# Optimal Hedging Levels and Hedging Effectiveness in Cattle Feeding

By Richard G. Heifner

Optimal hedging level, minimum-risk hedging level, and hedging effectiveness are defined in a manner consistent with portfolio theory and used to analyze hedging potential in cattle feeding. Estimated upper limits on optimal hedging levels ranged from 0.56 to 0.88 unit of short futures per unit of four types of slaughter cattle produced at five locations. When futures trading costs are taken into account, optimal hedging levels are depressed below these limits, depending upon the resource availabilities and profit expectations of individual firms. Location, grade, and sex of the cattle fed have small effects on optimal hedging levels and hedging effectiveness.

Key words: Futures trading; hedging; cattle feeding; risk; price analysis.

Within the last decade, the introduction and growth of active trading in live cattle futures has opened new hedging opportunities for cattle feeders. The hedging of inventories in commodity futures contracts is a well-established business practice, having been employed by grain merchants in the Midwest at least since the 1870's. But experience in trading cattle futures has been limited and the desirability of hedging remains subject to question—particularly among feeders who are not in position to deliver on the contract.

Cattle feeders have had a hedging market available in the Chicago Mercantile Exchange live beef cattle futures contract since November 30, 1964. Contracts are traded for delivery every second month beginning in February. The contract calls for delivery of Choice Grade steers weighing 1,050 to 1,250 pounds, but provision is made for substituting a limited number of high Good Grade steers at appropriate discounts. With the August 1969 future, the contract size was changed from 25,000 to 40,000 pounds. Par delivery was at Chicago until the August 1971 contract, with alternative delivery points at Omaha and Kansas City at discounts of 75¢ and $1 per hundredweight, respectively. For the August 1971 contract and subsequent contracts, par delivery is at Omaha with allowances at alternative delivery points as follows: Chicago, +50¢; Peoria, +50¢; Guymon, Okla., -$1.

Substantial numbers of feedlot cattle are hedged, but these represent a small fraction of the cattle on feed in the United States. The average number of open contracts in live beef futures was 13,638 during 1970-71 (*33*, p. 7).[1] With the change in contract size, this represented almost as many cattle as did open positions during the peak trading year, 1968-69. However, not all of the short open positions represent hedging. In a survey conducted on May 29, 1969, the Commodity Exchange Authority (*34*, p. 35) found about half of the total short positions classified as hedging. The 13,049 short hedging positions reported would have covered less than 5 percent of the over 11 million head of cattle on feed reported for April 1, 1969 (*38*, p. 6).

Economic theory suggests at least two types of benefits that may arise through futures trading. First, futures trading provides for shifting risks from production and marketing specialists to others who are willing to bear the risks at lower costs.[2] In this respect, it serves as an alternative to other types of capital markets such as the stock market or contracting.

Another type of benefit arises when the forward prices generated by futures trading enable producers and marketing firms to better coordinate their expectations and plans. This can result in an improved allocation of production resources over time. In this function, futures trading serves as an alternative or supplement to other coordinating arrangements such as integration through ownership, cooperatives, marketing orders and agreements, and Government control.[3]

This study focuses upon the risk-shifting aspect of futures trading. Its purpose was to measure the potential for hedging as a means for shifting the price risks associated with cattle feeding. To this end, the concepts of optimal hedging level, minimum risk hedging level, and hedging effectiveness are defined within the general framework for decisionmaking under risk that is provided by portfolio theory. Estimates of minimum risk hedging levels and hedging effectiveness for four types of

Footnotes are at end of article, p. 35.

cattle in five major cattle feeding regions are presented. Finally, the impact of futures trading costs and hedgers' profit expectations on optimal hedging levels is examined.

## The Importance of Risk Aversion

If futures trading is to result in improved distribution of risk, this benefit must be reflected in gains or potential gains to individual traders. Otherwise, individuals would have no motivation to use futures markets. For cattle feeders, as for other traders, the motives for trading in futures include making profits and reducing risk. By making profits, we mean obtaining long-run average returns which exceed costs. Reducing risk involves reducing profit variability, i.e., increasing profit stability. Businessmen buy insurance and hold liquidity reserves, and bankers require their borrowers to hold prescribed levels of equity, because risk is a factor in most business operations. Cattle feeding is no exception.[4]

Early students of futures trading concentrated on the risk-shifting aspects of hedging using several different measures of risk. For example, in his studies of the protection afforded by hedging, Howell employed tabular comparisons of the distribution of cash price changes and the distribution of basis changes.[5] Graf (*8*) measured hedging effectiveness in terms of percentage reduction in the gain or loss for holding grain stocks over selected intervals.

Emphasis was diverted from the risk-shifting aspect of hedging when Working (*40*) concluded that much hedging could not be explained simply as risk avoidance. He introduced a multipurpose concept of hedging and listed a variety of hedging categories. Taking note of Working's argument, Gray (*9*) proposed that lack of price bias be used as a criterion of hedging effectiveness.[6]

Recent students of futures trading have been concerned with the effect of hedging on both risk and expected return. D'Arge and Tomek (*4*) compared mean incomes and standard deviations of incomes for Long Island potato producers using various hedging and marketing strategies. Heifner (*15*) analyzed the impact of hedging on the mean and variance of returns from grain storage. Tomek and Gray (*32*) studied the effectiveness of hedging in reducing the variance of income from crop production and Gum and Wildermuth (*11*) have measured the effect of hedging in reducing price variability in cattle feeding.[7]

Modern developments in the general theory of risk bearing provide a framework for analyzing hedging decisions that simultaneously takes into account risk and expected return. The newer approach has its roots in the portfolio theory of Markowitz (*25*) and Tobin (*3*). Johnson (*22*) demonstrated how futures trading can be viewed as a problem of balancing risk against expected return. Ward and Fletcher (*39*) extended Johnson's theoretical framework to various special cases in hedging. In a previous article (*14*), I demonstrated application of the portfolio approach in managing seasonal grain inventories and Helmuth (*18*) has applied the approach to speculation in soybeans.

