Journal of Agricultural and Resource Economics 37(3):398–414

Acreage Response under Varying Risk Preferences

Carlos Arnade and Joseph Cooper

The assumption in standard expected utility model formulations that the coefficient of risk aversion is a constant is potentially unrealistic. This study takes the standard linear expected mean-variance problem and replaces the coefficient of risk aversion with a function of risk aversion, allowing risk to be depicted as a constraint that farmers face. Treating output prices as stochastic, the theoretical formulation measures the impact price variability itself has on risk preferences. Acreage response elasticities are also estimated as a function of prices and price variances using U.S. county-level data for corn, soybean, and wheat producers.

Key words: acreage response, price variability, risk function, risk preference

Introduction

The lag between planting and harvest guarantees that agricultural producers do not know in advance what price they will receive for their product, and the stochastic nature of production ensures that producers do not know what their output (yields) will be. Consequently, a wide body of literature addresses the effect of risk on agricultural producer decisions (e.g., Just, 1974; Love and Buccola, 1991; Pope and Just, 1991; Saha, Shumway, and Talpaz, 1994). A critical component of this literature is the impact of risk on producers' acreage decisions (Just, 1974; Chavas and Holt, 1990; Coyle, 1992; Lin and Dismukes, 2005). The volatility of agricultural prices ensures that price risk is a key component of agricultural risk (Ben-Jemma, 2007). Just (1974) showed that risk, measured by the variance of revenue, can even influence a producer's acreage decisions. Since then, expected utility models have been used to measure the impact of price risk on acreage decisions. These utility models range from linear mean-variance models and their dual (Coyle, 1992) to more general representations of utility (Saha, 1993) and models that use ARCH/GARCH methods to represent variances (Holt and Moschini, 1992).

We take the standard linear mean-variance representation of the acreage decision under price risk and replace the coefficient of risk aversion with a function of risk aversion.[1] Our theoretical formulation allows a producer's preference for risk to vary with respect to the level of price variability (i.e., risk preferences are defined at a risk margin). Replacing the risk coefficient with a function allows us to depict risk as a constraint and develop the indirect risk preference function (IRPF). Based on this formulation, which is consistent with safety-first models (Patrick et al., 1985; Roy, 1952), we specify compensated acreage equations whose arguments are output and input prices, price variances, and profits. We use this theoretical framework to estimate price and variance elasticities. Most importantly, the model allows us to evaluate the impact that price variability has on risk preferences.

Carlos Arnade and Joseph Cooper are economists with the Economic Research Service, USDA.

Review coordinated by Gary W. Brester.

[1] While this paper focuses on price risk, the technique introduced here could be made to handle yield risk with a few adjustments.

Acreage Decisions Under Price Risk

The mean-variance (E-V) function introduced by Markowitz (1952, 1959) and Tobin (1958) to analyze portfolio choice was adapted by Sandmo (1970) to model production decisions. Producers maximize a combination of income (profits) minus a weighted measure of its variance. Programming methods have used the linear E-V formulation to determine farmers' optimal acreage allocations, often by parametrically varying the coefficient of risk aversion (Simmons and Pomareda, 1975; Wiens, 1976; Brink and McCarl, 1978).[2] These papers represent a prescriptive approach to acreage choice; under certain assumptions these acreage choices can be considered efficient as described by the portfolio theory of finance (Du, 2005; Hanoch and Levy, 1969).

The alternative to programming methods is to use the E-V framework in an econometric model. Coyle (1992), who introduced dual methods to model risk, provides an example of econometric application of the E-V framework. In being descriptive of aggregate producer choices, the econometric approach allows for the possibility of inefficient behavior. That is, a host of factors beyond annual profits and risk may determine aggregate acreage allocations and lead to real-world departures from the efficient E-V frontier. In specifying econometric models, the mean-variance utility-optimization decision is a useful way to approximate real-world behavior, as long as it is nested in a specification that allows for deviations from this normative model. Under the assumption of constant absolute risk aversion (CARA) and a normal distribution of wealth, the linear E-V function serves as an approximation to more general utility functions (Ben-Jemma, 2007). The linear E-V model has gradually been replaced by these more general utility functions (e.g., Szpiro, 1986; Saha, 1993), which are less restrictive regarding the relationship between risk preferences and wealth. However, even the more general models do not explicitly portray risk preferences as varying with respect to a measure of risk itself, such as price variability.[3]

Thus, it is not surprising that there is a wide range of risk-preference estimates reported in the literature. Some variation arises from the measurement of different types of risk (i.e., yield risk, price risk) or risk measures that are based on models that make different assumptions about the relation between risk preferences and wealth (Pope and Just, 1991; Friend and Blume, 1975; Love and Buccola, 1991). Babcock, Choi, and Feinerman (1993) emphasize that different risk-aversion coefficients will appear reasonable depending on the amount of income at risk; that is, the size of a gamble matters. Saha, Shumway, and Talpaz (1994) present a method of testing the relationship between risk preferences and wealth. Even so, more than differences in wealth (and methodology) may drive the wide disparity in estimated risk preference measures. That is, given that variables that measure risk, such as price volatility, differ from period to period, it could be that reported risk-aversion estimates only reflect preferences at a particular risk margin.

The Standard Model Generalized

Suppose prices are stochastic, yields are known, and producers have non-neutral risk preferences. Producers allocate acreage to maximize profits in the presence of price risk by:

$$\begin{aligned} \max_{a_i} U^e(\pi, \sigma_\Pi) &= \max_{a_i} \pi - \lambda \sigma_\Pi \\ &= \max_{a_i} \sum_i^n p_i^e y_i a_i - C(w_1..w_m, y_1 a_1..y_n a_n) - \lambda \left(\sum_i^n (y_i a_i)^2 v p_i + Cov_{ij} \right), \end{aligned} \tag{1}$$

[2] By varying the coefficient of risk aversion, one can map out the optimal combinations of profits and variance of profits to determine the efficient frontier (Du, 2005). Hanoch and Levy (1969) discuss conditions for ensuring decision makers fall on the frontier.

[3] Friend and Blume (1975) show how a measure of portfolio allocation relates to price variances. However, they use this finding to relate risk preferences to changes in wealth. Serra et al. (2006) allow their risk-aversion measure of output risk to be a function of the variability of input prices, but they do not exploit this relationship.

where p_i^e, y_i, and a_i are expected output prices, yields, and acreages of the ith crop; vp_i is the variance of the ith crop price; $Cov_{ij} = (\sum_i \sum_j 2 \times vp_{ij} \times y_i \times y_j \times a_i \times a_j)$ represents price covariances; and w_k represents input prices. For expository purposes, we suppress covariances (for $i \neq j$) throughout this paper, though these terms could remain part of the analysis. One particularly unrealistic aspect of the utility function in equation (1) is the assumption that the coefficient of aversion, λ, remains constant. While this makes the problem tractable, it is unrealistic to assume price variability (risk itself) does not influence producers' aversion to risk. While we focus on the linear E-V for brevity, our criticism of the constant value for the risk-aversion coefficient also holds for utility functions that have decreasing absolute risk aversion (DARA) and increasing constant absolute risk aversion (ICARA) where the coefficient on the standard deviation of wealth is a constant.

