly reduces to TE_{12}.

5 An Example of Application

To illustrate the proposed approach, we utilise the data presented in Solow (1957) to measure cost savings due to technical change in the US non-farm sector during the period of 1909–1949. Total labour input L in million man-hour is recovered by dividing 'effective capital stock' K (column 3 in Solw's Table) by 'employed capital per man-hour' (column 6 in the Table). Given 'private non-farm GNP per man-hour' (column 5) and L, total output Y can be easily computed. The real price of capital or P_K, which is assumed to be equivalent to the marginal product of K, is obtained by multiplying the 'share of property in income' (column 4) by the ratio $\frac{Y}{K}$. In a similar way, wage or P_L can be calculated.

For the purpose of finding cumulated effects of technical change, 1909 is taken as the base period relative to each of the subsequent years (let us call them current periods). We first compare relative prices (P_K/P_L) in the base period with those in current periods in order to decide what measure, TE_{12} or TE^p_{12}, should be used. Column 6 in Table 1, which is presented here, shows the relative prices and it is not difficult to conclude that TE^p_{12} should be used. Subsequent calculations and the final results are tabulated in Table 1.

The results indicate that effects of technical changes are positive throughout the entire period except from 1909 to 1910. Looking at the cost savings under TE^p in the table, one can not help appreciating the *cumulative* effects of technology advance. Starting with a small number of 1721.21 in 1911, or about 4 per cent of that year's GNP, cost savings (in 1909 prices) due to technical progress had begun to exceed GNP since 1941. In other words, the non-farm sector would have to employ an extra 147,860.92 (= 102467.62/0.693) million man-hour labour input or an extra 632,516.17 (= 102467.62/0.162) million dollars capital stock, or any combinations of labour and capital stock totalled 102467.62 million dollars, in order to produce the output level of 1941, if the technologies prevailed three decades ago were used in 1941. The surprisingly sharp increases in technical effects from 1939 to 1945 reflect the fact that the pace at which technologies are invented and diffused in the United States, particularly in US manufacturing sector, were accelerated during World War II. Similar conclusion can be drawn for the time period of World War I. These become more apparent in Figure 4 where cost savings in 10^{10} US dollars are plotted against time.

In Figure 4, the technical change index or $A(t)$ as is presented in Solow (1957) was also drawn. The trends of the two curves are very similar indeed. Thus, Solow's arguments for the plausibility of his results and the associated methodology are also applicable to ours. Since Solow assumed neutrality and we did not, similarity of the results confirms Solow's assertion that technical change in the US non-farm sector was neutral from 1909 to 1949. The only incidences of noticable dissimilarity between the two curves occur for the periods 1936–38 and 1941–42. While our results indicate little technical change between 1936 and 1938, Solow reported a negative and then a positive technical change during the same period. From 1941 to 1942, our analysis shows a continuing and strong upward trend, Solow's recorded a constancy. We leave it to the experts on American economy to explore the significance and implications of these dissimilarities. Nevertheless, one should not judge tools simply based on the end products made using the tools; our framework is built on a much firmer theoretical ground than Solow's. As is argued earlier in the paper, our methodology is certainly more general than Solow's as well.

6 Concluding Remarks

Without any assumptions on properties of technical change, a remarkably simple and theoretically consistent framework is developed for deriving measures of technical change in this paper. The implementation of these measures requires no parametric specification or estimation of any production, cost or transformation functions. The methodology is operational as long as two or more input-output (time series and/or cross sectional) observations are available. Empirical results based on Solow's data support the applicability and usefulness of our approach.

Further research effort may be devoted to the relaxation of the maintained hypothesis of constant returns to scale. But whether this is possible or not has been and still is a question mark ever since the emergence of the well-known Solow Stigler controversy (Solow 1961; Stigler 1961).