## The Optimal Hedge

When we examine hedging in the context of portfolio analysis, we must conclude that the traditional illustration of hedging by holding one unit of the futures position for each unit of cash position can be misleading. This view of hedging is strictly applicable only when cash profits and futures profits are perfectly correlated. The portfolio approach provides a procedure for determining optimal hedging levels when this restrictive assumption is violated. It rests on the assumption that traders maximize expected profits relative to risk, or equivalently, minimize risk relative to expected profit, thereby avoiding arbitrary distinctions between hedgers and speculators. Furthermore, it leads to conclusions about hedging policies that are of considerable generality.

Like other problems of decisionmaking under risk, the hedging problem can be described as a problem of setting levels for activities with uncertain rates of return. In the hedging problem, the activities include cash activities and futures activities. The cash activities may involve holding a commodity in inventory over a prescribed time period or the transformation of one or more commodities into another commodity over time. The futures activities involve holding a long or short position in a specific futures contract over a designated period.

Let

$$R = \sum_k x_k r_k \tag{1}$$

be the total profit obtained by the firm in a particular time period where

$x_k$ = level of activity $k$, a constant set by the decisionmaker, and

$r_k$ = profit per unit of activity $k$, a random variable with mean $\mu_k$, variance $\sigma_{kk}$, and covariances $\sigma_{kh}$ for $h = 1,2, \ldots ,n$.

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$x_k$ = level of activity $k$, a constant set by the decisionmaker, and

$r_k$ = profit per unit of activity $k$, a random variable with mean $\mu_k$, variance $\sigma_{kk}$, and covariances $\sigma_{kh}$ for $h = 1,2,\ldots,n$.

Profit, $r_k$, is defined to equal revenue minus variable costs minus economic rents. Rents are imputed through the production process to those fixed resources which are limiting for the firm. In the absence of limiting resources, profit equals revenue minus variable costs.

For a particular activity, the mean profit rate, $\mu_k$, may be viewed as having two components, a market rate of return for bearing risk and a residual return to the nonlimiting resources of the firm. In long-run competitive equilibrium, profit would approach the market return for bearing risk and, therefore, be the same for all firms. In the short run, profit includes not only the market return for risk bearing but also a return on the firm's fixed resources which are not limiting. It is in this short-run situation when the firm has nonlimiting resources committed to production where hedging becomes important.

We shall assume that the hedger seeks the best possible combination of expected total profit and variance of total profit.[8] Mathematically, this may be described as a problem in maximizing

$$(2) \qquad \psi = \sum_k x_k \mu_k - \lambda \sum_k \sum_h x_k x_h \sigma_{kh}$$

where $\lambda$ is an unknown weight assigned to the variance of total profit relative to mean profit. In general, $\lambda$ may differ from individual to individual depending upon differences in the degree of risk aversion among individuals.

Without knowledge of $\lambda$, direct maximization of $\psi$ is impossible. However, in the hedging problem we are primarily concerned with the optimal level of the futures position relative to the cash position. We shall see that knowledge of $\lambda$ is not required in order to determine the relative levels of the various activities that will prevail when $\psi$ is maximized.[9]

When $\psi$ is at a maximum, the partial derivatives of $\psi$ with respect to the levels of the activities will be zero, i.e.,

$$(3) \qquad \frac{\partial \psi}{\partial x_k} = \mu_k - 2\lambda \sum_h x_h \sigma_{kh} = 0 \qquad k = 1,2,\ldots,n$$

Let $x_1$ represent the level of the futures position and let $x_2$ represent the level of the cash position. Combining the first two equations in (3) and eliminating $\lambda$ we obtain

$$(4) \qquad x_1 = \frac{(\mu_1 \sigma_{22} - \mu_2 \sigma_{12}) x_2 + \sum_{h>2} (\mu_1 \sigma_{2h} - \mu_2 \sigma_{1h}) x_h}{(\mu_2 \sigma_{11} - \mu_1 \sigma_{12})}$$

Equation (4) provides a general condition for specifying the optimal level of the futures position given the levels of the other activities of the firm.

The absence of $\lambda$ in equation (4) shows that the optimal level of the futures position is independent of the degree of risk aversion, so long as the levels of the other activities and the means, variances, and covariances of their profits remain constant. Thus, the optimal hedging level is the same for all risk-averse firms with the same mix of production activities and the same set of profit expectations and profit variances and covariances regardless of their differences in degree of risk aversion. Consequently, a single estimate of the optimal hedging level applies to a group of similar firms.