If we replace the risk-aversion coefficient with a function of risk aversion $g(\cdot)$, the acreage choice problem becomes:

$$\max_a \sum_i^n p_i^e y_i a_i - C(w_1..w_m, y_1 a_1 \ldots y_n a_n) - \sum_i^n g(y_1 a_1..y_n a_n, vp_1..vp_n)(y_i a_i)^2 vp_i. \tag{2}$$

We subsequently denote $g(\cdot)$ as the function of risk aversion whose arguments are yields, acreages, and the variance of prices.[4] Input prices, w, could be added to $g(\cdot)$ as well. We assume that most costs are known at planting and do not influence $g(\cdot)$. We do not restrict how producers weigh each price variance, but a risk-aversion function that explicitly has revenue variance as its sole argument represents a special case of our more general risk-aversion function.

Risk-Constrained Production Choice

Building on the previous section, we now represent risk as a constraint rather than a preference. That is, consider asking a group of producers whether risk represents a preference they have or is a constraint they face. Thus, we recast the producer decision as:

$$\max_{a_i} \sum_i^n p_i^e y_i a_i - C(w_1..w_m, y_1 a_1..y_n a_n) \quad s.t. \quad \sum_i^n g(y_1 a_1..y_n a_n, vp_1..vp_n)(y_i a_i)^2 vp_i \leq \overline{K}. \tag{3}$$

where "*s.t.*" denotes "subject to" and $\overline{K}$ represents the maximum level of risk that a producer is willing to take on. In equation (3), producers maximize profits while taking on a subjectively weighted amount of risk up to the level $\overline{K}$. In allowing the subjective level of risk to be determined outside the model—perhaps in an upper-stage optimization problem, which may take into account intertemporal considerations, debt, environmental preferences, or consumption choices—we allow for the possibility that producers' acreage choices may or may not land them on the efficient frontier, as depicted by portfolio theory.[5][6]

Note that the optimization problem in equation (3) is consistent with "safety-first" rules developed by Roy (1952) and applied to agriculture by Atwood (1985) and Berck and Hihn (1982), who argued that agricultural producers are likely to follow safety-first rules. In the safety-first model, the decision maker is concerned with the probability of failing to achieve his or her income goals (Shackle, 1969; Roy, 1952). In general, such a model is not consistent with the expected utility model (Pyle and Turnovsky, 1970). However, in a coordinated survey of 149 agricultural producers

[4] While the utility function is no longer linear, it remains linear in profits.

[5] To ensure efficiency one can always represent producer decisions as involving two steps: maximizing an objective subject to the constraint, as in equation (3) or (6), and then choosing the optimal levels of the constraining variable.

[6] Robison and Brake (1979) claim that a limitation of portfolio theory is that not all producers can find their preferred choice on the E-V set. They also point out that economists often lack the computational ability to map out the efficient frontier, indicating that farmers may also be unable to locate the frontier. While the E-V frontier is now relatively simple to estimate for a portfolio of stocks and bonds, the computational methods would be considerably more difficult for a farm portfolio because the model would include endogenously determined costs as well as revenues.

across twelve states, Patrick et al. (1985) found that many producers made decisions consistent with some of the safety-first considerations. The first-order conditions for equation (3) are:

$$p_i^e y_i - (\partial C(\cdot)/\partial a_i) - \gamma \times ((\partial g(\cdot)/\partial a_i \times (y_i a_i)^2 + g(\cdot) \times 2y_i^2 a_i) \times vp_i = 0 \tag{4}$$

for $= 1, \ldots, n$ crops, and:

$$\sum_i^n g_i(..) \times (y_i a_i)^2 vp_i = \overline{K}, \tag{5}$$

where γ is the Lagrange multiplier and represents the marginal impact of an increase in producer risk tolerance ($\overline{K}$) on profits. The solution to the system of first-order conditions is a set of producer acreage equations, denoted for the ith crop acreage as $A_i^*(p_1..p_n, w_1..w_m, vp_1..vp_n, \overline{K})$. Substitution of the acreage equations $A_i^*(\cdot)$ into the optimization problem in equation (3) produces the indirect utility function $U^*(p_1..p_n, w_1..w_m, vp_1..vp_n, \overline{K})$.

Up to this point, we have taken the standard linear E-V problem and replaced the coefficient of risk of aversion with a function of risk aversion. We then represent risk as a constraint that producers face rather than permitting risk to be subsumed within the objective function itself. This specification is consistent with safety-first rules and allows us to derive a system of acre equations that include producers' risk tolerance levels, $\overline{K}$, as an explanatory variable. However, for applied empirical analysis, it would be difficult to find a suitable variable to represent $\overline{K}$, either for each producer or as an aggregate measure. Thus, the above acreage functions, $A^*(..\overline{K})$, stand little chance of being estimated with data available to most economists.

Minimizing Risk: A Dual Approach

As an alternative, we set up a dual version of the optimization problem in equation (3) that allows us to derive a system of acreage equations that includes expected prices, variances, and profits as explanatory variables for which reliable proxy data are readily available. In particular, the producer's choice problem, in dual form to equation (3), is:

$$\min_{a_i} \sum_i^n g(y_1 a_1..y_n a_n, vp_1..vp_n) \times (y_i a_i)^2 vp_1 \quad s.t. \quad \sum_i^n p_i^e y_i a_i - C(w_1..w_m.y_1 a_1..y_n a_n) \geq \overline{\pi}. \tag{6}$$

In equation (6) producers minimize their subjective level of risk subject to the constraint that a predetermined level of profits is reached. For instance, a hired farm manager desires to reach a particular profit level and wants to attain it in the least risky way, or a producer has a target profit level necessary to meet anticipated household expenditures or debt payments.

The first-order conditions to the above problem are:

$$\partial g(\cdot)/\partial a_i (y_i a_i)^2 vp_i + g(\cdot) \times 2y_i^2 a_i vp_i = \theta(p_i^e y_i - (\partial C(\cdot)/\partial a_i)) \tag{7}$$

for $i = 1, \ldots, n$, and

$$\sum p_i y_i a_i - C(\cdot) = \overline{\pi}, \tag{8}$$

where θ, the Lagrange multiplier, represents the marginal impact that a change in profits has on the producer's subjective level of risk.

Solving equations (7) and (8) for the a_i's produces the acreage equation $\breve{A}_i(p_1..p_n, w_1..w_m, vp_1..vp_n, \overline{\pi})$ for $i = 1, \ldots, n$ crops. The acreage devoted to crop i is a function of output prices, input prices, price variances, and a predetermined level of profits. If the constraint in equation (6) is binding, then observed profits will be equivalent to predetermined profits.

These acreage functions could be called compensated acreage equations in that profits are held constant as acreage adjusts to compensate for changes in prices. Compensated acreage functions can be either upward- or downward-sloping in price. Regardless, these functions can be used to obtain uncompensated acreage elasticities.[7,8] Substituting each compensated acreage equation, $\breve{A}_j$, into the optimization problem in equation (6) produces the indirect risk function (IRPF). Relative to the choice problem in equation (3), the choice problem in equation (6) may or may not represent a more realistic view of how risk-averse producers make their decisions. But this point is irrelevant. In being dual to the constrained profit-maximization problem, the optimization problems in equations (3) and (6) are different representations of the same choice. The advantage of using the choice problem in equation (6) is that it produces acreage equations that are a function of prices, price variances, and the predetermined level of profits, which is observed if the constraint is binding. That is, data will most likely be available for estimating compensated acreage equations.