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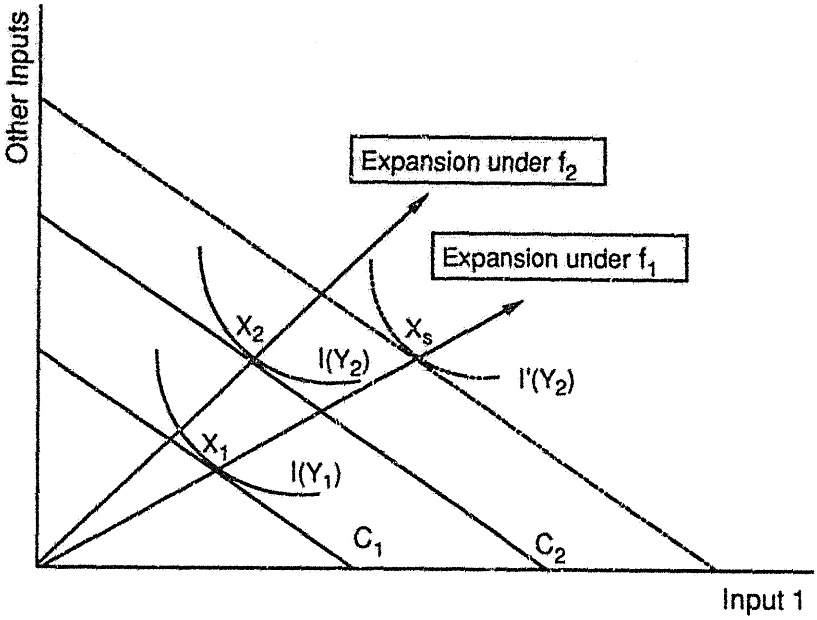