The second term in the numerator of equation (4) introduces the effects of other activities of the firm on the optimal level of hedging. Because the mix of production activities differs from firm to firm, the exact solution to equation (4) is specific to each individual firm. However, the second term vanishes if profits on the other activities are uncorrelated with profits on activities 1 and 2. This situation is approached for the cattle-feeding specialist who has no other production activities and for the feeder whose other activities, such as crop production, produce profits which are not highly correlated with cattle-feeding profits. In the empirical portions of this study, we assume that the effects of other activities on optimal hedging levels are negligible. Under this assumption, equation (4) reduces to the following expression for the optimal ratio of the futures position to the cash position:

$$(5) \qquad \frac{x_1}{x_2} = \frac{\mu_1 \sigma_{22} - \mu_2 \sigma_{12}}{\mu_2 \sigma_{11} - \mu_1 \sigma_{12}}$$

## Optimal Hedging Levels for 2 Types of Feeders

Cattle feeders can be divided between those that have fixed resources committed to cattle feeding and those that have no such fixed commitments. In the former category are individuals who own feedlot facilities or possess cattle-feeding skills which are not readily marketable. In the latter category are individuals who hire custom feeding services.

Optimal futures trading strategies differ markedly between the two types of individuals. Equations (4) and (5) apply to both types, but the value of $\mu_2$, the expected profit from cattle feeding, differs between the two groups. The expected cash profit, $\mu_2$, includes returns to fixed resources so long as these resources are not limiting. Hence, it tends to be larger for the feedlot owner feeding his own cattle than for the custom feeder

who must subtract the costs of feeding services in determining profits.

For the custom feeder, virtually all costs are subtracted from returns and $\mu_2$ tends to approach the market price for risk bearing. To the extent that the price risk in custom feeding is the same as the risk in holding a futures contract, competition would tend to force $\mu_1$ and $\mu_2$ to approach equality and the ratio $x_1/x_2$ would tend to approach 1. However, if at the same time the correlation between cash profits and futures profits approaches unity, the optimal hedging level becomes indeterminate. The data available and the methods used in this study do not permit sufficiently accurate estimates of these parameters to justify any conclusions about optimal futures positions as this situation is approached. Consequently, this study has little to say about futures trading for the custom feeder.

For the individual with fixed resources in cattle feeding, $\mu_2$ may exceed the competitive rate of return for risk bearing. In contrast, the expected futures profit, $\mu_1$, includes only the market return for risk bearing minus futures trading costs. This market return for risk bearing is commonly called a risk premium. Since both the risk premium and futures trading costs tend to be small, $\mu_2$ tends to dominate and the solution to equation (5) tends to be negative, implying a short position in futures. This is the situation with which we are concerned.

The empirical results derived in this study are strictly applicable to the feeder who has resources in cattle feeding which are not used to full capacity. If all of his feeding resources are used to full capacity, his optimal futures position lies somewhere between the position that would be optimal for the feeder with excess capacity and the position that would be optimal for the custom feeder. In this sense, the estimates presented represent upper limits on the optimal short futures positions for cattle feeders.

## The Minimum-Risk Hedge

The condition for the optimal hedge can be simplified if the market rate of return for risk bearing is zero and if futures trading costs are negligible. Under these assumptions, the profit rate on the futures activity, $\mu_1$, is zero and equation (5) reduces to

$$x_1/x_2 = -(\sigma_{12}/\sigma_{11}) \tag{6}$$

Equation (6) also defines the hedging ratio that minimizes risk given the level of the cash activity. This can be shown as follows: The variance of total profit for activities 1 and 2 is

$$V = x_1^2\sigma_{11} + 2x_1x_2\sigma_{12} + x_2^2\sigma_{22} \tag{7}$$

Differentiating with respect to $x_1$ we obtain

$$\frac{\partial V}{\partial x_1} = 2x_1\sigma_{11} + 2x_2\sigma_{12} \tag{8}$$

Noting that the second derivative is positive, indicating a minimum, we set (8) equal to zero and find that it reduces to equation (6). The hedging ratio specified by equation (6) will be referred to as the minimum-risk hedge. It is also the optimal hedge when the expected costs or returns from hedging are zero and profits from other activities are uncorrelated with profits from activities 1 and 2.

To estimate the minimum-risk hedge, the sample ratio of the covariance and variance, $s_{12}/s_{11}$, may be employed where these are calculated individually by the standard formulas. This is exactly equivalent to the standard procedure that one would use for calculating the regression of unit cash profits on unit futures profits. Therefore, the standard least-squares regression algorithm provides a convenient means to approximate the minimum-risk hedge.

Unfortunately, as is the case for many ratio estimates, the properties of $s_{12}/s_{11}$ as an estimator of $\sigma_{12}/\sigma_{11}$ are not easily specified. The estimate is consistent but apparently biased in small samples. Examination of the first few terms of the Taylor expansion of $s_{12}/s_{11}$ suggests that, when profits are from a bivariate normal distribution, the bias is positive and small with an upper limit of approximately 0.12 for the size of sample used here.[10]

## Hedging Effectiveness

Following Johnson (22, p. 144) we can define a measure of hedging effectiveness as the proportional reduction in profit variance obtained through hedging. Let $H = x_1/x_2$ represent the size of the futures position relative to the cash position. Assuming once again that the cash and futures profits are uncorrelated with profits from other activities, hedging effectiveness is represented as

$$Z = 1 - (\sigma_{22} + 2H\sigma_{12} + H^2\sigma_{11})/\sigma_{22} \tag{9}$$

This simplifies to

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With complete hedging, $H = -1$, we have

$$Z_c = (2\sigma_{12} - \sigma_{11})/\sigma_{22} \tag{11}$$

In this case, we note that hedging effectiveness exceeds zero, i.e., hedging reduces risk, if and only if the numerator is positive, i.e., if and only if

$$\sigma_{12}/\sigma_{11} > 0.5 \tag{12}$$

The term on the left is identical to the negative of the minimum-risk hedge as shown in equation (6).