The twin optimization problems in equations (3) and (6) represent decision making under uncertainty that takes into account safety-first considerations (Atwood, 1985; Shackle, 1969). For example, Roy (1952) proposed modeling safety-first by minimizing the probability of income falling below critical values, similar to equation (6). Telser (1955) proposed that expected income be maximized subject to satisfying probabilistic constraints upon the likelihood of low income level, analagous to equation (3). Kataoka (1963) discussed finding the maximum income level I for which the probability of income falling below I is below a prespecified level, as in equation (3). Our dual representations of a producer's choice in equations (3) and (6) highlight that these various representations of the safety-first problem are consistent with each other and are analogous to the dual representations (utility maximizing or expenditure minimizing) of consumer choice.

It also should be clear that it is possible to insert the indirect risk-preference function into the E-V objective function and, in a second step, maximize profits. Under certain assumptions, the solution would ensure that producers would lie on the efficient frontier.[9] Given the wide range of factors that go into farm decision making and given that our data aggregate across a wide range of producers, we choose not to impose these efficient portfolio restrictions on our model. Nonetheless, our model does not prevent the combination of observed profits ($\overline{\pi}$) and the variance of profits from being consistent with efficiency, assuming that the data reveal this to be the case.

Functional Form of the Acreage Equations Under the Dual Approach

We now derive compensated acreage equations from the dual IRPF that represents the solution to the producer choice problem in equation (6). Using an envelope relationship (see Appendix A for details) it is possible to show that:

$$\partial \breve{G}(\cdot)/\partial p_i = -\theta \times y_i \breve{A}_i(p_1^e..p_n^e, vp_1...vp_n, y_1..y_n, \overline{\pi}) \tag{9}$$

and $\partial \breve{G}(\cdot)/\partial \overline{\pi} = \theta$, such that:

$$\frac{-\partial \breve{G}(\cdot)/\partial P_i}{\partial \breve{G}(\cdot)/\partial \overline{\pi}} = y_i \breve{A}_i(p_1^e \ldots p_n^e, w_1 \ldots w_m, vp_1 \ldots vp_n, \overline{\pi}). \tag{10}$$

Dividing equation (10) by yield obtains the acreage equations. That is, a relationship similar to Roy's identity in consumer theory exists for recovering acreage equations from the IRPF function.

[7] A Slutsky-like relationship exists between these functions and the solution to the utility maximization problem in equation (4). This is shown in Appendix A.

[8] Chavas and Holt (1990) also estimate and discuss compensated acreage equations, but the means by which they derive such equations is entirely different.

[9] This is analogous to using input choice in obtaining a cost function and then using the cost function to determine the optimal level of output. Economists often estimate cost functions, accepting that producers may not operate at the profit-maximizing level of output.

We approximate the IRPF with the following flexible form:

$$
\begin{aligned}
\breve{G}(\cdot) \quad = \quad & \sum_i \gamma_{1i}(p_i y_i) + \sum_i \gamma_{2i} w_i + \sum_i \gamma_{3i} vp_i + \gamma_{4i}\pi + \frac{1}{2}\sum_i\sum_j \beta_{ij}(p_i y_i)(p_j y_j) + \\
& \frac{1}{2}\sum_i\sum_j \lambda_{ij} w_i w_j + \frac{1}{2}\sum_i\sum_j \alpha_{ij} vp_i vp_j \pi + \sum_i\sum_j v_{ij}(p_i) y_j w_j + \\
& \sum_i\sum_j \kappa_{ij}(p_i y_i) vp_j + \sum_i v_i p_i y_i^2 + \sum_i\sum_j \mu_{ij} w_i vp_j \pi + \sum_j d_{1j}(p_j y_i)\pi + \\
& \sum_j d_{2j} w_i \pi + \sum_j d_{3j} vp_j \pi + \frac{1}{2} d_4 \pi^2,
\end{aligned} \tag{11}
$$

where p_j represents the jth normalized output price (or its expectation), w_j is the jth normalized input price, vp_j is the jth normalized price variance, and $\overline{\pi}$ is expected profits. Any output or input price can serve as the numeraire; we have chosen wages. Taking the derivative of equation (11) with respect to price produces:

$$
\partial\breve{G}(\cdot)/\partial p_i = (\gamma_i + \sum_{j\neq i}\beta_{ij}(p_j y_i) + \beta_{ij} p_i y_i + \sum_j v_{ij} w_j + \sum_j \kappa_{ij} vp_j + d_{1i}\pi + \sum_j v_j y_j) y_i. \tag{12}
$$

Taking the derivative of equation (11) with respect to profits produces:

$$
\begin{aligned}
\partial\breve{G}(\cdot)/\partial\pi \quad = \quad & \gamma_4 + \sum\sum_j \mu_{ij} w_i vp_j + \alpha_{ij}\sum_i\sum_j vp_i vp_j + \sum_j d_{1j} p_j y d_j + \\
& \sum_{j\neq i} d_{2j} w_j + \sum_{j\neq i} d_{3j} vp_j + d_4\pi.
\end{aligned} \tag{13}
$$

Substituting equations (12) and (13) into equation (10) obtains the acreage equations:

$$
\begin{aligned}
(\partial\breve{G}(\cdot)/\partial p_i)(1/y_i)(-1/\partial\breve{G}/\partial\pi) = A_i = (\gamma_i + \sum_{j\neq i}\beta ij(p_j y_i) + \beta_{ii} p_i y d_i + \\
\sum_j v_{ij} w_j + \sum_j \kappa_{ij} vp_j + d_{1i}\pi + \sum_j v_j y_j)/(-1)[\gamma_4 + \sum_j d_{1j} p_j y_j + \\
\sum_{j\neq i} d_{2j} w_j + \sum_{j\neq i} d_{3j} vp_j + d_4\pi + \sum\sum_j \mu_{ij} w_i vp_j + \alpha_{ij}\sum_i\sum_j vp_i vp_j].
\end{aligned} \tag{14}
$$

The acreage equation (14) is nonlinear in parameters and may prove difficult to estimate. However, an acreage adding-up constraint can be used to simplify the equation. That is, the sum of the crop-acreage equations should equal the total area available for farming.[10] A convenient set of parameter restrictions that ensure that this "area" adding-up condition holds are:

$$
\begin{aligned}
\sum \gamma_{1i} = 1; \sum_i d_{1i} = 0; \text{and for each } j \\
\sum_i \beta_{ij} = 0, \sum_i v_{ij} = 0 + \sum_i \kappa_{ij} = 0
\end{aligned} \tag{15}
$$

and

$$
\begin{aligned}
-\Bigg[\gamma_4 + \sum_j d_{1j}(p_j y_j) + \sum_j d_{2j} w_j + \sum_j d_{3j} vp_j + d_4\pi + \\
\sum_i\sum_j \mu_{ij} w_i vp_j + \sum_i\sum_j \alpha_{ij} vp_i vp_j\Bigg] = (1/\overline{A}),
\end{aligned} \tag{16}
$$

[10] Pasture can be represented as a crop, whose output is hay. This holds both at the farm level and in the aggregate.