Figure 1: Technical Change without Relative Price Changes

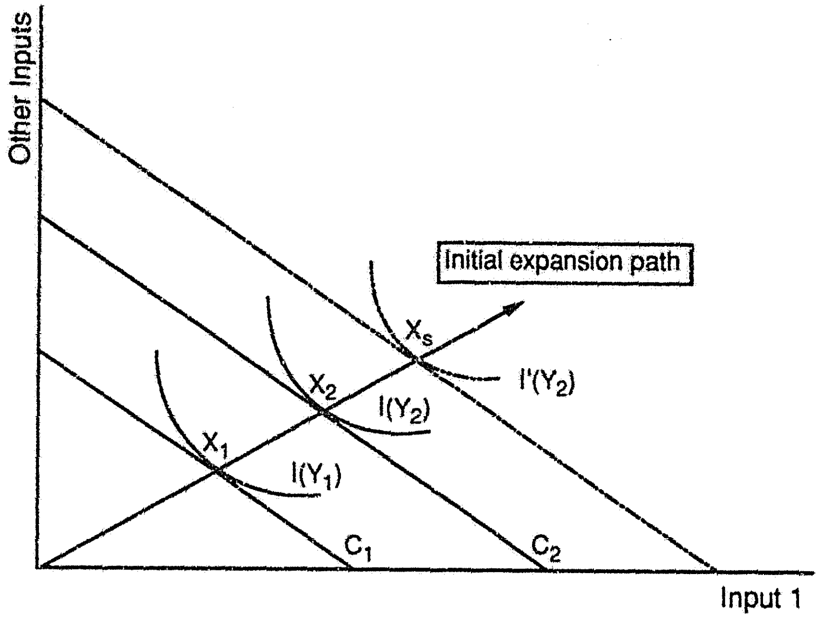

Figure 2: Neutral Technical Change without Relative Price Changes

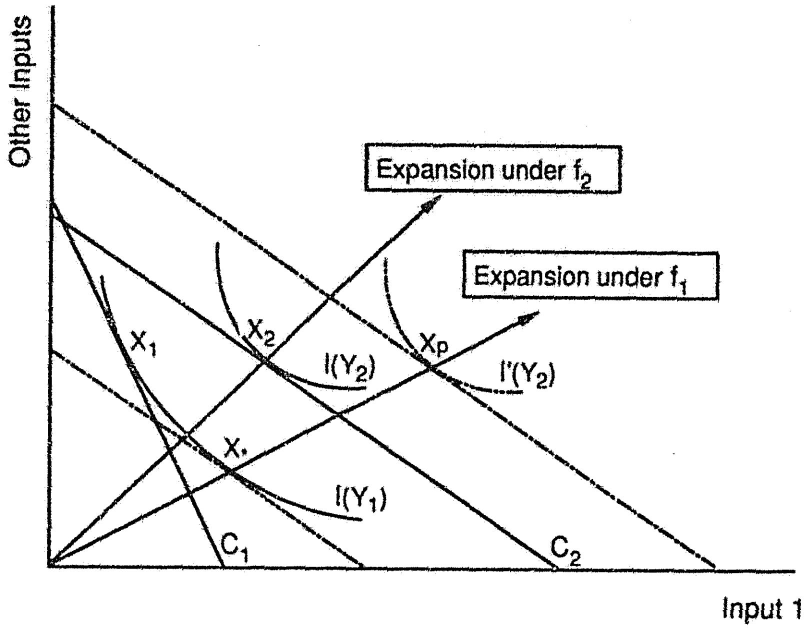

Figure 3: Technical Change under Relative Price Changes

Table 1: Data and Cost Savings due to Technical Change

Year	GNP[1]	Capital[1]	Labor[2]	P_K	P_L	P_K/P_L	λ	$P_2^t X_1$	$P_1^t X_2$	TE^p
1909	40263.64	133135	64628.64	0.101	0.414	0.245	1.00	40263.64	40263.64	0.00
1910	40842.27	139235	66302.38	0.097	0.413	0.235	1.0[illegible]	39561.00	41575.08	-1445.55
1911	42230.91	141640	65271.89	0.100	0.430	0.232	1.05	41104.65	41391.81	1721.21
1912	43891.40	148773	67318.10	0.097	0.437	0.223	1.09	41194.06	42962.21	1943.44
1913	46049.42	151015	67719.73	0.102	0.453	0.225	1.14	42828.49	4[illegible]355.74	5627.08
1914	44449.35	143385	65175.00	0.101	0.460	0.219	1.10	43165.15	41528.46	6124.03
1915	43866.27	148188	65569.91	0.102	0.439	0.232	1.09	41920.33	42178.67	3492.52
1916	49991.67	167115	71416.67	0.10[illegible]	0.449	0.238	1.24	43302.07	46518.51	7245.70
1917	52638.48	171327	77523.53	0.114	0.428	0.266	1.31	42780.81	49475.28	6454.00
1918	57929.89	176412	79164.86	0.112	0.480	0.234	[illegible].14	45952.98	50794.74	15320.75
1919	54922.48	176869	71606.88	0.110	0.495	0.222	1.36	46657.37	47585.52	16058.45
1920	50519.18	180776	70068.22	0.089	0.491	0.182	1.25	43601.30	47343.89	7363.08
1921	46787.92	154947	60763.53	0.111	0.486	0.229	1.16	46235.49	40872.19	12855.25
1922	52828.60	166933	67041.37	0.107	0.521	0.206	1.31	47945.97	44687.40	18220.92
1923	59939.46	193377	74090.80	0.104	0.536	0.195	1.49	48571.56	50287.07	22020.18
1924	59636.70	195460	71335.77	0.101	0.560	0.180	1.48	49604.64	49356.71	24115.46
1925	65539.02	211198	75159.43	0.104	0.579	0.180	1.63	51302.17	52535.30	30971.65
1926	68510.51	226266	78838.33	0.099	0.585	0.169	1.70	50979.12	55586.04	31157.36
1927	69331.60	233228	79600.00	0.096	0.590	0.163	1.72	50892.74	56606.94	31027.35
1928	70608.78	243980	80788.08	0.098	0.579	0.169	1.75	50416.43	58188.47	30224.85
1929	75669.62	258714	84547.06	0.097	0.598	0.162	1.88	51566.90	61238.54	35673.88
1930	67964.00	254865	77231.82	0.093	0.575	0.161	1.69	49457.63	57817.92	25665.29
1931	61363.95	226042	67880.48	0.088	0.610	0.145	1.52	51182.66	51023.56	26981.56

(Table 1 Continued)

Year	GNP	K	L	P_K	P_L	P_K/P_L	λ	$P_2^t X_1$	$P_1^t X_2$	TE^p
1932	51446.69	191974	58528.66	0.106	0.530	0.201	1.28	48419.96	43697.61	18170.77
1933	50458.06	180000	58064.52	0.101	0.554	0.183	1.25	49341.65	42292.20	19542.35
1934	57108.14	186020	62006.67	0.109	0.594	0.183	1.42	52902.04	44535.32	30498.56
1935	61837.47	188201	65575.26	0.115	0.612	0.188	1.54	54907.45	46234.73	38092.91
1936	71129.29	197018	72433.09	0.129	0.631	0.204	1.77	57967.64	49969.17	52435.80
1937	74610.06	208232	76838.38	0.122	0.641	0.190	1.85	57636.80	52930.39	53872.80
1938	69806.47	194062	69806.47	0.119	0.669	0.178	1.73	59088.25	48581.50	53861.84
1939	77218.03	198646	74678.95	0.135	0.675	0.200	1.92	61595.50	51064.56	67063.92
1940	85567.28	207987	79082.51	0.147	0.696	0.211	2.13	64517.66	53835.30	83276.00
1941	98369.76	228232	88462.02	0.162	0.693	0.235	2.44	66406.27	59772.26	102467.62
1942	108771.57	252779	95749.62	0.153	0.732	0.209	2.70	67675.95	65278.42	117547.04
1943	118336.44	262747	100285.11	0.154	0.776	0.198	2.94	70687.12	68167.34	139584.89
1944	125651.06	261235	99328.90	0.160	0.845	0.189	3.12	75872.58	67617.99	169158.15
1945	122934.86	252320	94857.14	0.153	0.889	0.172	3.05	77826.33	64862.16	172760.88
1946	118891.15	244632	97852.80	0.152	0.836	0.181	2.95	74211.90	65324.35	153809.77
1947	122493.89	256478	102591.20	0.156	0.804	0.194	3.04	72725.51	68487.60	152764.86
1948	126690.96	264588	103760.00	0.159	0.816	0.195	3.15	73877.32	69793.48	162664.09
1949	127077.36	269105	99668.52	0.154	0.859	0.179	3.16	76034.02	68556.03	171417.34

Data Source: Calculated based on Solow (1957).