The effectiveness of the minimum-risk hedge is

$$Z_m = -[2(-\sigma_{12}/\sigma_{11})\sigma_{12} + (-\sigma_{12}/\sigma_{11})^2\sigma_{11}]/\sigma_{22} \tag{13}$$

This reduces to

$$Z_m = (\sigma_{12})^2/(\sigma_{11}\sigma_{22}) \tag{14}$$

which is the square of the correlation between cash profits and futures profits. Thus, the $r^2$ between cash profits and futures profits measures the effectiveness of the minimum-risk hedge.

As we depart from the assumption of zero hedging costs under which the minimum-risk hedge is optimal, the notion of hedging effectiveness loses its usefulness. The minimum-risk hedge is the most effective hedge possible in that it minimizes the variance of total profit relative to the variance of cash profit. Thus, where they differ, the optimal hedge is less effective than the minimum-risk hedge. This situation arises because the definition of hedging effectiveness disregards expected losses or expected profits on the futures position.[11]

## Data Sources and Assumptions

Determination of optimal or minimum-risk hedging levels requires estimates of the variances and covariances of profits for the individual production and futures holding activities. The major source of profit variability in cattle feeding is the variation in prices of cattle and feed.[12] In this study, the variances and covariances in prices for feeder cattle, slaughter cattle, grain, and hay are taken into account. Prices on other inputs such as interest on borrowed capital, trucking, veterinary expense, etc., are assumed to be constant. Therefore, they do not enter into the calculation of the variances and covariances of profit.

The analysis is based upon profit observations for 18 consecutive 4-month feeding periods beginning in March 1965 and ending in March 1971. Profit from cattle feeding for each period is calculated by subtracting variable costs from returns. Costs include the value of the feeder, the grain, and the roughage priced at the beginning of the feeding period. Returns equal the value of the finished animal priced at the end of the feeding period. Risk is measured as the variance of profit after adjustment for seasonality. The adjustment for seasonality is accomplished by performing the calculations using a regression program and inserting dummy variables for two of the three seasonal feeding periods.

Table 1 lists the cattle-feeding locations analyzed. These were selected to represent the major cattle-feeding

Table 1.—Cattle and feed pricing points for the cattle feeding locations analyzed

| Feeding location | Slaughter cattle market | Feeder cattle market | Grain market | Hay price [a] |
|---|---|---|---|---|
| Eastern Corn Belt | Chicago | Kansas City | Corn, Chicago | Illinois |
| Western Corn Belt | Omaha | Omaha | Corn, Omaha | Iowa |
| Colorado | Denver, direct | Amarillo, auction | Corn, Denver | Colorado |
| High Plains | Clovis, N.Mex., direct | Clovis, N.Mex., auction | Sorghum, Ft. Worth | New Mexico |
| California | Visalia, direct | Visalia, auction | Barley, Stockton<br>Sorghum, Los Angeles | California |

[a] Hay prices are State averages as reported in Agricultural Prices (*36*).

regions in the United States and to take advantage of price data collected by Livestock Market News. Shown on the table for each location are the markets used as sources of price quotations for slaughter cattle, feeder cattle, grain, and hay.

To avoid confusing differences due to location with differences due to type of cattle, the same weight, sex, and grade categories were analyzed for each location so far as possible. Previous studies (*2, 3, 5, 12*) suggest that short-fed Good and Choice steers and heifers are among the most numerous types of fed cattle produced in each of the regions. Good Grade feeder cattle were assumed to finish out to Good Grade slaughter cattle and Choice feeders were assumed to finish as Choice slaughter cattle. Feed requirements, costs and rates of gain are assumed to be the same for Good Grade cattle as for Choice Grade cattle. The assumptions about buying and selling weights and feed consumption are shown in table 2.

Buying prices for feeders and selling prices for slaughter cattle are weekly averages reported by USDA's Market News Service for the markets selected.[13] These are calculated by Market News Service as a simple average of the daily prices for each week. The weeks selected are those that include the 15th of the month. The futures quotation used was the closing price on Wednesday.

Grain prices are Thursday prices for the weeks selected as reported in Grain Market News.[14] Hay prices are State estimates of monthly prices received by farmers as reported in Agricultural Prices (*36*).

For each feeding period, the futures contract selected for hedging was the one maturing the month after the cattle were sold. Since contracts mature only once every 2 months and a contract cannot be held beyond maturity, many cattle must be hedged in contracts maturing a month or more after the cattle are sold. This choice also avoids any sharp price movements that may tend to characterize delivery-month pricing in live cattle futures.

Profits from futures transactions were calculated under the assumption that hedging positions are terminated by buying back the futures rather than by delivery. Returns from futures trading equal the futures price change over the feeding period times the amount of the hedge. When futures trading costs were introduced, commissions and interest on margin deposits were included. The round term commission for trading live cattle futures is $36 per 40,000-pound contract. For this study, a margin of $500 per contract was assumed with interest at 7½ percent annually. On this basis, futures trading costs are 12¢ per hundredweight or $1.21 per head for 1,000-pound steers hedged over a 4-month period.

## Price Bias in Cattle Futures

Haverkamp (*13*) noted a tendency for cattle futures to sell at a discount below the ultimate cash price and suggested that this reflects a risk premium demanded by speculators.[15] The task of measuring price bias or risk premiums in live cattle futures is particularly difficult because of the cycle in cattle prices and the shortness of the data series available.