where $\overline{A}$ equals the total area available for farming. Imposing these restrictions leads to the following:

$$\begin{aligned}\sum_i A_i \quad = \quad \overline{A} \quad = \quad & -1/[\gamma_4 + \sum_j d_{1j}(p_j y_j) + \sum_j d_{2j} w_j + \sum_j d_{3j} vp_j + \\ & d_4 \pi + \sum_i \sum_j \mu_{ij} w_i vp_j + \sum_i \sum_j \alpha_{ij} vp_i vp_j].\end{aligned} \tag{17}$$

Substituting equation (17) into individual acreage equation (14) and rearranging terms produces:

$$A_i/\overline{A} = AS_i = [\gamma_{1i} + \sum_j \beta_{ij}(p_j y_j) + \sum_j v_{ij} w_j + \sum_j \kappa_{ij} vp_j + d_{1i}\pi + v_i y_i], \tag{18}$$

where AS is equal to the share of acreage devoted to the ith crop. Equation (18) is an acreage-share equation that is linear in the parameters. By exploiting the adding-up conditions, we can derive a system of compensated acreage-share equations that are linear in parameters and can be estimated with conventional econometric methods.

In using aggregate real-world data, our estimating equations may not represent efficient behavior. That is, the targeted level of profits ($\overline{\pi}$) of the optimization problem in equation (6) becomes the explanatory variable π in our econometric model. If farmers are on the efficient frontier of the E-V function, as described by portfolio theory, then the data will reflect that. If not, then the data imply that other considerations go into determining acreage levels. The model will be consistent with the data-generating process in either case.

Assessing Risk Preferences Given Equation Estimates

Risk preferences are allowed to vary at the margin, permitting us in principle to evaluate the impact of price variability (price risk) on risk preferences. Appendix A shows that a change in the variability of the ith price on the indirect risk function equals the impact it has on the direct risk function. That is:

$$\begin{aligned}\partial \breve{G}(\cdot)/\partial vp_i \quad &= \quad \partial G(\cdot)/\partial vp_i \\ &= \quad g_i(\cdot) \times (y_i a_i)^2 + \partial g(\cdot)/\partial vp \times \sum_j vp_j (y_j a_j)^2 \\ &= \quad \gamma_{3i} + \sum_i \boldsymbol{K}_{ij} p_i y + (\sum_j \alpha_{ij} vp_i + \sum_i \mu_{ij} w_i + \sum_j d_{3j})\pi.\end{aligned} \tag{19}$$

Rearranging terms, the derivative of risk preferences can be expressed in terms of observable data and the parameters of the indirect risk function as:

$$\begin{aligned}\partial g(\cdot)/\partial vp_i \quad = \quad & (\gamma_{3i} + \sum_i \boldsymbol{K}_{ij} p_i y_i + (\sum_j \alpha_{ij} vp_i + \sum_i \mu_{ij} w_i + \sum_j d_{3j}) - \\ & g(\cdot)(y_i a_i)^2)/(\sum_j vp_j (y_j a_j)^2).\end{aligned} \tag{20}$$

Equation (20) provides a means of evaluating the impact of price variances on risk preferences. The bold $\boldsymbol{K}$ parameters can be obtained from the estimated acreage-share equations. All other parameters except the intercept γ_{3i} can be obtained by estimating the total acreage equation in inverse form from equation (16). Homogeneity conditions for $g(\cdot)$ can be used to approximate the parameter γ_{3i}. Given these parameter estimates, an assumed level of $g(\cdot)$, and data on acreage, yields, prices, and price variances, equation (20) can be used to calculate the impact of a change in the ith price variance on risk preferences. While such a calculation may not be exact, it allows us to get some idea of how changing volatility levels influence producer risk preferences.

Data and Econometric Results

U.S. county-level data on corn, soybean, winter and spring wheat, and haying from 1975–2007 were obtained from the National Agricultural Statistics Service (NASS). Expected crop prices are drawn from futures markets. Price and yield densities are converted into within-season deviates (Cooper, 2009, 2010). The price deviate is equivalent to the harvest price minus planting price, divided by planting price, where these prices follow the same definitions used by the Risk Management Agency in pricing crop insurance products. The acreage decision is made annually. We assume targeted profits change from year to year to match changing conditions. For each year in the regression, prices from the previous ten years are used to generate the nonparametric price-density functions. These are converted to density functions for actual price, centered around the planting price in t. Hence, the density functions for price are forward looking with respect to the mean. Generated prices for each commodity are truncated by their respective loan rates. As the previous ten years of data are used to generate the means and variances of price and yields for each year, the time span for the econometric analysis covers 1985 to 2007. Other variables used in constructing the regression data include wages obtained from the *Economic Report of the President* and input-price indices for fertilizer, agricultural chemicals, and farm machinery.

All five crops (corn, winter wheat, spring wheat, soybeans, and hay) are grown at the national level. However, not every county grows every crop in our database, creating potential difficulties for estimation when we disaggregate down to county-level data. It is unlikely that available economic data explain why, for example, a county has never grown spring wheat. Hence, absent county-specific data on some crops, it would not be prudent to put data for all counties into a single regression. As such, we break the counties into four populations defined by the crops they have actually grown between 1985 and 2007:

- Population 1: 345 counties that grow only corn, soybeans, and winter wheat.
- Population 2: 79 counties that grow only corn and winter wheat.
- Population 3: 35 counties that grow only corn, soybeans, and spring wheat.
- Population 4: 91 counties that grow only corn and soybeans.

Haying acreage was also included in each population. The price of hay was used as a proxy for the return for not growing a crop.

Expected profit per acre is an explanatory variable in our model. We use relatively straightforward procedures to calculate county revenues based on existing price and yield data. However, construction of a profit variable requires making some assumptions due to data deficiencies on the cost side. While some extension services provide cost data for some crops at particular points in time, such data are rare at the county level and will not cover all years in our time period. Additionally, the USDA does not provide cost data at state-level aggregations or lower. Hence, for each year we construct an index (Wdx) of county costs composed of a weighted average of wages, fertilizer, and fuel prices. The prime (interest) rate was used to discount the index to reflect borrowing costs. Assuming that farming is a reasonably competitive industry, the Wdx index was then scaled up to approximately equal the average level of revenues for each year across all counties.

Dummy variables for ERS crop-reporting districts were created and multiplied by the cost index Wdx. For example, Population 1 (354 counties) contains fifty-six different crop-reporting districts, so fifty-five dummy interaction-term variables were included in each equation for Population 1. Profits for each county were represented as: $R_i - C_i \times D_i \times (Wdx)$, where R_i represents per acre revenue, D_i represents the dummy variable for the ith crop-reporting distinct, and C_i. represents a parameter that is jointly estimated with other parameters of the model.[11]

While not ideal, this approach is a reasonable way to construct the county-level profit variable in the model. It also gave us the added benefit of being able to derive the estimated level of profits from

[11] For example, if $R_i = \$400$, $Wdx = \$380$, and C_i is estimated to be 0.75, then profits per acre, when averaged over all farm uses, would equal $\$400 - 0.75 \times 380 = \$155$.

the model. Given that direct county-level cost estimates are not available, using crop-reporting-district dummies in lieu of county-level dummies provides a substantial decrease in the number of explanatory variables while still providing substantially less aggregation than state or larger regional aggregations. Estimated average profits constructed with this approach were \$154 per acre in Population 1 (corn, wheat, and soybeans), \$47 per acre in Population 2 (corn and wheat), \$53 per acre in Population 3 (corn, spring, wheat, and soybeans), and \$277 per acre in Population 4 (corn and soybeans). Remarkably, the average estimated profits (average all crops) seem reasonable and are no more than 30% different from USDA national estimates for specific crops, but these average profits mask several unreasonable county-specific estimates. For example, in Population 1, nineteen out of fifty-six crop-reporting districts had estimated negative costs, which is clearly incorrect. If one had an interest in comparing costs at the county level, a more flexible combination of the cost-index and crop-district dummy variables would probably be necessary.[12] Since comparing profits across counties is not the purpose of this model, average estimates of county profits were sufficient for our needs.