[1] in million dollars.

[2] in million man-hour.

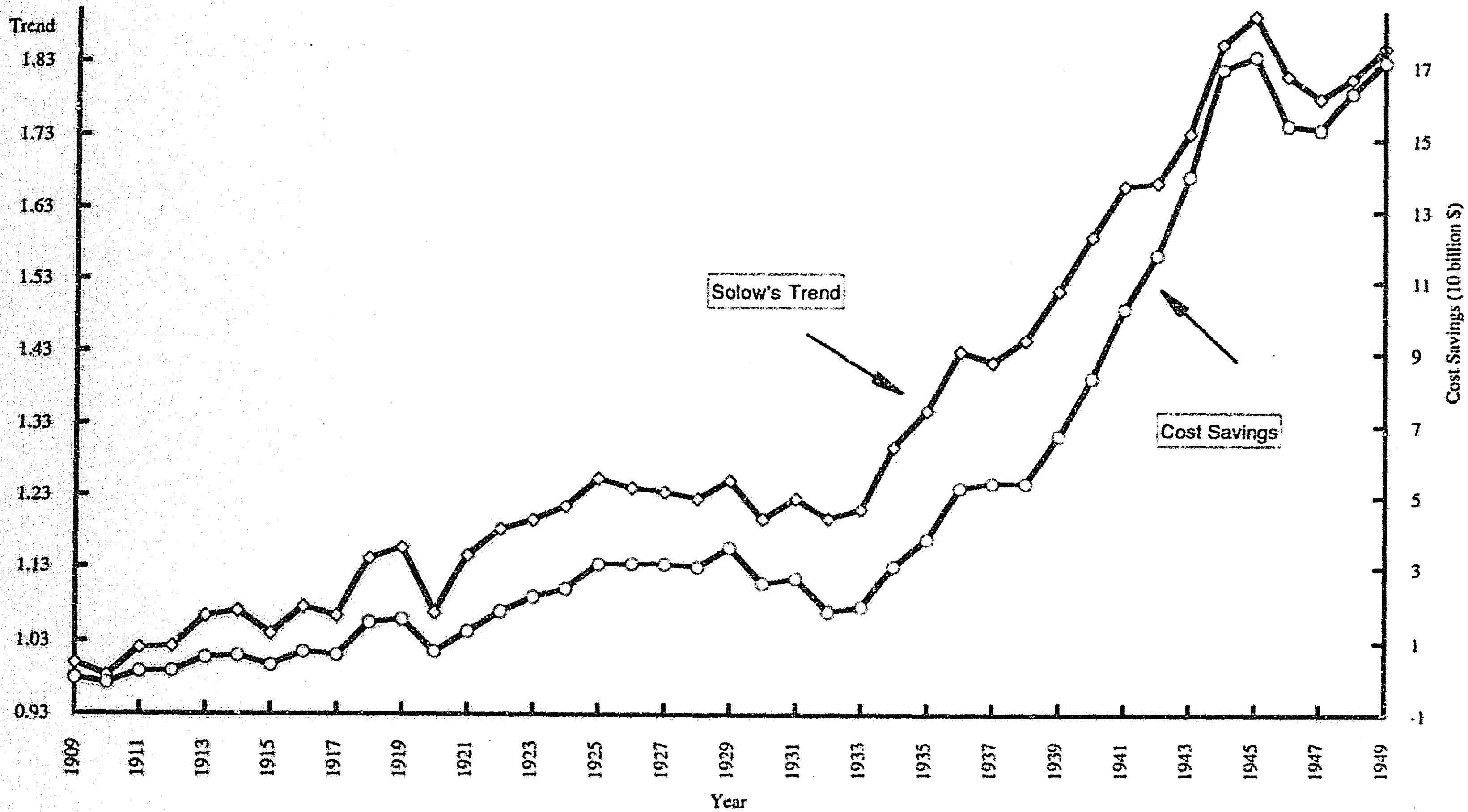

Figure 4: Cost Savings due to Technical Change in US Nonfarm Sector