Analysis of cattle futures price movements over the 6-year period, March 1965 to March 1971, shows that

Table 2.—Assumed buying and selling weights and feed consumption for Good and Choice short-fed steers and heifers

| Item | Steers | Heifers |
|---|---|---|
| Initial weight . . . . . .pounds | 692 | 667 |
| Days on feed . . . . . days | 122 | 122 |
| Daily gain [a] . . . . . .pounds | 2.87 | 2.65 |
| Total gain . . . . . .pounds | 350 | 323 |
| Finished weight . . . . . .pounds | 1,042 | 990 |
| Weight after shrink [b] . . . . . .pounds | 1,000 | 950 |
| Grain consumed per head: [a] | | |
| Corn Belt and Colorado, corn . . . . . .bushels | 37.9 | 35.6 |
| High Plains, grain sorghum . . . . . .bushels | 43.1 | 40.5 |
| California: | | |
| Grain sorghum . . . . . .bushels | 21.5 | 20.2 |
| Barley . . . . . .bushels | 24.5 | 23.0 |
| Hay consumed per head [a] . . . . . tons | .26 | .25 |

[a] Rates of gain are based on National Research Council data (*27*, p. 22). Feed consumption is based upon TDN requirements reported in the same publication.
[b] A 4 percent shrink is assumed.

holders of long positions have, indeed, gained on the average at the expense of holders of short positions. This gain is most pronounced during the last month of trading where the price increase has averaged 44¢ per hundredweight. Holding a short position in the near cattle futures contract over 18 successive 4-month feeding intervals from March 1965 to March 1971 would have resulted in an average loss of 59¢ per hundredweight, or about 15¢ per hundredweight per month. This amounts to $5.90 per head for hedging a 1,000-pound steer over a 4-month feeding period. However, the standard error of this estimate is $4.93 per head, so we are unable to reject the hypothesis that the bias is zero.

The estimated bias in the futures price can be adjusted for trend in the general level of cattle prices. The cash price of slaughter cattle increased approximately $6.50 per hundredweight over the 6-year period analyzed. This amounts to an average increase of 9¢ per month. The adjusted estimate of the bias is 15 − 9 = 6¢ per hundredweight per month, or about $2.30 per head, for 1,000-pound steers hedged over a 4-month feeding period.

To recapitulate, cattle futures price movements have favored the holders of long positions in the past, but this observed bias is not significantly different from zero from a statistical standpoint. Part of the observed bias can be attributed to the general rise in cattle prices that occurred over the period studied. The empirical evidence is simply insufficient to permit a firm conclusion about the existence or magnitude of the price bias in cattle futures. Consequently, in a subsequent section of this study, optimal hedging levels are reported for alternative assumptions about the bias.

## Minimum-Risk Hedging Levels for 4 Types of Cattle at 5 Locations

Table 3 summarizes the estimates of minimum-risk hedging levels and hedging effectiveness for four types of cattle at five locations. These estimates were based on observations for 18 consecutive 4-month feeding periods starting in March 1965 and ending in March 1971. They were obtained using a least-squares regression algorithm where cash profits were entered as the dependent variable and futures profits plus dummy variables representing two of the three seasons were used as independent variables. The regression coefficient associated with the futures price variable is reported as the estimate of the minimum-risk hedge. As noted earlier, this estimate probably has a slight upward bias. The standard error of the coefficient as provided by the least-squares algorithm is also shown to provide an indication of the precision of the estimate. The square of the corresponding partial correlation coefficient is presented as an estimate of the effectiveness of the minimum-risk hedge, and the Durbin-Watson statistic is shown as an indication of the degree of serial interdependence in the sample.

The estimated optimal hedging levels range from −0.56 to −0.88. These may be interpreted as 0.56 to 0.88 unit of short futures per unit of slaughter cattle produced. The corresponding estimates of hedging effectiveness range from 36 to 57 percent. All the correlation coefficients between cash profits and futures profits are significantly different from zero at the 1 percent level, except one which is significantly different from zero at the 5 percent level.[16] We conclude that hedging at the minimum-risk level can reduce profit risk in all of the situations studied.

The table shows that location, grade, and sex have little impact on hedging effectiveness. The highest correlation was 0.73 for Choice steers in the Eastern Corn Belt and the lowest correlation was 0.60 for Good heifers in the Western Corn Belt. Sample correlations differing by this amount can be expected to arise more than half the time in samples of this size when the parent populations have identical correlations.[17] Hence, the evidence examined here does not reveal any statistically significant differences in hedging effectiveness among the cattle feeding situations studied.

## Impact of Futures Trading Costs and Profit Expectations

The preceding results are applicable for the cattle feeder who expects his average profit from futures trading to be zero. This is a reasonable profit expectation if he is unable to forecast futures price changes and if he considers futures trading costs to be negligible. However, most hedgers will want to take futures trading costs into account and some may believe that futures price movements can be predicted. In this section, we explore how such variations in the hedger's profit expectations affect his optimal level of hedging.

When expected profits from futures trading are nonzero, we must resort to equation (5) to determine the optimal hedging level. In contrast to equation (6), we note that in equation (5) the expected returns for both the cash activity, $\mu_2$, and the futures activity, $\mu_1$, must be used to calculate the optimal hedging level. In other words, under these more general circumstances the optimal hedging level depends not only upon the variance and covariance of futures profits and cash profits, but also upon the levels of profit expected in both activities.