Parameter and Elasticity Estimates

Compensated acreage-share equations, as represented in equation (18), were specified as the system to be estimated. A lag acreage-share variable was included in the model to account for crop-rotation schedules. For each of the four population sets, the system of acreage equations was estimated with iterated SUR, which is equivalent to maximum likelihood.

Within each of the four data populations, there were years in which a particular crop was not grown. The standard Heckman two-step method was used to account for these observations. Within each population category, zero-one data represent whether a crop was grown in particular county in a particular year. Using this data, probit models were estimated and the inverse Mills Ratio was calculated and used as an explanatory variable in a subsequent model. Most exogenous variables in the probit models also served as exogenous variables in the subsequent share equations. Since share equations add up to 1 (and errors therefore add to 0), one equation must be dropped to allow for matrix inversion. In each case the hay-share equation was dropped. The inverse total-acreage function in equation (16) contains parameters (i.e., d_{1i}) common with the acreage-share equations. Initially, we tried estimating the total inverse acreage equation jointly with the system of share equations and imposed cross-equation restrictions. However, the approach produced poor results with elasticity estimates that made little sense.

As an alternative, we estimated the two sets of equations in sequence. Specifically, acreage-share equations for each of the four populations were estimated in a SUR system and common parameters in the total acreage equation were set equal to their estimated values in the share equations. The remaining parameters of the total acreage equation were estimated by OLS. While this procedure reduces potential efficiency, it allows us to obtain estimates of key parameters.

Having obtained estimates of the share equations for each of the four populations, uncompensated price elasticities were calculated at variable means using the formula in Appendix A.[13] These elasticities combine two effects: the change in acreage with respect to price when profits

---

[12] Estimated county cost coefficients were not consistent across equations (share and total acreage). Imposing this restriction requires estimating the acreage and share equations together. As noted, doing so created a poor-fitting model that produced nonsensical elasticities. One justification for allowing different county cost coefficients among share equations is that these cost-index coefficients may be picking up an input-price effect that is not completely captured by coefficients on input prices.

[13] Using equation (A2) from Appendix A and the definition of shares, the uncompensated elasticity with respect to price is:

$\partial a_i/dvp_j = (k_{ij}/s_i - g(\cdot) \times d_i \times a_j \times y_j^2) \times vp_j$

$\partial a_i/dp_j = (\beta_{ji}/s_i + d_i) \times p_j \times y_j$

where s_i equals the acreage share; d_i equals the coefficient of profits; y represents yields; and p_j ,vp_j, a_j represent price, price variances, and acreage. The coefficients β_{ij} and k_{ij} represent the coefficient on prices and variance terms.

Table 1. Uncompensated Acreage Price Elasticities

Population 1: Counties that Grow Corn, Soy, and Winter Wheat

	Corn	Soy	Winter Wheat
Corn	0.29[a]	−0.20	−0.72
Soy	0.01	0.10	0.64
Winter Wheat	−0.06	0.13	0.26
Hay	−0.10	0.42	−0.25

Population 2: Counties that Grow Corn, and Winter Wheat

	Corn	Winter Wheat
Corn	0.07	0.21
Winter Wheat	−0.01	0.14
Hay	0.04	−0.25

Population 3: Counties that Grow Corn, Soy, and Spring Wheat

	Corn	Soy	Spring Wheat
Corn	0.18	−0.08	0.07
Soy	0.04	0.35	−0.44
Spring Wheat	0.07	−0.14	0.81
Hay	0.08	−0.04	0.20

Population 4: Counties that Grow Corn and Soy

	Corn	Soy
Corn	0.31	−0.29
Soy	−0.12	0.29
Hay	0.03	0.002

Notes: [a] For example a 10% rise in corn price in Population 1 increases corn acreage by 2.9%.

are held constant and the remaining change in acreage as profits adjust. Output-price elasticities from the four models are reported in table 1. With the exception of the spring wheat elasticity in Population 3, all the own-price elasticities are less than 0.4, with seven (out of ten) elasticities less than 0.3. Overall, this is a strong price response for a short-run acreage-response model. Cross-price elasticities reveal a mix of substitution and complementarity among crops. Note that uncompensated elasticities are not symmetric and need not even be symmetric in sign.

Lin and Dismukes (2005), who also included price variances as explanatory variables, calculated acreage corn elasticities to be between 0.17 and 0.35. In our four models, we found corn elasticities to be between 0.07 and 0.31. Lin and Dismukes calculated soybeans to be 0.30, while ours lay between 0.10 and 0.35. Lin and Dismukes found wheat acreage elasticities to be between 0.250 and 0.234. Our winter wheat elasticities were 0.14 and 0.26, while our estimated spring wheat elasticity was 0.81. With the exception of the elasticity for high spring wheat, our estimates fall in the range summarized in the literature (Huang and Khanna, 2010).

Table 2 reports the uncompensated acreage elasticities with respect to price variances. The variance elasticity formula includes the function of risk aversion, whose value requires knowledge of one parameter outside the system of estimated equations. Variance elasticities can therefore be calculated over a range of risk-aversion values. Reported elasticities in table 2 are reported for a risk-aversion function set equal to 0.3 [14] Eight out of ten own-variance elasticities are negative. Price-variance elasticities for soybeans are quite small for two county populations. Corn variance elasticities are close to unity for two populations, and above 2 in magnitude for two populations.

[14] If the utility function is linear, as represented by the standard E-V problem, one can view the weight on profits as 1 and weight on risk as 0.3.

Table 2. Acreage Price-Variance Elasticities (Risk-Aversion Function set equal to 0.3)

Population 1: Counties that Grow Corn, Soy, and Winter Wheat

	Corn	Soy	Winter Wheat
Corn	−2.03[a]	−0.71	−0.16
Soy	−0.51	−0.21	0.23
Winter Wheat	−0.99	−0.36	0.14

Population 2: Counties that Grow Corn, and Winter Wheat

	Corn	Winter Wheat
Corn	−0.79	−0.74
Winter Wheat	−0.87	−2.52

Population 3: Counties that Grow Corn, Soy, and Spring Wheat

	Corn	Soy	Spring Wheat
Corn	−0.93	−0.03	0.29
Soy	−0.92	−0.07	−0.02
Spring Wheat	−0.44	−0.05	0.01

Population 4: Counties that Grow Corn and Soy

	Corn	Soy
Corn	−2.50	−1.82
Soy	−0.94	−0.76

Notes: [a] For example a 10% rise in the variances of corn prices in Population 1 counties will decrease acreage by approximately 20%

Wheat variance elasticities differ the most, being slightly positive in two models, and negative but greater than 2 in magnitude for another population.

Estimation of the Effect of Price Variances on the Function of Risk Preferences

Equations (19) and (20) relate the parameters of the indirect risk-aversion function to the impact price variability has on the direct function of risk aversion. However, for reasons discussed earlier, we were not able to estimate the γ_{3i} parameter. Assumptions about the degree of homogeneity of the risk-aversion function $g(\cdot)$ in price variances are used to approximate this coefficient instead. That is, the sum of the derivatives of $g(\cdot)$ with respect to variances must meet a homogeneity requirement (Euler's Theorem). Therefore, we calibrated the missing γ_{3i} coefficients to meet this requirement.

Table 3 reports the impact of a change in the level of price variability on the risk-aversion function $g(\cdot)$ under two assumptions: a) $g(\cdot)$ is homogeneous of degree one; and b), $g(\cdot)$ is homogeneous of degree zero in price variances. In several instances, the approximated γ_{3i} coefficient dominated the results. Therefore, these results should be viewed more as illustrative of the method introduced in this paper rather than a final say on how risk aversion changes at the margin. Assuming a homogeneous-of-degree-one risk-aversion function, an increase in every price variance increases the function of risk aversion for county populations 1, 3, and 4. That is, more risk makes producers more risk averse. For example, consider the case where the risk-aversion function is homogeneous of degree one in price variances. In Population 3, a 10% rise in soybean- and wheat-price variances increases the risk-aversion function by almost 5%, suggesting a strong relationship. For corn in Population 4, the risk-aversion-variance elasticity is almost 1. One the other hand, in Population 2, a rise in the variability of corn prices decreases risk aversion to the extent that it offsets the rise in price variability. While the results in table 3 are mostly illustrative, there could be an economic rationale for instances where an increase in price variability reduces risk aversion. For example, producers may become so accustomed to a volatile environment that they learn to adapt to it and

Table 3. Elasticities of Risk Preferences with Respect to Price Variances

	Homogeneity of:	
	Degree 1	**Degree 0**
Population 1: Counties that Grow Corn, Soy, and Winter Wheat		
Corn	0.38[a,b]	0.23
Soy	0.32	−0.21
Winter Wheat	0.30	−0.02
Population 2: Counties that Grow Corn, and Winter Wheat		
Corn	−1.96	−1.48
Winter Wheat	2.96	1.48
Population 3: Counties that Grow Corn, Soy, and Spring Wheat		
Corn	0.17	−0.30
Soy	0.47	0.13
Spring Wheat	0.46	0.17
Population 4: Counties that Grow Corn and Soy		
Corn	0.98	1.78
Soy	0.02	−1.78

Notes: [a] For example, a 10% increase in the variance of corn prices increases the risk-aversion function 3.8% in Population 1. These calculations are based on a risk-aversion function initially equal to .03.
[b] The first (second) column of risk preference elasticities assumes the risk-aversion function is homogeneous of degree one (zero) in price variances.

become less risk averse in the face of volatile prices. Or it may be that decreasing marginal risk aversion represents hidden factors not contained in the model. For example, as price become more volatile, producers may adopt safety nets (e.g., federal crop insurance) that ultimately make them less risk averse.

Conclusion

The theoretical model developed here uses as its starting point the expected mean-variance (E-V) model to specify and estimate farm-acreage equations. Assuming stochastic prices, we generalize the linear E-V model by replacing the coefficient of risk aversion with a function of risk aversion that includes price variances as arguments. This generalization allows a change in price variability to influence producer risk preferences. Appealing to the dual formulation for the sake of computational tractability, we then portray risk as a constraint that producers face. This formulation allows us to set up an indirect risk preference function (IRPF), whose envelope properties are used to derive compensated acreage equations. In doing so, we accept that safety-first considerations are taken into account by producers, who may nor my not lie on the efficient frontier as defined by portfolio theory.

Using county-level data for corn, soybeans, and wheat, we provide an example of how to implement our model by estimating systems of compensated acreage equations for four county-level populations. Each population contains all U.S. counties that traditionally grow the same mix of crops; by avoiding corner solutions, only observable economic variables are required to explain the amount of acreage planted to a particular crop. Estimated price and price-variance elasticities are generally found to be inelastic in the short run. We then illustrate our method for estimating the impact price variability has on risk references. In general, we find a rise in price variability makes producers more risk averse. However, in several instances we discover the opposite effect.

We see a variety of paths for future extensions of the model. For example, instead of estimating the impact price variability has on the function of risk aversion, future research might examine

methods for putting a value on the function of risk aversion. Therefore, we are in a position analogous to Friend and Blume (1975), who pioneered early efforts to measure the impact of wealth on risk preferences. Those authors were able to determine the direction of this impact but were unable to estimate actual risk preferences; later work building on their initial paper achieved this goal. In a similar vein, extensions of this paper may make it possible to estimate all the parameters in the IRPF. This paper also highlights the need to explore methods for testing real-world data for efficiency as defined by portfolio theory.

The methods described here can be used to compare how the impact of price variability on risk preferences differs across farmer typologies. One application could be to analyze commodity support payments provided under Title I of the U.S. Farm Act, which is scheduled for a rewrite in 2012. For instance, when correcting for differences in demographics, crop choices, and direct and counter-cyclical payments (DCCP) received, one may be interested in determining how the function of risk aversion differs among farmers enrolled in the Average Crop Revenue Election (ACRE) program, a new support option starting with the 2008 Farm Act, and those who elected to stay with traditional price-based support. Analysis of these differences may allow commodity support programs to be redesigned.

[Received May 2011; final revision received May 2012.]

References

Atwood, J. "Demonstration of the Use of Lower Partial Moments to Improve Safety-First Probability Limits." *American Journal of Agricultural Economics* 67(1985):787–793.

Babcock, B. A., E. K. Choi, and E. Feinerman. "Risk and Probability Premiums for CARA Utility Functions." *Journal of Agricultural and Resource Economics* 18(1993):17–24.

Ben-Jemma, M. M. "Yield Uncertainty and Risk Aversion in Duality Models: A Mean-Variance Approach." 2007. Working Paper, LEGI-Polytechnics of Tunisia.

Berck, P., and J. M. Hihn. "Using the Semivariance to Estimate Safety-First Rules." *American Journal of Agricultural Economics* 64(1982):298–300.

Brink, L., and B. McCarl. "The Tradeoff between Expected Return and Risk among Cornbelt Farmers." *American Journal of Agricultural Economics* 60(1978):259–263.

Chavas, J. P., and M. T. Holt. "Acreage Decisions Under Risk: The Case of Corn and Soybeans." *American Journal of Agricultural Economics* 72(1990):529–538.

Cooper, J. C. "The Empirical Distribution of the Costs of Revenue-Based Commodity Support Programs: Estimates and Policy Implications." *Review of Agricultural Economics* 31(2009):206–221.

———. "Average Crop Revenue Election: A Revenue-Based Alternative to Price-Based Commodity Payment Programs." *American Journal of Agricultural Economics* 92(2010):1214–1228.

Coyle, B. T. "Risk Aversion and Price Risk in Duality Models of Production: A Linear Mean-Variance Approach." *American Journal of Agricultural Economics* 74(1992):849–859.