Table 3.—Estimated minimum-risk hedging levels and hedging effectiveness for four types of short-fed cattle at five locations [a]

| Item | Eastern Corn Belt | Western Corn Belt | Colorado | High Plains | California |
|---|---|---|---|---|---|
| **Choice steers:** | | | | | |
| Min. risk hedge [b] | −0.88 | −0.80 | −0.84 | −0.74 | −0.76 |
| Standard error | 0.22 | 0.22 | 0.19 | 0.22 | 0.21 |
| Effectiveness [c] | 0.53 | 0.47 | 0.57 | 0.44 | 0.48 |
| Durbin-Watson | 2.30 | 2.63 | 1.85 | 1.85 | 2.50 |
| **Good steers:** | | | | | |
| Min. risk hedge | −0.82 | −0.64 | −0.66 | −0.71 | −0.72 |
| Standard error | 0.19 | 0.19 | 0.18 | 0.21 | 0.22 |
| Effectiveness | 0.57 | 0.44 | 0.49 | 0.44 | 0.44 |
| Durbin-Watson | 1.96 | 2.74 | 2.57 | 2.13 | 2.43 |
| **Choice heifers:** | | | | | |
| Min. risk hedge | −0.86 | −0.75 | −0.83 | −0.70 | [d] 0.76 |
| Standard error | 0.21 | 0.24 | 0.19 | 0.22 | 0.23 |
| Effectiveness | 0.54 | 0.42 | 0.57 | 0.42 | 0.45 |
| Durbin-Watson | 2.12 | 2.84 | 2.73 | 2.40 | 2.37 |
| **Good heifers:** | | | | | |
| Min. risk hedge | −0.68 | −0.56 | −0.69 | −0.63 | ([e]) |
| Standard error | 0.21 | 0.20 | 0.17 | 0.18 | ([e]) |
| Effectiveness | 0.43 | 0.36 | 0.53 | 0.45 | ([e]) |
| Durbin-Watson | 1.65 | 2.88 | 2.84 | 2.58 | ([e]) |

[a] Based upon observations for 18 consecutive 4-month feeding periods starting in March 1965 and ending in March 1971.

[b] The number of units of short futures per unit of slaughter cattle produced that minimizes price risk.

[c] The proportional reduction in the variance of profit obtained through hedging at the minimum-risk level.

[d] Prices for 700- to 900-pound heifers were used in the absence of a complete series of prices for 900- to 1,000-pound heifers.

[e] Prices not available.

Profits from cattle feeding are a residual after costs are subtracted from returns. Some of the costs are difficult to estimate and tend to vary from firm to firm depending upon the size of the feeding operation and the technology employed. Also, there is evidence that feeding is more profitable during certain seasons of the year than during other seasons, particularly in the Western Corn Belt, Colorado, and the High Plains. In these regions, the March-to-July feeding period has been most profitable, followed by the November-to-March feeding period and finally by the July-to-November feeding period.

Because expected profits from cattle feeding differ so much from firm to firm, no attempt is made here to prescribe optimal hedging levels for all of the situations studied. We shall instead illustrate how the optimal hedging level varies as expected profit varies for a particular situation. The situation selected is Choice steers in the Western Corn Belt fed from November to March.

For the 6-year period analyzed, the average return per head over costs of the feeder, grain, and roughage for Choice steers fed from November to March in the Western Corn Belt was just under $20. Other variable costs, including protein supplement, veterinary expense, labor, marketing expense, interest, and insurance on the cattle and feed probably amount to $10 to $15 per head for the typical feeder. Thus, the net profit tended to fall in the range of $5 to $10 on the average.

Table 4 shows calculated optimal hedging levels for Choice steers in the Western Corn Belt under two assumed levels of profit from cattle feeding and three assumed levels of futures trading costs. The three levels of futures trading costs are zero; $1.21 per head, representing only commissions and interest on margin deposits; and $3.51 per head, which includes these costs plus the adjusted estimate of the downward bias in futures prices reported previously.

Table 4.—Estimated optimal hedging levels and hedging effectiveness for short-fed Choice steers in the Western Corn Belt under alternative assumptions about expected profits and expected futures trading costs [a]

| Expected cost of hedging per head | Expected cash profit per head | |
|---|---|---|
| | $5 | $10 |
| 0 | -0.80 (0.47) | -0.80 (0.47) |
| $1.21 | -0.59 (0.44) | -0.70 (0.47) |
| $3.51 | [b] 0 (0) | -0.45 (0.39) |

[a] Figures in parentheses represent the proportional reduction in the variance of profits.

[b] In this situation, the gains from hedging are insufficient to cover hedging costs and the optimal solution calls for a zero position in futures.

The major conclusion from the table is that optimal hedging levels are quite sensitive to futures price bias, futures trading costs, and expected cash profits. Futures trading costs alone reduce optimal hedging ratios from -0.80 to -0.70 or -0.59 depending upon the cash profit level assumed. When the adjusted estimate of the downward bias in futures prices is added to trading costs, optimal hedging ratios drop to -0.45 in one case and to zero in the other case.

Because these results are based upon assumed levels of expected cash profits and expected futures profits, they are only illustrative. They show that the optimal level of hedging may be substantially smaller than the level of hedging that minimizes risk, and even a modest downward bias in futures prices markedly depresses the optimal hedging level.

## Conclusions

This analysis of cash future price relationships in cattle feeding shows that the concepts of optimal hedging level, minimum risk hedging level, and hedging effectiveness can be defined in a manner consistent with portfolio theory and used to provide meaningful estimates of hedging potential. The major conclusions from the study are as follows:

1. Short hedging in cattle futures is a management tool that can help the individual or firm with fixed resources in cattle feeding to obtain preferred combinations of expected profits and risks. In contrast, the individual without fixed resources in cattle feeding has no reason to hedge. To him, speculation and custom feeding represent alternative ways of investing in the cattle feeding business.

2. For firms with the same nonlimiting fixed resources in cattle feeding, the optimal level of hedging is independent of differences in the degree of risk aversion. Thus, a single optimal hedging level which applies to a group of firms can be estimated.

3. The squared correlation between cash profits and futures profits provides a meaningful measure of hedging effectiveness when hedging costs are negligible and futures prices are unbiased. In this case, the optimal level of hedging is the level that minimizes risk. When hedging costs are positive and/or futures prices are biased against the hedger, a lower level of hedging is optimal and the proportion of the risk shifted through hedging is less.

4. Although cattle futures price movements have, on the average, favored holders of long positions in the past, the large variability in these price movements makes it impossible to determine if cattle futures prices are, in fact, biased.

5. The upper limit on the optimal hedging level ranges between 0.56 and 0.88 unit of short futures per unit of slaughter cattle produced for the situations studied. However, optimal hedging levels are depressed below these levels when futures trading costs and possible futures price bias are taken into account.

6. In the cattle feeding situations studied, about one-third to one-half of the price risk can be shifted through hedging at the optimal level.

7. Location, grade, and sex of cattle fed have little effect on optimal hedging levels and hedging effectiveness. This suggests that one slaughter cattle futures contract may be sufficient to serve cattle feeders' hedging needs throughout the United States.

This study leaves many questions about hedging potential in cattle feeding unanswered or only partly answered. Only a limited number of cattle feeding situations were examined. The precision of the empirical results is restricted because only 6 years of data on cattle futures prices were available for analysis. Seasonal differences in hedging potential and differences between continuous feeding and feeding one or two lots of cattle per year were not examined. The effects of changing interest rates and variations in the costs of feedlot services deserve to be more carefully explored. The study does not probe the dynamic aspects of hedging, particularly the potential gains from basing production and hedging decisions on changing price expectations or price forecasts. These matters appear to represent promising areas for further exploration.

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## Footnotes

[1] Italic numbers in parentheses indicate items in the References.

[2] Theory suggests that the sharing of risks accomplished through efficient capital markets actually reduces the costs to society of bearing risks. Lintner (*24*) shows that the market price of risk declines as the size of perfect capital markets increases. He concludes: "Perfect capital markets are thus not merely an efficient *risk-sharing* mechanism: they are a remarkably efficient *risk-eliminating* mechanism. . ." (*24*, p. 98).

[3] Hicks (*19*) argued that forward trading contributes to efficiency by reducing inconsistencies in expectations and plans, but he noted that even with forward trading some sources of disequilibrium remain. In demonstrating how futures serve as a forward pricing mechanism in cattle feeding, Paul and Wesson (*28*) suggested that the spot-forward spread be viewed as the price of feedlot services. Ehrich (*6*) and others have attempted to determine how well cattle futures fulfill this forward pricing role, but experience to date does not permit a final conclusion.

[4] The prevalence of risk aversion can be supported theoretically by arguments based upon the decreasing marginal utility of wealth. The assumption of risk aversion does not imply that risk is minimized. It allows the possibility that the decisionmaker may be willing to accept large increases in risk for a small increase in expected returns. Furthermore, it does not rule out gambling when the stakes are sufficiently small. See Arrow (*1*, pp. 90-120) for a discussion of the theory of risk aversion.

[5] The term "basis" here refers to the difference between the futures price and a particular cash price at a point in time. Howell's work is reported in (*21*) and earlier publications cited therein.

[6] Price bias refers to the tendency for the futures price to lie below or above the cash price expected to prevail at the maturity of the future. A biased futures price will tend to move toward the expected cash price as contract maturity approaches.

[7] In each of these studies, the measure of profit variability employed was the variance or standard deviation of profit about its mean. A more general approach is to measure profit variability about its conditional expectation or forecasted value. This point is elaborated in my comment on the Tomek and Gray article (*17*). Furthermore, it is conceivable that a producer may be able to forecast profits more accurately with hedging than without hedging. He may thereby be able to obtain higher profits with hedging than without hedging by taking advantage of the more accurate forecasts and allocating resources to the activities with the highest predicted profits. An attempt to measure such gains is reported in my study of grain storage in Michigan (*16*). The development and testing of alternative profit forecasting schemes that might be used in hedging is beyond the scope of the current study.

[8] The assumptions and approach used here correspond to those employed by Markowitz (*25*) for the portfolio problem. However, inequality constraints imposed by resource limitations are not included since we are interested in situations where no constraints are binding. When the resource constraints are included, the model becomes somewhat more complicated, but the implications are unchanged for our purposes here.

[9] This result is a corollary to the "separation theorem" that Tobin (*31*, pp. 82-85) proved for portfolios. Johnson (*22*, p. 147) derived the result specifically for futures trading.