Du, S. *Commodity Futures In Asset Allocation*. Ph.D. thesis, Department of Agricultural Economics and Sociology, Pennsylvania State University, University Park, PA, 2005.

Economic Council of Advisors. *Economic Report of the President*. Washington, DC: U.S. Government Printing Office, 1995–2012.

Friend, I., and M. E. Blume. "The Demand for Risky Assets." *American Economic Review* 65(1975):900–922.

Hanoch, G., and H. Levy. "The Efficiency Analysis of Choices Involving Risk." *Review of Economic Studies* 36(1969):335–346.

Holt, M. T., and G. Moschini. "Alternative Measures of Risk in Commodity Supply Models: An Analysis of Sow Farrowing Decisions in the United States." *Journal of Agricultural and Resource Economics* 17(1992):1–12.

Huang, H., and M. Khanna. "An Econometric Analysis of U.S. Crop Yield and Cropland Acreage: Implications for the Impact of Climate Change." Denver, CO: Paper presented at the Agricultural and Applied Economics Association Annual Meeting, 2010. Available online at http://econpapers.repec.org/paper/agsaaea10/61527.htm.

Just, R. E. "An Investigation of the Importance of Risk in Farmers' Decisions." *American Journal of Agricultural Economics* 56(1974):14–25.

Kataoka, S. "A Stochastic Programming Model." *Econometrica* 31(1963):181–196.

Lin, W. W., and R. Dismukes. "Risk Considerations in Supply Response: Implications for Counter-Cyclical Payments' Production Impact." Providence, RI: Paper presented at the American Agricultural Economics Association Annual Meeting, 2005. Available online at http://econpapers.repec.org/paper/agsaaea05/19304.htm.

Love, H. A., and S. T. Buccola. "Joint Risk Preference: Technology Estimation with a Primal System." *American Journal of Agricultural Economics* 73(1991):765–774.

Markowitz, H. "Portfolio Selection." *Journal of Finance* 7(1952):77–91.

———. *Portfolio Selection: Efficient Diversification of Investments*. New York: Wiley, 1959.

Patrick, G. F., P. N. Wilson, P. J. Barry, W. G. Boggess, and D. L. Young. "Risk Perceptions and Management Responses: Producer-Generated Hypotheses for Risk Modeling." *Southern Journal of Agricultural Economics* 17(1985):231–38.

Pope, R. D., and R. E. Just. "On Testing the Structure of Risk Preferences in the Agricultural Supply Analysis." *American Journal of Agricultural Economics* 73(1991):743–748.

Pyle, D. H., and S. J. Turnovsky. "Safety-First and Expected Utility Maximization in Mean-Standard Deviation Portfolio Analysis." *Review of Economics and Statistics* 52(1970):75–81.

Robison, L. J., and J. R. Brake. "Application of Portfolio Theory to Farmer and Lender Behavior." *American Journal of Agricultural Economics* 61(1979):158–164.

Roy, A. D. "Safety First and the Holding of Assets." *Econometrica* 20(1952):431–449.

Saha, A. "Expo-Power Utility: A 'Flexible' Form for Absolute and Relative Risk Aversion." *American Journal of Agricultural Economics* 75(1993):905–913.

Saha, A., C. R. Shumway, and H. Talpaz. "Joint Estimation of Risk Preference Structure and Technology Using Expo-Power Utility." *American Journal of Agricultural Economics* 76(1994):173–184.

Sandmo, A. "The Effect of Uncertainty on Saving Decisions." *Review of Economic Studies* 37(1970):353–360.

Serra, T., D. Zilberman, B. K. Goodwin, and A. Featherstone. "Effects of Decoupling on the Mean and Variability of Output." *European Review of Agricultural Economics* 33(2006):269–288.

Shackle, G. L. S. *Decision, Order, and Time in Human Affairs*. Cambridge: Cambridge University Press, 1969.

Simmons, R. L., and C. Pomareda. "Equilibrium Quantity and Timing of Mexican Vegetable Exports." *American Journal of Agricultural Economics* 57(1975):472–479.

Szpiro, G. G. "Measuring Risk Aversion: An Alternative Approach." *Review of Economics and Statistics* 68(1986):156–159.

Telser, L. G. "Safety First and Hedging." *Review of Economic Studies* 23(1955):1–16.

Tobin, J. "Liquidity Preference as Behavior Towards Risk." *Review of Economic Studies* 25(1958):65–86.

Wiens, T. B. "Peasant Risk Aversion and Allocative Behavior: A Quadratic Programming Experiment." *American Journal of Agricultural Economics* 58(1976):629–635.

Appendix A

Acreage Elasticities

Set the compensated optimal acreage $\breve{A}$ for crop j equal to the uncompensated optimal acreage A^*:

$$\text{(A1)} \qquad \breve{A}_j(p_1..p_n, w_1 \ldots w_m, vp_1 \ldots vp_n, \overline{\pi}(p,w)) = A_j^*(p_1 \ldots p_n, w_1..w_m, vp_1..vp_n, \overline{K}).$$

Taking the derivative with respect to p_j results in:

$$\text{(A2)} \qquad \partial\breve{A}_j(\cdot)/\partial p_j + (\partial A_j(\cdot)/\partial\pi) \times (\partial\pi/\partial p_j) = \breve{A}_j(\cdot)/\partial p_j + (\partial A_j(\cdot)/\partial\pi) \times y_j a_j = \partial A^*/\partial p_j,$$

where the middle term uses Hotteling's Lemma to replace the derivative of profit with respect to price by output, and the last equality holds because $\overline{K}$ (the constraint in equation (3)) is not a function of output prices. The compensated area response has both substitution effect and profit effects. That is, a rise in the price of the jth product causes producers to supply more of the jth product at the expense of other products, but can lead to an increase in profits. Similarly, taking the derivative with respect to vp_j, it can be shown that:

$$\text{(A3)} \qquad \partial\breve{A}_j(\cdot)/\partial vp_j + (\partial A_j(\cdot)/\partial\pi) \times (\partial\pi/\partial vp_j) = \partial A_j^*(\cdot)/\partial vp_j + (\partial A_j^*/\partial K) \times \partial K/\partial vp_j$$

and

$$\text{(A4)} \qquad \begin{aligned} \partial K/\partial vp_j &= (yd_j a_j)^2 \times (g(\cdot) + (\partial g/\partial vp_j)vp_j) + (f.o.c.) \times (\partial a_j/\partial vp_j) \\ &= (y_j a_j)^2 (g(\cdot) + \partial g/\partial vp_j \times vp_j), \end{aligned}$$

where $f.o.c.$ refers to first-order conditions of the constrained profit-maximization problem in equation (4), which are set to zero. Rearranging terms leads to:

$$\text{(A5)} \qquad \partial\breve{A}_j(\cdot)/\partial vp_j - (\partial A_j^*(\cdot)/\partial K) \times (y_j a_j^*)^2((g(\cdot) + (\partial g/\partial vp_j)vp_j) = \partial A_j^*(\cdot)/\partial vp_j.$$

In equation (A5), an estimate of the risk-aversion function, $g(\cdot)$, and its derivative must be used to calculate variance elasticities. The derivative of acreage with respect to the risk-tolerance level, $\partial A_j^*(\cdot)/\partial K$, can be approximated by its inverse, the derivative of the risk function with respect to acreage.