[10] The Taylor series expansion of $z = s_{12}/s_{11}$ evaluated at $s_{11} = \sigma_{11}$ and $s_{12} = \sigma_{12}$ proceeds as follows:

$$\begin{aligned} z = {} & \sigma_{12}/\sigma_{11} + (1/\sigma_{11})(s_{12} - \sigma_{12}) \\ & - (\sigma_{12}/\sigma_{11}^2)(s_{11} - \sigma_{11}) \\ & + 1/2\,[-(2/\sigma_{11}^2)(s_{12} - \sigma_{12})(s_{11} - \sigma_{11}) \\ & + (2\,\sigma_{12}/\sigma_{11}^3)(s_{11} - \sigma_{11})^2] + \ldots \end{aligned}$$

Taking expected values of both sides of the equality we obtain,

$$\begin{aligned} E(z) = {} & \sigma_{12}/\sigma_{11} - (1/\sigma_{11}^2)\,\mathrm{Cov}\,(s_{12}, s_{11}) \\ & + (\sigma_{12}/\sigma_{11}^3)\,\mathrm{Var}\,(s_{11}) + \ldots \end{aligned}$$

Goldberger (*7*, pp. 97-99) shows that the variance of the sample variance is as follows:

$$\begin{aligned} \mathrm{Var}\,(s_{11}) &= E\,(s_{11}^2 - Es_{11}^2)^2 \\ &= T^{-1}(\mu_4 - \sigma_{11}^2) - 2T^{-2}(\mu_4 - 2\,\sigma_{11}^2) \\ &\quad + T^{-3}(\mu_4 - 3\,\sigma_{11}^2) \end{aligned}$$

where $T$ is the number of observations in the sample and $\mu_4$ is the fourth moment about the mean. Under normality $\mu_4 = 3\,\sigma_{11}^2$ and,

$$\mathrm{Var}\,(s_{11}) = 2\,(T^{-1} - T^{-2})\,\sigma_{11}^2$$

Assuming further that the variance of $s_{12}$ is not larger than the variance of $s_{11}$ and that $s_{11}$ and $s_{12}$ have nonnegative covariance, we have $0 \leqslant \mathrm{Cov}\,(s_{12}, s_{11}) \leqslant \mathrm{Var}\,(s_{11})$. Approximate limits on $E(z)$ are then

$$(1 + U)\,\sigma_{12}/\sigma_{11} - U \leqslant E(z) \leqslant (1 + U)\,\sigma_{12}/\sigma_{11}$$

where $U = 2(T^{-1} - T^{-2})$. For $T = 16$,

$$1.117\,\sigma_{12}/\sigma_{11} - 0.117 \leqslant E(s_{12}/s_{11}) \leqslant 1.117\,\sigma_{12}/\sigma_{11}$$

[11] Alternative measures of hedging effectiveness can be defined which take expected profits or losses from the futures activity into account. But any such measure is arbitrary in that the magnitudes defined have meaning only if the individual's preferences between expected profit and risk are specified.

[12] Technical risk or output risk is disregarded in this study. In cattle feeding, output risk takes the form of variation in rates of gain and death loss. These tend to be relatively small in magnitude and virtually independent of price variation for the individual feeder. Procedures for dealing with output risks in analyzing hedging decisions have been developed by McKinnon (*26*). He deals with the case where basis risk is absent and points out that when output and price are uncorrelated the optimal hedge is not affected by the output risk.

[13] To the extent feasible, daily prices were used in the analysis to avoid averaging out part of the price variation. For the auction markets, however, daily prices are generated only for certain days of the week and these differ from market to market. Weekly averages of cash prices were used for feeder cattle and slaughter cattle so that the same series of futures prices could be used for the various locations. Slaughter cattle prices for Chicago and Omaha and feeder cattle prices for Omaha and Kansas City were obtained from Livestock, Meat and Wool Market News, Weekly Summary and Statistics (*37*). Other spot prices for cattle were obtained from the records of the Livestock Division, Agricultural Marketing Service, USDA.

[14] The corn prices at Denver were obtained from the Denver office of Grain Market News. Prices for other grains and other locations are those reported in weekly issues of Grain Market News (*35*).

[15] The notion that hedgers would be willing to pay speculators a risk premium combined with the excess of short hedging over long hedging implies that the futures price would tend to be biased downward, a phenomenon that Keynes (*23*) termed "normal backwardation." Efforts to measure normal backwardation, risk premiums, or price bias have produced mixed results. For example, Gray (*10*) found little evidence of risk premiums whereas Houthakker (*20*) concluded that the idea of normal backwardation or risk premiums has substantial empirical support. Rockwell (*29*) found "no significant tendency toward normal backwardation," but he concluded that in some markets speculators tend to gain at the hedgers' expense due to the superior forecasting ability of the speculators. Lintner's theoretical analysis of capital markets (*24*) suggests that the price of risk declines as the size of the market increases. Under equilibrium conditions we might therefore expect the market price of risk to become very small. This perhaps helps to explain why students of futures markets have had difficulty in finding empirical evidence of risk premiums.

[16] The tests are based upon a table in Snedecor (*30*, p. 174). Critical values for $r$ at the 1 percent and 5 percent levels with 14 degrees of freedom are 0.623 and 0.497, respectively. The corresponding $r^2$ values are 0.388 and 0.247.

[17] This conclusion is based on the $Z$ test as outlined in Snedecor (*30*, p. 178). The calculated $Z$'s are 0.928 and 0.693. The standard error of their difference is $\sqrt{2/13} = 0.392$. The calculated $t$ is $(0.928 - 0.693)/0.392 = 0.60$. The probability of obtaining a $t$ larger than this in samples of this size from two populations with the same correlation is approximately 0.56.