The Envelope Relationship and Derivation of the Acreage Equation

Acreage equations are recovered from the IRPF function. To show this, substitute the optimal acreage equations ($\breve{A}(p_1..p_n, vp_i \ldots vp_n, w_i, \pi)$, into the Lagrangian problem in equation (6) and take the derivative:

$$\text{(A6)} \quad \partial\breve{G}(\cdot)/\partial p_i = \left[\sum_i \partial g(\cdot)/\partial\breve{A}_i\right)(\breve{A}_i y_i)^2 vp_i + g(\cdot) \times 2\breve{A}_i y_i^2 - \theta\left(p_i y_i - \partial C(\cdot)/\partial\breve{A}_i\right] \times \partial\breve{A}_i/\partial p_i - \theta\breve{A}_i y_i.$$

The second equality comes from using the first-order condition of the problem in equation (6) (which is set to equal zero) and eliminating the term in the bracket. Taking the derivative of the IRPF with respect to the level of profits produces:

$$\text{(A7)} \quad \partial\breve{G}/\partial\pi = [\sum_i \partial g(\cdot)/\partial\breve{A}_i)(\breve{A}_i y_i)^2 vp_i + g(\cdot) \times 2\breve{A}_i y_i^2 vp_i - \theta(p_i y_i - \partial C(\cdot)/\partial\breve{A}_i] \times \partial\breve{A}_i/\partial\pi + \theta = \theta.$$

Again, the bracketed term is zero by the first-order conditions. Combining these two relationships produces equation (10).

Measuring the Impact of Risk on Risk Preferences

With certain functional forms, it is possible to use the indirect risk-preference function (IRPF) to measure the impact of price variances on risk preferences. Taking the derivative of the IRPF with respect variance of the ith price:

$$\text{(A8)} \quad \partial\breve{G}/\partial vp_i = (g(a_i y_i)^2 + (\sum_j^n \partial g(\cdot)/\partial vp_i) \times vp_j \times (a_j y_j)^2 + \sum_j^n \partial a_j/\partial vp_i) \times (f.o.c.) = \partial G/\partial vp_i,$$

where *f.o.c.* represents the first-order conditions in equation (4), which are set equal to zero, which obtains equation (19) of the text.

Equation (A8) shows that the change in the IRPF function with respect to a price variance is the same as change in the direct objective function with respect to the same price variance. It also shows that this derivative contains the risk-aversion function and its first derivative.

Symmetry

Taking the cross-price derivatives of equation (A6), one obtains:

$$\text{(A9)} \quad \begin{aligned} -\partial\breve{G}(\cdot)/\partial p_j \partial p_i &= \theta y_j((\partial\breve{A}_j(\cdot)/\partial p_i) + \partial\breve{A}_j)(\cdot)/\partial\pi \times y_i) \\ &= -\partial\breve{G}(\cdot)/\partial p_i \partial p_j = \theta y_j((\partial\breve{A}_i(\cdot)/\partial p_j) + \partial\breve{A}_i/\partial\pi \times y_j). \end{aligned}$$

Acreage equations will be symmetric only in rare circumstances and only at one particular point. A similar situation holds for symmetry relationships with respect to price variances.

Homogeneity

Homogeneity cannot be established without restricting the properties of the original $g(\cdot)$ function. Acreage equations can be specified as homogeneous of degree zero in prices and their variances. However, this does not carry over to the indirect function of risk aversion. The key reason lies in the fact that the profit component of the objective function remains linear while the risk component of the objective function can be nonlinear.

Curvature

Acreage enters the objective function in nonlinear form. Combining suboptimal solution vectors for acreage and comparing these with a weighted average of solution vectors, as is typically done to establish curvature, does not establish a clear result. An indirect proof may lie in demonstrating that our indirect utility function in equation (3)—which can be viewed as a constrained profit function—is convex in prices. Inverting this function obtains the indirect-risk preference function (IRPF), which would be concave in prices.

To be concave, the IRPF requires positive acreage responses. To see this, note that in equation (9), the derivative of the IRPF with respect to price equals $-\theta A_j y_j$. Therefore, the second derivative of the IRPF equals $-[(\partial A_j^*/\partial p_j)\theta \times y_j = A_j y_j(\partial\theta/p_j)]$. If compensated acreage responses are positive and the value of the multiplier increases in price, the diagonals of the second-derivative matrix are negative, a necessary but insufficient condition for concavity.

Acreage Allocation and the Efficient Frontier

The IRPF function $G(\ldots,\overline{\pi})$ represents the minimum risk preference attainable given a target level of profits, the latter of which is determined outside the model, perhaps in a upper stage problem involving consumption choices and intertemporal considerations. Farmers' predetermined profits, $\overline{\pi}$, may or may not lie on the efficient frontier for the given level of chosen risk (as defined by portfolio theory). To determine optimal profits, one completes the optimization by substituting the IRPF into the risk component of the E-V problem and then reallocates acreage in order to obtain:

$$\text{(A10)} \quad \max_{\overline{a}_i} \overline{\pi}(\overline{a}) - G(vp, \overline{\pi}(\overline{a}), p, w),$$

where $\overline{a}$ represents the optimal acreage allocations from the risk-minimization problem in equation (6). In effect, the second-stage problem reallocates acreage to ensure producers lie on the efficient frontier. If producers were already on the frontier, having targeted the optimal level of profits in the first place, there would be no need to reallocate acreage. The solution for the ith acreage is:

$$\text{(A11)} \quad \left(1 - \left(\frac{\delta G(\cdot)}{\delta\pi}\right)\right) \times \delta\pi/\delta a_i = 0.$$

Profit-maximizing acreage allocation occurs where either the shadow price of each acreage allocation is zero or where the derivative of the IRPF with respect to profits equals 1. Casual observation indicates that land is scare and rents, and thus shadow prices, are positive. Therefore, at the profit-maximizing allocation, the derivative of the IRPF with respect to profits is most likely equal to 1. However, this derivative also represents the denominator of the envelope relation in equation (11). In turn, if that denominator equals 1, supply equations can be obtained by taking the derivative of the risk-preference function with respect to price alone. Thus, if producers are efficient profit maximizers, our envelope relationship reduces to a risk-inclusive version of Hoteling's lemma.

This distinction in envelope relationships provides a way to test whether or not the producer's choice of $\overline{\pi}$—a choice that is outside of the model—lies on the efficient frontier. If the derivative of $G(\cdot)$ with respect to profits is 1, then our envelope relationship produces acreage rather than acreage-share equations. If share equations are estimated when in fact acreage equations should be estimated, then the resulting model's coefficients would represent the true coefficients divided by acreage.

One way to test producer efficiency is to estimate share equations and then evaluate whether the coefficient on each equation's constant term changes from year to year and see if its change is related to the (inverse of) annual changes in total acreage. If so, it provides evidence that acreage rather than acreage share should be the left-hand-side variable. Additionally, equation (A11) is equal to zero, indicating that producers are on the E-V frontier. Nonetheless, such a test is imperfect in that other reasons exist for a changing constant besides those relating to annual changes in acreage. Acknowledging this limitation, we tested each region and found that while each equation's constant often changed, the change was not always related to annual changes in total acreage.